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Critical Nodes and Links Evaluation with Multi-Criteria Based on Entropy-Weighted Method

Zhe-Ming Lu* and Ya-Pei Feng

School of Aeronautics and Astronautics
Zhejiang University
Hangzhou, 310027, P. R. China
* Corresponding Author
zheminglu@zju.edu.cn

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Abstract. *The assessment of network vulnerability is of great importance in the presence of unexpected disruptive events or adversarial attacks targeting on critical nodes and links. So, the problem of evaluating node and link importance in complex networks has been an active area of research in recent years. In this paper, we study Critical Node Importance(CNI) and Critical Link Importance(CLI) and put forward a new multi-criteria ranking method using the entropy-weighted method to evaluate CNI and CLI in complex networks. As is well known, most of existing methods only consider one factor (e.g. betweenness, degree) , but not the integration of multiple factors in evaluating critical nodes and links, so each of those methods has a limited application range. We use the entropy-weighted method to evaluate the importance of each factor and obtain its weight. Then we compare our scheme with the manual-weighted method which set the weight of each factor randomly using several real networks. According to the evaluation results, our method has good performance on discrimination and precision to evaluate CNI and CLI.*

Keywords: Complex networks, Critical nodes and links, Multi-criteria, Entropy-weighted method.

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1. **Introduction.** Since the seminal papers by Wattts and Strogatz [1] on the small-world property and Barabasi and Albert [2] on the scale-free property were published, complex networks have become a hot topic and brought together researchers from many areas including mathematics, physics, biology, computer science, sociology, epidemiology, and others. Actually in large complex networks, not all nodes and links are equivalent, and some important nodes or links significantly affect the overall network performance. How to evaluate key nodes and links in complex networks has been a basic and vital issue in recent years. The non-homogeneous topology structure of complex networks essentially determines the importance of each node[3]. So evaluating the importance of nodes or links in the network would help us study the core issues of actual networks, such as, how to enhance the robustness of large-scale networks, how to improve the capacity of the network against virus attacks, and how to optimize network routing and so on.

There are two kinds of methods mostly applied in the research of node importance evaluation.

Method 1: The basic idea of the first kind of method is to measure the node centrality in the complex network [4], such as degree, betweenness and closeness [7] and so on. In the "degree" based method, the more edges a node is connected with, and the more important the node is. But this method is one-sided, some core nodes do not have large degree values, such as the "bridge nodes" connected with some important nodes. In the "betweenness" based method, the intermediary nodes have an interpersonal impact on members at the end of paths. So this method can not evaluate the importance of nodes effectively. In the "closeness" based method, the much closer a node is next to the center of the network, the more important the node is. This method can accurately discover the important nodes in centralized star networks, but not for other networks such as regular graphs and ER random graphs. For example, Reference [8] analyzed different centrality measures and proposed a measure which is based on the PageRank-algorithm.

Method 2: The theory of the second kind of method aims to evaluate the importance of a certain node by computing the destructive degree of the network if this node ceases to be effective, which is often called vulnerability. In References[5, 6], the authors proposed a measurement associated with the shortest path. In this method, if the removal of a node on the shortest path increases the distance between the source node and the target node, then the deleted node would be important. But it's a hard problem to find the k-most vital arcs on the shortest path, and the complexity is exponential.

However, the algorithms mentioned above only use a single metric measurement. For a real-world complex network, it has a lot of nodes with very complex relationships. Only one metric measurement can not fully reflect the characteristics of the complex network, and thus it is crucial to unite multiple measures. So we should pay attention to how to combine various factors to make comprehensive evaluation. Then another paper proposed a method choose eight indicators [9] (degree, closeness, beweenness, etc.) as the decision criteria to determine the node importance by fuzzy AHP and TOPSIS. So in another literature [10], the AHP method is used to get the best scenario by pairwise comparison with each factor and comprehensive evaluation. Experiment results show that the algorithm is effective. Different criteria often lead to significantly different results. Therefore, Reference [11] proposed a multi-criteria evaluation method (PCGRAE) based on principal component analysis (PCA) and grey relational analysis (GRA) specifically.

