

Paper Surface Topography Using Three-light Photometric Stereo

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Abstract

In paper industry, the measurement of paper surface topography is important for paper quality control. Non-contact and fast photometric stereo overcomes the disadvantages of the traditional surface topography measurement by the profilometer. In this paper, we present an improvement to three-light photometric stereo for paper surface topography reconstruction. Experiments comparing reconstructed surfaces to optical profilometer measured surfaces demonstrate that the improvement increases robustness with regard to parameter selection in surface reconstruction, and provides smaller reconstruction errors.

1 Introduction

Paper surface topography has an important role in printing and paper manufacturing, since printing defects can be frequently derived from paper surface variations, that is, paper surface roughness. At the present time, the standardized roughness rating methods in the paper industry are profilometers and airflow-based measurement devices. Both methods require expensive special equipment, and essential laboratory conditions. In addition, they are slow, and therefore unsuitable for on-line control purposes.

Photometric stereo [9] provides a fast and non-contact alternative to the standardized methods. At least three monochromatic images of a sample with non-coplanar illumination directions are required to determine the surface height gradients and the surface reflectance. A modern description of the photometric stereo algorithm can be found in [2]. The traditional method for integrating the surface height from gradient information is the Frankot-Chellappa algorithm [3]. The most recent developments in surface reconstruction from gradient fields can be found in [1]. These methods are suitable when both the x- and y-gradient fields are available. Recently Hansson [5, 4] has studied two- and three-light photometric stereo in paper surface reconstruction. In his methods, the paper surface topography is calculated from one and three gradient fields in two- and three-light methods, respectively.

In this work we study and improve Hansson's three-light photometric stereo method. The results are contrasted to optical profilometer measured surface topographies, whereas Hansson applied reference profiles measured by mechanical profilometer.

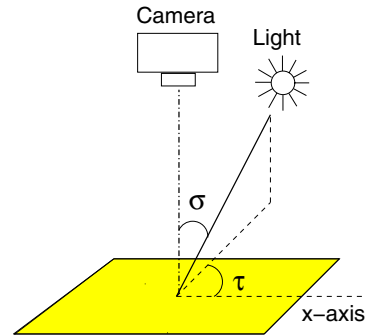


Figure 1: Geometry of the illumination.

2 Review of Photometric Stereo

In photometric stereo, the viewing direction is held constant while the direction of the illumination between successive images is varied. Thus, the correspondence between image points is known *a priori*. The usage of the radiance values at a single image location, in successive views, makes the technique photometric. The technique can be used to determine the surface orientation at each image point [9].

For the Lambertian surfaces the reflected intensity is independent of the viewing direction. However, the intensity depends on the direction of the light source. Lambert's Law [7] represents the image intensity at the point (x, y)

$$i(x, y) = \rho\lambda(\mathbf{l}^T \cdot \mathbf{n}), \quad (1)$$

where ρ is the surface albedo, λ is the intensity of the light source, $\mathbf{n} = [n_1, n_2, n_3]^T = \frac{[p, q, 1]^T}{\sqrt{p^2 + q^2 + 1}}$ is the unit normal to the surface and $\mathbf{l} = [\cos(\tau)\sin(\sigma), \sin(\tau)\sin(\sigma), \cos(\tau)]^T$ is the unit vector toward the light source. Elements p and q are surface partial derivatives measured along the x and y axes, respectively. τ is the tilt angle of illumination; the angle that the projection of the illuminant vector incident onto the test surface plane makes with an axis in that plane. σ is the slant angle that the illuminant vector makes with a normal to the test surface plane. Fig. 1 illustrates the tilt and slant angles. Lambert's Law assumes orthogonal projection and constant illumination over the surface. In orthogonal projection, light rays traveling from the object to the image are parallel, and the focal length is infinite.

3 Hansson's Three Light Photometric Stereo

Hansson and Fransson [4] extended a two light photometric stereo method in [5] into a three light method. They used tilt angles 0° , 120° and 240° and derived directed derivatives for the surface for the respective tilt angles. They modeled the signal as

$$s_0(x, y) = \frac{\partial f}{\partial x} * PSF + n(x, y), \quad (2)$$

where $s_0(x, y)$ is the measured directional derivative of slant angle 0° , $\frac{\partial f}{\partial x}$ is the directional derivative of surface height function f , $n(x, y)$ is the noise, $*$ represents a convolution, and PSF is a point-spread function. The Fourier transform of $s_0(x, y)$ is

$$S_0(u, v) = i2\pi u F_0(u, v) OTF(u, v) + N(u, v), \quad (3)$$

where u and v are spatial frequencies, $F(u, v)$ is the Fourier transform of surface height function, $OTF(u, v)$ (optical transfer function) is the Fourier transform of PSF , and $N(u, v)$ is the Fourier transform of $n(x, y)$. Hence the frequency response is

