

Improved Linear (hull) Cryptanalysis of Round-reduced Versions of KATAN

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Abstract. KATAN is a family of block ciphers published at CHES 2009. Based on the Mixed-integer linear programming (MILP) technique, we propose the first third-party linear cryptanalysis on KATAN. Furthermore, we evaluate the security of KATAN against the linear attack without ignoring the dependence of the input bits of the 2×1 S-box (the AND operation). Note that in previous analysis, the dependence is not considered, and therefore the previous results are not accurate. Furthermore, the mounted 131/120-round attack on KATAN32/48 respectively by our 84/90-round linear hull is the best single-key known-plaintext attack. In addition, a best 94-round linear hull attack is mounted on KATAN64 by our 76-round linear hull.

Keywords. KATAN, Mixed-integer linear programming, linear hull, dependence

INTRODUCTION

Demands for lightweight ciphers used in resource-constrained devices with low cost are increasing in recent years. Many lightweight block ciphers are published in recent years, such as LBlock [1], PRESENT [2], LED [3], PRIDE [4] and SIMON [5].

Related works

KATAN is a family of lightweight block ciphers published at CHES 2009 [6]. After its publication, KATAN receives extensive cryptanalysis. For instance, the conditional differential cryptanalysis by Knellwolf et al. [7] on 78/70/68-round KATAN32/48/64, differential cryptanalysis by Albrecht et al. [8] on 115-round KATAN32, meet-in-the-middle attack by Isobe et al. [9] on 110/100/94-round KATAN32/48/64, match box meet-in-the-middle cryptanalysis by Fuhr et al. [10] on 153/129/119-round KATAN32/48/64, and dynamic cube attack by Ahmadian et al. [11] on 118-round or 155-round on KATAN32. All results are presented in Table 1.

Linear cryptanalysis is an important cryptanalysis technique on modern block ciphers [12]. It aims at finding a non-random linear expression on bits

of plaintext, ciphertext, and subkey, where the expression has non-zero correlation. The extended linear hull cryptanalysis is presented by Nyberg [13] in 1995. No third-party linear cryptanalysis on KATAN has been proposed. Furthermore, the security evaluation of KATAN with respect to linear cryptanalysis proposed by the designers is not accurate owing to ignoring the dependence of the S-box, where the dependence of S-box means that different S-box share one same input.

Our contribution

In this paper, we first evaluate the linear security cryptanalysis on KATAN32 without ignoring the dependence of the S-box based on the Mixed-integer linear programming(MILP) technique [14, 15]. Furthermore, 84/90/76-round linear hulls on KATAN32/48/64 respectively are proposed. Moreover, 131/120/94-round attack on KATAN32/48/64 are mounted by these linear hulls. A comparison between this paper and other single-key attacks is listed in Table 1. Although, cryptanalysis provided by paper [10] can attack more rounds, their cryptanalysis is based on stricter chosen-plaintext model. As we know, the 131/120-round attacks on KATAN32/48 respectively in this paper are the best single-key known-plaintext attacks, and our 94-round attack on KATAN64 is the first linear attack on KATAN64.

The paper is organized as follows. Section 2 proposes the brief description of KATAN. Section 3 shows the searching method of linear masks. The results about the linear (hull) cryptanalysis are described in Section 4. Section 5 is the conclusion.