There are also two main kinds of methods for evaluating link importance, i.e., centrality and vulnerability. Reference [12] detected community structures based on edge betweenness, where the main idea is: if the betweenness of an edge is relative lower, a pair of nodes connected by that edge should be in the same community. In another literature [13], the authors studied the optimization problems of Critical Link Disruptor (CLD) and Critical Node Disruptor (CND) to identify critical links and nodes whose removals will maximally destroy the network function based on a metric called total pairwise connectivity. Their method is not only effective but also reveal the vulnerability degree of different real-world and synthetic networks. On the basis of these methods, we propose an entropy-weighted method to get better results.

In our paper, we choose ten indicators as the evaluation criteria to evaluate CNI, including degree, betweenness, closeness, eigenvector centrality, approximate eigenvector centrality, subgraph, flow betweenness, approximate flow betweenness, accumulated nomination and loss of node deletion. We also choose three indicators as the evaluation criteria for link importance, including edge betweenness, drop rate of the number of spanning trees, increase rate of average distance. Based on these indicators, we propose Multi-criteria Decision Making (MCDM) analysis based on the entropy-weighted method. Entropy, in information theory, is a criterion to evaluate the amount of uncertainty [14] , represented by a discrete probability distribution, in which there is agreement that a broad distribution represents more uncertainty than a sharply packed one.

2. **Preliminaries.** Networks discussed in this paper are undirected and unweighted that can be represented by a graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$,where $\mathbf{V} = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes and $\mathbf{E} = \{e_1, e_2, \ldots, e_n\}$ is the set of links[15].

Within the scope of graph theory and network analysis, there are various types of measures which determine the relative importance of nodes and links within the network. These measures can be defined as follows.

Definition 1: Degree

It is defined as the number of other nodes connected to a node, which indicates the ability among nodes in direct communication. Let $D(a)$ denotes the degree of a node v_a, we have

$$D(a) = \sum_{i=1}^{n} \delta(i, a) \tag{1}$$

$$\delta(i, a) = \begin{cases} 1, & \text{If Node } v_i \text{ is connected to Node } v_a \\ 0, & \text{If Node } v_i \text{ is not connected to Node } v_a \end{cases} \tag{2}$$

Definition 2: Betweenness

It is introduced as a measure to quantify the ability of a node in controlling the communication between other nodes in a complex network. Assume that p_{ij} is the number of shortest paths between Node v_i and Node v_j, $p_{ij}(v_a)$ is the number of shortest paths between Node v_i and Node v_j which also pass through Node v_a. Let $B(a)$ denote the betweenness of v_a, then we have

$$B(a) = \sum_{i \neq j \neq a} \frac{p_{ij}(v_a)}{p_{ij}} \qquad v_i, v_j, v_a \in \mathbf{V} \tag{3}$$

The larger the value $B(a)$ is, the more important the node v_a is.

Definition 3: Closeness

Closeness can be regarded as a measure of how fast a node spreads information to all other nodes sequentially. We use $C(i)$ to represent the closeness of Node v_i, and correspondingly d_{ij} denotes the length of the shortest paths between Node v_i and Node v_j. Then we have

$$C(i) = \frac{1}{\sum_{i \neq \jmath} d_{ij}} \tag{4}$$

The larger the value $C(a)$ is, the closer to center the node is , and the more important the node is.

Definition 4: Eigenvector centrality

Eigenvector centrality is a measure of the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more than connections to low-scoring nodes. In fact, Google's PageRank is a variant of the eigenvector centrality measure. We denote $E(a)$ as the Eigenvector centrality of Node v_a. Then we have

$$E(a) = \frac{1}{\lambda} \sum_{i=1}^{n} \delta(i, a) \times x_i \tag{5}$$

Here, $\mathbf{A}$ is the adjacency matrix, λ is the main eigenvalue of $\mathbf{A}$, satisfying $\mathbf{A}\boldsymbol{x} = \lambda \boldsymbol{x}$, $\delta(i, a)$ is defined in Eq.(2), and $\boldsymbol{x} = \{x_1, x_2, \ldots, x_n\}$ is the eigenvector corresponding to the main eigenvalue λ.

