BLT-sets over small fields

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Abstract

A BLT-set is a set X of $q+1$ points of the generalized quadrangle $Q(4,q)$, q odd, such that no point of $Q(4,q)$ is collinear with more than 2 points of X. BLT-sets are closely related to flocks of the quadratic cone, elation generalised quadrangles and certain translation planes. In this paper we report on the results of computer searches for BLT-sets for odd $q \leq 25$. We complete the classification for $q \leq 17$ finding one new BLT-set, and provide further examples for $q = 19$, $q = 23$ and $q = 25$ finding 10 new BLT-sets altogether. The relationship between BLT-sets and flocks of the quadratic cone in $PG(3,q)$ means that this work classifies flocks for $q \leq 17$ and finds new flocks for $q = 19$, $q = 23$ and $q = 25$. In total the 10 new BLT-sets yield 26 new flocks.

1 Introduction and Motivation

A BLT-set is a set X of $q+1$ points of the generalized quadrangle $Q(4,q)$, q odd, such that no point of $Q(4,q)$ is collinear with more than 2 points of X. (See Payne and Thas [7] for background on generalized quadrangles.)

They were introduced by Bader, Lunardon and Thas [1], although the nomenclature is due to Kantor [4]. Their introduction was motivated by the following connection with flocks of the quadratic cone. Given a BLT-set X, and a distinguished element x of X, the set of points of $Q(4,q)$ collinear with x forms a quadratic cone in the polar hyperplane H of x. Moreover, the intersections of the polar hyperplanes of the points of X (other than x) and H form a flock of this quadratic cone. Conversely, given a flock of a quadratic cone, one can obtain a BLT-set with a distinguished point by reversing this procedure. Thus each flock gives rise to a BLT-set, which is turn gives rise to q further flocks, corresponding to the different choices of x. These flocks are called *derived* flocks in [1].

A second motivation for studying BLT-sets is the connection with elation generalized quadrangles, first pointed out by Thas [9] but clarified by Knarr[5]. Knarr gives a geometric construction of an elation generalised quadrangle of order (q^2, q)

from each BLT-set in $Q(4,q)$, whereas previously the construction had proceeded via a group coset geometry.

A third motivation for studying BLT-sets of $Q(4,q)$ is the connection with translation planes of order q^2 and rank at most 2 over their kernel, that is, with spreads of $PG(3,q)$. By the Klein correspondence, a spread of $PG(3,q)$ corresponds to an ovoid of $Q^+(5,q)$. Given a BLT-set X of $Q(4,q)$, and a point x of X, embed $Q(4,q)$ as a hyperplane H in the Klein quadric $Q^+(5,q)$ and let y be the pole of that hyperplane. Then the intersection O with the Klein quadric of the union of the spans $\langle x,y,z\rangle$ with $z \in X$, z not equal to x is an ovoid of $Q^+(5,q)$. This was independently observed by Walker [10] and Thas.

A computer search for flocks of the quadratic cone of $PG(3,q)$ for small q was carried out by De Clerck and Herssens [3]. A computer free classification of the conical flocks for $q \leq 8$ was given by De Clerck, Gevaert and Thas [2]. The purpose of this paper is to extend the computer search by placing it in the more general context of finding BLT-sets rather than individual flocks.

2 Techniques

Consider the generalized quadrangle $Q(4,q)$ of order q. This quadrangle has $(q+1)(q^2+1)$ points and lines. Let $G(q)$ denote the collinearity graph of $Q(4,q)$. Then $G(q)$ has $(q+1)(q^2+1)$ vertices, is regular of valency q^2+q and has an automorphism group of size $h(q^4-1)(q^2-1)q^4$, where $q=p^h$ with p prime. Table 1 shows these values for $q \leq 25$ to give some idea of the sizes involved.

q	$(q+1)(q^2+1)$	$h(q^4-1)(q^2-1)q^4$
3	40	51840
5	156	9360000
7	400	276595200
9	820	6886425600
11	1464	25721308800
13	2380	137037962880
17	5220	2008994088960
19	7240	6114035779200
23	12720	41348052472320
25	16276	190429200000000

Table 1: Number of points and size of automorphism group for $Q(4,q)$

A BLT-set can then be viewed simply as a special kind of independent set in $G(q)$ and a search for BLT-sets can be structured in a similar way to a search for independent sets. The standard recursive action in a back-track search for an independent set in a graph extends an independent k-set X to an independent $(k+1)$-set X' by adding a new vertex x that is not adjacent to any vertex in X. This is usually implemented by maintaining a set of "live" points at all times, selecting x from the live points and then updating the set of live points by deleting the neighbours of x.

