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Array Calibration Method in Super-resolution Direction Finding for Wideband Signals

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Abstract. *Most super-resolution direction finding methods need to know the array manifold exactly, but there are usually various errors or perturbations in application, which directly lead to the performance degradation, and even failure. The paper proposed an array calibration method in super-resolution direction finding for wideband signals based on spatial domain sparse optimization when mutual coupling, gain/phase uncertainty, and sensor location errors exist simultaneously. First, the Fast Fourier Transformation (FFT) is employed to divide the wideband signals into several sub-bands; Then corresponding optimization functions are founded by the signals of every frequency; After that the error parameters are estimated by expectation maximization(EM) iteratively; Finally, the information of all frequencies is integrated to calibrate the array, consequently the actual directions of arrival (DOA) can be estimated.*
Keywords: Super-resolution direction finding, Array calibration, Wideband signals, Sparse optimization

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1. **Introduction.** Super-resolution direction finding is one of the major research contents in array signal processing, it is widely used in radio monitoring[1-5], internet of things[6,7] and electronic countermeasure fields[8,9]. At present, most direction finding methods are based on knowing the accurate array manifold, but there are often high frequency oscillation, amplifiers, channels which are not consistent, and sometimes accompanied with sensor position disturbance, lengths of channels are discordant in practical systems, which directly lead to performance deteriorates of direction finding methods, and even failure, so the array is often necessary to be calibrated.

Calibration methods in array signal processing can be classified into using source and self correction. The former are implemented by using the auxiliary source whose position is known, the latter are usually based on some optimization functions to estimate the directions and perturbation parameters of the array alternately. Most existing methods only adapt to one kind of array imperfection, for example, mutual couple among sensors[10-12], gain/phase uncertainty [13-15] and sensor location errors[16-18], they are based on eigenstructure and lack adaptation to the background of low signal to noise ratio(SNR) as well as small number of snapshots. Some of them have their unique advantage: Cao and Ye[19] proposed a calibration method for channel gain/phase uncertainty based on fourth-order cumulant technique, it adapts to the background of non-Gaussian signals and Gaussian noise. Mavrychev[20] studied partly calibrated array, it does not need the accurate position information among each subarray, consequently the error caused by position perturbation is avoided. The DOA estimation in the presence of more than one kind of array imperfection has also been studied in some literatures, Friedlander and Weiss[21] proposed a technique which alternately iterated to estimate the source directions, mutual coupling and gain/phase uncertainty based on subspace principle, but it needs to solve the high dimensional nonlinear optimization problem, which has a great computational complexity and slow convergence speed, meanwhile, perturbation parameters are fuzzy for the uniform linear array. Song[22] and See[23] calibrated more than one kind of error which exist in the array simultaneously, but both of the methods need many iterations; Ng[24] also studied the same problems, but the method he proposed needs to know the source directions in advance. All the methods above only adapt to narrowband signals, and need many snapshots, but there are rare published literatures of array calibration for wideband signals, especially for several errors existing simultaneously.

The paper proposed a novel array calibration method in super-resolution direction finding for wideband signals based on spatial domain sparse optimization, when mutual coupling, gain/phase uncertainty, and sensor location errors exist simultaneously, the corresponding optimization functions are founded by the signal of every frequency, then the error parameters are estimated by expectation maximization(EM) iteratively, at last, the information of all frequencies is integrated to calibrate the array, consequently the actual DOA can be acquired.

Notations used in this paper are shown as follows, for a given matrix $\boldsymbol{X}$, $\boldsymbol{X}^{\mathrm{T}}$ and $\boldsymbol{X}^{\mathrm{H}}$ denote transpose and conjugate transpose respectively, $\mathrm{diag}(\boldsymbol{X})$ means forming a diagonal matrix by taking the given vector as the main diagonal, $\mathrm{Re}(\boldsymbol{X})$ means taking the real part, $\mathrm{E}(\boldsymbol{X})$ is the expectation operator, $< \cdot >$ denotes solving conditional expectation, $[\cdot]$ means Hadamard product,$\boldsymbol{X}_{-k}$ donates removing the kth element from $\boldsymbol{X}$, $\boldsymbol{I}_M$ stands for the identity matrix with dimension $M \times M$.

2. Array signal model.

2.1. Ideal signal model. It is seen from Figure.1, suppose there are K far-field wideband signals $s_k(t)(k = 1, 2, \cdots, K)$ impinging on the uniform linear array composed of M omnidirectional sensors, the space of them is d, it is equal to half of the wavelength of the center frequency, DOAs of them are $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_k, \cdots, \alpha_K]$, the first sensor is defined as the reference, then output of the mth sensor can be written as

$$x_m(t) = \sum_{k=1}^{K} s_k(t - \tau_m(\alpha_k)) + n_m(t)(m = 1, 2, \cdots, M) \tag{1}$$

where $\tau_m(\alpha_k) = (m-1)(d/c)\sin(\alpha_k)$ is the propagation delay for the kth signal arriving at the mth sensor with respect to the reference of the array, c is the propagating speed of the signal, $n_m(t)$ is the Gaussian white noise on the mth sensor.

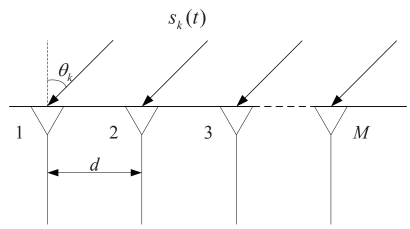

FIGURE 1. The structure of array

Before the processing, we divide the output vector into J nonoverlapping components, Fast Fourier Transformation (FFT) is performed on (1) and the array outputs of J frequencies can be represented as

$$\boldsymbol{X}(f_i) = \boldsymbol{A}(f_i, \boldsymbol{\alpha})\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i)(i = 1, 2, \cdots, J) \tag{2}$$

Z snapshots are collected at every frequency, then we have

$$\boldsymbol{X}(f_i) = [\boldsymbol{X}_1(f_i), \cdots, \boldsymbol{X}_m(f_i), \cdots, \boldsymbol{X}_M(f_i)]^{\mathrm{T}} \tag{3}$$

where

$$\boldsymbol{X}_m(f_i) = [X_m(f_i, 1), \cdots, X_m(f_i, z), \cdots, X_m(f_i, Z)] \tag{4}$$

and $\boldsymbol{A}(f_i, \boldsymbol{\alpha})$ is a $M \times K$ dimensional steering vector

$$\boldsymbol{A}(f_i, \boldsymbol{\alpha}) = [\boldsymbol{a}(f_i, \alpha_1), \cdots, \boldsymbol{a}(f_i, \alpha_k), \cdots, \boldsymbol{a}(f_i, \alpha_K)] \tag{5}$$

$$\boldsymbol{a}(f_i, \alpha_k) = [1, \exp(-\mathrm{j}2\pi f_i d/c\sin\alpha_k), \cdots, \exp(-\mathrm{j}(M-1)2\pi f_i d/c\sin\alpha_k)]^{\mathrm{T}} \tag{6}$$

and

$$\boldsymbol{S}(f_i) = [\boldsymbol{S}_1(f_i), \cdots, \boldsymbol{S}_k(f_i), \cdots, \boldsymbol{S}_K(f_i)]^{\mathrm{T}} \tag{7}$$

is the signal vector matrix after FFT to $s_k(t)(k = 1, 2, \cdots, K)$, where

$$\boldsymbol{S}_k(f_i) = [S_k(f_i, 1), \cdots, S_k(f_i, z), \cdots, S_k(f_i, Z)] \tag{8}$$

here, $S_k(f_i, z)$ is the zth snapshots of the kth signal at f_i,

$$\boldsymbol{N}(f_i) = [\boldsymbol{N}_1(f_i), \cdots, \boldsymbol{N}_m(f_i), \cdots, \boldsymbol{N}_M(f_i)]^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{N}_m(f_i) = [N_m(f_i, 1), \cdots, N_m(f_i, z), \cdots, N_m(f_i, Z)] \tag{10}$$

is the noise vector after performing FFT on $n_m(t)(m = 1, 2, \cdots, M)$, with mean 0 and variance $\mu^2(f_i)$.

