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A novel representation for Multi-Channel log-polar quantum images

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Abstract. *Mostly existing quantum image models represent image in Cartesian coordinates. Some complicated affine transformation can not be easily handled in Cartesian coordinates. In this paper, a new representation of Multi-Channel log-polar quantum image (MCLPQI) sampled in log-polar coordinates is proposed. This form contains the image RGB color information and transparency α of the corresponding pixel. Then we give the polynomial preparation process which proves that our quantum image representation state can be made in a feasible time complexity. Finally we discussed color transformation among different channels and single channel color transformations, transparency adjustments. Analysis demonstrates that the MCLPQI is a efficient quantum log-polar color image representation.*
Keywords: Quantum computation, Quantum image representation, Log-polar image, Multi-Channel swapping transformation

1. **Introduction.** Since quantum computation is proposed, many researches have been conducted in recent decades. Quantum mechanics [1] or quantum computer can efficiently solve many problems than classical computer owing to its inherent unique properties such as quantum parallel computation [1], quantum entanglement [2] and so on. Any quantum unitary transformation can be divided into elementary unitary quantum gates which mainly act on one or two qubits [3] i.e. that is CNOT gate or Toffoli gate. Now in quantum computation, whether can design a quantum circuit to realize corresponding quantum algorithm becomes evaluation standard.

Digital image processing plays an important role in practical applications [4]. Correspondingly, quantum image processing has been a hot topic in recent years. Quantum image processing is a part of quantum computation. And the discussion about the quantum image also needs the concern of the quantum gates. The first challenge for researches is how to use quantum state to represent a quantum image. Many quantum image models have been designed, i.e., Qubit Lattice[5], Entangled image[6], Real Ket [7], and Flexible Representation of Quantum Images(FRQI)[8]. An enhanced quantum representation (NEQR)[9], Multi-Channel representation of quantum image(MCRQI)[10], and quantum representation for log-polar images[11], have been proposed. All of the existing models can only represent Multi-Channel color image in Cartesian coordinates. However, in our daily life colorful image is common and some complex transformation is based on three channels. Log-polar coordinates is a well-known sampling method in the field of image processing. And some complicated affine transformations especially about rotation and scaling can be easily discussed in Log-polar coordinates.

In this paper, a Multi-Channel log-polar quantum image representation (MCLPQI) is proposed for storing and processing images sampled in log-polar coordinates for the first time. The quantum state of MCLPQI is prepared in polynomial time complexity about the image size and corresponding quantum circuits are constructed. The state mainly contains the information of RGB channel and transparency information.

The rest of this paper is organized as follows. Section 2 gives the concrete form of MCLPQI and the polynomial preparation theories are studied in Section 3. Furthermore, transformations mainly about single color channel, different color channels and transparency channel are discussed in Section 4. Finally, Section 5 concludes this paper.

2. **New representation for Multi-Channel log-polar quantum image(MCLPQI).** A novel quantum representation model MCLIPQI is proposed about log-polar color images in this paper. In this section, the new form will be described and we will prove it is a quantum normalized state.

The concrete form of MCLPQI is shown in Eq. (1)

$$|I(\theta)\rangle = \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{RGB\alpha}^{\rho\theta}\right\rangle \otimes |\rho\theta\rangle \tag{1}$$

Where $\left|R_{RGB\alpha}^{\rho\theta}\right\rangle$represents the RGB color and transparency αrepresents the corresponding pixel.

$$\begin{aligned}\left|R_{RGB\alpha}^{\rho\theta}\right\rangle &= \cos\theta_R^{\rho\theta}|000\rangle + \cos\theta_G^{\rho\theta}|001\rangle + \cos\theta_B^{\rho\theta}|010\rangle + \cos\theta_\alpha^{\rho\theta}|011\rangle \\ &+\ \sin\theta_R^{\rho\theta}|100\rangle + \sin\theta_G^{\rho\theta}|101\rangle + \sin\theta_B^{\rho\theta}|110\rangle + \sin\theta_\alpha^{\rho\theta}|111\rangle\end{aligned} \tag{2}$$

Here, $\theta_X^{\rho\theta} \in \left[0, \frac{\pi}{2}\right]$,$X \in \{R, G, B, \alpha\}$,$\rho = 0, 1, ..., 2^m - 1$,$\theta = 0, 1, ..., 2^n - 1$ encodes color information and $|\rho\theta\rangle$ encodes the corresponding position information. The sampling resolutions of the log-radius and the angular orientations of a log-polar image are assumed to be 2^m and 2^n respectively. Obviously above MCLPQI representation is a normalized state.

