MVA '96 IAPR Workshop on Machine Vision Applications, November. 12-14, 1996, Tokyo, Japan

MDL Mediated by Geometrical Redundancy Reduction and Complement Space

Zhiyong YANG, Songde MA and Qifa KE
National Laboratory of Pattern Recognition
Institute of Automation, Chinese Academy of Sciences
P. O. Box 2728, Beijing 100080, P. R. China
Email: yangzy@prlsun6.ia.ac.cn masd@prlsun2.ia.ac.cn

Abstract

Segmentation/binding/organization is a problem at the heart of machine and human vision. There are various approaches to this problem, such as local filtering[1], region competition[2], MDL (minimum description length) method[3], global energy optimi-zation[4], and correlation theory of the brain[5]. In this paper, we see visual representation as building descriptions of MDL and formulate segmentation /binding/organization as general functional optimi-zation. The main purpose of this paper is to elucidate how geometrical redundancy reduction (GRR) and complement space (Co-Space) of figure can mediate MDL. Several important binding examples are presented as the results of these mediating effects.

1 MDL Model

1.1 MDL For Figures

MDL for a solitary plane line segment of constant curvature is suggested as[6]

$$E_I = 2\,h_0 + \left(p + q \times |c|^{2\phi}\right) \times s^{\alpha} \tag{1}$$

In (1), $p > 0, q > 0$ are constants. s is the length of the segment. c is the curvature. $0 < \phi < 1$, $0 < \alpha < 1$, are exponents. $2h_0$ is the energy for two end points. Cells in the cat visual cortex can signal end points. The last two terms represent stretching and bending effects but multiplied by reduced length s^{α} $(0 < \alpha < 1)$. Up to the second order derivatives, Eq.(1) is complete and is invariant under coordinate transformation.

Let's consider the scaling property of Eq.(1). Consider a circle of radius R. Under scaling transformation, $R \rightarrow R' = \mu R,\ \mu > 1$, we have

$$E'_I = \left(p' + q' R^{-2\phi}\right) \times R^{\alpha}$$
$$p' = p \times \mu^{\alpha} \quad q' = q \times \mu^{\alpha - 2\phi} \tag{2}$$

So, scaling transformation is equal to changing the "interaction constants" and leaves Eq.(1) invariant. This is a good property and is absent in many other models. Under the limit $\mu \rightarrow \infty$, it is reasonable to ask $q' \rightarrow 0$. So we obtain a constraint on exponents $\alpha,\ \phi,\ \alpha - 2\phi < 0$ from the simple scaling analysis.

MDL for the interior of a region with area A is suggested as

$$E_S = \Phi \times A^{\beta} \quad (0 < \beta < 1) \tag{3}$$

Φ is a constant. β is an exponent.

1.2 MDL In Co-Space

The complement set of a figure or object is its complement space. The Co-Space of a figure can be filled with other figures, this is occlusions. We can describe a figure by

1 describing the figure itself,
2 describing its Co-Space, or
3 describing both the figure and its Co-Space.

Since MDL for figure itself is not always globally optimal, describing Co-Space and describing both figure and Co-Space become interesting important cases. The MDL for a line of constant curvature in Co-Space is suggested to be

$$\overline{E}_I = 2\,\overline{h}_0 + \left(\overline{p} + \overline{q} \times |c|^{2\phi}\right) \times s^{\alpha} \tag{4}$$

$\overline{h}_0 \approx h_0,\ \overline{p} \approx p,\ \overline{q} \approx q$ are constants. Eq.(4) has the same scaling property as Eq. (1). The MDL for the interior of a region with area A in Co-Space is supposed to be

$$\overline{E}_S = \overline{\Phi} \times A^{\beta} \quad (0 < \beta < 1) \tag{5}$$

$\overline{\Phi} \approx \Phi$ is a constant.