$$H_{I,0}(u, v) = i2\pi u OTF(u, v). \quad (4)$$

For the directional derivatives of tilt angles 120° and 240° , the frequency responses can be written as

$$H_{I,120}(u, v) = i\pi(-u - \sqrt{3}v) OTF(u, v), \quad (5)$$

and

$$H_{I,240}(u, v) = i\pi(-u + \sqrt{3}v) OTF(u, v). \quad (6)$$

Hansson utilized the Wiener filter as a restoration filter, and in integration of the filtered signal. The applied Wiener filter is given as follows:

$$H_{R,k} = \frac{H_{I,k}^*}{|H_{I,k}|^2 + SNR(u, v)^{-1}}, \quad k \in \{0, 120, 240\}, \quad (7)$$

where $SNR(u, v) = |F(u, v)|^2 / |N(u, v)|^2$ is the signal-to-noise ratio in the frequency domain.

The Fourier transform of the reconstructed surface height functions is given by

$$\hat{F}_k(u, v) = S_k(u, v) H_{R,k}(u, v), \quad k \in \{0, 120, 240\}. \quad (8)$$

Hansson proposed using the following weight functions for summation of the reconstructed surface height functions

$$w_k(u, v) = \frac{\hat{F}_k}{\hat{F}_0 + \hat{F}_{120} + \hat{F}_{240}}, \quad k \in \{0, 120, 240\}. \quad (9)$$

The weights are proportional to the absolute spectrum of the surface height functions at each point, and therefore the strongest signal will affect the final results most.

Using the weight functions, the Fourier transformed surface height functions can be integrated to common surface height function in the frequency plane

$$\hat{F} = \hat{F}_0 w_0 + \hat{F}_{120} w_{120} + \hat{F}_{240} w_{240}. \quad (10)$$

By using the inverse Fourier transform on \hat{F} , the reconstructed topography of the surface is obtained.

Experimentally we have found that Hansson's proposed weighting functions for surface height functions do not perform well. The Wiener filter is optimal in reconstruction of stochastic surfaces with known parameters. However, an accurate approximation of OTF and SNR functions is not trivial. Therefore we propose two new weighting functions, Average and Symmetrical weighting functions, which are less dependent on Wiener filtering parameters.

Our first proposal, average weighting functions, are constant weighting functions, and given as

$$w_0 = w_{120} = w_{240} = \frac{1}{3}. \quad (11)$$

The Fourier transformed surface height functions are averaged, and therefore each of the surface height functions has the same weight. This corresponds to having the least amount of prior knowledge. In Hansson's weighting functions, the strongest signal is emphasized, because both the weight and strength of the signal are high. This can be crucial for imperfectly restored signals, since the noise can have a significant influence on the final result.

Our second proposal is to use the following symmetrical weight functions

$$w_0 = \begin{cases} 1 & -30^\circ \leq \theta < 30^\circ \vee 150^\circ \leq \theta < 210^\circ \\ 0 & \text{otherwise} \end{cases}, \quad (12)$$

$$w_{120} = \begin{cases} 1 & 90^\circ \leq \theta < 150^\circ \vee 270^\circ \leq \theta < 330^\circ \\ 0 & \text{otherwise} \end{cases}, \quad (13)$$

and

$$w_{240} = \begin{cases} 1 & 210^\circ \leq \theta < 270^\circ \vee 30^\circ \leq \theta < 90^\circ \\ 0 & \text{otherwise} \end{cases}, \quad (14)$$

where θ is the angle with respect to the x-axis in the test surface plane, see Fig. 2 (a-c). Symmetrical weighting functions are not dependent on the surface height functions, but the signals from illumination direction are assumed to provide the most correct information from the respective direction.

4 Experimental Results

The purpose of the experiments was to study the accuracy of the weighting functions in surface reconstruction with different SNR values. Six paper and cardboard samples were measured using the three-light photometric stereo method and a laser profilometer. Correlations and surface reconstruction errors were calculated to the profilometer measurements.

The images for photometric stereo were acquired using a 3CCD camera, Hamamatsu C7780-10 with resolution of 1344×1024 pixels with 12 bits per pixel. In the experiments, the image area was $13.7 \text{ mm} \times 10.4 \text{ mm}$. A fiber optic halogen light source was utilized as a white light source. The images were acquired using slant angle of 60° for

illumination. Topographies were reconstructed using the three weighting functions: 1) Hansson's proposed weighting functions according to signal strengths of surface height functions, 2) Average of the three surface height functions, and 3) Symmetrical weight functions, denoted as Hansson, Average, and Symmetrical, respectively. The Hansson and Symmetrical weighting functions are presented in Fig. 2.