Definition 5: Approximate eigenvector centrality

Although eigenvector centrality is not bound by multipath propagation, it is too complex to calculate the eigenvalue λ. Considering that the centric value of a node can be calculated from the centric values of its surrounding nodes. Assume the number of a network is n and the iterative times up to now is m. Initially, we set the centric value of each node v_i as 1, that is, $E_0(v_i) = 1$. Then the centric value of each node $E_m(v_i)$ equals the sum of the original centric value of all its surrounding nodes. So, the eigenvector centrality can be calculated approximately as follows:

$$E_m(v_i) = \sum_{j \in U_i} \frac{E_{m-1}(v_j)}{\sum_{l=1}^{n} E_{m-1}(v_l)} \qquad m = 1, 2, \dots D \tag{6}$$

Here, D is the diameter of the network, and U_i is the set of the surrounding (neighboring) nodes of v_i.

Definition 6: Subgraph

A subgraph of a network $\mathbf{G}$ is a graph whose node set is a subset of $\mathbf{G}$, and whose adjacency relation is a subset of that of $\mathbf{G}$ restricted to this subset. It can be calculated as the number of closed loops which starts from one node and ends to the same node. One closed loop indicates one subgraph of the network. We denote $C_s(v_i)$ as the subgraph indicator of Node v_i. Then we have

$$C_s(v_i) = \sum_{i=0}^{\infty} \frac{A_{ii}^n}{n!} \tag{7}$$

Here, $\mathbf{A}$ is the adjacency matrix, $\mathbf{A}^n$ is the n-th power of $\mathbf{A}$, and A_{ii}^n is the i-th diagonal elements of $\mathbf{A}^n$. The longer the closed loop is, the less important the node is.

Definition 7: Flow betweenness

Let g_{jk} be the amount of flow between v_j and v_k, and $g_{jk}(v_i)$ denotes the amount of flow between v_j and v_k which must pass through v_i for any maximum flow. The flow betweenness is therefore a measure of the contribution of a node to all possible maximum flows. We denote $C_f(v_i)$ as the flow betweenness of Node v_i. Then we have

$$C_f(v_i) = \sum_{j<k} \frac{g_{jk}(v_i)}{g_{jk}} \tag{8}$$

Here, v_i , v_j and v_k are distinct nodes.

Definition 8: Approximate flow betweenness

Flow betweenness can test the geometric center of a network, but it is too complex and the computational complexity is very high. For a network with N nodes, the complexity is $O(N^3)$. In this paper, we use an approximate algorithm to calculate the flow betweenness with the complexity $O(N^2)$. We denote $C_{af}(v_i)$ as the approximate flow betweenness of Node v_i. Then we have

$$C_{af}(v_i) = \sum_{n=1}^{D} \sum_{j \in U_i} \lambda(v_i) C_n(v_j) \tag{9}$$

$$C_n(v_j) = \sum_{m \in U_j} \lambda(m) C_{n-1}(m) \qquad m \neq j \tag{10}$$

Here, U_i is the set of surrounding nodes of v_i, m is the iterative times up to now, and D is the diameter of the network. $\lambda(v_i) = \frac{1}{D(i)}$, where $D(i)$ is the degree of Node v_i.

Definition 9: Accumulated nomination

Accumulated nomination can be described as follows. Initially we give a certain nomination value to each node, and in every subsequent cycle, for each node, a new nomination value which is affected by not only its original nomination value but also the nomination values of the other nodes connected to it. After a certain number of cycles, the accumulated nomination value of a node will be close to a constant. The larger the obtained constant is, the more important the node is.

Definition 10: Loss of node deletion

The loss of node deletion [17, 18] of a node can be defined as the drop rate in performance when the node and all its edges are removed from the network. Assume N is the number of nodes, and V_i is denoted as deletion loss of Node v_i, then we have

$$V_i = \frac{L - L_i}{L} \tag{11}$$

Here, $L = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}}$ is the global efficiency of the network which quantifies the efficiency of the network sending information between nodes. L_i is the global efficiency after the removal of Node v_i and all its surrounding edges.

Definition 11: Edge betweenness

Edge betweenness is a measure to quantify the ability of an edge in controlling the communication between nodes in a complex network. Let p_{kl} denote the number of shortest paths between Node v_k and Node v_l, $p_{kl}(e_{ij})$ be the number of shortest paths between Node v_k and Node v_l which must pass through the edge e_{ij} , and $B_{e_{ij}}$ denote the edge betweenness of e_{ij}. Then we have

$$B_{e_{ij}} = \sum_{k \neq l} \frac{p_{kl}(e_{ij})}{p_{kl}} \qquad v_k, v_l \in \mathbf{V}, e_{ij} \in \mathbf{E} \tag{12}$$

The larger the value $B_{e_{ij}}$ is, the more important the edge e_{ij} is.