In usual computational theory, $\phi = 1,\ \alpha = 1,\ \beta = 1$, the various MDL items are simply additive and thus MDL per unit length or area is constant. Here, we suggest $0 < \phi < 1,\ \ 0 < \alpha < 1,\ 0 < \beta < 1$ for segments or regions of same geometrical redundancy. So with the increase of length S or area A, MDL distributed on unit length or area decreases. This property represents a kind of binding effect: points on a line segment or a region are not independent of each other but interact with each other. These interactions lead to the decrease of representing length per unit length or area with the increase of length or area(See Fig.1). Retinotopic mapping suggests the magnification factor to be $M = c\,r^{-\delta}$, $\delta = 1.13,\ 1.59,\ 2$ (c is a constant and r is the eccentricity)[7]. One may identify the exponents as $\alpha = 1 - \delta/2 = 0.435, 0.205, 0$, $\beta = \alpha$. $\alpha = 0$ refers to the logarithmic mapping. So our MDL model means a strategy for visual representation: doing geometrical redundancy reduction and representing shape in a global, correlating way, representing figure /background in a complement way.

With these items, our MDL model is quite different from those in regularization theory[8], active contour models[9] and the region competition model[2].

2 Solution to MDL : Collinear Binding

All Gestalt binding rules[10] can be formulated as optimal conditions of MDL. In fact, collinear condition, parallelism, proximity, closeness, good continuation and figure/ground segmentation serve to define "good" Co-Space. See Fig.2. Here we only consider collinear binding and illusory figures. More comprehensive analysis will be presented elsewhere in detail. First consider collinear binding. For simplicity, we only consider how two line segments of length a and curvature c aligned in a line of curvature c with a gap of length b can be grouped into the perception of a line.

For ungrouped two line segments, we have energy

$$H_1 = 4\,h_0 + 2 \times \left(p + q \times |c|^{2\phi}\right) \times a^{\alpha} \tag{6}$$

For the grouped state, we have energy

$$H_2 = 2\,h_0 + 2\,h_1 + \left(p + q \times |c|^{2\phi}\right) \times (2a+b)^{\alpha} + \left(\overline{p} + \overline{q} \times |c|^{2\phi}\right) \times b^{a} \tag{7}$$

$2\,h_1$ is due to the two near end points. Hereafter we simply set $h_1 = h_0$. The last term $\left(\overline{p} + \overline{q} \times |c|^{2\phi}\right) \times b^{\alpha}$ is for segment in Co-Space. A simple explanation is that visual system represents the whole line and then erases the gap. A line contour emerges from the line segments when the MDL mediated by GRR and Co-Space is the global minimum

$$\Delta H = \left(p + q \times |c|^{2\phi}\right) \times (2a+b)^{\alpha} + \left(\overline{p} + \overline{q} \times |c|^{2\phi}\right) \times b^{a} - 2 \times \left(p + q \times |c|^{2\phi}\right) \times a^{\alpha} \le 0 \tag{8}$$

Let $K(c) = p + q \times |c|^{2\phi}$, $\overline{K}(c) = \overline{p} + \overline{q} \times |c|^{2\phi}$

$\sigma(c) = \overline{K}(c)/K(c)$. Given α, σ solving (8), we can obtain critical gap $b_c(\sigma, a, \alpha)$(See Fig.3). $b < b_c$, the grouped state corresponds to real perception. When the curvature is zero, critical gap is scaling invariant, that is , if (a, b) is a solution, then $(\mu a, \mu b)(\mu > 0)$ is also a solution.

Generalizing to the case where n segments of length a and $(n-1)$ gaps of length b in between aligned in a line of curvature c, we obtain the critical gap $b_c(n, a, \sigma, \alpha)$. (Fig.3).

We have used this MDL to do line structure extraction in many synthesized and real images. Its power is really impressive[6].