$$\begin{aligned}&\||I(\theta)\rangle\| \\ &= \frac{1}{\sqrt{2^{n+m+2}}}\sqrt{\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left[\begin{array}{l}\left(\cos\theta_R^{\rho\theta}\right)^2 + \left(\cos\theta_G^{\rho\theta}\right)^2 + \left(\cos\theta_B^{\rho\theta}\right)^2 + \left(\cos\theta_\alpha^{\rho\theta}\right)^2 + \\ \left(\sin\theta_R^{\rho\theta}\right)^2 + \left(\sin\theta_G^{\rho\theta}\right)^2 + \left(\sin\theta_B^{\rho\theta}\right)^2 + \left(\sin\theta_\alpha^{\rho\theta}\right)^2\end{array}\right]} \\ &= \frac{1}{\sqrt{2^{n+m+2}}}\cdot\sqrt{2^m\cdot 2^n\cdot 2^2} = 1\end{aligned} \tag{3}$$

3. **Polynomial preparation for multi-channel log-polar quantum image.** In quantum computation, the preparation process transforms an initialized quantum state to the desired quantum image state required for further processing. This transformation involves unitary transforms described by unitary matrices.

Step1: we will construct an empty Quantum image with size $2^m \times 2^n$ from initial state $|I\rangle_0$.$|I\rangle_0 = |0\rangle^{\otimes 3+m+n}$

Two single quantum gates are shown, which will be utilized to build the quantum operation U_1.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U_1 = I \otimes H^{\otimes 2+m+n}$$

$$\begin{aligned} & U_1 \left(|I\rangle_0\right) = |0\rangle \otimes \left(\tfrac{1}{\sqrt{2^2}} \sum_{i=0}^{3} |i\rangle\right) \otimes \left(\tfrac{1}{\sqrt{2^m}} \sum_{\rho=0}^{2^m-1} |\rho\rangle\right) \otimes \left(\tfrac{1}{\sqrt{2^n}} \sum_{\theta=0}^{2^n-1} |\theta\rangle\right) \\ & = \tfrac{1}{\sqrt{2^{2+m+n}}} \left(|0\rangle \sum_{i=0}^{3} |i\rangle\right) \otimes \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} |\rho\theta\rangle \\ & = |I\rangle_1 \end{aligned} \tag{4}$$

The middle state $|I\rangle_1$ is an empty quantum image with size$2^m \times 2^n$, every pixel in which is stored in a normalized quantum superposition.

Step 2: we should set the RGB color of all pixels in the middle state $|I\rangle_1$.Because the log-polar image resolution is $2^m \times 2^n$, 2^{m+n} sub-operation are needed to set RGB color for every pixel individually in this procedure.

We define the following transform:

$$R_R^{\rho\theta} = I \otimes \sum_{i=0, i\neq 0}^{3} |i\rangle\langle i| + R_y(2\theta_R^{\rho\theta}) \otimes |0\rangle\langle 0|$$

$$R_G^{\rho\theta} = I \otimes \sum_{i=0, i\neq 1}^{3} |i\rangle\langle i| + R_y(2\theta_G^{\rho\theta}) \otimes |1\rangle\langle 1|$$

$$R_B^{\rho\theta} = I \otimes \sum_{i=0, i\neq 2}^{3} |i\rangle\langle i| + R_y(2\theta_B^{\rho\theta}) \otimes |2\rangle\langle 2|$$

$$R_\alpha^{\rho\theta} = I \otimes \sum_{i=0, i\neq 3}^{3} |i\rangle\langle i| + R_y(2\theta_\alpha^{\rho\theta}) \otimes |3\rangle\langle 3|$$

$$R_y(2\theta_R^{\rho\theta}) = \begin{pmatrix} \cos\theta_R^{\rho\theta}, -\sin\theta_R^{\rho\theta} \\ \sin\theta_R^{\rho\theta}, \cos\theta_R^{\rho\theta} \end{pmatrix}, \quad R_y(2\theta_G^{\rho\theta}) = \begin{pmatrix} \cos\theta_G^{\rho\theta}, -\sin\theta_G^{\rho\theta} \\ \sin\theta_G^{\rho\theta}, \cos\theta_G^{\rho\theta} \end{pmatrix}$$

$$R_y(2\theta_B^{\rho\theta}) = \begin{pmatrix} \cos\theta_B^{\rho\theta}, -\sin\theta_B^{\rho\theta} \\ \sin\theta_B^{\rho\theta}, \cos\theta_B^{\rho\theta} \end{pmatrix}, \quad R_y(2\theta_\alpha^{\rho\theta}) = \begin{pmatrix} \cos\theta_\alpha^{\rho\theta}, -\sin\theta_\alpha^{\rho\theta} \\ \sin\theta_\alpha^{\rho\theta}, \cos\theta_\alpha^{\rho\theta} \end{pmatrix}$$

