

On the Security of Non-Linear HB (NLHB) Protocol Against Passive Attack

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Abstract. As a variant of the HB authentication protocol for RFID systems, which relies on the complexity of decoding *linear* codes against passive attacks, Madhavan et al. presented Non-Linear HB(NLHB) protocol. In contrast to HB, NLHB relies on the complexity of decoding a class of *non-linear* codes to render the passive attacks proposed against HB ineffective. Based on the fact that there has been no passive solution for the problem of decoding a random non-linear code, the authors have claimed that NLHB's security margin is very close to its key size.

In this paper, we show that passive attacks against HB protocol can still be applicable to NLHB and this protocol does not provide the desired security margin. In our attack, we first linearize the non-linear part of NLHB to obtain a *HB equivalent* for NLHB, and then exploit the passive attack techniques proposed for the HB to evaluate the security margin of NLHB. The results show that although NLHB's security margin is relatively higher than HB against similar passive attack techniques, it has been overestimated and, in contrary to what is claimed, NLHB is vulnerable to passive attacks against HB, especially when the noise vector in the protocol has a low weight.

Key words: : RFID, Authentication, LPN problem, HB protocols

1 Introduction

Radio Frequency Identification (RFID) tags are forming the next generation technology for identifying objects, and are poised to supplant barcodes in near future. Their advantages such as: more storage and ease of use have caused a universal proliferation of RFID tags in many commercial as well as national security applications; [1] ranging from electronic passports [4, 3], contactless credit cards [2], to supply chain management [6, 5, 7].

This widespread deployment of RFID tags has raised some concerns about their security. On the other hand, RFID tag constraints in processing power and memory make them tougher to deal with in security. These kinds of constraints dictate a paradigm shift in security provision for RFIDs which is known as *lightweight cryptography*.

Lightweight authentication protocol is a subset of lightweight cryptography which

tackles providing authentication in highly constrained environments (e.g RFID systems) as well as security provision to a reasonable extent [8, 9].

1.1 Notations Used

- $G_{a \times b}$: $a \times b$ binary matrix.
- $h_{1 \times b}$: $1 \times b$ binary vector.
- $A \otimes B$: matrix multiplication of A and B.
- \oplus : XOR operation.
- x_i : i^{th} bit of binary vector x .
- $Hwt(\cdot)$: hamming weight function.
- $h \otimes G$: matrix multiplication of a vector h into matrix G .
- R, T : *Reader* and *Tag* respectively.

1.2 HB Family Protocols

The HB lightweight authentication protocol proposed by Hopper and Blum in 2001 [10] is the first in the *HB family* of protocols. An overview of a paralleled r -round of the HB protocol is given in Figure 1. This protocol aims at unilateral authenticating of an RFID tag to a reader only by lightweight operations. The operations used in this protocol are one matrix multiplication and some XORs. On the other hand, The security of this algorithm and some others in this family against passive attacks is reduced to a well-known NP-hard problem called *Learning with Parity Noise* (LPN) problem [11].

Z The other members of this family emerged as a result of proposing an at-

<p>Specifications</p> <p>- r, η, ϵ: Public parameters.</p> <p>- ν: d-bit noise vector where: $Prob(\nu_i = 1) = \eta, i = 1, \dots, r$</p> <p>HB Protocol</p> <p>- Secret parameter $x \in \{0, 1\}^k$ is shared between R and T.</p> <p>(1) R: Chooses a random $A_{k \times r}$ matrix.</p> <p>(2) $R \Rightarrow T : A$</p> <p>(3) T: Computes $z_{1 \times r} = (x \otimes A) \oplus \nu_{1 \times r}$</p> <p>(4) $T \Rightarrow R : z$</p> <p>(5) R: ACCEPTS iff $Hwt(z \oplus (x \otimes A)) \leq \epsilon r$</p>

Fig. 1. Paralleled version of an r -round HB protocol

tack on the previous one in order to eliminate the weaknesses and render the prior proposed attacks ineffective. Some of other members of this family are: HB⁺ [12], HB⁺⁺ [13], HB* [14], HB-MP [16], HB[#] [21] and NLHB [17]. Attacks

which have targeted these authentication protocols consists both *passive* [20, 18, 19] and *active* types [21, 22]. In an active attack, the adversary is able to eavesdrop the transcripts between a reader and a tag as well as being able to interact with them and manipulate the messages exchanging in between [23] in order to impersonate either of them. It should be noted that active attacks involve a broad spectrum of attacks which differ in adversary's capabilities (e.g. DET [23] and GRS [21] attack models). On the contrary, in a passive attack, the adversary has only access to the transcripts from an arbitrary number of authentication sessions between a tag and a reader and aims at impersonating either of them.