A search for BLT-sets can be structured in precisely the same way except that we have to modify the definition of live points. Suppose that X is a partial BLT-set (expressed as a set of vertices in $G(q)$). Then a point is *not* live if either

(1) it is a neighbour of a vertex in X, or
(2) it is a neighbour of a common neighbour of two vertices in X.

With this observation it is straightforward to write a naive back-track program to construct BLT-sets.

However there are significant difficulties with the *isomorphism problem* as the naive algorithm produces many isomorphic copies of each BLT-set. The graph isomorphism program **nauty** [6] can be used to determine the isomorphism classes of the resulting BLT-sets, but there are practical limitations. For reasons that are not well understood, graphs relating to geometries are often pathological cases for graph isomorphism programs such as **nauty**. Although the graphs $G(q)$ are not among the most pathological, they are fairly large, so isomorphism checking rapidly becomes very difficult.

However, for $q \leq 17$ it is feasible to use an *orderly algorithm* (see Royle [8] for details) to compute the precise numbers of partial BLT-sets of all sizes. Although this computation took several months of computer time, the techniques are well-understood and the programming effort minimal so we have a high degree of confidence in the results.

For $q > 17$ it becomes too expensive to use **nauty** and to do a fully exhaustive search. For $q = 19$ we used a hybrid method, whereby the orderly algorithm was used to construct all the "partial BLT-sets" of size five, which were then completed to BLT-sets in all possible ways. These were partitioned by using a combinatorial invariant termed the *F-profile* (see below for details), rather than exact isomorphism. Hence any new BLT-set must have the same F-profile as one of the BLT-sets on our list (an event which we believe is unlikely to occur). For $q > 19$ we abandoned any sort of completeness in the search and simply aimed to construct as many BLT-sets as possible, in a variety of ad-hoc ways. One technique that proved to be fruitful was "guessing" a possible group of automorphisms — usually by taking a modestly sized subgroup of the group of automorphisms of one of the known BLT-sets — and then constructing BLT-sets stabilised by that group of automorphisms. The resulting BLT-sets were again partitioned by F-profile. The results of these computations are a list of BLT-sets that are guaranteed to be different, but there is no guarantee that the list is complete.

3 BLT-sets and flocks

Recall that for q odd, a *flock* of the quadratic cone in $PG(3, q)$ is a set of q planes whose intersections with the cone partition the points of the cone (other than the vertex of the cone). If we fix the cone to have equation $X_0X_1 = X_2^2$, then each plane Π_i can be given by an equation of the following form:

$$\Pi_i : a_iX_0 + b_iX_1 + c_iX_2 + X_3 = 0.$$

and we can represent the flock just by giving a set of q triples:

$$\mathcal{F} = \{(a_i, b_i, c_i) \mid i \in GF(q)\}.$$

It is straightforward to check that Π_i and Π_j do not have any common points on the cone if and only if

$$(c_i - c_j)^2 - 4(a_i - a_j)(b_i - b_j) \in \not\Box$$

where $\not\Box$ is the set of non-squares in $GF(q)$.

As described above, a BLT-set is equivalent to a collection of $(q + 1)$ flocks, with each point of the BLT-set being associated to a different flock. We would like to be able to easily compute the $q + 1$ flocks associated with each BLT-set, and, conversely, to compute the BLT-set associated with a flock. These operations are simple provided the appropriate models are chosen.