Definition 12: Drop rate of the number of spanning trees

First we calculate the number of spanning trees of a network, according to the Matrix-Tree theorem: for an undirected network, let b_{ij} denote the associated number between Node v_i and Edge e_j, if Node v_i and Edge e_j are connected, $b_{ij} = 1$; otherwise, $b_{ij} = 0$. Thus, $\mathbf{B}$ denotes the incidence matrix, and the number of spanning trees can be calculated as follows:

$$\tau(\mathbf{G}) = |\mathtt{det}(\mathbf{C}_r)| \tag{13}$$

Here, $\tau(\mathbf{G})$ is the number of spanning trees, $\mathbf{C}_r$ is the $(n-1)th$ order principal minor of Kirchhoff matrix, and Kirchhoff matrix can be calculated as $\mathbf{B}\mathbf{B}^T$.When we delete Edge e_i, recalculate the number of spanning trees of the new network $\tau_{e_i}(\mathbf{G})$. Then we use $T(e_i)$ to denote the drop rate of the number of spanning trees when Edge e_i is deleted, and $T(e_i)$ can be calculated as follows

$$T(e_i) = \frac{\tau(\mathbf{G}) - \tau_{e_i}(\mathbf{G})}{\tau(\mathbf{G})} \tag{14}$$

The larger the drop rate is, the more important the edge is.

Definition 13: Increase rate of average distance

To obtain the increase rate of the average distance, we first calculate the average distance of the network, denoted as $L(\mathbf{G})$. For an undirected graph, the average distance is defined as the average value over all the distances between every two nodes. When an edge e_i is deleted, we recalculate the average distance of the new network, denoted as $L_{e_i}(G)$. Thus, the increase rate of the average distance $D(e_i)$ can be calculated as follows:

$$L(\mathbf{G}) = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \tag{15}$$

$$D(e_i) = \frac{L_{e_i}(\mathbf{G}) - L(\mathbf{G})}{L(\mathbf{G})} \tag{16}$$

Here, d_{ij} denotes the length of the shortest paths between Node v_i and Node v_j, and N is the number of nodes. The larger the increase rate is, the more important the node is.

3. Multi-Criteria Analysis Based on the Entropy-weighted Method.

3.1. **Entropy-weighted Method.** The Multi-Criteria analysis problem with m alternatives and n criteria can be expressed in the matrix format as follows [16]:

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \tag{17}$$

Here, $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_m$ are feasible alternatives with $\mathbf{A}_m = \{x_{m1}, x_{m2}, \dots, x_{mn}\}$, and $\mathbf{C}_1$, $\mathbf{C}_2$, ..., $\mathbf{C}_n$ are evaluation criteria with $\mathbf{C}_n = \{x_{1n}, x_{2n}, \dots, x_{mn}\}$. x_{ij} is the performance rating of alternative $\mathbf{A}_i$ under criterion $\mathbf{C}_j$, and w_j is the weight of criterion $\mathbf{C}_j$, satisfying $\sum_{j=1}^{n} w_j = 1$.

Then we calculate the normalized matrix $\mathbf{X}^*$ as follows:

$$\mathbf{X}^* = \begin{pmatrix} x^*_{11} & x^*_{12} & \dots & x^*_{1n} \\ x^*_{21} & x^*_{22} & \dots & x^*_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x^*_{m1} & x^*_{m2} & \dots & x^*_{mn} \end{pmatrix} \tag{18}$$

Here $x^*_{ij} = \frac{x_{ij} - x_{min}(j)}{x_{max}(j) - x_{min}(j)}$.

