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Road Network Modeling with Layered Abstraction for Path Discovery in Vehicle Navigation Systems

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Abstract. *Algorithms for path planning have recently drawn renewed attention from researchers due to the advances of intelligent transportation applications. The difference on path planning between past and nowadays is the complexity of dynamic transportation network. This paper focuses on the road network modeling to speed up real time path discovery in dynamic and complex road networks. On the basis of community detection (partitioning), an agglomerative hierarchical clustering algorithm with greedy strategy is proposed in this paper. The algorithm performs a hierarchical network abstraction to make a layered road network with largest community modularity. Static and dynamic information of the real time road network conditions is concerned in the process of network abstraction. Based on the road network layering, the performance of path discovery can be improved due to the reduction of unnecessary searching. The performance of the proposed algorithm was evaluated and compared with the well-known Dijkstra and A-star algorithms. The result shows that the proposed algorithm performs better than the other two algorithms.*

Keywords: Path planning, Community modularity, Agglomerative hierarchical clustering, Road network abstraction, Vehicle navigation.

1. Introduction.

In recent years, the advances of smart car applications and the progress on vehicle navigation systems have led to a resurgence of the interest in shortest path problems. The shortest path problem involves finding the optimum path between a current position and a destination. An optimum path found in vehicle navigation systems usually pursues a minimum travel distance or minimum travel time. The crucial point in this research area is developing increasingly efficient optimal algorithms. In other words, it is quite important to study and find a more effective approach to discover an optimal route in the road network. Earlier researches in the area of shortest path problem mostly revolve around the following two problem variants: the shortest paths problem of single origin node to all destination nodes for a given time, and the problem of all nodes to single destination node for all possible departure times [1]. Dissimilarly, the subject in this paper is about the shortest path problem of a single origin node to single destination node, which is based on a directed graph and a pair of source and destination is given. There are two most well-known algorithms, the Dijkstra algorithm [2] and the A-star algorithm [3], proposed for solving this kind of problem. However, they are effective only in a static network. On the other hand, the quality of a driving path recommendation depends on the route planning strategy used in a vehicle navigation system. Traditional distance-based route planning approaches have been not suitable for the environment of a modern traffic network due to the complex and time-dependent traffic conditions, e.g. rainy environment [4]. There are two key points must be concerned in modern vehicle navigation systems. The first is to provide a quality-oriented path discovery result of driving from a source point to a destination point. The second is to provide a quick response to the path planning request. Accordingly, this paper firstly considers the dynamic traffic conditions in the proposed road network modeling approach, and secondly uses a layered abstraction approach to model the road network and reduce the path discovery time for real-time responses to path planning requests. This paper, in sum, focuses on proposing a road network modeling approach for shortest path discovery in vehicle navigation applications. The traffic road network gives not merely static but also dynamic information which is required for the discovery of an optimal path. The proposed approach uses the concepts of community detection and hierarchical clustering to model the road network and to facilitate a faster path discovery. The rest of this paper is organized as follows. In section 2, brief introduction of related literatures are given. Section 3 describes the proposed approach of road network modeling in this paper for shortest path discovery in vehicle navigation. The evaluations of the proposed approach is present in section 4, and a conclusion is given in the last section.

2. **Related Works.** A road network can be represented as a weighted directed graph consisting of vertices and edges. A path from an origin to a destination may be defined as a sequential list of edges. The shortest path problem is to find the path that has a minimum cost from the origin to the destination vertex. This problem has been studied in various fields of computer science and transportation [5]. Dijkstra algorithm is one of the well-known algorithms for solving this shortest path problem. It can find the shortest path between a given node and every other in a graph based on a greedy strategy. It can also be used to find the shortest path between a given pair of nodes (not all nodes) by stopping the algorithm once the destination node has been tracked. Performance comparison between Dijkstra and Bellman-Ford algorithms was studied and presented in [6]. It was verified that Dijkstra algorithm performs better than the other algorithms. Afterwards three speed-up techniques [7] were proposed to improve the performance of Dijkstra algorithm. They are bi-directional searching technique, heuristic value conducted technique, and hierarchical searching technique. A combination of the three techniques given above was also proposed in the work for the performance improvement further. The A-star algorithm is the representative work belongs to the second technique. So far A-star is the most popular heuristic algorithm of shortest path discovery. The survey of several heuristic shortest path algorithms, including A-star, is provided in [8]. In A-star algorithm, a global variable is introduced to estimate the distance between current