The construction of a BLT-set from a flock can be found in Bader, Lunardon & Thas [1]. Given a flock

$$\mathcal{F} = \{(a_i, b_i, c_i) \mid i \in GF(q)\},$$

define the following set of points of $Q(4, q)$ (given by the form $X_0X_1 - X_2^2 + X_3X_4$):

$$X = \{(0, 0, 0, 1, 0)\} \cup \{(b_i, a_i, -c_i/2, c_i^2/4 - a_ib_i, 1) \mid i \in GF(q)\}.$$

Then X is a BLT-set, with distinguished point $(0, 0, 0, 1, 0)$.

Given a BLT-set X containing the point $(0, 0, 0, 1, 0)$ it is easy to reverse the above construction to get a flock $\mathcal{F}$. It is easy to guarantee that X contains $(0, 0, 0, 1, 0)$ by construction, or if necessary to map X to an isomorphic BLT-set containing this point.

It is also straightforward to obtain the other q flocks associated with X (the derived flocks) — Bader, Lunardon & Thas [1] explain how derivation can be performed entirely in the flock model. Given

$$\mathcal{F}_0 = \{(a_i, b_i, c_i) \mid i \in GF(q)\}$$

define for each i the set

$$\mathcal{F}_i = \{(0, 0, 0)\} \cup \{((a_i - a_j)/\Delta, (b_i - b_j)/\Delta, (c_i - c_j)/\Delta) \mid j \neq i\}$$

where

$$\Delta = (c_i - c_j)^2/4 - (a_i - a_j)(b_i - b_j).$$

Then $\mathcal{F}_1$, $\mathcal{F}_2$, ..., $\mathcal{F}_q$ are the flocks *derived* from $\mathcal{F}_0$.

3.1 Isomorphism and Profiles

As explained above, determining isomorphism of BLT-sets is a difficult problem in practice, so it is necessary to develop a cheaper way to distinguish different BLT-sets. To accomplish this we associate a purely combinatorial invariant with each BLT-set, which (for want of a better name) we call the F-profile. The F-profile is based on the $q+1$ flocks associated with the BLT-set. Given a flock $\mathcal{F}$, we can define the *profile* of $\mathcal{F}$ in the following fashion:

$$\mathrm{prof}(\mathcal{F}) = (x_0, x_1, \ldots, x_q)$$

where x_i is the number of points of $PG(3,q)$ that lie on precisely i planes of the flock. This is therefore a vector of length $q+1$ with entries summing to q^3+q^2+q+1. Clearly flocks with different profiles are not isomorphic, but the converse is not necessarily true. As a BLT-set is equivalent to a collection of $q+1$ flocks, we then associate a multiset of $q+1$ profiles to each BLT-set. This multiset is the F-profile of X.

If we are just interested in the stabiliser group of the BLT-set X then it is possible to use the package MAGMA to compute the stabiliser of X in $\mathrm{Aut}(G(q))$. For all the known BLT-sets with $q \leq 25$ the F-profile distinguishes all the orbits.

4 Results

The results of the search are summarised in Table 2. These results confirm those of De Clerck and Herssens [3] showing that they are complete for $q \leq 13$. For $q = 17$ the list is completed by the addition of one new BLT-set that yields two new flocks. For $q = 19$ we find three new BLT-sets, yielding six new flocks, for $q = 23$ we find four new BLT-sets, yielding 11 new flocks and for $q = 25$ we find two new BLT-sets yielding 7 new flocks.

We explicitly give each of the new BLT-sets found as a set of $q+1$ points of $Q(4,q)$, together with their F-profiles. Given the explicit details presented above, it should be straightforward for any researcher to obtain the associated flocks for further analysis. This data is also available from `<http://www.cs.uwa.edu.au/~gordon/data.html>`

4.1 The previously known BLT-sets

Most BLT-sets were discovered as flocks, hence we shall present them as such, and give the associated BLT-set the same name as the flock. Occasionally flocks associated with the same BLT-set were discovered separately — we simply concatenate the names of the flocks to yield the name of the BLT-set.

The known families of BLT-sets for odd q are as follows (this list is based on that given by De Clerck and Herssens [3]). For many of the details see the paper by Thas [9].