In general, evaluation criteria can be classified into two types: benefit and cost. Benefit criteria means that a larger value is more valuable whilst cost criteria are just the reverse. The data in matrix $\mathbf{X}^*$ have different dimensions, thus it needs to be normalized in order to transform various criterion dimensions into the non-dimensional criterion, which allows comparison across the criteria. In this paper, matrix $\mathbf{X}^*$ is normalized for each criterion $\mathbf{C}_j$ as:

$$p_{ij} = \frac{x^*_{ij}}{\sum_{i=1}^{m} x^*_{ij}} \qquad 1 \le j \le n \tag{19}$$

As a consequence, a normalized decision matrix representing the relative performance of the alternatives is obtained as:

$$\mathbf{P} = (p_{ij})_{m \times n} \qquad 1 \le i \le m, 1 \le j \le n \tag{20}$$

The amount of decision information in Eq.(3) emitted from each criterion $\mathbf{C}_j$ can be measured by the entropy value e_j as follows:

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} p_{ij} \ln(p_{ij}) \qquad 1 \le i \le m, 1 \le j \le n \tag{21}$$

The degree of diversity of the information contained by each criterion can be calculated as:

$$d_j = 1 - e_j \qquad 1 \leq j \leq n \tag{22}$$

Thus, the objective weight for each criterion is given by:

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \qquad 1 \leq j \leq n \tag{23}$$

3.2. **Critical Nodes and Links Selected with Entropy-weighted Method.** Based on the concepts of above entropy-weighted method, our scheme consists of the following steps:

Step 1: Determine the problem of important node and link selection. Here m denotes the number of criteria for evaluating CNI or CLI, and n denotes the number of nodes or links in the network;

Step 2: Construct the criteria for evaluating CNI or CLI, denoted as $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_m$;

Step 3: Collect the data of decision matrix, denoted as $\mathbf{X}^*$ in Eq. (18), and x_{ij} can be calculated with Eqs. (1) to (11) or Eqs. (12) to (16). Then we calculate the normalized decision matrix, denoted as $\mathbf{P}$ in Eq. (20) obtained from Eq. (19);

Step 4: Elicit the weights of criteria using Eqs. (21) to (23), that is, we have calculated the weight of each criteria we choose;

Step 5: Calculate the comprehensive score S_i of each node or link as follows

$$S_i = \sum_{j=1}^{n} w_j * p_{ij} \qquad 1 \leq j \leq n \tag{24}$$

Step 6: Rank the nodes or links according to all the scores S_i. The larger the score value is, the better the performance of the node or link is[19].

4. **Simulation Results.** In our experiments, our entropy-weighted method is compared with the manual-weighted method in estimating the critical nodes and links in different networks. The manual-weighted method set the weights of different indices subjectively, and the weights of different criteria can be changed according to different cases. And the sum of all the weights is 1.

Experiment 1: Firstly, we choose the Zachary karate club network (Zachary network), which is widely used as a research example. There are 34 members in this network, and the members always communicate with each other in the creative activities. In order to get the accurate comparative result, we obtain data from different methods, including questionnaire, the manual-weighted method and the entropy-weighted method. The data collected by questionnaire are subjective.

First we do experiments to test the performance on Critical Node Importance(CNI): we set the weights of ten different criteria in two different methods, i.e., the entropy-weighted method and the manual-weighted method, the CNI test results are as shown in Table 1 and Table 2. From Fig. 1 and Table 2, we can see that Node 8 and Node 24 are ranked different by different methods. According to Fig. 1 , Node 24 has bigger degree than Node 8, but Node 8 is connected to Node 1, Node 3 and Node 2, and these three nodes are very important in the network , while Node 24 is connected to the less important nodes. That is to say, Node 8 is closer to the center of the whole network and any two nodes are able to more quickly reach via it. So Node 8 is more important than Node 24. In the same way, Node 20 is more important than Node 28.

Then we do experiments for Critical Link Importance(CLI). There are 78 links in the test network, we set the weights of three different criteria using two different methods, i.e., the entropy-weighted method and the manual-weighted method, the results are shown in

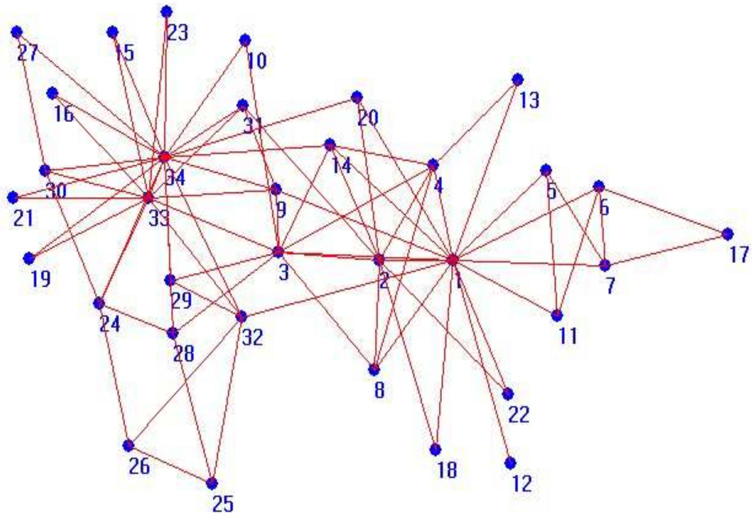

FIGURE 1. Zachary karate club network

TABLE 1. Weights of ten criteria in two methods for CNI evaluation of the Zachary network.