node and destination node as the evaluation of the possibility of an optimal route while choosing the next node. The difference between A-star and Dijkstra algorithms is that A-star not only considers the labeled distance of an examining node but also takes into consideration of the closeness between the examining and destination nodes. The closeness is an estimated cost of moving from current position to a destination. This heuristic ensures that A^* only tries to select nodes that mostly likely directly lead to the direction towards the goal node. The heuristic A-star algorithm can facilitate the reduction of computation time in its searching process. Accordingly, A-star algorithm can be faster than Dijkstra algorithm. However, A-star still does not lead to the optimal path of a path discovery. This is because of that A-star is a best-first searching algorithm and the searching scope is reduced by the heuristic strategy. Afterwards there were several A-star based approaches proposed [9, 10, 11]. Several A-star based approaches concerning the applications of transportation systems in the shortest path discovery were also proposed [12, 13, 14, 15]. There is a network modeling method for urban road network [16], however it is a distance-based approach based on taxi trajectories. A method called Approximate Path Searching (APS) was proposed and it is based on the Next Region (NR) approach instead of providing a set of regions for the path between a source and a destination [17]. But quick response is not its first concern and a few percentages delay in arrival time will be caused. The two methods [18, 19] are based on Branch & Bound method and theoretical analysis to find near-optimal path. The former one utilizes linear programming method and the later one can provides the sequence of regions that must be visited in fixed order. But they did not adopt layered road network modeling for the purpose of quick responses to real time path discovery. In this paper, a multi-layer road network modeling approach is proposed for the purpose of facilitating path discovery with a performance improvement.

3. **Proposed Road Network Modeling Approach.** A road network model is the fundamental and essential basis of a traffic route path planning. The quality of road network model abstraction has an influence on the speed and accuracy of the path planning. A road network model contains a network topology and corresponding traffic information. The network topology mainly represents the connections between traffic nodes and lines while the traffic information represents the weights on the nodes and lines. Traffic information can be categorized into static information and dynamic information. Static traffic information is that of changeless or not much varied information such as the road types, section lengths, number of lanes, passing directions, speed limits, etc. On the contrary, dynamic traffic information is about traffic conditions such as traffic flows, controls, accidents, roadworks, etc. In a traffic path planning of vehicle navigation, the selection of a path with minimum traveling time is mostly the major objective. It is closely related to the real time traffic conditions and the factor of time should be introduced into the traffic model namely a dynamic road network model.

In this paper, the time-dependent road network model is defined as follows. The directed graph G represent a road network model consists of the sets of V, E, D, C, and Q. V and E are the sets of nodes (vertices) and road sections (edges) in the road network respectively. D and C are the sets of the distances and classes of road sections respectively. $Q(t)$ is the set of the real time traffic conditions of road sections at the time t.