From the construction of the above transform, we can construct a useful transform$U_{\rho\theta}$.

$$U_{\rho\theta} = I^{\otimes 3} \otimes \sum_{j=0}^{2^m-1} \sum_{i=0, ji\neq\rho\theta}^{2^n-1} |ji\rangle\langle ji| + \Omega_{\rho\theta} \otimes |\rho\theta\rangle\langle\rho\theta| \tag{5}$$

$$\Omega_{\rho\theta} = R_B^{\rho\theta} \cdot R_G^{\rho\theta} \cdot R_R^{\rho\theta} \cdot R_\alpha^{\rho\theta} \tag{6}$$

When we apply every sub-operation on the middle state$|I\rangle_1$, we can get the following equation.

$$
\begin{aligned}
&U_{\rho\theta}\left(|I\rangle_1\right)=\left(I^{\otimes 3}\otimes\sum_{j=0}^{2^m-1}\sum_{i=0,ji\neq\rho\theta}^{2^n-1}|ji\rangle\langle ji|+\Omega_{\rho\theta}\otimes|\rho\theta\rangle\langle\rho\theta|\right)\\
&\left(\tfrac{1}{\sqrt{2^{2+m+n}}}\left(|0\rangle\sum_{i=1}^{3}|i\rangle\right)\otimes\left(\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}|\rho\theta\rangle\right)\right)\\
&=\tfrac{1}{\sqrt{2^{2+m+n}}}\left\{\left(I^{\otimes 3}\left(|0\rangle\sum_{i=1}^{3}|i\rangle\right)\right)\otimes\left(\sum_{j=0}^{2^m-1}\sum_{i=0,ji\neq\rho\theta}^{2^n-1}|ji\rangle\langle ji|\left(\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}|\rho\theta\rangle\right)\right)+\right.\\
&\left.\left(\Omega_{\rho\theta}\left(|0\rangle\sum_{i=1}^{3}|i\rangle\right)\right)\otimes\left(|\rho\theta\rangle\langle\rho\theta|\left(\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}|\rho\theta\rangle\right)\right)\right\}\\
&=\tfrac{1}{\sqrt{2^{2+m+n}}}\left(|0\rangle\sum_{i=1}^{3}|i\rangle\right)\otimes\sum_{j=0}^{2^m-1}\sum_{i=0,ji\neq\rho\theta}^{2^n-1}|ji\rangle+R_{RGB\alpha}^{\rho\theta}\otimes|\rho\theta\rangle
\end{aligned}
$$

From the above equation, every sub-operation can set the RGB color for the corresponding pixel. Next, the whole operation U_2 is consisted of all the aforementioned sub-operation.

$$U_2=\prod_{\rho=0}^{2^m-1}\prod_{\theta=0}^{2^n-1}U_{\rho\theta} \tag{7}$$

Through the operationU_2, all pixels have been set RGB colors and the quantum state $|I\rangle_2$ is the final quantum image.

$$
\begin{aligned}
U_2\left(|I\rangle_1\right)&=\prod_{\rho=0}^{2^m-1}\prod_{\theta=0}^{2^n-1}U_{\rho\theta}\left(|I\rangle_1\right)\\
&=\tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{RGB\alpha}^{\rho\theta}\right\rangle\otimes|\rho\theta\rangle=|I\rangle_2
\end{aligned} \tag{8}
$$

After these two steps, the whole quantum image preparation has been finished. Next we will give a Lemma and Corollary to analyze and summarize this preparation procedure. Simultaneously, we discuss the time complexity of the preparation procedure.

Theorem 3.1. *Given 4 vectors of angles,*

$$\theta_X=\left(\theta_{X_0},\theta_{X_1},\cdots,\theta_{X_{2^{n+m}-1}}\right),X\in\{R,G,B,\alpha\}$$

Satisfying $R_{RGB\alpha}^{\rho\theta}$ where $n+m$ is the number of qubits carrying the position information. Assumed the initialized state is $|0\rangle^{3+n+m}$,there is a unitary transform U that turns the quantum state to the targeted state $|I(\theta)\rangle$ composed by Hadamard and controlled rotation transformations.

Proof: there are two steps to achieve the unitary transformation U, Hadamard transformations U_1 are used in step 1 and then controlled rotation transformations U_2 are used in step 2 to change from $|I\rangle_1$to $|I\rangle_2$.From the preparation process we can get the conclusion $U=U_2U_1$,therefore we can get the Lemma results.