1.3 LPN Problem

If we see from a passive adversary perspective, who has only access to s number of parallelized r -round HB protocol transcripts (i.e. $A_{k \times n}, z_{1 \times n}, \eta$ where $n = s \times r$) and his goal is to recover secret parameter x , it will be obvious that she faces a decoding problem of a codeword ($x \otimes A$) generated by a random linear block code A in presence of noise ν [25]. This problem is called *LPN problem* with parameters k, n, η and has been shown to be NP-hard in worst case [25].

1.4 LPN Solvers

In addition to worst case complexity results of the LPN problem, there are numerous studies on average case complexity [26, 20]. These studies has led to finding some algorithms to solve the LPN problem under certain assumptions (*LPN solvers*). Proposition of these algorithms paved the way for applying passive attack against some of HB family protocols.

In [20], the BKW algorithm has been reported which can be considered as an instance of the generalized birthday paradox [27]. In [18], another algorithm (FMICM) has been proposed inspired by fast correlation attack [24] on ciphers. The solution proposed by the FMICM algorithm is under the assumption of having low bit rate ($\frac{k}{n}$) and high η . Besides some deterministic LPN solvers such as the two aforementioned algorithms, there are some probabilistic algorithms such as CTIN [19] which accomplish their goal even when the adversary has access to less amount of transcripts comparing to deterministic ones.

As said, applying any passive attack on HB protocol requires to utilize an LPN solver algorithm to solve the LPN problem. Thus, the terms *LPN solver* and *passive attack* against HB protocol point to the same notion and are used interchangeably hereafter.

Using LPN solvers caused a dramatic decrease in security margin of some of HB family protocols against passive attacks [18, 19]. As an attempt to search for a variant of the HB, which relies on the complexity of decoding *linear* codes against passive attacks, Madhavan et al. presented Non-Linear HB (NLHB) protocol. In contrast to HB, NLHB relies on the complexity of decoding a class of *non-linear* codes to render the passive attacks proposed against HB ineffective. Based on this fact that there has been no passive solution for the problem of decoding a random non-linear code, the authors have claimed that NLHB's security margin

is very close to its key size.

Our Contribution. In this paper, we present a passive attack on the NLHB protocol. The idea of our attack is the linearization of the non-linear part of the NLHB protocol to convert it to an equivalent of conventional HB protocol. This method has been adopted in order to be able to deploy the passive attack techniques used against HB on NLHB.

Outline. The remainder of this paper is organized as follows. In section 2, we give a brief description of the NLHB protocol and section 3 elaborates on our attack method on it. In section 4, we display the results of applying our attack on NLHB compared to similar attacks on HB and eventually, section 5 concludes the paper.

2 Description of the NLHB Protocol

Figure 2 shows one session of a parallelized r -round version of NLHB protocol. The tag and reader share a k -bit secret x in advance. The reader transmits a random $k \times r$ challenge matrix A to the tag. Having A received, the tag computes $f(x \otimes A)$. Subsequently, it also computes $z = f(x \otimes A) \oplus \nu$, where ν is a noise-vector whose bits are all independently distributed according to the Bernoulli distribution with parameter η , just like the noise vector in the HB protocol. $x \otimes A$ is also an r -bit vector similar to HB, but z differs in size. It is a d -bit vector ($d = r - p$). On receiving z , the reader checks whether $Hwt(z \oplus f(x \otimes A)) \leq \epsilon d$ Where $0 < \epsilon < \eta < 0.5$. If this is true, reader accepts and this means that the tag has been authenticated successfully.

<p>Specifications</p> <ul style="list-style-type: none"> - r, η, p, ϵ: Public parameters - $d = r - p$ - ν : d-bit noise vector where: $Prob(\nu_i = 1) = \eta, i = 1, \dots, d$ <p>NLHB Protocol</p> <ul style="list-style-type: none"> - Secret parameter $x \in \{0, 1\}^k$ is shared between R and T. (1) R : Chooses a random $A_{k \times r}$ matrix. (2) $R \Rightarrow T : A$ (3) T : Computes $z_{1 \times d} = f(x \otimes A) \oplus \nu_{1 \times d}$ (4) $T \Rightarrow R : z$ (5) R : ACCEPTS iff $Hwt(z \oplus f(x \otimes A)) \leq \epsilon d$

Fig. 2. Parallelized version of an r -round NLHB protocol

2.1 Function f

The function f used in the protocol is a non-linear function which maps $\{0, 1\}^r$ to $\{0, 1\}^d$. Specifically, in [17], the function f is defined as following:

$$y = f(x); y \in \{0, 1\}^d, x \in \{0, 1\}^r \quad (1)$$

and

$$y_i = x_i \oplus g(x_{i+1}, \dots, x_{i+p}) \quad (2)$$

where $g : \{0, 1\}^p \rightarrow \{0, 1\}$ is a non linear boolean function.