2.2. **Array error model.** For convenience, we only discuss the information at frequency f_i for the moment.

(1) Mutual Coupling: The degree of mutual coupling is closely related to signal frequency, when there is only mutual coupling among sensors, perturbation matrix can be expressed by $\boldsymbol{W}_{(1)}(f_i)$, we define Q as the freedom degree of the array, according to the property of uniform linear array, $\boldsymbol{W}_{(1)}(f_i)$ can be expressed as:

$$\boldsymbol{W}_{(1)}(f_i) = \begin{bmatrix} 1 & c_1(f_i) & \cdots & c_Q(f_i) & \\ c_1(f_i) & 1 & c_1(f_i) & \ddots & \\ & c_1(f_i) & & & c_Q(f_i) \\ \vdots & \ddots & \ddots & \ddots & \\ c_Q(f_i) & & & & \\ & \ddots & & 1 & c_1(f_i) \\ & c_Q(f_i) & & c_1(f_i) & 1 \end{bmatrix} \tag{11}$$

where $c_q(f_i)(q = 1, 2, \cdots, Q)$ is the mutual coupling coefficient, when the distance between two sensor is q, signal frequency is f_i, the steering vector of the array can be revised to

$$\boldsymbol{a}_{(1)}'(f_i, \alpha_k) = \boldsymbol{W}_{(1)}(f_i)\boldsymbol{a}(f_i, \alpha_k), (k = 1, 2, \cdots, K) \tag{12}$$

corresponding array manifold is

$$\boldsymbol{A}_{(1)}'(f_i, \boldsymbol{\alpha}) = [\boldsymbol{a}_{(1)}'(f_i, \alpha_1), \cdots, \boldsymbol{a}_{(1)}'(f_i, \alpha_k), \cdots, \boldsymbol{a}_{(1)}'(f_i, \alpha_K)] = \boldsymbol{W}_{(1)}(f_i)\boldsymbol{A}(f_i, \boldsymbol{\alpha}) \tag{13}$$

for the sake of simplicity, we define the mutual coupling perturbation vector between sensors as $\boldsymbol{w}_{(1)}(f_i) = [c_1(f_i), \cdots, c_Q(f_i)]^{\mathrm{T}}$.

(2) Gain/Phase Uncertainty: When there is only gain/phase uncertainty among array channels, $\boldsymbol{W}_{(2)}(f_i)$ is defined as perturbation matrix, it is

$$\boldsymbol{W}_{(2)}(f_i) = \mathrm{diag}([1, W_2(f_i), \cdots, W_m(f_i), \cdots, W_M(f_i)]^{\mathrm{T}}) \tag{14}$$

where

$$W_m(f_i) = \rho_m(f_i)\exp(\mathrm{j}\varphi_m(f_i)), (m = 1, 2, \cdots, M) \tag{15}$$

is the gain and phase perturbation of the mth sensor, $\rho_m(f_i)$, $\varphi_m(f_i)$ are respectively the gain and phase of the mth sensor with respect to the reference sensor, so the perturbed steering vector is

$$\begin{aligned} \boldsymbol{a}_{(2)}{}'(f_i, \alpha_k) &= [1, W_2(f_i)\mathrm{e}^{\mathrm{j}2\pi f_i \tau_2(\alpha_k)}, \cdots, W_m(f_i)\mathrm{e}^{\mathrm{j}2\pi f_i \tau_m(\alpha_k)}, \cdots, W_M(f_i)\mathrm{e}^{\mathrm{j}2\pi f_i \tau_M(\alpha_k)}]^{\mathrm{T}} \\ &= \mathrm{diag}([1, W_2(f_i), \cdots, W_m(f_i), \cdots, W_M(f_i)]^{\mathrm{T}})\boldsymbol{a}(f_i, \alpha_k) \\ &= \boldsymbol{W}_{(2)}(f_i)\boldsymbol{a}(f_i, \alpha_k) \end{aligned} \tag{16}$$

so the corresponding array manifold matrix is

$$\boldsymbol{A}_{(2)}{}'(f_i, \boldsymbol{\alpha}) = [\boldsymbol{a}_{(2)}{}'(f_i, \alpha_1), \cdots, \boldsymbol{a}_{(2)}{}'(f_i, \alpha_k), \cdots, \boldsymbol{a}_{(2)}{}'(f_i, \alpha_K)] = \boldsymbol{W}_{(2)}(f_i)\boldsymbol{A}(f_i, \boldsymbol{\alpha}) \tag{17}$$

for the sake of simplicity, we also define the gain/phase uncertainty vector among sensors as $\boldsymbol{w}_{(2)}(f_i) = [\rho_2(f_i)\mathrm{e}^{\mathrm{j}\varphi_2(f_i)}, \cdots, \rho_m(f_i)\mathrm{e}^{\mathrm{j}\varphi_m(f_i)}, \cdots, \rho_M(f_i)\mathrm{e}^{\mathrm{j}\varphi_M(f_i)}]^{\mathrm{T}}$.

(3) Sensor Location Error: When there is only sensor location error, it can be equivalent to introduce to an orientation dependent phase perturbation, that is

$$\boldsymbol{a}_{(3)}{}'(f_i, \alpha_k) = [1, \mathrm{e}^{\mathrm{j}2\pi f_i \Delta\tau_2(\alpha_k)}, \cdots, \mathrm{e}^{\mathrm{j}2\pi f_i \Delta\tau_m(\alpha_k)}, \cdots, \mathrm{e}^{\mathrm{j}2\pi f_i \Delta\tau_M(\alpha_k)}]^{\mathrm{T}} \cdot \boldsymbol{a}(f_i, \alpha_k) \tag{18}$$

here

$$\Delta\tau_m(\alpha_k) = \frac{\Delta d_m}{c}\sin\alpha_k \tag{19}$$

is the propagation delay error introduced by sensor location error when the kth signal arriving at the mth sensor, Δd_m is the error between actual and measured positions of the mth sensor, suppose the reference of them are coincide, we can define

$$\boldsymbol{D}(f_i, \alpha_k) = [1, \mathrm{e}^{\mathrm{j}2\pi f_i \Delta\tau_2(\alpha_k)}, \cdots, \mathrm{e}^{\mathrm{j}2\pi f_i \Delta\tau_m(\alpha_k)}, \cdots, \mathrm{e}^{\mathrm{j}2\pi f_i \Delta\tau_M(\alpha_k)}]^{\mathrm{T}} \tag{20}$$

so the corresponding array manifold matrix is

$$\begin{aligned} \boldsymbol{A}_{(3)}{}'(f_i, \boldsymbol{\alpha}) &= [\boldsymbol{a}_{(3)}{}'(f_i, \alpha_1), \cdots, \boldsymbol{a}_{(3)}{}'(f_i, \alpha_k), \cdots, \boldsymbol{a}_{(3)}{}'(f_i, \alpha_K)] \\ &= [\boldsymbol{D}(f_i, \alpha_1) \cdot \boldsymbol{a}(f_i, \alpha_1), \cdots, \boldsymbol{D}(f_i, \alpha_k) \cdot \boldsymbol{a}(f_i, \alpha_k), \cdots, \boldsymbol{D}(f_i, \alpha_K) \cdot \boldsymbol{a}(f_i, \alpha_K)] \\ &= \boldsymbol{W}_{(3)}(f_i, \boldsymbol{\alpha}) \cdot \boldsymbol{A}(f_i, \boldsymbol{\alpha}) \end{aligned} \tag{21}$$

where the perturbation matrix is

$$\boldsymbol{W}_{(3)}(f_i, \boldsymbol{\alpha}) = [\boldsymbol{D}(f_i, \alpha_1), \cdots, \boldsymbol{D}(f_i, \alpha_k), \cdots, \boldsymbol{D}(f_i, \alpha_K)] \tag{22}$$

for the sake of simplicity, we also define the sensor position error perturbation vector as $\boldsymbol{w}_{(3)} = [\Delta d_2, \cdots, \Delta d_M]^{\mathrm{T}}$, it has no relation with the frequency, so we omit symbol f_i.