Table 1. The analysis results of KATAN based on single-key

Version	Cryptanalysis method	Model	Rounds	Data	Time	Reference
KATAN32	Differential	CP	78	2^{22}	2^{22}	[7]
	Differential	CP	115	2^{32}	2^{79}	[8]
	Match box MITM	CP	153	2^5	$2^{78.5}$	[10]
	Dynamic cube attack	CP	118/155	$2^{19}/2^{32}$	$2^{78.3}/2^{78.3}$	[11]
	MITM	KP	110	138	2^{77}	[9]
	Match box MITM	KP	121	4	$2^{77.5}$	[10]
	Linear hull	KP	131	$2^{28.93}$	$2^{78.93}$	Section 0.7
KATAN48	Differential	CP	70	2^{31}	2^{78}	[7]
	Match box MITM	CP	129	2^5	2^{76}	[10]
	MITM	KP	100	128	2^{78}	[9]
	Match box MITM	KP	110	4	$2^{77.5}$	[10]
	Linear hull	KP	120	$2^{47.22}$	$2^{75.22}$	Section 0.7
KATAN64	Differential	CP	68	2^{32}	2^{78}	[7]
	Match box MITM	CP	119	2^5	$2^{78.5}$	[10]
	MITM	KP	94	116	$2^{77.68}$	[9]
	Match box MITM	KP	102	4	$2^{77.5}$	[10]
	Linear hull	KP	94	2^{57}	2^{78}	Section 0.7

CP: chosen-plaintext attack; KP: known-plaintext attack

BRIEF DESCRIPTION OF KATAN

KATAN is a family of block ciphers with 32, 48, or 64-bit block length, denoted by KATAN32, KATAN48 or KATAN64 respectively. All versions share the same 80-bit master key. For each version, the plaintext is loaded in two registers L_1 and L_2 , where the lengths of L_1 and L_2 for each version are listed in Table 2. In the first place, the round function for KATAN32 is illustrated. For KATAN32, the registers L_1 and L_2 are shifted to the left by 1 position, and two new computed bits by two nonlinear functions $f_a(\cdot)$ and $f_b(\cdot)$ are loaded in the least significant bits of L_1 and L_2 , where the least significant(rightmost) bit for each register will be denoted as 0-th bit. The ciphertext is obtained after 254 rounds. The f_a and f_b are defined as follows

$$\begin{aligned} f_a(L_1) &= L_1[x_1] \oplus L_1[x_2] \oplus (L_1[x_3] \wedge L_1[x_4]) \oplus \\ &\quad (L_1[x_5] \wedge IR) \oplus k_a, \\ f_b(L_2) &= L_2[y_1] \oplus L_2[y_2] \oplus (L_2[y_3] \wedge L_2[y_4]) \oplus \\ &\quad (L_2[y_5] \wedge L_2[y_6]) \oplus k_b, \end{aligned}$$

where IR is round constant, k_a and k_b are two subkey bits. The index x_i and y_i are listed in Table 2.

Table 2. The parameters for KATAN

version	$ L_1 $	$ L_2 $	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	y_4	y_5	y_6
KATAN32	13	19	12	7	8	5	3	18	7	12	10	8	3
KATAN48	19	29	18	12	15	7	6	28	19	21	13	15	6
KATAN64	25	39	24	15	20	11	9	38	25	33	21	14	9

For KATAN48, the shift update of the registers and the nonlinear function f_a , f_b are applied twice with same round subkey in each round, while the nonlinear functions and update of the register are applied three times for KATAN64.

Since we only consider single-key cryptanalysis in this paper, and therefore the key schedule is omitted here. More details on KATAN can be found in paper [6].

THE LINEAR CRYPTANALYSIS OF KATAN

0.1 Notations

$L_1^r[i]$: the i -th bit of the register L_1 in r -th round

$L_2^r[i]$: the i -th bit of the register L_2 in r -th round

k_a^r : the r -th round subkey used in f_a

k_b^r : the r -th round subkey used in f_b

α_t : the masks of the variable t

0.2 Definition of the linear cryptanalysis

Denote f be a boolean function, the correlation ϵ_f of f is defined by

$$Pr(f(x) = 0) - Pr(f(x) = 1).$$

The linear cryptanalysis is evaluated by the correlation.