The authors have also proposed some specific functions for g corresponding to parameter p to achieve maximum entropy and lower the complexity of the protocol (see Table 1). In [17], the authors have shown that for a general function

p	g
2	$x_{i+1}x_{i+2}$
3	$x_{i+1}x_{i+2} \oplus x_{i+1}x_{i+3}$ $x_{i+1}x_{i+3} \oplus x_{i+2}x_{i+3}$ $x_{i+1}x_{i+2} \oplus x_{i+2}x_{i+3} \oplus x_{i+3}x_{i+1}$
4	$x_{i+1}x_{i+4} \oplus x_{i+2}x_{i+3}$ $x_{i+1}x_{i+4} \oplus x_{i+2}x_{i+4} \oplus x_{i+3}x_{i+4}$ $x_{i+1}x_{i+4} \oplus x_{i+2}x_{i+3} \oplus x_{i+3}x_{i+4}$

Table 1. Proposed g function for NLHB protocol

of f , the existing passive attacks on the HB protocol family (discussed in section 1.4) do not work on their protocol.

3 Proposed Attacking Method

3.1 Description

In this section, we present our three-phase passive attack on the NLHB protocol. In this passive attack, we assume that the attacker has access to n rounds of the NLHB protocol where $n = s \times r$ (i.e. s sessions of an r -round protocol) and thus can form matrix A according to (3).

$$A_{k \times n} = (A_{k \times r}^1 || \dots || A_{k \times r}^s) \quad (3)$$

where $A_{k \times r}^i$ is random matrix in i^{th} session.

Exploiting the passive attack techniques proposed for the HB protocol, we require to find an *HB equivalent* of the NLHB protocol. This implies that we should first find a linear approximation of its non-linear part and then update its parameters accordingly. Hence, phase I and II of our attack tackle the former and latter implications and phase III is the utilization of passive attack techniques on the equivalent HB protocol.

Phase I: Linearization

Our objective in this phase is to find a relatively good linear approximation for non-linear part of NLHB to convert the problem of decoding a non-linear random code to LPN problem. To do so, we should find a matrix B such that the probability q in (4) is relatively high.

$$\text{prob}(f(x \otimes A) = (x \otimes A)_{1 \times n} \otimes B_{n \times n^*}) = q; \quad n^* = n - s \times p \quad (4)$$

To construct matrix B , we require to linearize the whole system and according to (2), the non-linear part of the algorithm is the function g which will be our target for linearization hereafter. We can use the Walsh-Hadamard technique [28] to find the best linear approximation for the boolean function g such that:

$$g(x_{i+1}, \dots, x_{i+p}) \approx \sum_{j=i+1}^{i+p} c_j x_j \quad (5)$$

According to table 2, all functions proposed for NLHB can be linearly approximated with a relatively high probability. A linear approximation of all g functions with their probabilities q are shown in table 2.

Having c_j s from linear approximation of g , we can conclude this phase by cal-

p	g	$\approx \mathbf{g}$	q
2	$x_{i+1}x_{i+2}$	x_{i+1}	0.75
		x_{i+2}	0.75
3	$x_{i+1}x_{i+2} \oplus x_{i+1}x_{i+3}$	x_{i+1}	0.75
	$x_{i+1}x_{i+3} \oplus x_{i+2}x_{i+3}$	x_{i+3}	0.75
	$x_{i+1}x_{i+2} \oplus x_{i+2}x_{i+3} \oplus x_{i+3}x_{i+1}$	x_{i+2}	0.75
4	$x_{i+1}x_{i+4} \oplus x_{i+2}x_{i+3}$	x_{i+1}	0.62
	$x_{i+1}x_{i+4} \oplus x_{i+2}x_{i+4} \oplus x_{i+3}x_{i+4}$	x_{i+4}	0.75
	$x_{i+1}x_{i+4} \oplus x_{i+2}x_{i+3} \oplus x_{i+3}x_{i+4}$	$x_{i+2} \oplus x_{i+3} \oplus x_{i+4} \oplus 1$	0.75

Table 2. Best linear approximation of function g in NLHB protocol and their probabilities

culating matrix B . Similar to matrix A in (3), matrix B for s sessions has the following structure:

$$B_{n \times n^*} = (B_{n \times d}^1 || \dots || B_{n \times d}^s) \quad (6)$$

in which,

$$b_{ij}^l = \begin{cases} 1 & \text{if } i = j \\ c_j & \text{for } j = i + 1, \dots, i + p \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where $i = 1, \dots, n$; $j = 1, \dots, n^*$; $l = 1, \dots, s$.