Criteria	**Entropy-weighted method**	**Manual-weighted method**
`Degree`	0.0969	0.1
`Subgraph`	0.0928	0.1
`Betweenness`	0.0824	0.1
`Eigenvector centrality`	0.1064	0.1
`Approximate eigenvector centrality`	0.1040	0.1
`Closeness`	0.1320	0.1
`Loss of node deletion`	0.0934	0.1
`Flow betweenness`	0.0869	0.1
`Approximate flow betweenness`	0.0981	0.1
`Accumulated nomination`	0.1064	0.1

Table 3 and Table 4. From Fig. 1, we can see that the Zachary network is divided into two parts, Node 1 and Node 34 are the corresponding centers of the two parts. So, the links surrounded with the two centers should be more important. From Table 4, we can see that the CLI is ranked differently with different methods. However, in the entropy-weighted method, the critical links overall prefer to the links around Node 1 and Node 34, such as Link 1-7, Link 1-6 and Link 1-12. Besides, in the entropy-weighted method, if a link is connected with nodes of greater importance, the link will be ranked ahead.

Therefore, for the Zachary network, from above ranking results, we can see that the entropy-weighted method is more reasonable and effective than the manual-weighted method in most cases.

Experiment 2: In this experiment, we use the Advanced Research Projected Agency network (ARPA network) in Fig. 2, which is widely used as an example in many researches.

First we do experiments for Critical Node Importance (CNI). The weights of ten different criteria and the CNI results by different methods are shown in Table 5 and Table 6. We can see that, Node 12 and Node 15 in Fig. 2, which have the same degree, are ranked differently by different methods. Node 12 is closer to the center of network and

Table 2. Comparison between two methods in CNI ranking for the Zachary network.

CNI Rank	Entropy-weighted method	Manual-weighted method	questionnaire
1	1	1	1
2	34	34	34
3	33	33	33
4	3	3	3
5	2	2	2
6	9	9	9
7	32	32	32
8	14	14	14
9	4	4	4
10	31	31	31
11	8	24	8
12	24	8	24
13	20	28	20
14	28	20	28

Table 3. Weights of three criteria in two methods for CLI evaluation of the Zachary network.

Criteria	Entropy-weighted method	Manual-weighted method
`Edge betweenness`	033817	0.3
`Drop rate of number of spanning trees`	0.32548	0.4
`Increase rate of average distance`	0.33633	0.3

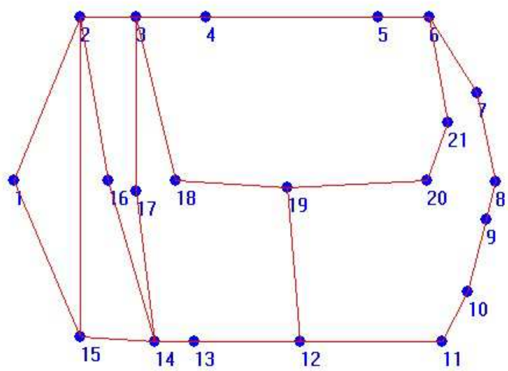

Figure 2. ARPA network

reaches other nodes more easily than Node 15. So Node 12 is more important as obtained by the entropy-weighted method. We can also see that, Node 2 have larger degree than Node 14, but Node 14 have larger betweenness and flow-betweenness, that means there are more information flows through Node 14, so Node 14 takes more important position than Node 2.

Then we do experiments for Critical Link Importance (CLI). There are 26 links in the test network, the weights of three different criteria and the CLI results by two different methods are shown in Table 7 and Table 8. From Table 8, we can see that there are

Table 4. Comparison between two methods in CLI ranking for the Zachary network.