1. Linear — for all q the linear flock is given by

   $$\{(t, -mt, 0) \mid t \in GF(q)\},$$

   where m is a fixed nonsquare. The associated BLT-set is transitive.

2. FTW — for $q \equiv -1 \pmod 3$, the flock FTW due to Fisher, Thas and Walker is given by

   $$\{(t, 3t^3, 3t^2) \mid t \in GF(q)\}.$$

q	Name	Group Size	Orbit structure
5	Linear	960	$\{6\}$
5	FTW=Fi=K3	720	$\{6\}$
7	Linear	5376	$\{8\}$
7	Fi = K2	384	$\{8\}$
9	Linear	28800	$\{10\}$
9	Fi	400	$\{10\}$
9	K1 = G	5760	$\{10\}$
11	Linear	31680	$\{12\}$
11	FTW	1320	$\{12\}$
11	Fi	288	$\{12\}$
11	DCHT	144	$\{12\}$
13	Linear	61152	$\{14\}$
13	Fi	392	$\{14\}$
13	K2/JP	48	$\{2, 12\}$
17	Linear	176256	$\{18\}$
17	FTW	4896	$\{18\}$
17	Fi	648	$\{18\}$
17	K2/JP	32	$\{2, 16\}$
17	DCH1/2	144	$\{6, 12\}$
17	New	24	$\{6, 12\}$
19	Linear	273600	$\{20\}$
19	Fi	800	$\{20\}$
19	New	40	$\{20\}$
19	New	20	$\{20\}$
19	New	16	$\{2^2, 8^2\}$
23	Linear	582912	$\{24\}$
23	FTW	12144	$\{24\}$
23	Fi	1152	$\{24\}$
23	K2/JP	44	$\{2, 22\}$
23	DCH1/2	72	$\{6, 18\}$
23	New	1152	$\{24\}$
23	New	24	$\{24\}$
23	New	16	$\{4^2, 8^2\}$
23	New	6	$\{3^2, 6^3\}$
25	Linear	1622400	$\{26\}$
25	Fi	2704	$\{26\}$
25	K1	124800	$\{26\}$
25	K3/BLT	100	$\{1, 25\}$
25	New	8	$\{2, 8^3\}$
25	New	16	$\{2, 8, 16\}$

Table 2: The known BLT-sets for $q \leq 25$, complete up to $q \leq 17$

The associated BLT-set is transitive.

3. K1 — for all q, the flock K1 due to Kantor is given by

$$\{(t, -mt^\sigma, 0) \mid t \in GF(q)\},$$

where m is a fixed non-square and σ is an automorphism of $GF(q)$. It is linear if and only if $\sigma = 1$. The associated BLT-set is transitive.

4. K2/JP — for $q \equiv \pm 2 \pmod 5$ the flock K2 due to Kantor is given by

$$\{(t, 5t^5, 5t^3) \mid t \in GF(q)\}.$$

Johnson and Payne observed that a different flock can be derived from K2.

5. K3/BLT — for $q = 5^h$, the flock K3 due to Kantor is given by

$$\{(t, k^{-1}t + 2t^3 + kt^5, t^2) \mid t \in GF(q)\},$$

where k is a given non-square. Bader, Lunardon and Thas observed that a different flock can be derived from K3.

6. Fi — for all q the flock Fi due to Fisher has the following complicated construction. Let ξ be a primitive element of $GF(q^2)$ so $w = \xi^{q+1}$ is a primitive element of $GF(q)$; put $i = \xi^{(q+1)/2}$, so $i^2 = w$, $i^q = -i$; put $z = \xi^{q-1} = a + bi$, so z has order $q+1$ in the multiplicative group of $GF(q^2)$; then the triples of Fi are

$$\{(t, -wt, 0) \mid t \in GF(q), t^2 - 2(1+a)^{-1} \in \square\}$$

together with

$$\{(-a_{2j}, -wa_{2j}, 2b_{2j}) \mid 0 \le j \le (q-1)/2\}$$

where $a_k = (z^{k+1} + z^{-k})/(z+1)$ and $b_k = i(z^{k+1} - z^{-k})/(z+1)$. The associated BLT-set is transitive.