(4) Multiple Errors: When the three errors above exist simultaneously, the output of the array at frequency f_i can be expressed as

$$\begin{aligned} \boldsymbol{X}'''(f_i) &= \boldsymbol{A}'''(f_i, \boldsymbol{\alpha})\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i) \\ &= \boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)\boldsymbol{W}_{(3)}(f_i, \boldsymbol{\alpha}) \cdot \boldsymbol{A}(f_i, \boldsymbol{\alpha})\boldsymbol{S}(f_i) + \boldsymbol{N}(f_i) \end{aligned} \tag{23}$$

we can establish the following equation so as to make clearer the relation between $\boldsymbol{X}'''(f_i)$ and the three errors

$$\begin{aligned}\boldsymbol{X}'''(f_i) &= \boldsymbol{A}'''(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)+\boldsymbol{N}(f_i)\\ &= \boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)\boldsymbol{W}_{(3)}(f_i,\boldsymbol{\alpha})\cdot\boldsymbol{A}(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)+\boldsymbol{N}(f_i)\\ &= \boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)\boldsymbol{A}(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)+\boldsymbol{\Lambda}_{(3)}(f_i)\boldsymbol{w}_{(3)}+\boldsymbol{N}(f_i)\\ &= \boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(3)}(f_i)\boldsymbol{A}(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)+\boldsymbol{\Lambda}_{(2)}(f_i)\boldsymbol{w}_{(2)}(f_i)+\boldsymbol{N}(f_i)\\ &= \boldsymbol{W}_{(2)}(f_i)\boldsymbol{W}_{(3)}(f_i)\boldsymbol{A}(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)+\boldsymbol{\Lambda}_{(1)}(f_i)\boldsymbol{w}_{(1)}(f_i)+\boldsymbol{N}(f_i)\end{aligned} \tag{24}$$

where $\boldsymbol{A}'''(f_i,\boldsymbol{\alpha})$ is the array manifold matrix along with the three errors simultaneously, $\boldsymbol{\Lambda}_{(1)}(f_i)$, $\boldsymbol{\Lambda}_{(2)}(f_i)$ and $\boldsymbol{\Lambda}_{(3)}(f_i)$ are respectively the coefficient vectors related to the mutual coupling, gain/phase uncertainty and sensor location errors, they are determined by

$$[\boldsymbol{\Lambda}_{(1)}(f_i)]_{:,u}=\frac{\partial}{\partial\boldsymbol{w}_{(1)}(f_i,u)}[(\boldsymbol{W}_{(1)}(f_i)-\boldsymbol{I}_M)\boldsymbol{W}_{(2)}(f_i)\boldsymbol{W}_{(3)}(f_i,\boldsymbol{\alpha})\cdot\boldsymbol{A}(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)] \tag{25}$$

$$[\boldsymbol{\Lambda}_{(2)}(f_i)]_{:,u}=\frac{\partial}{\partial\boldsymbol{w}_{(2)}(f_i,u)}[\boldsymbol{W}_{(1)}(f_i)(\boldsymbol{W}_{(2)}(f_i)-\boldsymbol{I}_M)\boldsymbol{W}_{(3)}(f_i,\boldsymbol{\alpha})\cdot\boldsymbol{A}(f_i,\boldsymbol{\alpha})\boldsymbol{S}(f_i)] \tag{26}$$

$$[\boldsymbol{\Lambda}_{(3)}(f_i)]_{:,u}=\frac{\partial}{\partial\boldsymbol{w}_{(3)}(u)}[\boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)(\boldsymbol{W}_{(3)}(f_i,\boldsymbol{\alpha})\cdot\boldsymbol{A}(f_i,\boldsymbol{\alpha})-\boldsymbol{A}(f_i,\boldsymbol{\alpha}))\boldsymbol{S}(f_i)] \tag{27}$$

where $\boldsymbol{w}_{(1)}(f_i,u)$, $\boldsymbol{w}_{(2)}(f_i,u)$ and $\boldsymbol{w}_{(3)}(u)$ is respectively the uth element of $\boldsymbol{w}_{(1)}(f_i)$, $\boldsymbol{w}_{(2)}(f_i)$ and $\boldsymbol{w}_{(3)}$.

3. **Estimation theory.** The searching space can be divided into several discrete angle grids $\boldsymbol{\Omega}=[\overline{\alpha}_1,\cdots,\overline{\alpha}_l,\cdots,\overline{\alpha}_L]$, and $K\ll L$, take $\boldsymbol{\Omega}$ into (23), we have

$$\overline{\boldsymbol{X}}'''(f_i)=\boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i)+\boldsymbol{N}(f_i),(i=1,2,\cdots,J) \tag{28}$$

then corresponding covariance matrix is

$$\overline{\boldsymbol{R}}'''(f_i)=E[\overline{\boldsymbol{X}}'''(f_i)(\overline{\boldsymbol{X}}'''(f_i))^{\mathrm{H}}] \tag{29}$$

in (28), $\overline{\boldsymbol{S}}(f_i)=[\overline{\boldsymbol{S}}(f_i,1),\cdots,\overline{\boldsymbol{S}}(f_i,z),\cdots,\overline{\boldsymbol{S}}(f_i,Z)]$, where $\overline{\boldsymbol{S}}(f_i,z)=[\overline{S}_1(f_i,z),\cdots,\overline{S}_l(f_i,z),\cdots,\overline{S}_L(f_i,z)]^{\mathrm{T}}$ is a sparse matrix, it only contains K non-zero elements, they are non-zero if and only if $\overline{\alpha}_l=\alpha_k(l=1,2,\cdots,L;k=1,2,\cdots,K)$, and $\overline{S}_l(f_i,z)=S_k(f_i,z)(l=1,2,\cdots,L;k=1,2,\cdots,K)$, so $\overline{\boldsymbol{S}}(f_i)$ can be regarded as $\boldsymbol{S}(f_i)$ jointed many zero elements.