CLI Rank	Entropy-weighted method	Manual-weighted method	Questionnaire
1	2-20	2-20	2-20
2	1-32	1-32	1-32
3	3-33	3-33	3-33
4	1-9	1-9	1-9
5	20-34	20-34	20-34
6	1-7	1-12	1-7
7	1-6	3-28	1-6
8	1-12	14-34	1-12
9	14-34	9-34	14-34
10	1-3	1-7	1-3
11	27-34	1-6	27-34
12	1-11	26-32	1-11
13	3-28	27-34	3-28
14	9-34	1-11	9-34
15	26-32	25-32	26-32
16	25-32	1-3	25-32
17	1-13	2-31	1-13
18	21-34	21-34	21-34
19	23-34	23-34	23-34
20	2-31	1-13	2-31
21	32-34	30-33	32-34
22	1-20	28-34	1-20

Table 5. Weights of ten criteria in two methods for CNI evaluation of the ARPA network.

Criteria	Entropy-weighted method	Manual-weighted method
`Degree`	0.1078	0.1
`Subgraph`	0.1049	0.1
`Betweenness`	0.0587	0.1
`Eigenvector centrality`	0.0779	0.1
`Approximate eigenvector centrality`	0.0836	0.1
`Closeness`	0.1270	0.1
`Loss of node deletion`	0.1257	0.1
`Flow betweenness`	0.0996	0.1
`Approximate flow betweenness`	0.1095	0.1
`Accumulated nomination`	0.0779	0.1

little difference between the entropy-weighted method and the manual-weighted method. Because the ARPA network is an approximate symmetrical network, some links are very similar. We can see that, Link 12-13 is more close to the center of the network, and Node 12 is connected to Node 19,so there are more information flows through Link 12-13. That is to say, Link 12-13 is more important than Link 5-6. Thus, for the ARPA network, from above ranking results, we can see that the entropy-weighted method is more reasonable and effective to evaluate critical nodes and critical links.

Table 6. Comparison between two methods in CNI ranking for the ARPA network.

CNI Rank	Entropy-weighted method	Manual-weighted method	Questionnaire
1	3	3	3
2	14	2	14
3	2	14	2
4	12	15	12
5	15	12	15
6	19	19	19
7	6	6	6
8	13	13	13
9	4	17	4
10	17	4	17
11	18	18	18
12	16	16	16
13	5	1	5
14	1	5	1
15	11	11	11
16	20	20	20

Table 7. Weights of three criteria in two methods for CLI evaluation of the ARPA network.

Criteria	Entropy-weighted method	Manual-weighted method
`Edge betweenness`	0.3384	0.3
`Drop rate of number of spanning tree`	0.3247	0.4
`Increase rate of average distance`	0.3368	0.3

Experiment 3: In this experiment, we use a simple nine nodes network as shown in Fig. 3. There are 9 nodes and 15 links, and the network can be divided into two parts, and Node 4 connects the two parts.

The CNI results are listed in Table 9 and Table 10. We can see that, Node 3, Node 4 and Node 5 are ranked differently by different methods. In fact, when we choose the most important node, we should consider all of the parameters. Once Node 4 is deleted, it will cause the other nodes out of connect, which leads to performance degradation. Thus, Node 4 is more important than Node 3 and Node 5.

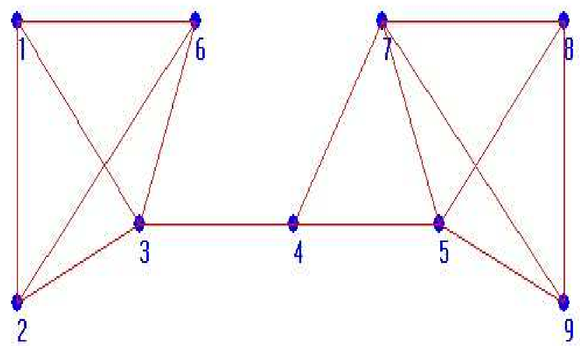

Figure 3. Nine nodes network

Table 8. Comparison between two methods in CLI ranking for the ARPA network.