Corollary 3.1. *the unitary transformation U described in the Lemma 1, for four given vectors of angles,*

$$\theta_X=\left(\theta_{X_0},\theta_{X_1},\cdots,\theta_{X_{2^{n+m}-1}}\right),X\in\{R,G,B,\alpha\}$$

,can be implemented by Hadamard, CNOT and $C^{2+m+n}\left(R_y\left(\frac{2\theta_i}{2^{m+n}-1}\right)\right)$ gates, where $R_y\left(\frac{2\theta_i}{2^{m+n}-1}\right)$ are the rotations about y axis by the angle $\frac{2\theta_i}{2^{m+n}-1}$,$i=0,1,\cdots 2^{m+n}-1$.

Proof: From the proof of Lemma 1, the transform U is composed of U_2U_1.the transform U_1 can be directly implemented by one identity matrix and (2+m+n) Hadamard gates; and $U_2$is constructed by $\prod_{\rho=0}^{2^m-1}\prod_{\theta=0}^{2^n-1} U_{\rho\theta}$, Where

$$U_{\rho\theta} = I^{\otimes 3} \otimes \sum_{j=0}^{2^m-1} \sum_{i=0, ji\neq\rho\theta}^{2^n-1} |ji\rangle \langle ji| + \Omega_{\rho\theta} \otimes |\rho\theta\rangle \langle\rho\theta|$$

$$\Omega_{\rho\theta} = R_B^{\rho\theta} \cdot R_G^{\rho\theta} \cdot R_R^{\rho\theta} \cdot R_\alpha^{\rho\theta}$$

$$R_X^{\rho\theta} = C^2 \left(R_y \left(2\theta_X^{\rho\theta}\right)\right), X \in \{R, G, B, \alpha\}$$

Hence, $U_{\rho\theta}$ can be implemented by $C^{m+n+2}\left(R_y\left(2\theta_X^{\rho\theta}\right)\right)$ and NOT operation [9].

And in [10] implies that $C^{m+n+2}\left(R_y\left(2\theta_X^{\rho\theta}\right)\right)$ operations can be broken down into $2^{m+n+2} - 1$ simple operations,

$$R_y\left(\frac{2\theta_X^{\rho\theta}}{2^{m+n}-1}\right), R_y\left(-\frac{2\theta_X^{\rho\theta}}{2^{m+n}-1}\right)$$

, and $2^{m+n+2} - 2$ CNOT operations.

The total number of simple operations used to prepare Multi-Channel log-polar quantum state is

$$\begin{aligned} &2 + m + n + 4 \times 2^{m+n} \times (2^{m+n+2} - 1 + 2^{m+n+2} - 2) \\ &= 32 \times 2^{2(m+n)} - 12 \times 2^{m+n} + (m + n + 2) \end{aligned}$$

This number is quadratic to the total 2^{m+n} angle and radius values. this indicates the efficient of the preparation process.

Theorem 3.2. *(Polynomial Preparation Theorem)Given four vectors,*

$$\theta_X = \left(\theta_{X_0}, \theta_{X_1}, \cdots, \theta_{X_{2^{n+m}-1}}\right), X \in \{R, G, B, \alpha\}$$

of angles, there is a unitary transform U that turns quantum computers from the initial state $|0\rangle^{3+n+m}$ *to the MCLPQI state,*

$$|I(\theta)\rangle = \frac{1}{\sqrt{2^{2+m+n}}} \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} \left|R_{RGB\alpha}^{\rho\theta}\right\rangle \otimes |\rho\theta\rangle$$

Composed of polynomial number of simple unitary matrix.

Proof: From Lemma 1and Corollary 1, we can easily get the conclusion.

4. **Elementary transformations for MCLPQI.** Many image processing method and operations have been proposed about classical image such as color transformations, geometric transformations. However, these parallel operations for quantum image are not too complete. Designing these corresponding quantum circuits would not be easy task. Based on MCRQI, channel swapping operation and one channel operation is proposed in [10]. In this section, channel swapping and one channel swapping operations are discussed, respectively.

4.1. **Channel swapping operation.** Channel Swapping operation (CSO) on MCRQI is defined in [12], corresponding, three sorts of CSO,U_{RG} ,U_{GB} andU_{RB} on MCLPQI are designed which is useful in color transformation.

