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A Multi-Scale Regularity Measure as Geometric Criterion for Image Segmentation

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Abstract

Recent experimental results [1] on human vision show that low fractal dimension lines are highly capable to evocate namable objects. In other terms, regular lines are recognized by human vision as object edges. In this paper, a regularity measure of discrete lines geometry is presented. This quantitative measure based on a ratio between lines lengths at different scale is analyzed in the framework of brownian motion theory. The measure at a given scale is always computed from the maximum precision image, so that it doesn't introduce any sub-resolution assumption. A scale choice determines the quantity of global information vs. local information one wants to measure. We show how this quantitative measure leads to a relevant shape information. To illustrate this, an image segmentation application example is realized. The segmentation based essentially on geometry criteria uses a region growing process which depends on a single parameter that can be fixed in a natural way, comparing contour regularity to a geometric model regularity. We present experimental results performed on real-scene images, including indoor and outdoor images.

1 Introduction

Rogowitz and Voss [1] have presented in a recent article a series of experiments which shows that a human eye easily sees and "understands" curves of low fractal dimension. Curves of that class are highly capable to "evocate namable objects". In other terms, *regular* lines are recognized by human vision as object edges. And in a wide variety of images, the interesting objects are man-made, so that they possess the quality of a high geometric regularity. For indoor images, objects will be walls, doors, chairs or tables; for outdoor images, they will be roads or buildings. It is therefore quite interesting to be able to quantify the regularity of an object. We present in this paper a new regularity measure for digitized curves, which is an attempt to formulate what we call regularity. In section 2, we define this measure and we study its theoretic ability to discriminate irregular curves from regular curves. Section 3 is devoted to show how this measure can be applied to the segmentation of images in which the intersting objets are manmade. In section 4, we present experimental results performed on real-scene images, including indoor and outdoor images.

2 The Regularity Measure

Fractal theory introduces a nice notion of curve regularity with the fractal dimension [2], which describes the evolution of the curve length as the length measurement takes more and more curve details into account. To compute the fractal dimension of a curve, one typically counts the number N of balls of radius ε which are needed to cover that curve, and estimates the limit slope of the graph $ln\varepsilon \mapsto -lnN$ when ε is close to 0. But a crucial problem arises when one wants to study any local property P in a digitized curve, which is that all accessible informations are quantized. What sense can have an expression such as $\lim_{\varepsilon \to 0} P(\varepsilon)$? As distances are lower bounded by the pixel size, ε should always be greater that one pixel. To estimate the fractal dimension, it is needed to compute the number N at many low scales $\varepsilon = 1, 2..n$ pixels, and assume that the graph $ln\varepsilon \mapsto -lnN$ is well described by a straight line $lnN = lnL - s \cdot ln\varepsilon$, that is to say the curve is *self-similar* enough. In other words, the fractal dimension is representative of a scale range, not of one scale. So how can we determine regularity at a specific scale, especially when the scale is chosen for a close-to-the-pixel analysis?

As image processing occurs in a discrete world where the pixel size may be not considered as negligible compared to the size of the objects represented in the image, we think it's necessary to find out a discrete definition of regularity instead of a continuous one: we do not want to deal with limits when an ε tends to zero. The fractal idea of comparison between lengths at different scales seems to us to prefectly correspond to the notion of regularity. In a discrete framework, it is natural to compare the length at a detail level (or scale) of s pixel to the one at $k \cdot s$ pixels, where k is an integer. So let $C = (p_i)_1^n$ be a digitized curve, where p_{i+1} is connected to p_i. We define the k-regularity at scale s of the curve C at the pixel p_i as:

$$r_{s,k}(C)(i) = \frac{\| p_{i+ks} - p_i \|}{\Sigma_{j=1}^{k} \| p_{i+js} - p_{i+(j-1)s} \|}$$

and the k-regularity at scale s of the curve C as its average value along C:

$$R_{s,k}(C) = \overline{r_{s,k}(C)(i)}$$

We can see k as a smoothing parameter.