The *potential* introduced by Nyberg[13] is used to evaluate the linear hull cryptanalysis. Give the input and output masks α and β for a block cipher $C = f(P, K)$, the *potential* $ALH(\alpha, \beta)$ is defined by

$$ALH(\alpha, \beta) = \sum_{\gamma} (Pr(\alpha \cdot P + \beta \cdot C + \gamma \cdot K = 0) - 1/2)^2.$$

0.3 Dependence of S-box

For simplicity, each AND operation \wedge is treated as a 2×1 S-box. The S-box is active if the output mask is non-zero. Since IR is constant, $L_1[x_5] \wedge IR$ is a linear operation in each round, not a S-box. In this paper, the dependence of two S-box illustrates that the two S-box share one input. Owing to the fact that only few bits are registered in each round, S-box for KATAN are dependent.

Usually, the correlation of a linear characteristic for a block cipher is obtained from the correlation of the approximation of round function by pilling-up lemma. Whereas, the pilling-up lemma is not suitable for KATAN due to the dependence of the S-box.

For instance, suppose two approximations of two S-box of L_1 in 0-th round and 4-th round, both with zero input mask and non-zero output mask, are $L_1^0[5] \wedge L_1^0[8]$ and $L_1^4[5] \wedge L_1^4[8]$. Clearly, each approximation has the same correlation (absolute) 2^{-1} . The correlation of XOR-ed function $L_1^0[5] \wedge L_1^0[8] + L_1^4[5] \wedge L_1^4[8]$ of the two approximations is 2^{-2} if applying pilling-up lemma. However, the correlation of $L_1^0[5] \wedge L_1^0[8] + L_1^4[5] \wedge L_1^4[8] = L_1^0[5] \wedge (L_1^0[8] + L_1^4[5])$ is 2^{-1} due to $L_1^4[8] = L_1^0[5]$.

The above example shows that the dependence of the S-box should be taken into consideration when computing the correlation. Consequently, the correlation of the linear characteristic will be computed directly instead of applying the pilling-up lemma in this paper. The computing method in the following is similar to paper[14, 15].

Obviously, the XOR-ed function of all approximations for active S-box is a quadratic function. Denote quadratic boolean function $f(t) = Q(t) + L(t)$, where $t = (t[1], t[2], \dots, t[n]) \in \mathbb{F}_2^n$, $Q(t) = t[i_1] \wedge t[i_2] + t[i_3] \wedge t[i_4] + \dots + t[i_{m-1}] \wedge t[i_m]$ is the sum of quadratic term $t[i] \wedge t[j]$, and $L(t) = t[j_1] + t[j_2] + \dots + t[j_{n-1}] + t[j_n]$ is linear combination of $t[i]$. This kind of function satisfying the property that $i_1, i_2, i_3, i_4, \dots, i_{m-1}, i_m$ are not coincident is called *the standard quadratic function* in the following. Most important, the correlation ϵ_f of the standard function can be obtained directly as follow:

$$- \{j_1, j_2, \dots, j_n\} \subseteq \{i_1, i_2, \dots, i_m\}: \epsilon_f = 2^{-m/2}.$$

– others: $\epsilon_f = 0$.

In other words, if the correlation of the standard function is non-zero, there is a negative correlation between the correlation and the amount of the variables existing in the quadratic terms. Moreover, for any quadratic function, there exists a non-singular transform $s = A \cdot t$ such that $g(s) = f(A^{-1} \cdot s) = Q(s) + L(s)$ is the standard form of f . What is more, the correlation of the standard function g equals to that of f .

For instance, $f(t) = t[1] \wedge t[2] + t[1] \wedge t[3] + t[2] \wedge t[4] + t[2]$. Suppose non-singular transform $s[1] = t[1] + t[4], s[2] = t[2] + t[3], s[3] = t[3], s[4] = t[4]$, therefore the standard form $g(s) = s[1] \wedge s[2] + s[3] \wedge s[4] + s[2] + s[3]$. Hence, the correlation of f is obtained from g by the above method, which is 2^{-2} , due to $\{2, 3\} \subseteq \{1, 2, 3, 4\}$.

In brief, three steps for computing the correlation of a linear characteristic are applied. Firstly, obtain the XOR-ed function of all approximations of each active S-box. Secondly, derive the stand form of the XOR-ed function. Finally, calculate the correlation of the standard form by the above method. The calculating method is also suitable for other ciphers with the similar S-box of KATAN, such as SIMON.