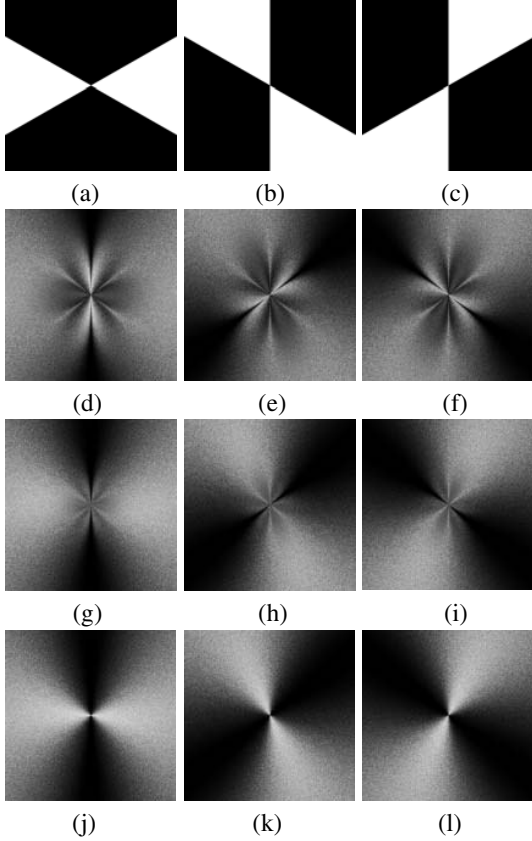


Figure 2: Weighting functions for 0°, 120°, and 240° tilt angles. (a-c) Symmetrical weighting functions, and Hansson's proposed weighting functions for different SNR factors: (d-f) $4 \cdot 10^{-4}$, (g-i) $4 \cdot 10^{-5}$, and (j-l) $4 \cdot 10^{-7}$ for Cardboard 1 sample.

An optical profilometer was used as a reference topography measurement device. The profilometers are too slow for on-line use, but in laboratory conditions and for small measurement areas, good accuracy compensates the weaknesses of the profilometers. The advantages to the mechanical profilometers are non-contact measurement, better spatial resolution, and possibility to acquire 2D images [8]. The applied profilometer was Rodenstock RM-600 3-D/C laser profilometer with resolution of $5 \mu m$ in profile direction, and $5 \mu m$ between profiles.

Experiments on real profilometer data were performed using a sample set, which included a variety of paper samples and cardboard samples. The sample set consists of two light weight coated (LWC) paper samples, two supercalendered (SC) paper samples, and two base cardboard samples. The first two sample types, LWC and SC are similar in roughness, while the cardboard is significantly rougher. Each sample was measured with the laser profilometer, and the imaged area was $15 mm \times 15 mm$, which

corresponds to image size of $3000 \text{ pixels} \times 3000 \text{ pixels}$.

The surfaces reconstructed using photometric stereo were registered with the profilometer measurements using a cross-correlation based method [6]. The profilometer measurements were point-wise aligned to the photometric stereo measurements using geometric affine transformation and interpolation.

Hansson [5] proposed the following functions for the OTF and SNR:

$$OTF = \frac{4000}{4000 + 4\pi^2(u^2 + v^2)}, \quad (15)$$

where u and v are given in mm^{-1} , and

$$SNR = \frac{4 \cdot 10^{-4}}{\Delta x^2(u^2 + v^2)}, \quad (16)$$

where Δx is the image resolution in millimeters. The Wiener filter performs the inverse of PSF, that is it restores the signal from the noise originated from the acquisition device, optics and light scattering from paper surface. The proposed Wiener filter attenuates higher frequencies. The effect of the filter response of the Wiener filter, $H_{R,k}$ in Eq. 7 to the three weighting functions was studied in means of correlation, and RMS error to the profilometer measurements. The numerator in signal-to-noise ratio in Eq. 16, denoted as SNR factor, was altered from $4 \cdot 10^{-3}$ to $4 \cdot 10^{-7}$. Hansson's proposed OTF was adopted directly. The Hansson weighting functions for three SNR factors are presented in Fig. 2. As the SNR factor decreases, the shape of the Hansson weighting functions becomes closer to Symmetrical weighting functions. In Figure 3 are presented the RMS errors to the profilometer measurements on Cardboard 1 sample, and LWC 1 sample when different SNR factors were used. For both samples, the RMS error of the Symmetrical weighting functions is nearly constant over all the SNR factors, whereas for the Average and Hansson weighting functions, the RMS error increases as the SNR factor increases. Table 1 shows correlations, r , and RMS errors

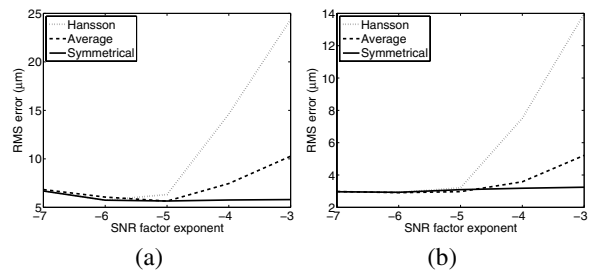


Figure 3: RMS errors for weighting functions for different SNR factors. Samples: (a) Cardboard 1, and (b) LWC 1.