4.1.1. *Swapping between R and G channel.* Obviously, our targeted state can be defined as the following form:

$$\begin{aligned} & U_{RG}\left(\left|I(\theta)\right\rangle\right) = \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1} U_{RG}\left(\left|R_{RGB\alpha}^{\rho\theta}\right\rangle\right)\otimes\left|\rho\theta\right\rangle \\ & = \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{GRB\alpha}^{\rho\theta}\right\rangle\otimes\left|\rho\theta\right\rangle \end{aligned} \tag{9}$$

Where,

$$\begin{aligned} & \left|R_{RGB\alpha}^{\rho\theta}\right\rangle = \cos\theta_R^{\rho\theta}\left|000\right\rangle + \cos\theta_G^{\rho\theta}\left|001\right\rangle + \cos\theta_B^{\rho\theta}\left|010\right\rangle + \cos\theta_\alpha^{\rho\theta}\left|011\right\rangle \\ & + \sin\theta_R^{\rho\theta}\left|100\right\rangle + \sin\theta_G^{\rho\theta}\left|101\right\rangle + \sin\theta_B^{\rho\theta}\left|110\right\rangle + \sin\theta_\alpha^{\rho\theta}\left|111\right\rangle \end{aligned}$$

$$\begin{aligned} & \left|R_{RGB\alpha}^{\rho\theta}\right\rangle = \cos\theta_G^{\rho\theta}\left|000\right\rangle + \cos\theta_R^{\rho\theta}\left|001\right\rangle + \cos\theta_B^{\rho\theta}\left|010\right\rangle + \cos\theta_\alpha^{\rho\theta}\left|011\right\rangle \\ & + \sin\theta_G^{\rho\theta}\left|100\right\rangle + \sin\theta_R^{\rho\theta}\left|101\right\rangle + \sin\theta_B^{\rho\theta}\left|110\right\rangle + \sin\theta_\alpha^{\rho\theta}\left|111\right\rangle \end{aligned}$$

The quantum circuit to realize this operation U_{RG} is described in 1

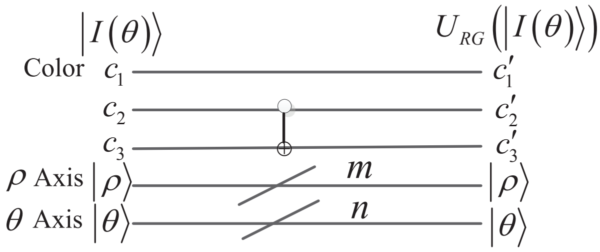

FIGURE 1. Quantum circuit for swapping between channels R and G

4.1.2. *Swapping between R and B channel.* Similar with the U_{RG}, the swapping operation and the corresponding quantum circuit between R and B channel is shown in Eq.(9) and 1, separately,

$$\begin{aligned} & U_{RG}\left(\left|I(\theta)\right\rangle\right) = \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1} U_{RG}\left(\left|R_{RGB\alpha}^{\rho\theta}\right\rangle\right)\otimes\left|\rho\theta\right\rangle \\ & = \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{GBR\alpha}^{\rho\theta}\right\rangle\otimes\left|\rho\theta\right\rangle \end{aligned} \tag{10}$$

Where,

$$\begin{aligned} & \left|R_{BGR\alpha}^{\rho\theta}\right\rangle = \cos\theta_B^{\rho\theta}\left|000\right\rangle + \cos\theta_G^{\rho\theta}\left|001\right\rangle + \cos\theta_R^{\rho\theta}\left|010\right\rangle + \cos\theta_\alpha^{\rho\theta}\left|011\right\rangle \\ & + \sin\theta_B^{\rho\theta}\left|100\right\rangle + \sin\theta_G^{\rho\theta}\left|101\right\rangle + \sin\theta_R^{\rho\theta}\left|110\right\rangle + \sin\theta_\alpha^{\rho\theta}\left|111\right\rangle \end{aligned}$$

The corresponding quantum circuit is shown in 2

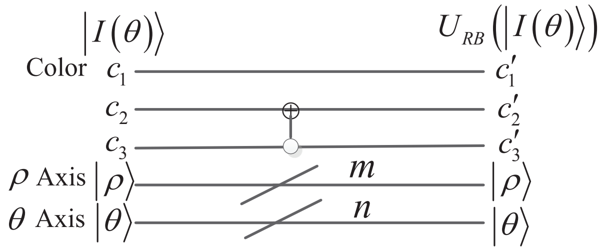

FIGURE 2. Quantum circuit for swapping between channels R and B

4.1.3. *Swapping between G and B channel.* Just like theU_{RG} , the swapping operation and the corresponding quantum circuit between G and B channel is shown in and , separately,

$$\begin{aligned} U_{GB}\left(|I(\theta)\rangle\right) &= \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1} U_{GB}\left(\left|R_{RGB\alpha}^{\rho\theta}\right\rangle\right)\otimes|\rho\theta\rangle \\ &= \frac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{RBG\alpha}^{\rho\theta}\right\rangle\otimes|\rho\theta\rangle \end{aligned} \tag{11}$$