$$G = (V, E, D, C, Q) \tag{1}$$

$$V = \{v_i|\ i = 1,\ 2,\ 3,\ \ldots,\ n\} \tag{2}$$

$$E = \{e_{ij}|v_i,\ v_j \in V\} \tag{3}$$

Table 1. Road class vs. vehicle speed

Road Section Class	C-1	C-2	C-3	C-4	C-5
Speed (km/h)	100	80	60	40	30

Table 2. Congestion coefficient

Condition	Heavy jam	Jam	General	Unobstructed
Coefficient	0	0.2	0.7	1

$$D = \{d_{ij} | e_{ij} \in E\} \tag{4}$$

$$C = \{c_{ij} | e_{ij} \in E\} \tag{5}$$

$$Q(t) = \{q_{ij}(t) | e_{ij} \in E\} \tag{6}$$

The optimal path defined in this paper is the least traveling time path. The traveling time means the total time a vehicle travels all the road sections from a starting point to a destination point. As shown in equation (7), the traveling time $h_{ij}(t)$ for a single road section from vertex i to vertex j at the time t is defined with distance d_{ij}, vehicle speed $s_{(c_{ij})}$, and traffic condition $q_{ij}(t)$. The vehicle speed $s_{(c_{ij})}$ is the best speed without traffic congestions. The relationship between the speed and road section class is shown in Table 1 where road sections are divided into five classes. The real time traffic condition $q_{ij}(t)$ is acquired and transformed from the data provided by Baidu Map system. A congestion coefficient is used to represent the traffic condition. Table 2 shows the congestion coefficient which has an influence on the original designed vehicle speed of a road section in four different congestion conditions. Accordingly, the object function $f_{ab}(t)$ is defined as equation (8) where P_{ab} is the set of all possible route paths from starting vertex a to destination vertex b.

$$h_{ij}(t) = \frac{d_{ij}}{s_{c_{ij}}.q_{ij}(t)} \tag{7}$$

$$f_{ab}(t) = \min_{\forall p \in P_{ab}} \sum_{\forall e_{ij} \in p} (h_{ij}(t)) \tag{8}$$

Before the route path planning, a preliminary processing of road network will be launched and an approach of community detection is utilized as the pre-process. A community is a group of nodes with more similarity to each other. The nodes in the same community are densely linked and the connection among communities are loosely. Figure 1 illustrates an example of network community structure. The evaluation of community robustness can be represented and quantified by the modularity of the community. In this paper, the calculation of a community modularity Q is shown as equations (9)-(11). The wij is defined by the reciprocal of the least passing time of a road section. A higher weight means a deeper connectivity between the two vertices of a road section. In addition, some of the roads are one-way streets, therefore a two-way road is represented by two one-way road sections. A traffic road network can be treated as a weighted directed graph. Using the concept of modularity in a road network, then wij represents the weight of a road section directed from vertex i to j, and m is the sum of weights on all road sections. The w_i^{in} and w_j^{out} are the sums of all road sections go into vertex i and leave from vertex j respectively. The Boolean function $\delta(v_i, v_j)$ indicates whether vi and vj are in the same community.