3 Solution to MDL : Illusory Figures

The Kanizsa map in Fig.4 represents a kind of binding in Co-Space. Suppose that the MDL for a sector in the Co-Space is

$$\overline{E} = \overline{\Phi} \times \left(\mu\, a^2\right)^{\beta} \tag{9}$$

Then, we have binding condition

$$\Delta H = \overline{\Phi} \times (2a+b)^{2\beta} - 4 \times \overline{\Phi} \times \left(\mu\, a^2\right)^{\beta} \le 0 \tag{10}$$

$\mu \sim 0.5$ is a constant. μa^2 is a part of the area of the triangle. We obtain critical gap $b_c(a, \beta, \mu)$

$$b_c = \left(2^{1/\beta} \times \mu^{1/2} - 2\right) \times a \qquad (11)$$

Notice that critical gap is linearly scaled with the radius of the inducing circle. For example, $\beta = 0.435$, $\mu = 0.35$, $b_c = 0.911a$; $\beta = 0.435$,, $\mu = 0.3$, $b_c = 1.48a$. So, our model gives specific and meaningful results which can be tested by psychophysical experiments. Rewriting Eq.(11) as $b_c/a = \left(2^{1/\beta} \times \mu^{1/2} - 2\right)$, one can see critical gap curve is collapsed into a point in parameter space.

We notice that for binding in Co-Space, there usually are apparent brightness filled in the binding area. We also develop a network dynamics for binding in Co-Space. The main results resemble the prediction (11): a Kanizsa map emerges when $b \le b_c$ and b_c/a is scaling invariant.

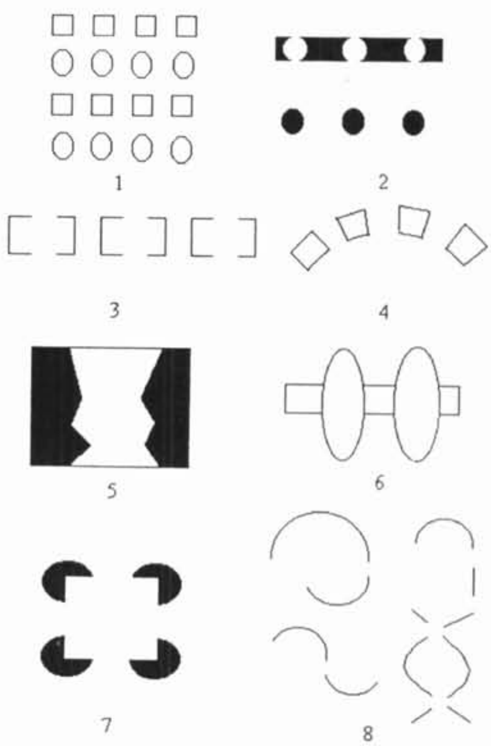


Fig.2 Example of good Co-Space
1 Rectangles and circles are perceived as two groups which define different Co-Space. 2 Co-Space defined by circles in a straight line. 3 Closed Co-Space induced by line segments is more preferred. 4 Co-Space of blocks is aligned along a arc. 5 Co-Spaces are closed and complement to each other. So they are alternatively perceived as figure or ground. 6 Figures in Co-Space of other figures. They possess relative depth. 7 Closed Co-Space induced by four packmen. 8 Collinear condition induces good Co-Space between pairs of various line segments.

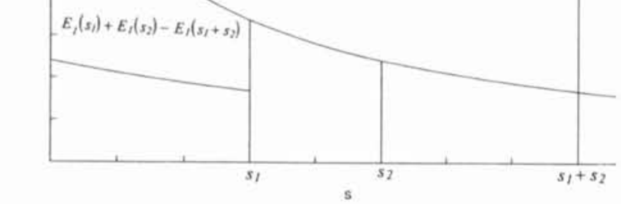


Fig.1 MDL for line segments with constant curvature. $d E_1/ds \sim s^{-(1-\alpha)}$, $d^2 E_1/d^2 s \sim -s^{-(2-\alpha)} < 0$. This property induces two kinds of binding effect, $\Delta_1(s) = E_1(s) - (K_0 \times s) < 0$

$\Delta_2(s_1 + s_2) = E_1(s_1 + s_2) - E_1(s_1) - E_1(s_2) < 0$.