Where

$$\begin{aligned} \left|R_{RBG\alpha}^{\rho\theta}\right\rangle &= \cos\theta_R^{\rho\theta}|000\rangle + \cos\theta_B^{\rho\theta}|001\rangle + \cos\theta_G^{\rho\theta}|010\rangle + \cos\theta_\alpha^{\rho\theta}|011\rangle \\ &+ \sin\theta_R^{\rho\theta}|100\rangle + \sin\theta_B^{\rho\theta}|101\rangle + \sin\theta_G^{\rho\theta}|110\rangle + \sin\theta_\alpha^{\rho\theta}|111\rangle \end{aligned}$$

And the quantum circuit is shown in

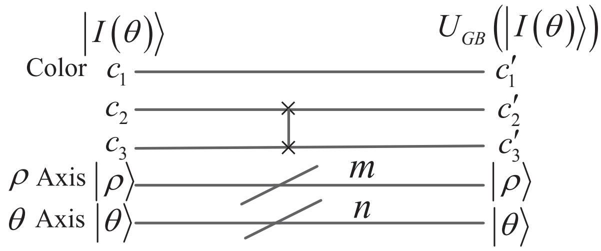

FIGURE 3. Quantum circuit for swapping between channels G and B

4.2. **One channel swapping operation.** From MCLPQI representation, we known that all the color and position information is entangled together. So the method trying to change the pixels with different color scales is not so easy to realize. On the other hand, one channel swapping operation mainly focused on changing the angles of the fixed channel. So in this section, three channel angle operations will be given.

Firstly, we can use the following transformation

$$R_y(2\theta_R') \otimes |00\rangle\langle 00|$$

to act on $\left|R_{RGB\alpha}^{\rho\theta}\right\rangle$, through this way we can change the R color from $\theta_R^{\rho\theta}$ to $\theta_R^{\rho\theta} + \theta_R'$. That is to say

$$\begin{aligned}U_R\left(|I(\theta)\rangle\right) &= \tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left(R_y(2\theta'_R)\otimes|00\rangle\langle 00|\right)\left(\left|R_{RGB\alpha}^{\rho\theta}\right\rangle\right)\otimes|\rho\theta\rangle \\ &= \tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{R''GB\alpha}^{\rho\theta}\right\rangle\otimes|\rho\theta\rangle\end{aligned} \tag{12}$$

Here,

$$\begin{aligned}\left|R_{R''GB\alpha}^{\rho\theta}\right\rangle &= \cos\left(\theta_R^{\rho\theta}+\theta'_R\right)|000\rangle+\cos\theta_G^{\rho\theta}|001\rangle+\cos\theta_B^{\rho\theta}|010\rangle+\cos\theta_\alpha^{\rho\theta}|011\rangle \\ &+\sin\left(\theta_R^{\rho\theta}+\theta'_R\right)|100\rangle+\sin\theta_G^{\rho\theta}|101\rangle+\sin\theta_B^{\rho\theta}|110\rangle+\sin\theta_\alpha^{\rho\theta}|111\rangle\end{aligned}$$

The realization of this circuit is illustrated in

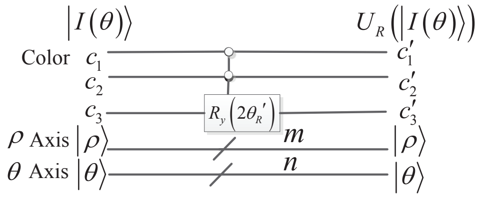

FIGURE 4. Quantum circuit for changing R channel by angle θ'_R

In a similar way, we can give the change angle transformation about G color from $\theta_G^{\rho\theta}$ to $\theta_G^{\rho\theta}+\theta'_G$ and corresponding quantum circuit.

$$\begin{aligned}U_G\left(|I(\theta)\rangle\right) &= \tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left(R_y(2\theta'_G)\otimes|01\rangle\langle 01|\right)\left(\left|R_{RGB\alpha}^{\rho\theta}\right\rangle\right)\otimes|\rho\theta\rangle \\ &= \tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R_{RG''B\alpha}^{\rho\theta}\right\rangle\otimes|\rho\theta\rangle\end{aligned} \tag{13}$$

where,

$$\begin{aligned}\left|R_{RG''B\alpha}^{\rho\theta}\right\rangle &= \cos\theta_R^{\rho\theta}|000\rangle+\cos\left(\theta_G^{\rho\theta}+\theta'_G\right)|001\rangle+\cos\theta_B^{\rho\theta}|010\rangle+\cos\theta_\alpha^{\rho\theta}|011\rangle \\ &+\sin\theta_R^{\rho\theta}|100\rangle+\sin\left(\theta_G^{\rho\theta}+\theta'_G\right)|101\rangle+\sin\theta_B^{\rho\theta}|110\rangle+\sin\theta_\alpha^{\rho\theta}|111\rangle\end{aligned}$$