or,

$$B_{n \times d}^l = \begin{pmatrix} 1 & 0 & 0 \\ c_1 & 1 & 0 \\ c_2 & c_1 & 0 \\ \vdots & c_2 & \vdots \\ c_p & \vdots & \dots \\ 0 & c_p & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & c_p \end{pmatrix}; \quad l = 1, \dots, s \quad (8)$$

Phase II: Finding a new linear equivalent protocol

In this phase, we attempt to find an equivalent HB protocol for NLHB using the linear approximation obtained in previous phase. Since our approximation is valid with probability q , we can rewrite (4) as following to formulate the HB equivalent of our NLHB protocol with new parameters denoted by $*$.

$$\begin{aligned} y &= f(xA) = (x \otimes A) \otimes B \oplus e \\ &= (x \otimes A^*) \oplus e \end{aligned} \quad (9)$$

where $A_{k \times n}^* = A_{k \times n} \otimes B_{n \times n}$, $Prob(e_i = 1) = (1 - q)$; $i = 1, \dots, n$
Now, by adding the noise of protocol to both side of (9) we have:

$$y = f(x \otimes A) \oplus \nu = (x \otimes A^*) \oplus \nu \oplus e = (x \otimes A^*) \oplus \nu^* \quad (10)$$

where $\nu^* = \nu \oplus e$.

As ν and e are independent, the probability of error for the new noise vector can be calculated by (11).

$$\begin{aligned} Prob(\nu_i^* = 1) &= \eta^* \\ &= Prob(\nu_i = 1) + Prob(e_i = 1) - Prob(\nu_i = 1) \times Prob(e_i = 1) \\ &= \eta + (1 - q) - (1 - q)\eta. \end{aligned} \quad (11)$$

As it is apparent from (11), the noise of the equivalent HB protocol (ν^*) is more than the noise in NLHB protocol. Therefore, in general, the NLHB protocol is more resistant against the passive attacks comparing to the HB protocol with the same parameters. Nevertheless, according to our results in section 4, this strength is far lower than it has been claimed and desired.

Phase III: Recovering secret parameter x

Up to here, we have accomplished to find an equivalent HB form for the NLHB protocol. From now on, the problem of recovering secret parameter x is an LPN problem with random matrix A^* and parameters k, n^*, η^* (equivalent HB parameters) and therefore can be achieved by using any of LPN solvers discussed in section 1.4.

3.2 Complexity of the Attack

Complexity of our attack consists of three parts corresponding to each phase. For phase I, we need to find the best linear approximation for boolean function g with p variables. This can be done by finding Walsh-Hadamard coefficients of g with complexity of $O(p2^p)$.

In phase II, we just have a matrix multiplication of $A_{k \times n}$ and $B_{n \times n^*}$ to form A^* . This process has the complexity of $O(knn^*)$ in general. But due to sparse form of matrix B in (8), this complexity is reduced to $O(kpn^*)$. Finally, the complexity of phase III relies on the complexity of the LPN algorithm solver used (e.g. table ??). So, the complexity of our attack is calculated by (12) in which the complexity of phases I, II and III are denoted by C_I, C_{II} and C_{III} respectively.

$$\begin{aligned} C &= C_I + C_{II} + C_{III} = O(p2^p) + O(kpn^*) + C_{III} \\ &\approx O(kpn^*) + C_{III}; n^* \gg p \end{aligned} \quad (12)$$

It should be noted that the complexity which computed in (12) is the *time complexity* of our attack. To be more precise, we should calculate the *data complexity* of our attack in terms of the amount of protocol rounds required to apply the attack(n^*) as well. Phase I and II are applied on the number of rounds of the protocol which are determined in phase III and these two phases do not impose any additional data complexity to our attack. Therefore, data complexity of our attack only relies on the data complexity of LPN solvers discussed in [20, 18, 19].

4 Results

In this section, we demonstrate the results of applying our passive attack using three LPN solvers BKW, FMICM and CTIN on NLHB and compare the security margins of NLHB (i.e. its equivalent HB) with HB protocol with the same parameters. Our motivation to do such an unfair comparison is to demonstrate that security margin of the NLHB is not far more than the HB protocol with the same parameters.