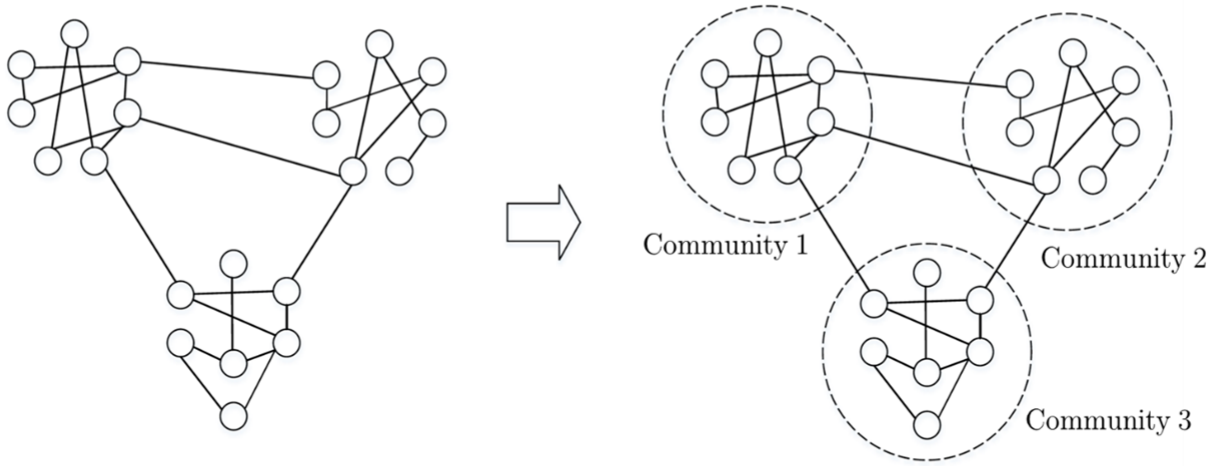

FIGURE 1. An example of network community structure

$$Q = \frac{1}{m} . \sum_{\forall v_i,\ v_j \in V} \left(w_{ij} - \frac{w_i^{\text{in}} . w_j^{\text{out}}}{m} \right) . \delta\left(v_i,\ v_j\right) \tag{9}$$

$$\mathrm{m} = \sum_{\forall v_i,\ v_j \in V} w_{ij} \tag{10}$$

$$\delta\left(v_i,\ v_j\right) = \begin{cases} 1,\ v_i,\ v_j \text{ in the same community} \\ 0,\ \text{otherwise} \end{cases} \tag{11}$$

The methods of community partitioning can be divided into graph partitioning, hierarchical clustering, and heuristic algorithms. The hierarchical clustering method can obtain a hierarchical community structure. This paper utilizes a hierarchical clustering strategy for the road network layering. The hierarchical clustering strategies generally fall into two types, an agglomerative approach and a divisive approach. The agglomerative one is a bottom-up approach that firstly treats each node as a single community and then merges adjacent nodes to form a larger community. The step is iterated until the modularity of the road network is expected. On the contrary, the divisive approach is a top-down manner that firstly treats all the nodes in a road network as a single community and gradually divides the community into smaller ones until a termination condition. Both of the approaches, in sum, can partition a road network into layered communities, but in general, the performance of agglomerative approach is better the one of divisive approach.

Figure 2 shows an example of road network abstraction. The lower layer is a real road network and the higher layer is an abstracted network which consists of virtual nodes. The virtual nodes can indicate the common characteristics of the nodes in the lower-layer network. Once a route path planning is requested, an approximate searching scope can be obtained by the corresponding higher-layer virtual nodes of the lower-layer nodes (e.g. starting and destination nodes in real road network). After mapping the higher-layer searching scope to the lower-layer network, the scale of searching in lower-layer network will be significantly reduced. As a result of the reduced searching scale, shortest path discovery approaches such as A-star or ant colony optimization algorithms are satisfiable for the vehicle navigation.

This paper uses an agglomerative hierarchical clustering approach and greedy strategy to make the road network layered with largest modularity. The algorithm details is described as follows: (1) In the initial phase, each node (intersection) in the physical road