7. G/PTJLW — for $q = 3^h$, the flock G due to Ganley is given by

$$\{(t, -(nt + n^{-1}t^9), t^3) \mid t \in GF(q)\},$$

where n is a fixed nonsquare. Payne and Thas, and also Johnson, Lunardon and Wilke observed that a different flock can be derived from G.

Three seemingly sporadic BLT-sets were previously known. De Clerck, Herssens and Thas found a flock for $q = 11$ called DCHT, and De Clerck and Herssens found two pairs of flocks (related by derivation) for $q = 17$ and $q = 23$. We call both the associated BLT-sets DCH1/2. (The BLT-sets that we give were found by our search, and then identified, hence are not directly related to the specific flocks as presented in De Clerck and Herssens [3].)

DCHT	(0,0,0,0,1)	(0,0,0,1,0)	(10,10,0,10,1)	(8,8,0,2,1)
$q = 11$	(9,7,7,8,1)	(7,3,1,2,1)	(6,1,4,10,1)	(1,4,10,8,1)
	(5,2,3,10,1)	(4,6,9,2,1)	(3,5,1,8,1)	(2,9,9,8,1)
DCH1/2	(0,0,0,0,1)	(0,0,0,1,0)	(6,6,12,6,1)	(12,12,7,7,1)
$q = 17$	(13,13,9,14,1)	(16,16,15,3,1)	(14,11,8,12,1)	(10,3,5,12,1)
	(11,5,1,14,1)	(7,14,13,3,1)	(5,15,11,12,1)	(1,7,6,12,1)
	(15,1,16,3,1)	(9,4,13,14,1)	(4,10,16,12,1)	(8,2,8,14,1)
	(2,9,2,3,1)	(3,8,11,12,1)		
DCH1/2	(0,0,0,0,1)	(0,0,0,1,0)	(22,22,0,22,1)	(21,21,0,19,1)
$q = 23$	(19,19,7,10,1)	(2,2,7,22,1)	(7,14,18,19,1)	(1,3,5,22,1)
	(11,10,20,14,1)	(9,13,1,22,1)	(14,1,14,21,1)	(15,6,9,14,1)
	(17,4,15,19,1)	(4,17,6,14,1)	(3,16,6,11,1)	(20,7,14,10,1)
	(8,12,1,20,1)	(13,8,16,14,1)	(6,15,14,14,1)	(10,9,20,11,1)
	(16,5,3,21,1)	(18,20,5,10,1)	(5,18,6,15,1)	(12,11,20,15,1)

Table 3: The three known sporadic BLT-sets

4.2 The new BLT-sets

For each value of q we shall simply identify the BLT-sets by groupsize, and call them $X_{\mathbf{groupsize}}$.

For $q = 17$, $q = 19$ and $q = 23$ the field has prime order and hence is simply represented as the integers modulo q. For $q = 25$ we the field we use has primitive polynomial $\omega^2 + 4\omega + 2$ and we use the convention that 0 represents 0, 1 represents 1, 2 represents ω, 3 represents w^2 and so on.

X_{24}	(0,0,0,0,1)	(0,0,0,1,0)	(6,6,12,6,1)	(12,12,7,7,1)
	(2,2,4,12,1)	(9,9,5,12,1)	(8,16,15,12,1)	(10,3,5,12,1)
	(15,7,10,12,1)	(5,8,9,7,1)	(11,4,13,6,1)	(3,1,7,12,1)
	(13,10,12,14,1)	(16,11,3,3,1)	(14,15,16,12,1)	(1,14,0,3,1)
	(4,5,0,14,1)	(7,13,1,12,1)		