Let us present the behavior of the k-regularity on two particular digitized curves: the Brownian motion and the digitized straight line. These are our models of random irregularity and extreme regularity. The brownian motion is a curve where each point p_{i+1} has the same probability to be one of four points connected to p_i, while the digitized straight line is the digitization of an underlying continuous straight line, where the position p_{i+1} is highly constrained by the positions of $p_{k \leq i}$. Figure 1 presents the means and variances of the k-regularity applied to these two curves, for different values of the scale parameter s and the smoothing parameter k. Even when s and k are unreasonably close to the pixel size, $R_{s,k}$ reveals a pertinent information on the curve structure. Although variance is quite important, the line regular structure is visible even through $R_{1,2}$, which should a priori reflect nothing but a digitization noise. Of course, as s and k increase, the digitization effects are less and less important, so that $R_{s,k}$ discriminates more and more between the line and the random path.

So $R_{s,k}$ is a tool which is able to compute regularity at an arbitrary scale, even when the scale is very close to the pixel size. Let us see with the next section an application for which a close-to-the-pixel analysis is extremely useful.

3 An application: a regularity criterion for image segmentation

A region of a digitized image has two different signatures: the radiometric one depends on the pixel intensity inside the region, and the geometric one depends only on the line in $\mathbb{N}^2$ that is

		Brownian Motion		Straight Line	
k	s	Mean	Variance	Mean	Variance
2	1	0.6036	0.1357	0.8284	0.0209
	2	0.7937	0.0925	0.9628	0.0015
	3	0.6732	0.0870	0.9810	0.0006
	4	0.7504	0.0853	0.9892	0.0001
3	1	0.5295	0.0530	0.8055	0.0117
	2	0.6455	0.0965	0.9547	0.0011
	3	0.5634	0.0553	0.9774	0.0004
	4	0.6053	0.0770	0.9871	0.0002
4	1	0.4383	0.0579	0.7967	0.0094
	2	0.5497	0.0795	0.9518	0.0009
	3	0.4828	0.0514	0.9762	0.0003
	4	0.5187	0.0616	0.9864	0.0001
5	1	0.4039	0.0369	0.7926	0.0084
	2	0.4852	0.0637	0.9505	0.0007
	3	0.4349	0.0407	0.9756	0.0003
	4	0.4611	0.0504	0.9860	0.0003

Figure 1: $R_{s,k}$ - Brownian Motion vs. Straight Line

the region boundary. Numerous papers deal with segmentation based on radiometric models, and these models have been studied for a long time [3, 4, 5, 6, 7]. Authors [8, 9] have also studied the introduction of geometric models in the segmentation process. They consider the geometric analysis as a balance to the radiometric analysis. Formalizing the segmentation problem as an energy minimization problem, this leads to minimize an energy which takes the form $E = R + \lambda G$, where R computes the current partition radiometric deviation from the expected segmentation, and G the geometric one. The geometric factor, if used alone, leads to the trivial segmentation, where the whole image is a single region. For Leclerc [8], the geometric part computes the length of the region boundaries. Fua and Hanson [9] propose more sophisticated criteria, each one dedicated to segment a particular class of objects. For example, the geometric part of a criterion devoted to find buildings out of aerial images takes the form:

$$G = \alpha\theta + \frac{2L}{s}$$

where α is a constant parameter, θ is the average deviation from $\pi/2$ multiples of the angle formed by the contour tangent and an fixed direction, L is the contour length and s is a scale parameter. So the energy's geometric part is minimum when regions boundaries are composed of segments in two orthogonal directions.

Pure geometrics models occur mainly in shape analysis, where objects boundaries are considered as already defined and when the segmentation process is over [10, 11, 12]. Our regularity measure may be used without being mixed to any radiometric criterion. Before we show how it is possible, let us explain why it is difficult to base a segmentation process on a pure geometric criterion. Following Zucker [7], a segmentation of an image X depends on a boolean predicate P, and consists of a set of X-subsets $\{X_i\}_1^N$ such that:

(i) $\{X_i\}_1^N$ is a partition of X.