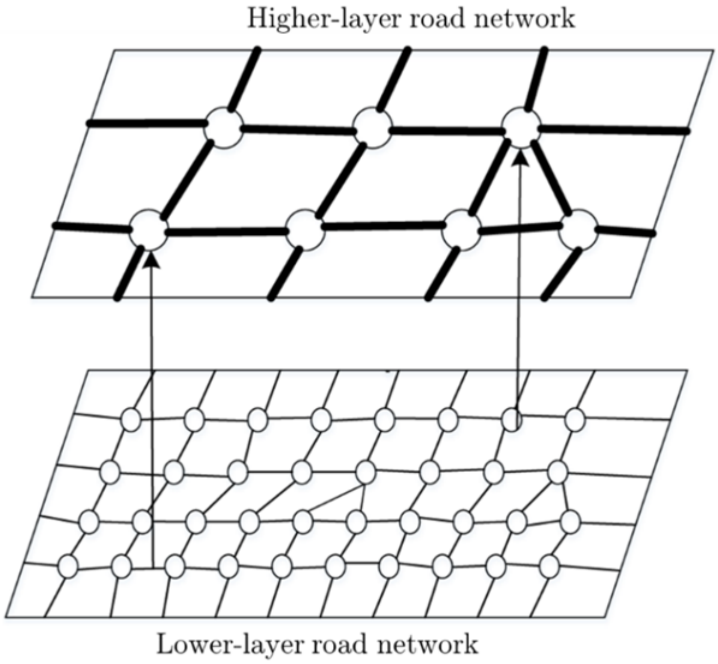

FIGURE 2. Two-layer example of road network abstraction

network will be treated as a single community and the weight of the edge between two nodes is defined by the reciprocal of the least passing time of a road section primarily calculated with the parameters of road section length and vehicle speed. In addition, to reflect the influence of real time traffic conditions, the congestion conditions are concerned in the calculation of least passing time. The conditions are categorized into four congestion levels: heavy jam, jam, general, unobstructed, which information can be obtained from Baidu traffic map system. Each level has a corresponding congestion coefficient used in the calculation of least passing time, as shown in equation (7). Regarding to the connectivity, it is defined that if there exists a road section between two nodes, the two nodes are adjacent. And, a two-way road is represented as two connections with different directions. It is also defined that if two nodes are adjacent and not belong the same community, the respective communities of these two nodes are also adjacent.

(2) For a node i, try to virtually assign the node to one of its adjacent communities and recalculate the community modularity of the traffic network. After that, reassign the node i to its another adjacent community and recalculate the community modularity again. This process should be repeated until the node i has been virtually assigned to each of its adjacent communities and each of the corresponding community modularity has also been calculated. After that, compare all of the modularity values with the original one to obtain the modularity increments, thus the value of community modularity after the assignment minus the one before the assignment. Then find the assignment which the corresponding increment of modularity is largest. Finally, if the largest increment is positive, merge the original community with the new one into one community. This indicates that the number of the communities decreased and the modularity of the road network increased. Otherwise, if the largest increment is negative, keep the network communities unchanged. The following equation (12) is used for the calculation of modularity increment and it is derived from equations (9)-(11).

$$Q = \frac{\sum_{\forall v_j \in C} (w_{ij} + w_{ji})}{m} - \frac{\left(\sum_{\forall v_j \in C, v_k \in V-C} w_{kj}\right).w_i^{\text{out}} + \left(\sum_{\forall v_j \in C, v_k \in V-C} w_{jk}\right).w_i^{\text{in}}}{m^2} \quad (12)$$

(3) For each of the other nodes in the road network, repeat the process of the step (2) given above until there is no more change in the communities.

(4) Once the statuses of the communities are no more changed, each of the communities in the traffic network will be treated as a new virtual node and the coordinates of the virtual node will be set to the coordinate average of all the original nodes in the corresponding community. If adjacent nodes exist in two different communities, the corresponding virtual nodes formed respectively by these two communities are also defined as adjacent nodes and the distance between the adjacent virtual nodes is a Euclidean distance.

(5) A new virtual traffic network different from the original road network will be generated with all the new virtual nodes and their connections, thus a road network abstraction is completed. This abstracted virtual traffic network is treated as an upper-layer network on the original road network. Step (2) to (5) should be repeated until there is an enough number of network layers or the requirement of network modularity is satisfied.

Finally, the layered road network model will be used in hierarchical path discoveries, and the top-down hierarchical search can reduce searching unnecessary scopes for fast path discovery.

Figure 3 illustrates a layered community partitions with hierarchical clustering and road network abstraction. Figure 4 shows an example of top-down hierarchical path discovery. Firstly, the optimal path can be found fast in higher-layer virtual nodes and consequently, a reduced searching scope (i.e. the next lower-layer virtual nodes) can be determined.