Define $\boldsymbol{\delta}(f_i)=[\delta_1(f_i),\cdots,\delta_l(f_i),\cdots,\delta_L(f_i)]^{\mathrm{T}}$ as the vector formed by variances of the elements in $\overline{\boldsymbol{S}}(f_i)$, it reflects the energy of the signal, that is

$$\overline{\boldsymbol{S}}(f_i)\sim N(0,\boldsymbol{\Sigma}(f_i)) \tag{30}$$

where $\boldsymbol{\Sigma}(f_i)=\mathrm{diag}(\boldsymbol{\delta}(f_i))$, as $\overline{\boldsymbol{S}}(f_i)$ is $\boldsymbol{S}(f_i)$ jointed many zero elements, $\boldsymbol{\delta}(f_i)$ contains K non-zero elements too. It can be seen from (24) and (28), probability density of the output signal at f_i along with the three errors simultaneously is

$$\begin{aligned} P1 &= P(\overline{\boldsymbol{X}}'''(f_i)|\overline{\boldsymbol{S}}(f_i);\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i)) \\ &= P((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z}|(\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i)) \\ &= |\pi\mu^2(f_i)\boldsymbol{I}_M|^{-Z}\exp(-\mu^2(f_i)\sum_{z=1}^{Z}\|\overline{\boldsymbol{X}}'''(f_i,z)-\overline{\boldsymbol{A}}'''(f_i,\boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i,z)\|_2^2) \\ &= |\pi\mu^2(f_i)\boldsymbol{I}_M|^{-Z}\exp(-\mu^2(f_i)\sum_{z=1}^{Z}\|\overline{\boldsymbol{X}}'''(f_i,z)-\boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)\times \\ &\quad \boldsymbol{W}_{(3)}(f_i,\boldsymbol{\Omega})\cdot\boldsymbol{A}(f_i,\boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i,z)\|_2^2) \end{aligned} \tag{31}$$

where $\boldsymbol{W}_{(3)}(f_i,\boldsymbol{\Omega})=[\boldsymbol{D}(f_i,\overline{\alpha}_1),\cdots,\boldsymbol{D}(f_i,\overline{\alpha}_l),\cdots,\boldsymbol{D}(f_i,\overline{\alpha}_L)]$, combining (28), (30) and (31), probability density of $\overline{\boldsymbol{X}}'''(f_i)$ is

$$\begin{aligned} P2 &= P(\overline{\boldsymbol{X}}'''(f_i);\boldsymbol{\delta}(f_i),\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i)) \\ &= P((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z};\boldsymbol{\delta}(f_i),\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i)) \\ &= \int\cdots\int P((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z}|(\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i))\times \\ &\quad P((\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{\delta}(f_i))\mathrm{d}\overline{\boldsymbol{S}}(f_i,1)\cdots\mathrm{d}\overline{\boldsymbol{S}}(f_i,Z) \\ &= \prod_{z=1}^{Z}\int P((\overline{\boldsymbol{S}}(f_i,z)|\overline{\boldsymbol{S}}(f_i,z);\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i))\times((\overline{\boldsymbol{S}}(f_i,z)_{z=1}^{Z}; \\ &\quad \boldsymbol{\delta}(f_i))\mathrm{d}\overline{\boldsymbol{S}}(f_i,z) \\ &= |\pi(\mu^2(f_i)\boldsymbol{I}_M+\boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\boldsymbol{\Sigma}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\mathrm{H}})|^{-Z}\exp(-Z\times\mathrm{tr}((\mu^2(f_i)\boldsymbol{I}_M+ \\ &\quad \boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\boldsymbol{\Sigma}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\mathrm{H}}))^{-1}\overline{\boldsymbol{R}}'''(f_i))) \end{aligned} \tag{32}$$

then we can employ Expectation Maximization(EM) method [25-28] to iteratively estimate these unknown parameters, compute distribution function of $P(\overline{\boldsymbol{X}}'''(f_i);\boldsymbol{\delta}(f_i),\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i))$, in the E-step:

$$\begin{aligned} F1 &= F(\overline{\boldsymbol{X}}'''(f_i),\overline{\boldsymbol{S}}(f_i);\boldsymbol{\delta}(f_i),\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i)) \\ &= F((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z},(\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{\delta}(f_i),\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i)) \\ &= <\mathrm{In}P((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z},(\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{\delta}(f_i),\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i))> \\ &= <\mathrm{In}P((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z}|(\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{w}_{(1)}(f_i),\boldsymbol{w}_{(2)}(f_i),\boldsymbol{w}_{(3)},\mu^2(f_i))+ \\ &\quad \mathrm{In}P((\overline{\boldsymbol{S}}(f_i,z))_{z=1}^{Z};\boldsymbol{\delta}(f_i))> \\ &= <(-MZ\mathrm{In}\mu^2(f_i)-\mu^{-2}(f_i)\sum_{z=1}^{Z}\|\overline{\boldsymbol{X}}'''(f_i,z)-\boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i,z)\|_2^2- \\ &\quad \sum_{l=1}^{L}(Z\mathrm{In}\delta_l(f_i)+\frac{\sum_{z=1}^{Z}|\overline{S}_l(f_i,z)|^2}{\delta_l(f_i)}))> \\ &= <(-MZ\mathrm{In}\mu^2(f_i)-\mu^{-2}(f_i)\sum_{z=1}^{Z}\|\overline{\boldsymbol{X}}'''(f_i,z)-\boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)(\boldsymbol{W}_{(3)}(f_i,\boldsymbol{\Omega})\cdot \\ &\quad \boldsymbol{A}(f_i,\boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i,z)\|_2^2-\sum_{l=1}^{L}(Z\mathrm{In}\delta_l(f_i)+\frac{\sum_{z=1}^{Z}|\overline{S}_l(f_i,z)|^2}{\delta_l(f_i)}))> \end{aligned} \tag{33}$$

In the M-step, solve derivatives of $F(\overline{\boldsymbol{X}}'''(f_i), \overline{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))$ for each parameter, that is

$$\begin{aligned} F2 &= \frac{\partial F(\overline{\boldsymbol{X}}'''(f_i), \overline{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \boldsymbol{w}_{(1)}(f_i)} \\ &= -2\mu^{-2}(f_i)[< {\boldsymbol{\Lambda}_{(1)}}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}_{(1)}(f_i) > \boldsymbol{w}_{(1)}(f_i) - < {\boldsymbol{\Lambda}_{(1)}}^{\mathrm{H}}(f_i)(\overline{\boldsymbol{X}}'''(f_i) - \boldsymbol{W}_{(2)}(f_i) \times \\ & \boldsymbol{W}_{(3)}(f_i, \boldsymbol{\Omega}) \cdot \boldsymbol{A}(f_i, \boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i)) >] \end{aligned} \tag{34}$$

$$\begin{aligned} F3 &= \frac{\partial F(\overline{\boldsymbol{X}}'''(f_i), \overline{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \boldsymbol{w}_{(2)}(f_i)} \\ &= -2\mu^{-2}(f_i)[< {\boldsymbol{\Lambda}_{(2)}}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}_{(2)}(f_i) > \boldsymbol{w}_{(2)}(f_i) - < {\boldsymbol{\Lambda}_{(2)}}^{\mathrm{H}}(f_i)(\overline{\boldsymbol{X}}'''(f_i) - \boldsymbol{W}_{(1)}(f_i) \times \\ & \boldsymbol{W}_{(3)}(f_i, \boldsymbol{\Omega}) \cdot \boldsymbol{A}(f_i, \boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i)) >] \end{aligned} \tag{35}$$

$$\begin{aligned} F4 &= \frac{\partial F(\overline{\boldsymbol{X}}'''(f_i), \overline{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \boldsymbol{w}_{(3)}} \\ &= -2\mu^{-2}(f_i)[< {\boldsymbol{\Lambda}_{(3)}}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}_{(3)}(f_i) > \boldsymbol{w}_{(3)} - < {\boldsymbol{\Lambda}_{(3)}}^{\mathrm{H}}(f_i)(\overline{\boldsymbol{X}}'''(f_i) - \boldsymbol{W}_{(1)}(f_i) \times \\ & \boldsymbol{W}_{(2)}(f_i, \boldsymbol{\Omega})\boldsymbol{A}(f_i, \boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i)) >] \end{aligned} \tag{36}$$

$$\begin{aligned} F5 &= \frac{\partial F(\overline{\boldsymbol{X}}'''(f_i), \overline{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \mu^2(f_i)} \\ &= \frac{\partial((\overline{\boldsymbol{X}}'''(f_i, z))_{z=1}^{Z}, (\overline{\boldsymbol{S}}(f_i, z))_{z=1}^{Z}; \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \mu^2(f_i)} \\ &= -\frac{MZ}{\mu^2(f_i)} + \frac{1}{(\mu^2(f_i))^2} < \sum_{z=1}^{Z} \|\overline{\boldsymbol{X}}'''(f_i, z) - \boldsymbol{A}'''(f_i, \boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i, z)\|_2{}^2 > \end{aligned} \tag{37}$$