(ii) X_i is connected.

(iii) $P(X_i) = \text{TRUE}$ for each i.

(iv) $P(X_i \bigcup X_j) = \text{FALSE}$ for $i \neq j$, where X_i and X_j are adjacent.

The predicate P determines whether a subset X_i is a part of a scene object or not. We can interpret (iii) as "each region belongs to at most one object", and (iv) as "each object contains at most one region". An algorithm can be naturally deduced from these points: from an original *over-segmentation*, that is a partition such that $P(X_i) = \text{TRUE}$ for each i, loop while possible:

$$\text{if } \exists(i,j), P(X_i \bigcup X_j) = \text{TRUE}, \text{then merge } X_i \text{ and } X_j$$

Let us take a simple example. Suppose we have to segment an image composed by two perfectly homogeneous objects: a black square on a white background. A "good" predicate would be:

$$P(X_i) : \int_{X_i} |I(x) - \bar{I}| dX_i = 0$$

The algorithm applied on any over-segmentation of X will lead to the true segmentation. But can we now imagine a pure geometric predicate that will lead to the true segmentation, whatever the original over-segmentation is? Can we geometrically decide whether a region belongs to a single scene object or not ? It is clear that the regions geometry is highly dependent on the initial over-segmentation. A region of the over-segmentation can have any arbitrary shape, so that the answer of the last two questions is NO, there is no over-segmentation independent geometric criterion which can lead to the perfect segmentation. Thus, if one wants to perform segmentation according to a geometric criterion, one must whether combine it with a radiometric one, in a $R + \lambda G$ style for example, or restrain the set of over-segmentation to a particular class, a class of *admissible partitions*. In this paper, we have choosen the latter solution, in order to show the effects of our regularity measure only. We have constrained the over-segmentation to be grey-level subsamplings. Figure 2 presents such an over-segmentation for an indoor image. Let us show now that the regularity measure is able to perform the segmentation as soon as the initial partition is a grey-level subsampling.

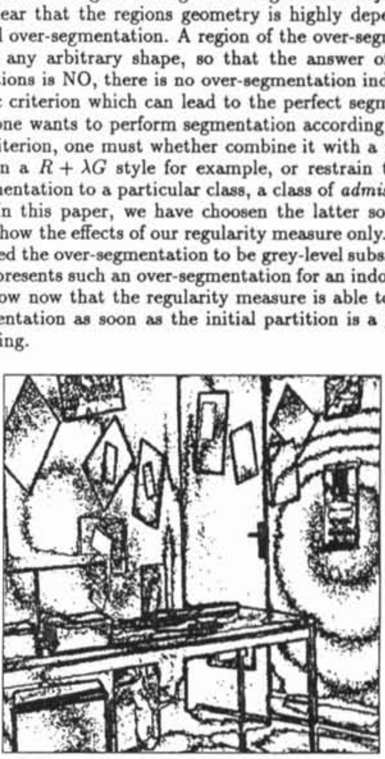

Figure 2: intensity subsampling

Regions boundaries in this kind of partition can belong to one of the two following classes:

- the region boundary is also an scene object boundary. In this case, the region boundary inherits the geometric regularity of the object boundary. We call this class *geometric boundaries*.
- the region boundary is *not* an scene object boundary. Here, the region is stopped *arbitrary*, depending on its texture or surface noise. We call this class *textural boundaries*.

Textural boundaries are irregular according to a close-to-the-pixel analysis, while geometric boundaries are regular. Our regularity measure should thus be able to distinguish the two classes of boundaries: textural boundaries are less regular than a certain threshold, geometric boundaries are more regular than this threshold.