In regard to the practical applications, the proposed network modeling method can be used in an on-line or off-line vehicle navigation system. In an on-line navigation application, the system in the vehicle should have a wireless or telecommunication connection to the traffic center server and the on-line Baidu map system to acquire the information of static and dynamical real time traffic conditions. The vehicle navigation system will finish the layered road network abstraction and provide it to the path discovery subsystem. The path discovery subsystem then will perform a top-down searching with the network model according to the request of a source-to-destination path planning. If on-line connection is not available, the vehicle navigation system will turn into an off-line operation mode. The off-line operation mode can utilize the latest information or analyze the history information recorded beforehand to construct a layered road network model and then provides it for path discovery.

4. **Performance Evaluations.** This section presents the evaluation results of the proposed approach of road network modeling based on the concepts of hierarchical clustering and road network abstraction. As shown in Figure 5, the road network e-map of Fuzhou City was used in the evaluations and provided by Fuzhou Investigation and Surveying Institute. The data structure of the road map contains the detailed and essential static information of physical road network for the uses in the evaluations. Regrading to the dynamical traffic information, the experiments used the history records of vehicle speed monitored and recorded by the Fuzhou Investigation and Surveying Institute. In addition, the dynamical traffic conditions of congestion status were obtained from Baidu traffic map system by the Baidu Open Platform SDK and APIs. Before the experiments, the obtained results of dynamical traffic conditions were also recorded in the proposed system for the uses in experiments. The parameter of congestion coefficient corresponding to the four

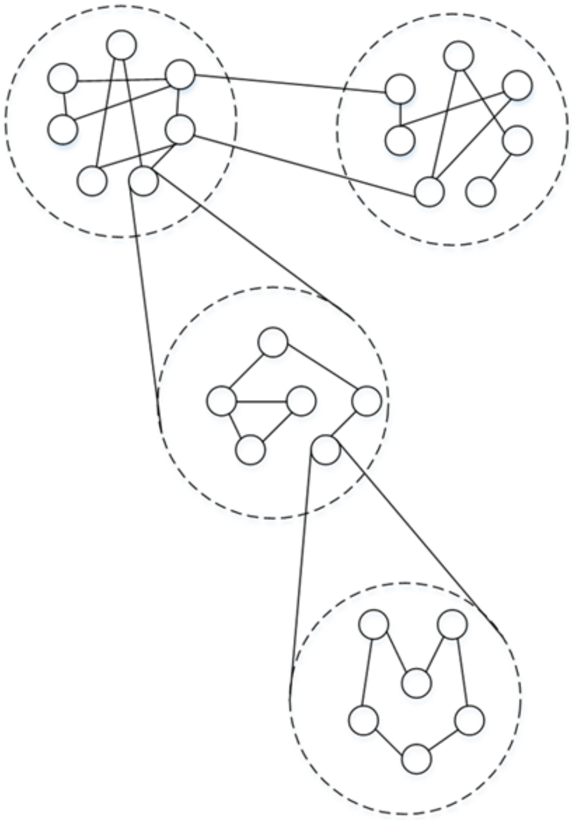

Figure 3. Layered community partitions with hierarchical clustering

Table 3. Number of nodes in each layer after road network abstraction

Layer	Number of nodes
4	433
3	1379
2	5827
1	22269

categorized congestion conditions is shown in Table 2. There are 22269 physical nodes, 27864 road sections, 5 kinds of road classes. Table 3 shows the result of road network abstraction. The network was virtually abstracted into four layers where layer 1 is the lowest layer and layer 4 is highest. The number of nodes in a higher layer is less than the one in a lower layer.

The performance results of path discovery for a vehicle navigation based on the proposed road network abstraction approach (hereinafter called RNA) were compared with the A-star and Dijkstra algorithms. Firstly, thirty paths with different distances are randomly selected in the experiments of path discovery. Ten of the selected paths range between 1 and 5 kilometers, ten of the others range between 5 and 10 kilometers, and the remainder paths range between 10 and 20 kilometers. Table 4 shows the number of nodes searched. It can be found that the number of nodes required to be searched is less than the A-star and Dijkstra algorithms in the different distance ranges. This can significantly reduce the path discovery time in a large and complex road network for vehicle navigation requirements.