$$\begin{aligned} F6 &= \frac{\partial F(\overline{\boldsymbol{X}}'''(f_i), \overline{\boldsymbol{S}}(f_i); \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \delta_l(f_i)} \\ &= \frac{\partial F((\overline{\boldsymbol{X}}'''(f_i, z))_{z=1}^{Z}, (\overline{\boldsymbol{S}}(f_i, z))_{z=1}^{Z}; \boldsymbol{\delta}(f_i), \boldsymbol{w}_{(1)}(f_i), \boldsymbol{w}_{(2)}(f_i), \boldsymbol{w}_{(3)}, \mu^2(f_i))}{\partial \delta_l(f_i)} \\ &= -\frac{Z}{\delta_l(f_i)} + \frac{1}{(\delta_l)^2(f_i)} < \sum_{z=1}^{Z} |\overline{S}_l(f_i, z)|^2 > \end{aligned} \tag{38}$$

set them to be 0 respectively, then estimation values of every parameter of the pth iteration can be solved

$$\begin{aligned} \boldsymbol{w}_{(1)}^{(p)}(f_i) &= < {\boldsymbol{\Lambda}_{(1)}}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}_{(1)}(f_i) >^{-1} < {\boldsymbol{\Lambda}_{(1)}}^{\mathrm{H}}(f_i)(\overline{\boldsymbol{X}}'''(f_i) - \boldsymbol{W}_{(2)}(f_i)\boldsymbol{W}_{(3)}(f_i, \boldsymbol{\Omega}) \cdot \boldsymbol{A}(f_i, \boldsymbol{\Omega}) \times \\ & \overline{\boldsymbol{S}}(f_i)) > \end{aligned} \tag{39}$$

$$\begin{aligned} \boldsymbol{w}_{(2)}^{(p)}(f_i) &= < {\boldsymbol{\Lambda}_{(2)}}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}_{(2)}(f_i) >^{-1} < {\boldsymbol{\Lambda}_{(2)}}^{\mathrm{H}}(f_i)(\overline{\boldsymbol{X}}'''(f_i) - \boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(3)}(f_i, \boldsymbol{\Omega}) \cdot \boldsymbol{A}(f_i, \boldsymbol{\Omega}) \times \\ & \overline{\boldsymbol{S}}(f_i)) > \end{aligned} \tag{40}$$

$$\boldsymbol{w}_{(3)}^{(p)} = < {\boldsymbol{\Lambda}_{(3)}}^{\mathrm{H}}(f_i)\boldsymbol{\Lambda}_{(3)}(f_i) >^{-1} < {\boldsymbol{\Lambda}_{(3)}}^{\mathrm{H}}(f_i)(\overline{\boldsymbol{X}}'''(f_i) - \boldsymbol{W}_{(1)}(f_i)\boldsymbol{W}_{(2)}(f_i)\boldsymbol{A}(f_i,\boldsymbol{\Omega})\overline{\boldsymbol{S}}(f_i)) > \tag{41}$$

$$(\mu^2(f_i))^{(p)} = \frac{1}{MZ} < \sum_{z=1}^{Z} \|\overline{\boldsymbol{X}}'''(f_i,z) - (\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{(p)}\overline{\boldsymbol{S}}(f_i,z)\|_2{}^2 > \tag{42}$$

$${\delta_l}^{(p)}(f_i) = \frac{1}{Z} < \sum_{z=1}^{Z} |\overline{S}_l(f_i,z)|^2 > \tag{43}$$

where (p) denotes number of iterations, after several times, the variations of $\boldsymbol{w}_{(1)}(f_i)$,$\boldsymbol{w}_{(2)}(f_i)$, $\boldsymbol{w}_{(3)}$,$\mu^2(f_i)$ and $\delta_l(f_i)$ tend to be zero, then they are deemed to be convergent, we can acquire their final estimation results:$\hat{\boldsymbol{w}}_{(1)}(f_i)$,$\hat{\boldsymbol{w}}_{(2)}(f_i)$, $\hat{\boldsymbol{w}}_{(3)}$,$\mu^2(f_i)$ and $\hat{\delta}_l(f_i)$, combining $\hat{\boldsymbol{\delta}}(f_i) = [\hat{\delta}_1(f_i),\cdots,\hat{\delta}_l(f_i),\cdots,\hat{\delta}_L(f_i)]^{\mathrm{T}}$ and $\hat{\boldsymbol{\Sigma}}(f_i) = \mathrm{diag}(\hat{\boldsymbol{\delta}}(f_i))$, we can use them for array calibration. Define $\boldsymbol{X}$ as the vector composed by sum of signal of all frequencies, as the signal of every frequency is independent of one another, the joint probability density of $\boldsymbol{X}$ is

$$\begin{aligned}
P(X) &= \prod_{i=1}^{J} P(\overline{\boldsymbol{X}}'''(f_i);\hat{\boldsymbol{\delta}}(f_i),\hat{\boldsymbol{w}}_{(1)}(f_i),\hat{\boldsymbol{w}}_{(2)}(f_i),\hat{\boldsymbol{w}}_{(3)},\hat{\mu}^2(f_i)) \\
&= \prod_{i=1}^{J} P((\overline{\boldsymbol{X}}'''(f_i,z))_{z=1}^{Z};\hat{\boldsymbol{\delta}}(f_i),\hat{\boldsymbol{w}}_{(1)}(f_i),\hat{\boldsymbol{w}}_{(2)}(f_i),\hat{\boldsymbol{w}}_{(3)},\hat{\mu}^2(f_i)) \\
&= |\pi|^{-JZ}\prod\nolimits_{i=1}^{J}|\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\mathrm{H}}|^{-Z}\times\exp(-Z\times \\
&\quad \sum_{i=1}^{J}\mathrm{tr}((\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\mathrm{H}})^{-1}\overline{\boldsymbol{R}}'''(f_i)))
\end{aligned} \tag{44}$$

perform logarithm operation on both sides of the (44), we have

$$\begin{aligned}
\mathrm{In}(P(X)) &= -JZ\mathrm{In}\pi - Z(\sum_{i=1}^{J}\mathrm{In}|\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\mathrm{H}}|) - Z\times \\
&\quad \sum_{i=1}^{J}\mathrm{tr}((\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\mathrm{H}})^{-1}\overline{\boldsymbol{R}}'''(f_i))
\end{aligned} \tag{45}$$

maximize (45), that is

$$\frac{\partial\mathrm{In}(P(\boldsymbol{X}))}{\partial\boldsymbol{\alpha}} = 0 \tag{46}$$

take (45) into (46) and we can infer

$$\begin{aligned}\hat{\alpha}_k \;=\; & \text{argmax}_{\alpha_k}|\text{Re}(\sum_{i=1}^{J}((\boldsymbol{a}'''(f_i,\alpha_k))^{\text{H}}((\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega})\hat{\boldsymbol{\Sigma}}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}))^{\text{H}}))^{-1})\times \\ & (\sum_{i=1}^{J}(\boldsymbol{a}'''(f_i,\alpha_k)(\boldsymbol{a}'''(f_i,\alpha_k))^{\text{H}}\times((\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k})\hat{\boldsymbol{\Sigma}}_{-k}(f_i)\times \\ & (\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k}))^{\text{H}}))^{-1}\overline{\boldsymbol{R}}'''(f_i))) - \sum_{i=1}^{J}(\overline{\boldsymbol{R}}'''(f_i)((\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k})\hat{\boldsymbol{\Sigma}}_{-k}(f_i)\times \\ & (\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k}))^{\text{H}}))^{-1}(\boldsymbol{a}'''(f_i,\alpha_k)(\boldsymbol{a}'''(f_i,\alpha_k))^{\text{H}}\times\sum_{i=1}^{J}((\hat{\mu}^2(f_i)\boldsymbol{I}_M + \boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k})\times \\ & \hat{\boldsymbol{\Sigma}}_{-k}(f_i)(\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k}))^{\text{H}}))^{-1}\frac{\partial \boldsymbol{a}'''(f_i,\alpha_k)}{\partial \alpha_k}))|^{-1}\end{aligned} \tag{47}$$