Table 4: New BLT-set for $q = 17$

X_{40}	(0,0,0,0,1)	(0,0,0,1,0)	(18,18,0,18,1)	(17,17,0,15,1)
	(14,14,9,18,1)	(9,9,13,12,1)	(1,2,1,18,1)	(4,8,14,12,1)
	(15,7,16,18,1)	(12,3,15,18,1)	(6,11,17,14,1)	(10,13,9,8,1)
	(2,16,5,12,1)	(11,4,4,10,1)	(5,12,10,2,1)	(8,1,11,18,1)
	(16,15,18,8,1)	(3,10,10,13,1)	(7,5,9,8,1)	(13,6,10,3,1)
X_{20}	(0,0,0,0,1)	(0,0,0,1,0)	(18,18,0,18,1)	(16,16,0,10,1)
	(10,10,1,15,1)	(9,9,8,2,1)	(5,5,1,14,1)	(8,8,1,13,1)
	(6,12,6,2,1)	(11,14,9,3,1)	(14,4,11,8,1)	(12,17,16,14,1)
	(15,3,14,18,1)	(13,2,5,18,1)	(17,7,2,18,1)	(7,6,11,3,1)
	(2,1,15,14,1)	(1,15,2,8,1)	(3,13,16,8,1)	(4,11,15,10,1)
X_{16}	(0,0,0,0,1)	(0,0,0,1,0)	(18,18,0,18,1)	(17,17,0,15,1)
	(16,16,0,10,1)	(8,8,13,10,1)	(4,12,1,10,1)	(7,9,0,13,1)
	(13,2,14,18,1)	(5,11,14,8,1)	(14,3,14,2,1)	(9,6,8,10,1)
	(12,1,14,13,1)	(6,10,7,8,1)	(3,14,6,13,1)	(10,7,8,13,1)
	(1,15,2,8,1)	(2,13,6,10,1)	(11,5,12,13,1)	(15,4,14,3,1)

Table 5: New BLT-sets for $q = 19$

X_{1152}	(0,0,0,0,1)	(0,0,0,1,0)	(22,22,0,22,1)	(21,21,0,19,1)
	(20,20,0,14,1)	(10,10,0,15,1)	(7,14,7,20,1)	(17,11,1,21,1)
	(14,5,13,7,1)	(4,8,11,20,1)	(3,9,22,20,1)	(13,6,16,17,1)
	(18,3,8,10,1)	(19,2,5,10,1)	(15,4,10,17,1)	(12,7,20,17,1)
	(6,15,10,10,1)	(8,13,3,20,1)	(5,16,13,20,1)	(1,18,18,7,1)
	(16,12,12,21,1)	(9,19,10,21,1)	(2,17,15,7,1)	(11,1,13,20,1)
X_{24}	(0,0,0,0,1)	(0,0,0,1,0)	(22,22,0,22,1)	(20,20,0,14,1)
	(5,5,20,7,1)	(15,6,9,14,1)	(11,9,1,17,1)	(2,12,0,22,1)
	(18,16,11,17,1)	(8,2,7,10,1)	(7,3,13,10,1)	(17,21,2,15,1)
	(4,13,4,10,1)	(13,15,13,20,1)	(10,18,19,20,1)	(9,11,7,19,1)
	(12,7,11,14,1)	(19,1,19,20,1)	(6,10,6,22,1)	(14,8,15,21,1)
	(1,19,6,17,1)	(16,14,4,22,1)	(21,4,12,14,1)	(3,17,5,20,1)
X_{16}	(0,0,0,0,1)	(0,0,0,1,0)	(22,22,0,22,1)	(21,21,0,19,1)
	(5,10,5,21,1)	(20,17,5,7,1)	(14,5,13,7,1)	(2,6,16,14,1)
	(3,9,22,20,1)	(6,7,1,5,1)	(10,1,20,22,1)	(17,4,3,10,1)
	(15,13,19,5,1)	(9,12,17,20,1)	(11,18,14,21,1)	(13,15,4,5,1)
	(16,3,17,11,1)	(12,20,13,21,1)	(18,2,20,19,1)	(8,14,2,7,1)
	(1,19,17,17,1)	(4,11,13,10,1)	(19,8,1,10,1)	(7,16,8,21,1)
X_{6}	(0,0,0,0,1)	(0,0,0,1,0)	(22,22,0,22,1)	(21,21,0,19,1)
	(11,11,21,21,1)	(5,5,1,22,1)	(15,7,14,22,1)	(20,14,16,22,1)
	(6,13,7,17,1)	(3,1,6,10,1)	(17,3,5,20,1)	(14,16,3,15,1)
	(16,15,12,19,1)	(8,4,11,20,1)	(19,17,12,5,1)	(12,18,22,15,1)
	(13,8,20,20,1)	(4,10,13,14,1)	(10,12,1,19,1)	(9,6,8,10,1)
	(7,20,22,22,1)	(2,9,9,17,1)	(18,2,3,19,1)	(1,19,1,5,1)