0.4 Automatic enumeration of characteristic with MILP

Similar with paper [14–16], we obtain the linear characteristic by the automatic enumeration with Mixed-integer linear Programming Modelling(MILP). The method denotes each mask bit by a 0-1 variable, then describes the propagation of the masks by linear constraints and optimizes a objective function. Constrains for linear operations are similar to paper [14–16]. Following is the MILP modelling for searching the linear characteristic, where α_t denotes the mask for variable t .

Constraints for linear operations

Constraints for bitwise XOR and branching structure are same with paper [14, 17] in the following.

1. For XOR operation $z = x \oplus y$, their masks satisfy $\alpha_x = \alpha_y = \alpha_z$.
2. For three branching structure $z = x = y$, their masks satisfy

$$\begin{cases} \tau \geq \alpha_x, \tau \geq \alpha_y, \tau \geq \alpha_z, \\ \alpha_x + \alpha_y + \alpha_z \geq 2\tau, \\ \alpha_x + \alpha_y + \alpha_z \leq 2, \end{cases}$$

where τ is the introduced new dummy variable.

Constraints for S-box

For S-box $z = x \wedge y$, their masks satisfy $2\alpha_z \geq \alpha_x + \alpha_y$.

Constraints dealing with dependence of S-box

In order to consider the dependence of the S-box, the $|L_1| + |L_2|$ initial variables of registers and the two new registered variables each round loaded in the

LSB of registers are treated as original variables. In this case, the XOR-ed function of approximations for each active S-box can be expressed as a quadratic function of these original 0-1 variables. Furthermore, there is a negative correlation between the correlation (non-zero) of the standard form and the number of the variables existing in the quadratic terms as shown in Section 0.3. Usually, the more variables exist in the quadratic terms of a boolean function, the more variables exist in that of its standard form. As a consequence, the amount of the variables in the quadratic terms is chosen as the preliminary measure of the correlation. On the other hand, the fact that one original variable exists in the quadratic terms is equivalent to the thing that this variable is the input of active S-box. Accordingly, the amount of all original variables as inputs of active S-box is the our preliminary measure of the correlation.

For each original variable t , denote a new 0-1 variable V_t to indicate whether the variable t is the input of one active S-box, where $V_t = 1$ if it is. In this case, $\sum_{t \in \mathbb{A}} V_t$ is our preliminary measure, where \mathbb{A} denotes the set consists of all original variables.

Furthermore, each variable t may be one input of several S-box (suppose n_t), which means these n_t S-box are dependent as previous shows. For instance, the original variable $L_1^0[5]$ for KATAN32 affects 2 S-box, 0-th and 4-th round S-box of L_1 . What is more, if all the n_t S-box are not active, $V_t = 0$; otherwise $V_t = 1$. This property for each original variable can be described by following constraints:

$$\begin{aligned} n_t \cdot V_t &\geq \beta_1 + \beta_2 + \cdots + \beta_{n_t}, \\ \beta_1 + \beta_2 + \cdots + \beta_{n_t} &\geq V_t, \end{aligned}$$

where $\beta_i, i \in 1, \dots, n_t$, are output masks of the n_t S-box, and also express whether these S-box are active.

Objective function

As previous shows, $\sum_{t \in \mathbb{A}} V_t$ is chosen as our preliminary measure of the correlation. Usually, the more variables exist in the quadratic terms, the smaller the correlation is. Therefore the objective function is to minimize $\sum_{t \in \mathbb{A}} V_t$.

0.5 The computation of *potential*

For each version, we will obtain the linear hull by previous methods. In the first place, obtain a linear characteristic with high correlation by Gurobi software, with MILP modeling presented in above Section 0.4. Secondly, search again to obtain as many as possible suitable characteristics with additional constraints of fixing the input and output masks equaling to that of the obtained linear characteristic with high correlation. Finally, obtain the correlation for each characteristic by the computing method shown in previous Section 0.3, then give the *potential*.