CLI Rank	Entropy-weighted method	Manual-weighted method
1	11-12	11-12
2	3-4	3-4
3	6-7	6-7
4	4-5	4-5
5	10-11	10-11
6	12-13	5-6
7	5-6	12-13
8	13-14	13-14
9	12-19	12-19
10	19-20	19-20
11	2-3	2-3
12	9-10	7-8
13	7-8	9-10
14	3-18	3-18
15	18-19	18-19
16	8-9	8-9
17	20-21	20-21
18	6-21	6-21
19	3-17	3-17
20	14-17	14-17
21	14-15	14-15
22	2-16	2-16

Table 9. Weights of ten criteria in two methods for CNI evaluation of the nine nodes network.

Criteria	Entropy-weighted method	Manual-weighted method
Degree	0.0921	0.1
Subgraph	0.0276	0.1
Betweenness	0.3354	0.1
Eigenvector centrality	0.0842	0.1
Approximate eigenvector centrality	0.0178	0.1
Closeness	0.0361	0.1
Loss of node deletion	0.1454	0.1
Flow betweenness	0.2384	0.1
Approximate flow betweenness	0.0209	0.1
Accumulated nomination	0.0844	0.1

Then we do experiments for Critical Link Importance (CLI), and the results are in Table 11 and Table 12. From Table 12, we can see that Link 5-8, Link 5-9, Link 7-8, and Link 7-9 are ranked differently with different methods. From Fig. 3, we can see that Node 5 is more important than Node 7, and thus the links start with Node 5 will be more important than the links start with Node 7. That is to say, Link 5-8 and Link 5-9 should be ranked before Link 7-8 and Link 7-9, as obtained by the entropy-weighted method.

According to Table 10 and Table 12, we get the critical graphic with CNI and CLI shown in Fig. 4. Here, the blue node denotes the most critical node, the green node is an auxiliary node, the grey nodes are the less important nodes; the red link is the most critical link and the yellow link is an auxiliary link, the grey dotted lines are the less important

Table 10. Comparison between two methods in CNI ranking for the nine nodes network.

Entropy-weighted method		Manual-weighted method	
CNI Rank	**Aggregative indicator**	**CNI Rank**	**Aggregative indicator**
4	0.9174	3	0.8739
3	0.8999	5	0.8457
5	0.6476	4	0.8440
7	0.5373	7	0.7995
8	0.3241	8	0.5958
9	0.3241	9	0.5958
1	0.2518	1	0.4970
2	0.2518	2	0.4970
6	0.2518	6	0.4970

Table 11. Weights of three criteria in two methods for CLI evaluation of the nine nodes network.

Criteria	Entropy-weighted method	Manual-weighted method
Edge betweenness	0.5333	0.3
Drop rate of number of spanning tree	0.1654	0.4
Increase rate of average distance	0.3013	0.3

Table 12. Comparison between two methods in CLI ranking for the nNine nodes network.

Entropy-weighted method		Manual-weighted method	
CLI Rank	**Aggregative indicator**	**CLI Rank**	**Aggregative indicator**
3-4	1.0000	3-4	1.0000
4-5	0.6613	4-5	0.3429
4-7	0.6613	4-7	0.3429
1-3	0.6312	1-3	0.2555
2-3	0.6312	2-3	0.2555
3-6	0.6312	3-6	0.2555
5-8	0.6146	7-9	0.1975
5-9	0.6146	5-8	0.1975
7-8	0.6146	7-8	0.1975
7-9	0.6146	5-9	0.1975
1-2	0.6057	1-6	0.1675
1-6	0.6057	1-2	0.1675
2-6	0.6057	8-9	0.1675
8-9	0.6057	2-6	0.1675
5-7	0.5981	5-7	0.1375

lines. Obviously, the critical graphic with CNI and CLI by the entropy-weighted method is more stable and more practical.

5. **Conclusion.** This study mainly considers the multi-criteria based comprehensive ranking method for evaluating CNI and CLI for complex networks using the entropy-weighted method. We characterize CNI and CLI based on several key criteria with different weights. The criteria include not only centrality but also vulnerability. We do experiments to compare the entropy-weighted method with the manual-weighted method using three different

Figure 4. Comparison of critical graphic with CNI and CLI for a nine nodes test network: (a)using the entropy-weighted method, (b)using the manual-weighted method

networks and get persuasive results. The entropy-weighted method gets a more reasonable ranking result, and it is an objective method which can avoid artificial deviation. However, further studies are still necessary to understand how to make reasonable weights in more complex and special networks.

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