Table 6: New BLT-sets for $q = 23$

X_8	(0,0,0,0,1)	(0,0,0,1,0)	(21,21,22,8,1)	(2,2,6,10,1)
	(24,1,14,14,1)	(3,5,15,8,1)	(18,20,6,14,1)	(8,10,22,8,1)
	(10,15,0,12,1)	(4,9,15,14,1)	(17,23,9,6,1)	(13,22,5,14,1)
	(9,19,13,4,1)	(6,16,22,22,1)	(20,7,9,24,1)	(23,12,1,8,1)
	(5,18,7,20,1)	(22,11,1,10,1)	(1,14,9,4,1)	(15,4,7,14,1)
	(11,3,14,18,1)	(7,24,23,4,1)	(14,8,19,8,1)	(12,6,8,18,1)
	(16,13,9,14,1)	(19,17,5,12,1)		
X_{16}	(0,0,0,0,1)	(0,0,0,1,0)	(14,14,22,14,1)	(9,10,19,14,1)
	(17,18,7,24,1)	(24,1,14,14,1)	(6,7,2,10,1)	(18,20,6,14,1)
	(3,6,11,18,1)	(22,3,7,10,1)	(4,9,15,14,1)	(11,17,13,4,1)
	(8,15,8,24,1)	(5,13,5,4,1)	(12,22,24,14,1)	(10,21,10,16,1)
	(15,2,6,12,1)	(16,4,5,24,1)	(23,11,16,10,1)	(19,8,11,22,1)
	(21,12,13,10,1)	(13,5,16,22,1)	(7,24,24,8,1)	(1,19,21,20,1)
	(20,16,22,18,1)	(2,23,10,20,1)		

Table 7: New BLT-sets for $q = 25$

5	Linear	$6 \times (25, 125, 0, 0, 0, 6)$
5	FTW=Fi=K3	$6 \times (51, 65, 30, 10, 0, 0)$
7	Linear	$8 \times (49, 343, 0, 0, 0, 0, 0, 8)$
7	Fi=K2	$8 \times (125, 186, 60, 26, 0, 3, 0, 0)$
9	Linear	$10 \times (81, 729, 0, 0, 0, 0, 0, 0, 0, 10)$
9	Fi	$10 \times (270, 349, 150, 40, 5, 6, 0, 0, 0, 0)$
9	K1 = G	$10 \times (225, 486, 0, 108, 0, 0, 0, 0, 0, 1)$
11	Linear	$12 \times (121, 1331, 0, 0, 0, 0, 0, 0, 0, 0, 0, 12)$
11	FTW	$12 \times (496, 638, 165, 165, 0, 0, 0, 0, 0, 0, 0, 0)$
11	Fi	$12 \times (469, 652, 252, 78, 0, 9, 0, 4, 0, 0, 0, 0)$
11	DCHT	$12 \times (481, 641, 237, 81, 15, 9, 0, 0, 0, 0, 0, 0)$
13	Linear	$14 \times (169, 2197, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 14)$
13	Fi	$14 \times (777, 1021, 441, 126, 0, 0, 7, 8, 0, 0, 0, 0, 0, 0)$
13	K2/JP	$12 \times (795, 1024, 379, 145, 23, 14, 0, 0, 0, 0, 0, 0, 0, 0)$ $2 \times (801, 1014, 372, 166, 12, 15, 0, 0, 0, 0, 0, 0, 0, 0)$
17	Linear	$18 \times (289, 4913, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 18)$
17	FTW	$18 \times (1769, 2363, 408, 680, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
17	Fi	$18 \times (1692, 2249, 972, 288, 0, 0, 0, 0, 9, 10, 0, 0, 0, 0, 0, 0, 0, 0)$
17	K2/JP	$2 \times (1777, 2223, 752, 408, 32, 28, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $16 \times (1811, 2129, 839, 371, 51, 19, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
17	DCH1/2	$6 \times (1689, 2363, 792, 298, 24, 51, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0)$ $12 \times (1783, 2132, 957, 269, 33, 45, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)$
17	X_{24}	$6 \times (1785, 2125, 976, 232, 74, 20, 6, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $12 \times (1788, 2157, 880, 303, 73, 14, 4, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
19	Linear	$20 \times (361, 6859, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 20)$
19	Fi	$20 \times (2321, 3168, 1320, 410, 0, 0, 0, 0, 0, 15, 0, 6, 0, 0, 0, 0, 0, 0, 0, 0)$
19	X_{40}	$20 \times (2518, 2932, 1197, 501, 50, 42, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
19	X_{20}	$20 \times (2501, 2961, 1221, 419, 115, 21, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
19	X_{16}	$8 \times (2455, 3050, 1193, 417, 85, 31, 8, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $2 \times (2467, 3000, 1258, 400, 64, 44, 4, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $8 \times (2502, 2938, 1266, 410, 88, 30, 5, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $2 \times (2549, 2829, 1326, 416, 94, 24, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