Three paths with different starting and destination nodes are selected for the other experiments of path discovery. The theoretical shortest distances of the paths (S1, D1),

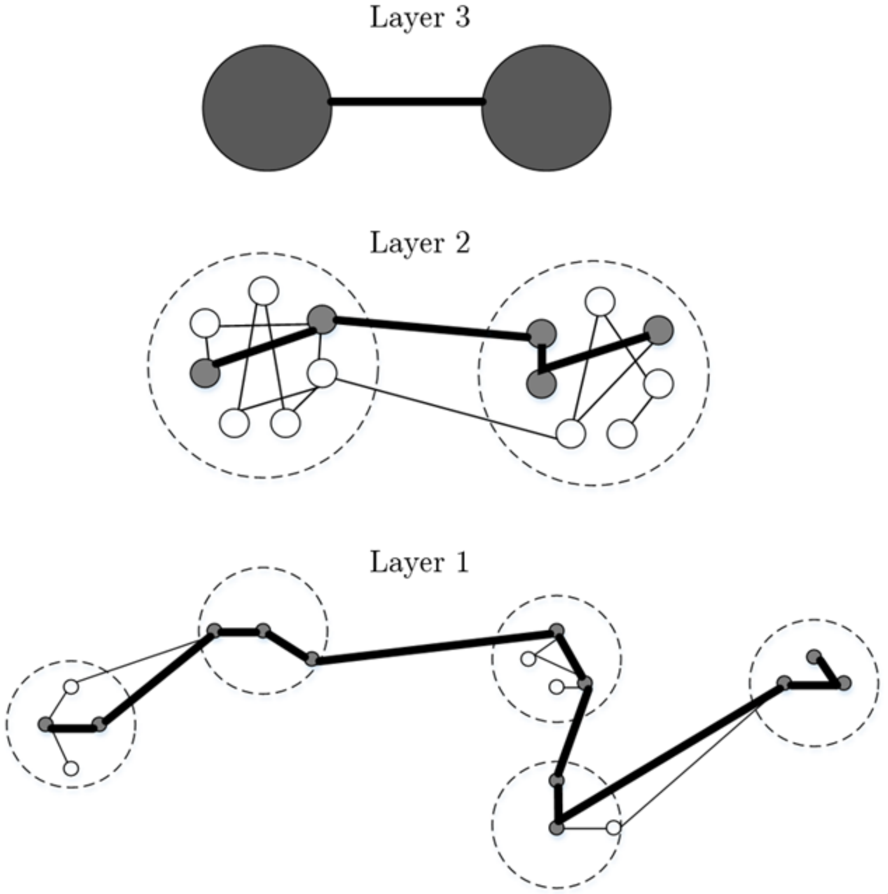

Figure 4. Top-down hierarchical path discovery

Table 4. Comparison of performance with varied distances

Distance (starting node to destination node)	**Number of nodes searched**		
	RNA	**A***	**Dijkstra**
1 5 km	237	1243	4032
5 10 km	423	2092	8509
10 20 km	1035	4053	15346

Table 5. Comparison of performance with definite starting and destination nodes

Starting	**Destination**	**Theoretical shortest path (m)**	**Number of nodes searched**			**Operation time (s)**		
			RNA	**A***	**Dijkstra**	**RNA**	**A***	**Dijkstra**
S1	D1	5251	213	992	7243	0.57	0.9	7.21
S2	D2	6780	440	588	4595	0.58	0.91	7.24
S3	D3	3740	114	687	5723	0.55	0.89	7.15