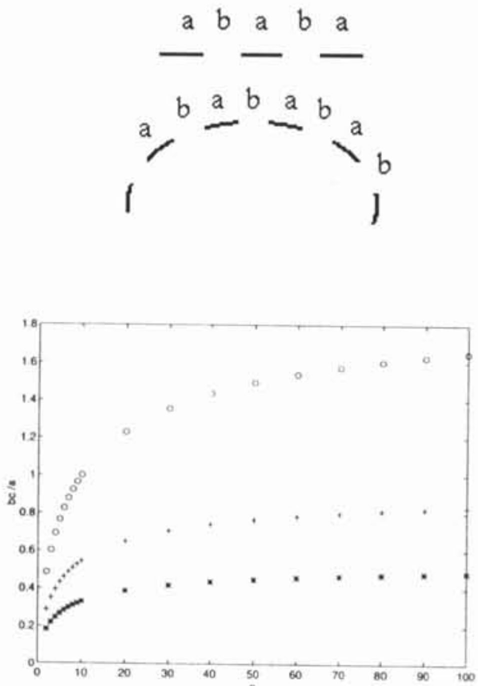
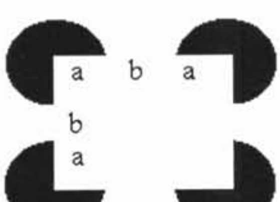


Fig. 3 Solution to MDL : Collinear binding. Top, examples of collinear binding. Middle, critical gap plot $b_c/a - -\overline{K}/K$ for two bars of length a with a gap of length b in between on a line. $b<b_c$, collinear binding occurs. $\alpha = 0.1$, '*' ; $\alpha = 0.2$, 'o' ; $\alpha = 0.4$, '+' ; $\alpha = 0.5$, '-' . Bottom, critical gap plot $b_c/a - -n$ for n segments of length a with $(n-1)$ gaps of length b in between on a line. $b<b_c$, collinear binding emerges. $\sigma = \overline{K}/K = 0.75$,'o'; $\sigma = \overline{K}/K = 1$, '+'; $\sigma = \overline{K}/K = 1.25$, '*' .



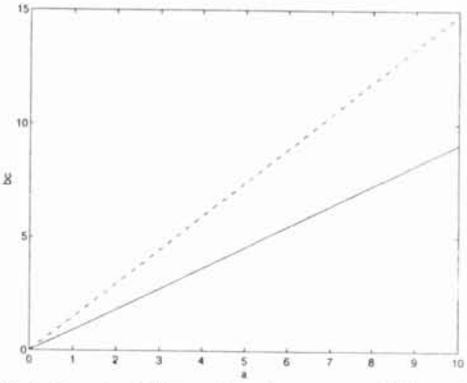


Fig.4 Solution to MDL : Kanizsa map. When $b \le b_c$, binding in Co-Space occurs and a Kanizsa map emerges. A square appears to occlude the four discs.

References

[1] J Canny, A computational approach to edge detection, *IEEE Trans PAMI* **8**, 679-698(1986).

[2] S C Zhu, T S Lee & A L Yuille, Region Competition: unifying snakes, region growing, and Bayes/MDL for Multi-band image segmentation, *Proc 5th ICCV'95*, 416-423.

[3] Y G Leclerc, Constructing simple stable descriptions for Image partitioning, *Int'l J Comp Vis*.**3**, 73-102(1989).

[4] J D McCaffery, Human and Machine Vision: Computing Perceptual Organisation. *Ellis Horwood Limited* 1990.

[5] DeLiang Wang, Emergent synchrony in locally coupled neural oscillators, *IEEE Trans Neural Networks* **6**, 941-948(1995).

[6] Z Y Yang & S D Ma, A phenomenological approach to salient maps and illusory contours, *Network: Comp. in Neural Sys*. (In the press).

[7] H A Mallot, An overall description of retinotopic mapping in the cat's visual cortex areas 17,18, and 19, *Biol Cybern* **52**, 45-51(1985).

[8] T Poggio, V Torre,& C Koch, Computational Vision and Regularization Theory, *Nature* **317**, 314-319(1985).

[9]L D Cohen 1991, On active contour models, *CVGIP, Image Underst* **53**, 211-218(1991).

[10] I Rock & S Palmer, The legacy of Gestalt psychology, *Sci Am* **263**, 84-90(1990).