4 Experimental Results

We have involved our regularity measure in a region-growing algorithm. Since we are interested in the close-to-the-pixel behavior

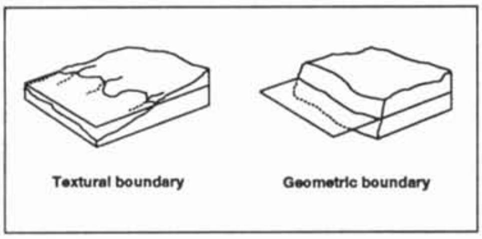

Figure 3: The two classes

of the curves, we have chosen $R_{2,2}$ to discriminate textural boundaries from geometric boundaries. We have fixed the threshold value to 0.92, which is a numeric approximation of the minimum regularity of the straight line. That means that the straight line is our model of regularity. The algorithm is inspired by the *sub-optimal segmentation algorithm* proposed by Monga [13]. The problem is that the formalism used by Monga, which follows the one presented by Zucker in [7], is not strictly applicable to a segmentation based on geometric criteria. Recall that a segmentation of an image X following a boolean predicate P should be a set of regions $\{X_i\}_1^N$ that is constraint by the four properties ((i) to (iv)) exposed in section 3. The choice of same predicate P to determine whether a subset of X belongs to a single region or not, and to determine whether two subsets of X should merge or not, disables the possibility of using pure geometric predicates. To be more precise, each predicate that takes care only of what happens at the border of a subset is prohibited. In our case, two distinct objects may be merged, even if our criterion has decided that they were actually distinct.

These considerations lead us to adopt the following formalism:

(i) $\{X_i\}_1^N$ is a partition of X.

(ii) X_i is connected.

(iii) $P_r(X_i) =$ TRUE for each i.

(iv) $P_m(X_i, X_j) =$ FALSE for $i \neq j$, where X_i and X_j are adjacent.

We have to use two different predicates, a region predicate P_r which tests the ability of a subset to belong to a single object, and a merging predicate P_m which tests whether two subsets can belong to the same object or not. $P_m(X_i, X_j)$ must be FALSE if $P_r(X_i \bigcup X_j)$ is. For example, if we want to process a segmentation according to the regularity criterion, P_r would always be TRUE, and $P_m(X_i, X_j)$ would stand for: " the common frontier between X_i and X_j is a textural boundary". Note that one can choose $P_m(X_i, X_j) = P_r(X_i \bigcup X_j)$, in which case one come back to the Zucker formalism.

Monga proposes the use of a function that controls the ***quality*** of a segmentation. This function formalizes the ***strategic*** aspect of region growing. We use a function named $dQ(X_i, X_j)$ which quantifies the quality increasing of the segmentation when merging X_i and X_j.

Given an initial segmentation, the algorithm loops while possible on the three following points:

- determine the merging list, i.e. the list of P_m-mixable adjacent pair of subsets;
- sort the merging list according to dQ;
- merge the best independent pairs.

We present now the application of the algorithm on several grey-level images. Beginning with an intensity sub-sampled image, segmentation is performed in two steps:

- small regions merging: small regions cannot be efficiently treated by the regularity criterion, since their boundaries contain a very poor geometric information. At this step, $P_m(X_i, X_j)$ is TRUE if the length of the boundary of X_i is less than N pixels, where N has been arbitrary fixed to 10. $P_r(X_i)$ is always TRUE, and $dQ(X_i, X_j)$ increases as the average gradient along the common frontier decreases. Since the best *independent* pairs merge, a small region merge with exactly one of its neighbors.
- regularity merging: $P_m(X_i, X_j)$ is TRUE if the regularity $R_{2,2}$ of the common frontier is lower than a regularity threshold. The threshold's value is 0.92, as discussed previously. $P_r(X_i)$ is TRUE whatever X_i is, and $dQ(X_i, X_j)$ is equal to $P_m(X_i, X_j)$.