then final result of DOA can be estimated. We can get $c_1(f_i),\cdots,c_Q(f_i)$ according to $\hat{\boldsymbol{w}}_{(1)}(f_i)$, then $\boldsymbol{W}_{(1)}(f_i)$ can be acquired by (11); and $\rho_2(f_i)\text{e}^{\text{j}\varphi_2(f_i)},\cdots,\rho_M(f_i)\text{e}^{\text{j}\varphi_M(f_i)}$ can be estimated by $\hat{\boldsymbol{w}}_{(2)}(f_i)$, thus we can calculate $\boldsymbol{W}_{(2)}(f_i)$ based on (14) and (15); we can also solve $\Delta d_2,\cdots,\Delta d_M$ according to $\hat{\boldsymbol{w}}_{(3)}$,then $\boldsymbol{a}'''(f_i,\alpha_k)$ and $\boldsymbol{A}'''(f_i,\boldsymbol{\Omega}_{-k})$ can be acquired. Thus, we will get DOA estimation based on (47) and the parameters above.

It can be seen from the deduction above, there is a large amount of calculation for the proposed algorithm, which will limit its application in actual system, so it is necessary to improve the computation speed. The signal of J frequencies can be divided into W groups, each group contains J/W bins, then corresponding J/W groups of errors can be solved by (39)∼(43), that is to say the errors of every group can be estimated simultaneously. In actual application, W processors can be used to dispose the W groups of observed data simultaneously, the calculation speed can almost increase W times. The method is used for wideband signal, and has employed spatial domain sparse optimization, so we can call it WSDSO for short, and the method is also suitable for the circumstances of the calibration of any two or only one kind of error.

4. **Simulations.** In order to verify the effective of the method, some simulations are presented with matlab below, consider some wideband chirp signals impinge on a uniform linear array with 6 omnidirectional sensors from directions $(25^\circ, 45^\circ, 65^\circ)$, the center frequency of the signals is 3GHz, width of the band is 15% of the center frequency, the band is divided into 9 frequency bins, and spacing d between adjacent sensors is equal to half of the wavelength of the center frequency. The array errors are very complex, it is difficult to establish accurate function, so we will simplify the process in the simulations. When there are mutual coupling, gain/phase uncertainty, and sensor location errors in the array, suppose the freedom degree of sensors Q=2, mutual coupling perturbation vector $\boldsymbol{w}_{(1)}(f_i) = [c_1(f_i), c_2(f_i)]^{\text{T}} = [a_1(f_i) + b_1(f_i)\text{j}, a_2(f_i) + b_2(f_i)\text{j}]^{\text{T}}$, $a_1(f_i)$ and $b_1(f_i)$ is selected between $(-0.5\sim0.5)$ randomly, $a_2(f_i)$ and $b_2(f_i)$ is selected between $(-0.25\sim0.25)$ randomly; The gain relatively to the first channel of the other five ones are selected between $(0\sim2)$ randomly, and that of the phase are selected between $(-30^\circ\sim30^\circ)$ randomly; Sensor location error is selected between $(-0.25d\sim0.25d)$ randomly, the searching space is $(0^\circ\sim90^\circ)$, number of grid L is 181, then the set $\boldsymbol{\Omega} = [0^\circ, 0.5^\circ,\cdots,90^\circ]$, the EM method is finished when the update ratio is smaller than 0.001, i.e. $\|\boldsymbol{\delta}^{(p+1)}(f_i) - \boldsymbol{\delta}^{(p)}(f_i)\|_2/\|\boldsymbol{\delta}^{(p)}(f_i)\|_2 < 0.001$, and we have known the number of the signal, estimation error of DOA is defined as

$\sum_{k=1}^{K} |\alpha_k - \hat{\alpha}_k|$, and the normalized spectrum is the ratio of the spatial spectrum to the absolute of the maximum one.

In the first simulation, suppose SNR is 12dB, the number of snapshots at every frequency is 30, WSDSO method is employed for estimating the three errors above, 300 Monte-Carlo simulations are repeated, their average values are deemed as the final results, FIGURE 2 and FIGURE 3 show the mutual couple estimation of $c_1(f_i)$ and $c_2(f_i)$ at every frequency bin, where f_i-A means actual value at the ith $(i = 1, \cdots, 9)$ frequency bin, and f_i-E means the corresponding estimated value; FIGURE 4 and FIGURE 5 show gain and phase uncertainty estimation of different channels at every frequency bin, the first channel is defined as the reference, where ch$i$-A means actual value of the ith $(i = 2, \cdots, 6)$ channel relative to the first one, and ch$i$-E means the corresponding estimated value; FIGURE 6 shows the relative error (a ratio of the error and the spacing d) estimation of sensor location at every frequency bin, the first sensor is defined as the reference, where sensor$i$-A means the relative error between the actual position and the measured one of the ith sensor, and sensor$i$-E means the corresponding estimated value.

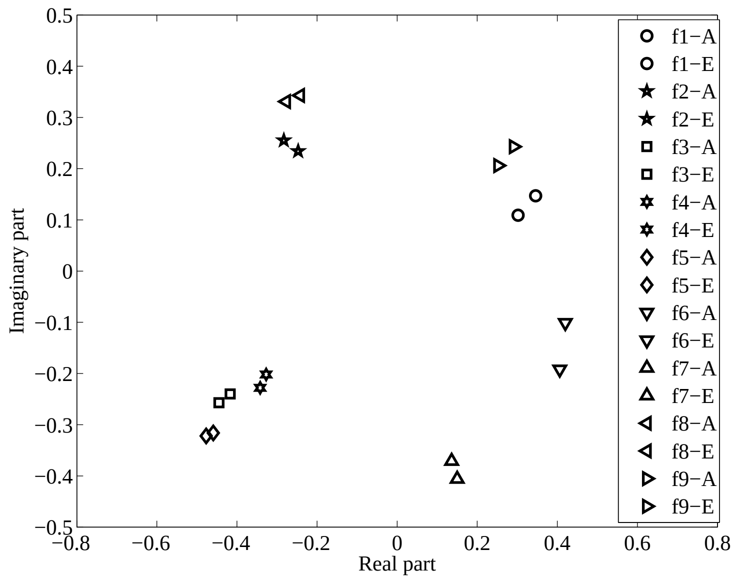

FIGURE 2. Mutual couple error estimation of c1

It can be seen from FIGURE 2∼6, the method can effectively estimate the three error perturbations existing in the array simultaneously, especially when the frequency is near to the center bin, we can use these results to calibrate the array and acquire the actual DOA of the wideband signals.

In the second simulation, traditional two-sided correlation transformation (TCT)[33] and WSDSO methods are employed for estimating DOA of wideband signals along with the three errors above. Here, TCT is performed without calibration(TCTWC), it is used for observing the improvement of the array by the proposed method, other conditions are the same with the first simulation, their normalized spectrums are shown in FIGURE 7, it can be seen that WSDSO method can relatively accurately estimate the DOA of the signals.

In the third simulation, the conditions are the same with the first simulation, FIGURE 8 presents the DOA estimation error as a function of SNR when number of snapshots is 30; while FIGURE 9 shows that of number of snapshots when SNR is 12dB.