Corresponding circuit is shown in

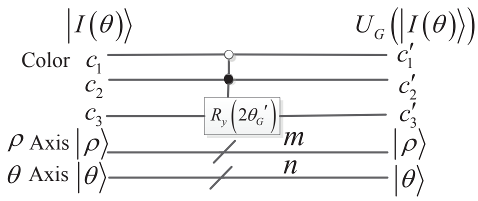

FIGURE 5. Quantum circuit for changing G channel by angle θ'_G

Parallel B channel color angle transformation from $\theta_B^{\rho\theta}$ to $\theta_B^{\rho\theta}+\theta'_B$is shown

$$
\begin{aligned}
&U_B\left(|I(\theta)\rangle\right) = \tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left(R_y(2\theta'_B)\otimes|10\rangle\langle 10|\right)\left(\left|R^{\rho\theta}_{RGB\alpha}\right\rangle\right)\otimes|\rho\theta\rangle \\
&=\tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R^{\rho\theta}_{RGB''\alpha}\right\rangle\otimes|\rho\theta\rangle
\end{aligned}
\tag{14}
$$

where,

$$
\begin{aligned}
&\left|R^{\rho\theta}_{RGB''\alpha}\right\rangle = \cos\theta^{\rho\theta}_R|000\rangle + \cos\theta^{\rho\theta}_G|001\rangle + \cos\left(\theta^{\rho\theta}_B+\theta'_B\right)|010\rangle + \cos\theta^{\rho\theta}_\alpha|011\rangle \\
&+\sin\theta^{\rho\theta}_R|100\rangle + \sin\theta^{\rho\theta}_G|101\rangle + \sin\left(\theta^{\rho\theta}_B+\theta'_B\right)|110\rangle + \sin\theta^{\rho\theta}_\alpha|111\rangle
\end{aligned}
$$

Quantum circuit is illustrated in the following

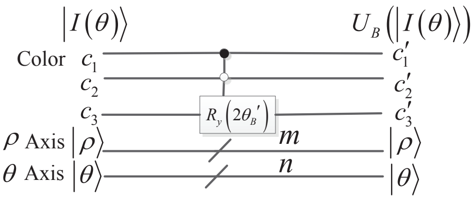

FIGURE 6. Quantum circuit for changing B channel by angle θ'_B

Finally, α channel color transformation from $\theta^{\rho\theta}_\alpha$ to $\theta^{\rho\theta}_\alpha+\theta'_\alpha$ is given

$$
\begin{aligned}
&U_\alpha\left(|I(\theta)\rangle\right) = \tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left(R_y(2\theta'_\alpha)\otimes|11\rangle\langle 11|\right)\left(\left|R^{\rho\theta}_{RGB\alpha}\right\rangle\right)\otimes|\rho\theta\rangle \\
&=\tfrac{1}{\sqrt{2^{2+m+n}}}\sum_{\rho=0}^{2^m-1}\sum_{\theta=0}^{2^n-1}\left|R^{\rho\theta}_{RGB\alpha''}\right\rangle\otimes|\rho\theta\rangle
\end{aligned}
\tag{15}
$$

Where

$$
\begin{aligned}
&\left|R^{\rho\theta}_{RGB\alpha''}\right\rangle = \cos\theta^{\rho\theta}_R|000\rangle + \cos\theta^{\rho\theta}_G|001\rangle + \cos\theta^{\rho\theta}_B|010\rangle + \cos\left(\theta^{\rho\theta}_\alpha+\theta'_\alpha\right)|011\rangle \\
&+\sin\theta^{\rho\theta}_R|100\rangle + \sin\theta^{\rho\theta}_G|101\rangle + \sin\theta^{\rho\theta}_B|110\rangle + \sin\left(\theta^{\rho\theta}_\alpha+\theta'\right)|111\rangle
\end{aligned}
$$