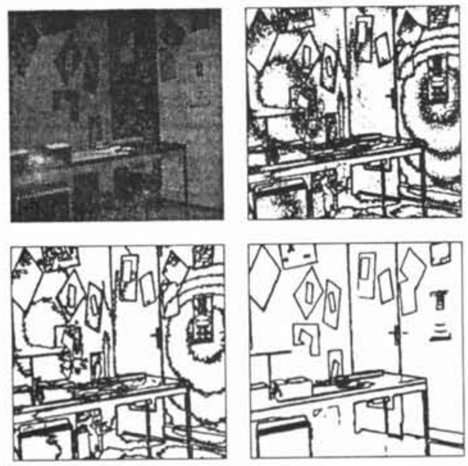

Figure 4: indoor processing

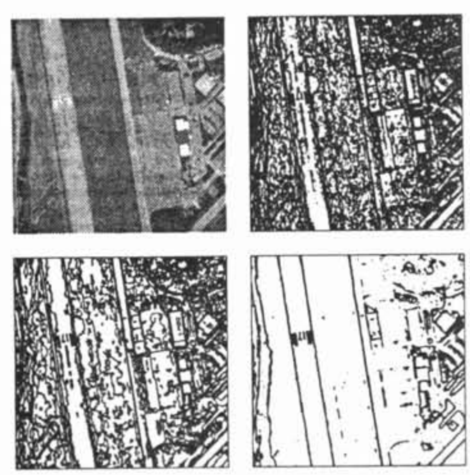

Figure 5: airport processing

In the following figures, the upper-left image (A) is the grey-level image, the upper-right image (B) is the grey-level subsampling, the lower-left image (C) is obtained from (B) by removing small regions, and the lower-right image (D) is obtained from (C) by applying the regularity merging. Figure 4 shows the segmentation processing on an indoor image. This first result shows our regularity criterion's ability to detect irregularity in a practical case. One can see that each irregular boundary in image (C) has been removed in image (D). Some other boundaries seem to have also disappear, in the poster on the right wall for example. In fact, this is not the case. Merging occurs here because there are regions of the intensity subsampling which belong to both the poster and the wall. These regions present irregularity against the wall, and irregularity against the poster, so that the poster is finally merged to the wall. This kind of problem is actually due to an incompatibility between our definition of acceptable partitions and the definition of over-segmentation (see section 2).

Figure 5 shows the segmentation processing on the aerial image an airport. The airport shows another direct application of the method described in this paper. Main structures have been correctly segmented, but some objects such as the group of trees on the upper-left corner have disappeared. This is due to the simplicity of the strategic function dQ we have used. Better results are obtained with the same merging predicate, but with more sophisticated strategic functions which quantify a confidence we can have in the regularity measure.

5 Conclusions

We have presented in this paper a set of geometric measures that quantify regularity of digitized curves. These measures have been theoretically tested on models of irregular and regular curves, the Brownian Motion and the digitized straight line. One of our regularity measure has been involved in a region-growing segmentation algorithm, and has shown its practical ability to perform segmentation on images where interesting features are man-made objects. In this paper, we have only use the regularity criterion as the decisive criterion. But it is possible to have it cooperate with other criteria, and in particular with radiometric criteria. Cooperation with radiometry may occur in two ways:

- the strategic function decides which mergings should come first;
- the initial radiometric over-segmentation gives its input data to the geometric criterion;

We have found two limitations on the use of the regularity mesure. The first one is that the regularity measure takes a pratical sense only if the length of the curve is large enough compared to the scale parameter. Our current work is to precise the confidence one can have in the regularity measure, depending on the length of the analysed curve. The second one is that in the segmentation process we proppose, the initial segmentation is highly constrained by the fact it must be an admissibility partition, so that a few radiometric criteria can be used here. In some cases, the admissibility criterion may be in conflict with the over-segmentation criterion. We have to find now is another and larger class of admissible partitions which eliminates that contradiction. One possible way is admissible frontiers instead of admissible partitions. The admissible frontiers would be the only frontiers to be allowed to disappear in a region merging. This would lead to a larger class of geometric-unstable over-segmentations, and would allow many radiometric criteria to be used in the presegmentation process.

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