Tables 3, 4 and 5 show a comparative time and data complexity of applying passive attacks on NLHB and HB protocol for three different but low noise probability ($\eta = 0.15, 0.10, 0.05$ for HB and correspondent $\eta^* = 0.36, 0.32, 0.29$ in NLHB respectively) as well as the number of rounds of the protocol required to apply the attack (data complexity). As the results show, not only are the passive attacks on HB applicable to NLHB, but also the security margin of NLHB protocol is not far more than HB protocol. It is manifest that the results of our attack using FMICM are remarkably better in comparison with BKW and CTIN. Furthermore, we can have better results when the noise vector in the protocol has a lower weights (table 5).

Key Length	Time Complexity						Data Complexity					
	CTIN		BKW		FMICM		CTIN		BKW		FMICM	
	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB
32	2^{21}	2^{26}	2^3	2^{15}	2^8	2^{12}	2^3	2^{11}	2^3	2^{15}	2^8	2^{12}
64	2^9	2^{36}	2^{31}	2^{40}	2^{19}	2^{22}	2^{13}	2^{14}	2^{31}	2^{40}	2^{19}	2^{22}
128	2^{23}	2^{78}	2^{47}	2^{62}	2^{35}	2^{45}	2^{13}	2^{15}	2^{47}	2^{62}	2^{35}	2^{45}
192	2^{39}	2^{118}	2^{63}	2^{83}	2^{52}	2^{67}	2^{13}	2^{16}	2^{63}	2^{83}	2^{52}	2^{67}
256	2^{56}	2^{162}	2^{76}	2^{99}	2^{71}	2^{88}	2^{14}	2^{16}	2^{76}	2^{99}	2^{71}	2^{88}

Table 3. Time complexity and Data complexity passive attacks on HB and NLHB $\eta = 0.15, \eta^* = 0.36$

Key Length	Time Complexity						Data Complexity					
	CTIN		BKW		FMICM		CTIN		BKW		FMICM	
	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB
32	2^1	2^{13}	2^{20}	2^{25}	2^8	2^{10}	2^{10}	2^{13}	2^{20}	2^{25}	2^8	2^{10}
64	2^4	2^{30}	2^{28}	2^{37}	2^{17}	2^{19}	2^{10}	2^{13}	2^{28}	2^{37}	2^{17}	2^{19}
128	2^{13}	2^{66}	2^{44}	2^{59}	2^{35}	2^{38}	2^{13}	2^{15}	2^{44}	2^{59}	2^{35}	2^{38}
192	2^{24}	2^{102}	2^{57}	2^{78}	2^{54}	2^{63}	2^{13}	2^{15}	2^{57}	2^{78}	2^{54}	2^{63}
256	2^{31}	2^{140}	2^{70}	2^{94}	2^{71}	2^{85}	2^{14}	2^{16}	2^{70}	2^{94}	2^{71}	2^{85}

Table 4. Time complexity and Data complexity passive attacks on HB and NLHB $\eta = 0.1, \eta^* = 0.32$

Key Length	Time Complexity						Data Complexity					
	CTIN		BKW		FMICM		CTIN		BKW		FMICM	
	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB	HB	NLHB
32	2^1	2^{10}	2^{17}	2^{23}	2^6	2^8	2^{11}	2^{13}	2^{17}	2^{23}	2^6	2^8
64	2^2	2^{26}	2^{24}	2^{35}	2^{12}	2^{16}	2^{11}	2^{13}	2^{24}	2^{35}	2^{12}	2^{16}
128	2^5	2^{57}	2^{37}	2^{57}	2^{25}	2^{36}	2^{14}	2^{15}	2^{37}	2^{57}	2^{25}	2^{36}
192	2^9	2^{88}	2^{50}	2^{73}	2^{42}	2^{54}	2^{14}	2^{16}	2^{50}	2^{73}	2^{42}	2^{54}
256	2^{14}	2^{120}	2^{60}	2^{89}	2^{58}	2^{76}	2^{14}	2^{16}	2^{60}	2^{89}	2^{58}	2^{76}

Table 5. Time complexity and Data complexity passive attacks on HB and NLHB $\eta = 0.05, \eta^* = 0.29$

5 Conclusions

We presented a passive attack against NLHB protocol by finding an HB equivalent of it and then using some LPN solver techniques. Our results not only negate the authors claim that their protocol is resistant to passive attacks on the HB protocol but also show that the NLHB has not elevated the security margin of the HB remarkably and this is mainly due to the poor design of the non-linear part of the NLHB.

In summary, what we did is as follows. We:

- targeted Non-Linear HB protocol for passive attack.
- found a linear approximation of the non linear part of the protocol and converted the protocol to an equivalent HB protocol with higher noise.
- applied three well-known LPN solver techniques as a passive attack to the equivalent protocol.
- calculated the complexity of our attack on NLHB.

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