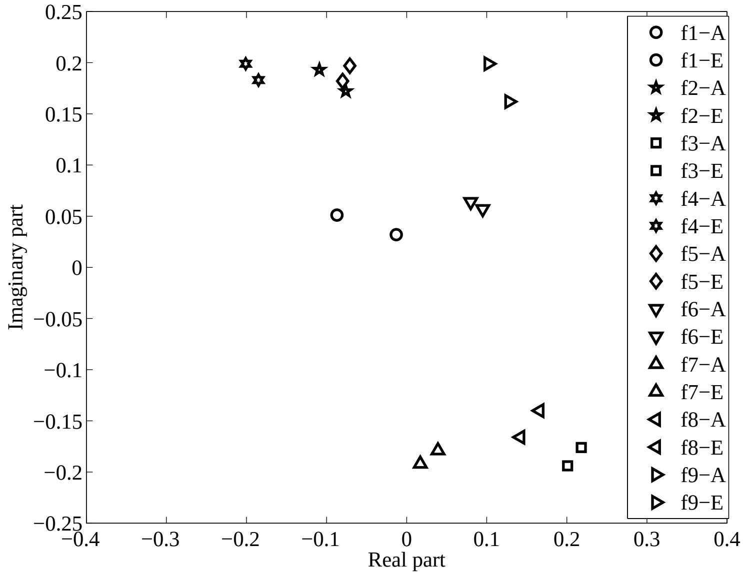

FIGURE 3. Mutual couple error estimation of c2

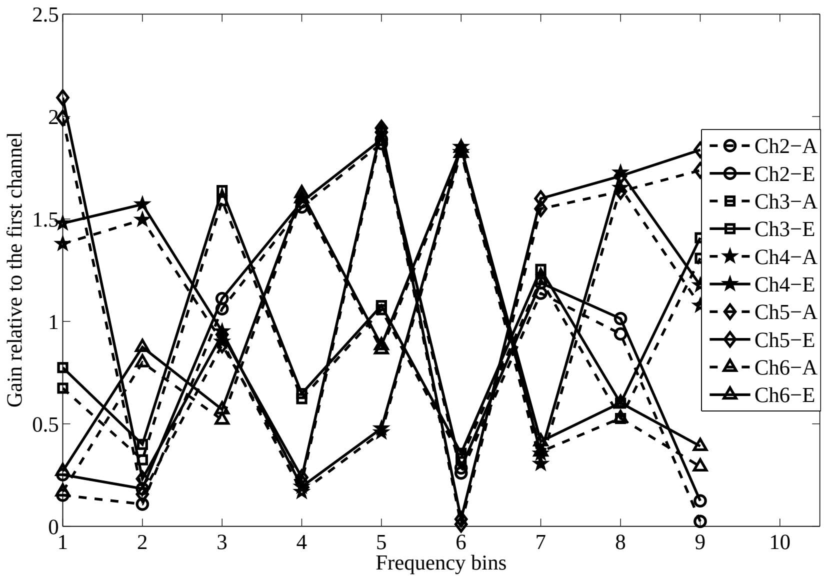

FIGURE 4. Gain uncertainty estimation

It can be seen from FIGURE 8 and 9, WSDSO method can effectively estimate the DOA of wideband signals along with the three errors existing in the array simultaneously, when the SNR or snapshots increase to some threshold, the estimation error approximately converges to 1.3°.

In the fourth simulation, TCTWC, Quasi-Blind Calibration (QBC)[20] and WSDSO are respectively employed for the DOA estimation along with gain/phase uncertainty and sensor location error perturbations. Here, QBC method is performed at every narrowband frequency, their average value is deemed as the wideband results, other conditions are the same with the third simulation, FIGURE 10 presents the estimation error as a function

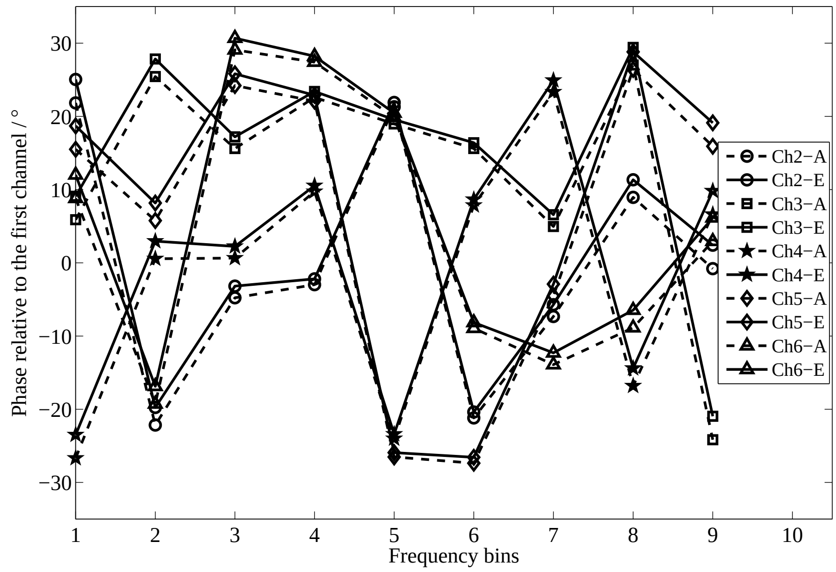

Figure 5. Phase uncertainty estimation

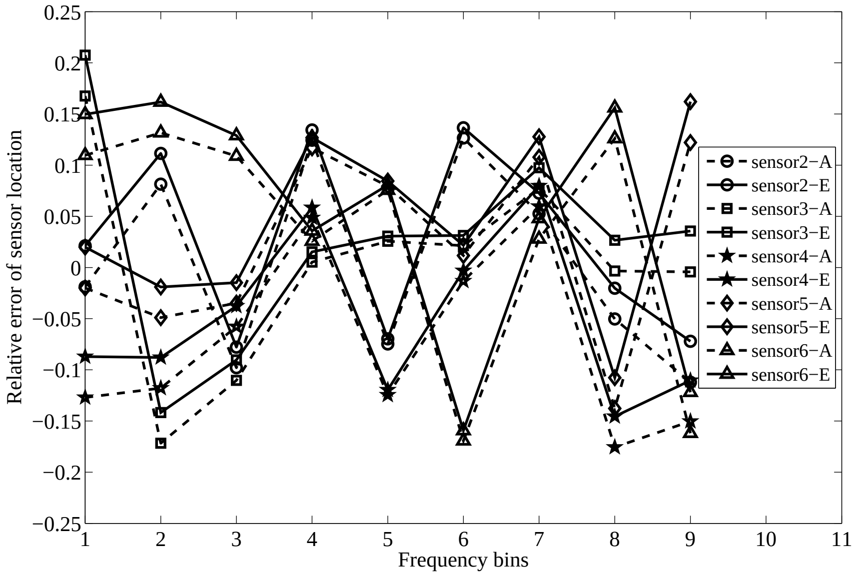

Figure 6. Sensor location error estimation

of SNR when number of snapshots is 40; while FIGURE 11 shows that of number of snapshots when SNR is 12dB.

FIGURE 10 and FIGURE 11 have respectively shown the calibration accuracy of the two methods when the gain/phase uncertainty and sensor location error perturbations exist in the array simultaneously. Obviously, whether the estimation accuracy or convergent rate, WSDSO is better than QBC method, WSDSO approximately converges to 0.8° , and QBC converges to 1.2° under the same condition at last.