Table 8: F-profiles for $q \leq 19$

Linear	$24 \times (529, 12167, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 24)$
FTW	$24 \times (4302, 5888, 759, 1771, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
Fi	$24 \times (4069, 5554, 2340, 732, 0, 0, 0, 0, 0, 0, 18, 0, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
K2/JP	$2 \times (4467, 5074, 2112, 946, 22, 99, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $22 \times (4485, 5027, 2166, 886, 78, 78, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
DCH1/2	$18 \times (4308, 5377, 2078, 756, 129, 46, 14, 10, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $6 \times (4310, 5300, 2259, 640, 123, 57, 21, 6, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
X_{1152}	$24 \times (3959, 6168, 1620, 788, 30, 141, 0, 8, 0, 0, 0, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
X_{24}	$24 \times (4472, 5038, 2229, 761, 160, 60, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
X_{16}	$4 \times (4361, 5295, 2078, 767, 138, 76, 0, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $8 \times (4369, 5267, 2124, 710, 190, 51, 6, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $8 \times (4386, 5225, 2145, 729, 172, 54, 6, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $4 \times (4473, 4999, 2311, 722, 153, 56, 4, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
X_6	$6 \times (4443, 5072, 2279, 687, 176, 55, 7, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $6 \times (4451, 5089, 2199, 746, 189, 39, 6, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $3 \times (4457, 5058, 2240, 748, 158, 49, 10, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $3 \times (4460, 5036, 2292, 700, 165, 63, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $6 \times (4473, 5019, 2265, 743, 171, 39, 8, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

Table 9: F-profiles for $q = 23$

Linear	$26 \times (625, 15625, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 26)$
Fi	$26 \times (5226, 7045, 3042, 936, 0, 0, 0, 0, 0, 0, 0, 0, 13, 14, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
K1	$26 \times (3025, 12500, 0, 0, 0, 750, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$
K3/BLT	$25 \times (5736, 6446, 2766, 1126, 91, 111, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $1 \times (5901, 6050, 3000, 1150, 100, 75, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
X_8	$8 \times (5678, 6546, 2826, 907, 251, 60, 6, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $8 \times (5701, 6473, 2903, 884, 249, 49, 16, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $2 \times (5714, 6454, 2882, 932, 221, 64, 8, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $8 \times (5735, 6427, 2864, 946, 246, 52, 4, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
X_{16}	$2 \times (5649, 6609, 2800, 904, 224, 86, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $8 \times (5699, 6507, 2827, 938, 233, 65, 6, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ $16 \times (5727, 6439, 2867, 948, 223, 67, 4, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

Table 10: F-profiles for $q = 25$

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