Then we give the corresponding quantum circuit Fig.7

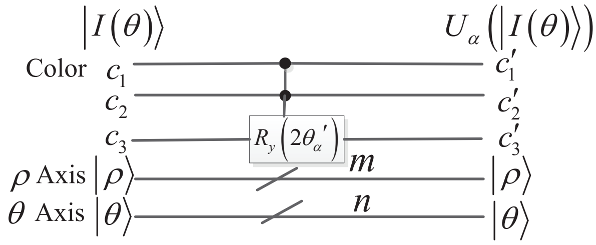

FIGURE 7. Quantum circuit for changing B channel by angle θ'_α

From the polynomial preparation process, the transform aiming to change only one value on some position and keep the others unchangeable can be realized. Then we give the concrete description about transform form. Suppose we want to change X

channel($X \in \{R, G, B, \alpha\}$) on the position $|\rho\theta\rangle$ from $\theta_X^{\rho\theta}$ to θ'_X , we can design the following transform,

$$C_X = I^{\otimes 3} \otimes \sum_{j=0}^{2^m-1} \sum_{i=0}^{2^n-1} |ji\rangle \langle ji| + C'_X \otimes |\rho\theta\rangle \langle \rho\theta| \tag{16}$$

Where

$$C'_X = \sum_{j=0, j\neq f(X)}^{3} |j\rangle \langle j| \otimes I + |f(X)\rangle \langle f(X)| \otimes R_y(2\theta_X) \tag{17}$$

When$X = R, G, B, \alpha$, corresponding function value is$f(X) = 0, 1, 2, 3$.

5. **Conclusions.** A new representation about Multi-Channel Log-polar quantum image has been proposed. This model considered three color channel and transparency information as a four channel sampled in Log-Polar coordinates by using three qubit dedicate channel information. From the polynomial preparation theory, only using the Hadamard gate and control rotation gate can turn the initial quantum state to the MCLPQI state. That is to say, unitary transformation can be used to complete the preparation process.

Moreover, different channel swapping operations, single channel swapping operations and corresponding circuits for designing transformations are studied. The corresponding transformations matrices and quantum circuits are given and they can be easily implemented by basic quantum gates. However, some different complex geometric transformations such as scaling and rotation is not discussed in this paper at the form of MCLPQI. The concrete applications of this form will be our future work.

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REFERENCES

[1] M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, 2000.

[2] J. I. Latorre, Image compression and entanglement, *arXiv: quant-ph/0510031*, 2005.

[3] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. W. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter,*Elementary gates for quantum computation*, Phys. Rev. Part A, vol. 52, pp. 3457, 1995.

[4] R.C. Gonzalez, R.E. Woods, Eddins,S.L.,*Digital Image Processing*,Publishing House of Electronics Industry, Beijing, 2002.

[5] S.E. Venegas-Andraca ,S. Bose, Storing, processing and retrieving an image using quantum mechanics, *Proc. SPIE Conf. Quantum Information and Computation*, vol.5105,pp. 137-147, 2003.

[6] S. E. Venegas-Andraca, J. L. Ball, Processing images in entangled quantum systems, *Quantum Information Processing*, vol.9 ,pp. 1-11, 2010.

[7] J. I. Latorre, Image compression and entanglement, *arXiv:quant-ph/0510031*, 2005.

[8] P. Q. Le, F. Dong, K. Hirota, A flexible representation of quantum images for polynomial preparation, image compression, and processing operations. *Quantum Information Processing*, vol.10 , pp.63-84, 2010.

[9] Y. Zhang, K. Lu, Y.H. Gao,and M. Wang, NEQR: a novel enhanced quantum representation of digital images, *Quantum Information Processing*, vol. 12, no. 8, pp. 2833-2860, 2013.

[10] B. Sun, P. Q. Le, A.M. Iliyasu, J. Adrian Garcia, F. Yan, J., F. Dong, and K. Hirota, A multi-channel representation for images on quantum computers using the RGBαcolor space, *Proc. of the IEEE 7th International Symposium on Intelligent Signal Processing*, pp. 160-165, 2011.

[11] Y. Zhang, K. Lu, Y. H. Gao, K. Xu, A novel quantum representation for log-polar images, *Quantum Information Processing*, pp. 1-24, 2013.

[12] B. Sun, P. Q. Le, A.M. Iliyasu, J. Adrian Garcia, F. Yan, J., F. Dong, and K. Hirota, A multi-channel representation for images on quantum computers using the RGBαcolor space. *Proc. of the IEEE 7th International Symposium on Intelligent Signal Processing*, pp. 160-165, 2011.

[13] P.Q. Le, A. M. Iliyasu, F. Dong, and K. Hirota, Efficient color transformations on quantum images, *Journal of Advanced Computational Intelligence and Intelligent Informatics*, vol. 15, no. 6, pp. 698-706, 2011.