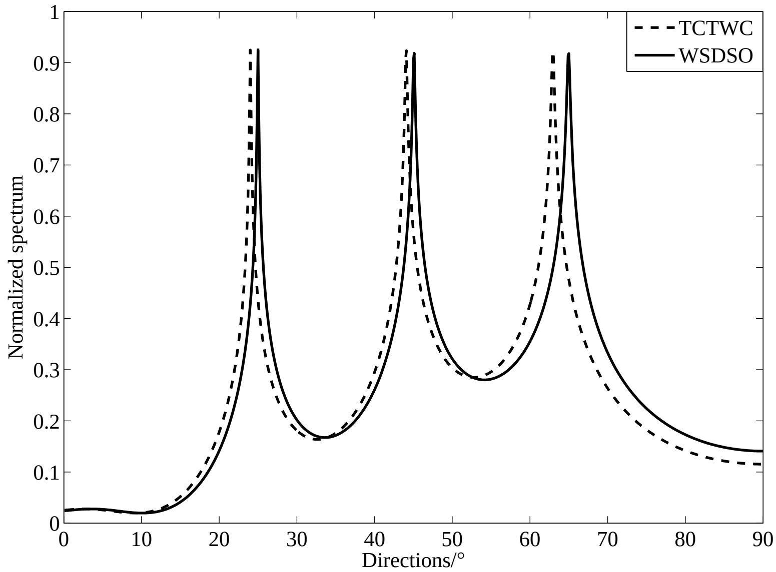

Figure 7. Normalized spectrums

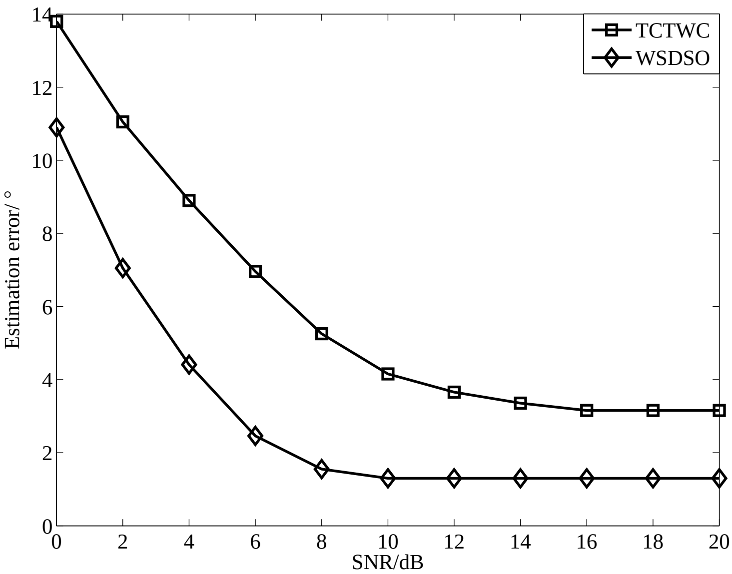

Figure 8. Calibration accuracy versus SNR with three errors

5. **Conclusion.** The paper proposed a novel array error calibration method in super-resolution direction finding for wideband signals based on spatial domain sparse optimization to the mutual coupling, gain/phase uncertainty, and sensor location perturbation errors existing in the array, it can estimate and calibrate the three errors simultaneously. The optimization functions are founded by the signal of every frequency, then the functions are optimized iteratively, after that information of all frequencies is integrated to solve the errors. Thus, the actual directions of arrival(DOA) can be acquired. In view of the large amount of calculation of the method, we put forward the use of multiple processors for the implementation so as to improve the execution efficiency. It can be seen from

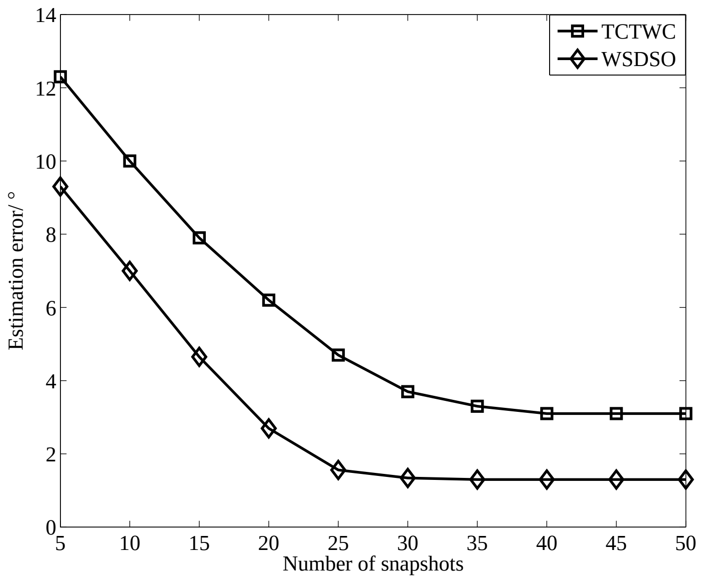

Figure 9. Calibration accuracy versus number of snapshots with three errors

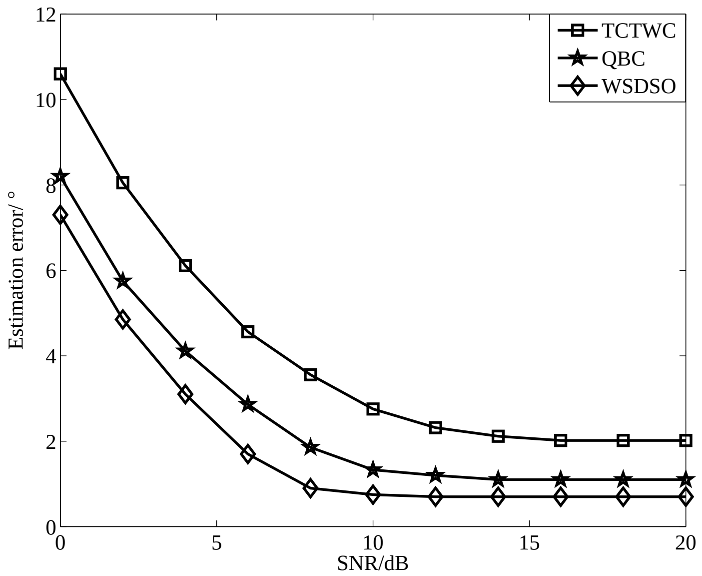

Figure 10. Calibration accuracy versus SNR with two errors

these simulations, as the effect of these errors are very difficult to separate from the array manifold thoroughly, the array calibration and the DOA estimation still can not reach actual value at high SNR or large number of snapshots, our work will be committed to optimizing the method to improve the estimation precision and broaden the bandwidth further in future.

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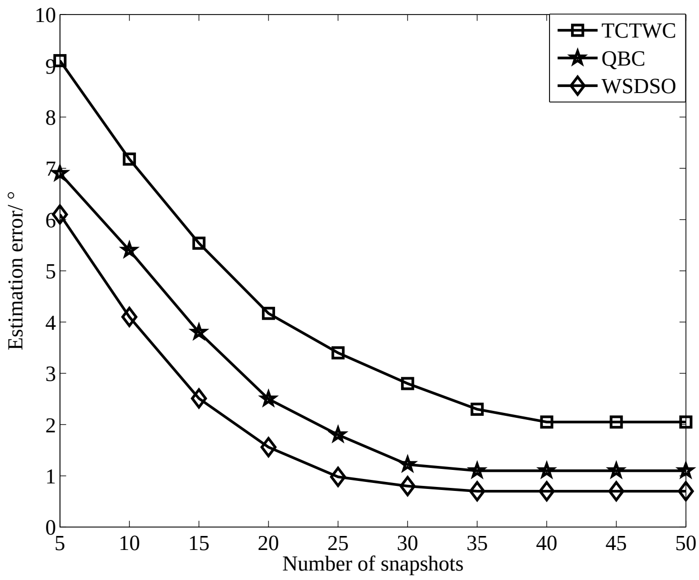

FIGURE 11. Calibration accuracy versus number of snapshots with two errors

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