for the three weighting functions to profilometer measured surface for all the samples. The results are calculated from topographies, which were scaled to the same mean and variance as profilometer topographies, since gradient fields do not provide information on the scale of the measurements. For the cardboard samples, the correlations are significantly higher than for the two other sample sets. RMS error is lower for SC and LWC samples than for cardboard samples,

Table 1: Correlations, r , and RMS errors, RMSE, to profilometer measured surface for weighting functions. The applied SNR factors was $4 \cdot 10^{-5}$.

Sample	Hansson		Average		Symmetric	
	r	RMSE	r	RMSE	r	RMSE
Cardboard 1	0.53	6.32	0.59	5.65	0.60	5.65
Cardboard 2	0.42	7.44	0.49	6.83	0.50	6.91
SC 1	0.18	4.23	0.24	3.56	0.25	3.98
SC 2	0.28	4.61	0.31	3.59	0.31	4.08
LWC 1	0.26	3.21	0.26	2.96	0.25	3.09
LWC 2	0.23	2.57	0.29	2.25	0.28	2.44

since the RMS error decreases as the surface variation, that is roughness, decreases.

Figure 4 presents profiles, which are calculated using the Symmetrical and Hansson weighting functions in addition to reference profile measured with profilometer. The profiles are from Cardboard 1 sample. In the figure titles are the coefficients of determination, r^2 , which are calculated between the reference profile and the photometric stereo reconstructed profile. Figure 4 demonstrates the effect of SNR factor. In Fig. 4 (a) and (b) SNR factor was $4 \cdot 10^{-4}$, which was originally recommended by Hansson. As the SNR factor decreases, the Wiener filter attenuates the higher frequencies more, and the variation of the reconstructed surface becomes smaller. The SNR factor $4 \cdot 10^{-5}$ produces reasonable results, but the factor $4 \cdot 10^{-7}$ attenuates nearly all the variation from the surface. The coefficients of determination are high for small SNR factors, but at the same time the RMS error and surface shape deteriorate.

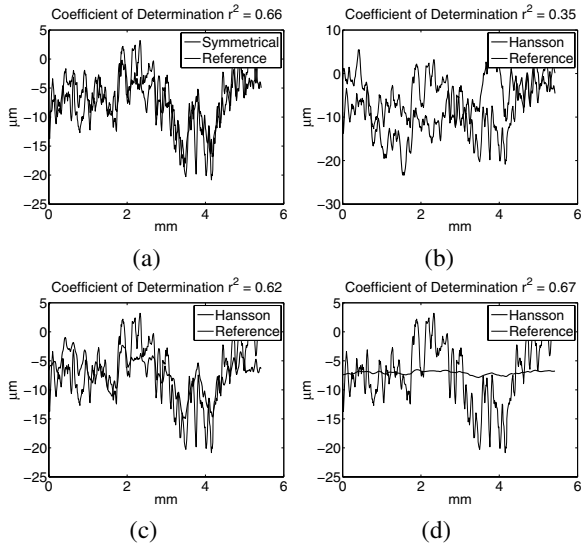


Figure 4: Profiles calculated with (a) Symmetrical weighting functions (SNR factor $4 \cdot 10^{-4}$), and Hansson weighting functions using SNR factors: (b) $4 \cdot 10^{-4}$, (c) $4 \cdot 10^{-5}$, and (d) SNR factor $4 \cdot 10^{-7}$. In (b) the y-axis has been scaled to focus on a region of interest.

5 Conclusions

In this work, Hansson's three-light photometric stereo method was studied and further developed. The reconstructed topographies were aligned to profilometer measurements, and the linear correlations and RMS errors

were calculated between measurement methods. Two new weighting functions for surface reconstruction were proposed, which performed better than the originally proposed functions on six different paper and cardboard samples.

The applied Wiener filter in surface reconstruction is optimal for a stochastic surface with known parameters. If the parameters are known, and there is not much noise present, the proposed weighting functions in the previous work [4] can be suitable. However, if the SNR is overestimated, the noise is not attenuated by the Wiener filter. Therefore, the energy of the noisy signal spreads over the spectra of the reconstructed topographies, and the Hansson weighting functions are disturbed, see Fig. 2. The proposed sample independent symmetrical weighting functions appeared to be more robust in paper surface reconstruction.

This work is part of a research project aiming to develop a machine vision system for on-line measurement of paper surface topography. The photometric stereo methods are fast and non-contact, and therefore suitable for on-line measurements. The future challenges of photometric stereo include faster, more robust and more accurate surface reconstruction.

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