(S2, D2) and (S3, D3) are 5251, 6780 and 3740 meters, respectively. The number of nodes searched and the operation time consumed are shown in Table 5. It is also quite obvious that the number of required nodes for searching is less than both of the A-star and Dijkstra algorithms in each pair of the starting and destination nodes. This indicates that the proposed RNA can significantly reduce the path discovery time in the complex road

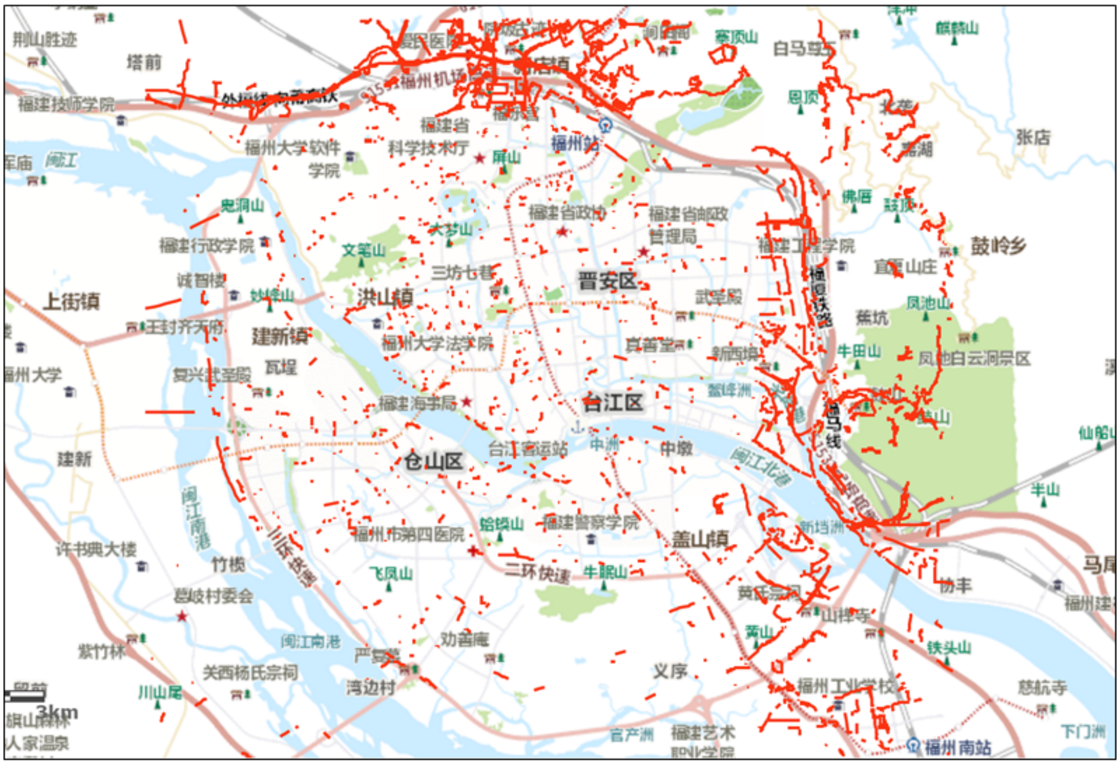

FIGURE 5. The traffic road network of Fuzhou City in China

network for vehicle navigation requirements. It is also verified by the results of operation time shown in Table 5.

5. **Conclusion.** The shortest path discovery for vehicle navigation should not merely consider the distance between the source and destination nodes, but also the time-dependent road network conditions. This paper proposes a road network modeling algorithm that concerns both of the static network information and dynamic real-time network conditions. Physical road networks can be abstracted into hierarchical network layers with the measurement of community modularity. The road network abstraction will be finished by an agglomerative hierarchical clustering approach. The path discovery will be performed by a top-down searching of the layered network. This can reduce unnecessary searches and the searching scope for a path discovery. The performance evaluations show that the proposed approach can obtain a better result in terms of the number of searching nodes and the operation time than the other two compared algorithms of Dijkstra and A-star.

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