RESULTS

The linear cryptanalysis shown by the designers [6] did not consider the dependence of the S-box. The 126-round linear characteristic eliminated by 42-round linear approximation is not accurate, due to the dependence of the S-box. With taking the dependence of S-box into consideration, we obtain some new results for the security cryptanalysis. Furthermore, some linear hulls with high *potential* are obtained by the MILP modelling shown in Section 0.4. What is more, we mount some best attacks by these linear hulls.

0.6 Results for linear characteristic

For KATAN32, the correlation for the best 42-round linear approximation is 2^{-5} according to the designers [6], with ignoring the effect of the dependence of S-box. In the case of taking the dependence of S-box into consideration, we evaluate the security of the linear cryptanalysis again. The correlation of the best 42-round linear approximation is still 2^{-5} by our method. At the same time, a best 84-round linear characteristic with correlation 2^{-15} is presented in Appendix, while the previous 84-round linear characteristic is directly eliminated as no more than 2^{-5*2} by pilling-up lemma according to the designers. The obtained 84-round linear characteristic demonstrates that KATAN32 is secure against linear cryptanalysis based on one linear characteristic, however we can mount linear hull attack on KATAN in the following.

For KATAN48/64, a best 43/37-round respectively linear characteristic with correlation $2^{-8}/2^{-10}$ is also obtained in this paper under the consideration of the dependence of the S-box, while the previous 43/37-round linear characteristic for KATAN48/64 provided by designers has correlation $2^{-9}/2^{-10}$. Due to the limitation of computing resources, the more accurate security analysis for KATAN48/64 are not obtained. The masks are listed in Appendix.

0.7 Results for linear hull

By setting the input and output masks presented in Table 3, linear hulls for some versions are obtained with some additional constraints owing to the limitation of computing resource. Moreover, some best attacks are mounted by these linear hulls.

For KATAN32, a 84-round linear hull consisting of 98264 linear characteristics with *potential* $2^{-27.93}$ is obtained with additional constraints $\sum_{t \in \mathbb{A}} V_t \leq 44$. In addition, a 131-round attack with 21-round forward and 26-round backward is mounted by this linear hull. In the attack process, 50-bit subkey require to be guessed, while another 10-bit subkey with linear effect to the linear approximation do not. The involved subkey bits in the attack process are listed in Table 4, and the involved bits of the registers are listed in Table 5.

For KATAN48, a 90-round linear hull consisting of 99434 linear characteristics with *potential* $2^{-46.22}$ is obtained with additional constraints $\sum_{t \in \mathbb{A}} V_t \leq 65$. Furthermore, we mount a 120-round attack with 16-round forward and 14-round

Table 3. The input and output masks for linear hull

version	Input masks of register L_1	Input masks of register L_2
KATAN32	1000010001000	0000000000000010000
KATAN48	000000000000000000	0000000000000000100000000100
KATAN64	0100000000100000100000000	0000000000000000000100000000100010000000
version	output masks of register L_1	output masks of register L_2
KATAN32	0010000000000	100000000000000000
KATAN48	1000000001000000001	0000000010000000000000000000
KATAN64	1000001001001000100000000	100010000100000100101000100100000100100

backward by this linear hull. In the attack process, 28-bit subkey require to be guessed, while another 8-bit subkey do not. The involved subkey bits in the attack process are listed in Table 4, and the involved bits of the registers are listed in Table 6.

For KATAN64, a 76-round linear hull consisting of 82908 linear characteristics with *potential* 2^{-57} is obtained with additional constraints $\sum_{t \in \mathbb{A}} V_t \leq 77$. What is more, we mount a 94-round attack with 12-round forward and 6-round backward by this linear hull. In the attack process, 20-bit subkey require to be guessed, while another 13-bit subkey do not. The involved subkey bits in the attack process are listed in Table 4, and the involved bits of the registers are listed in Table 7.

The constraints $\sum_{t \in \mathbb{A}} V_t \leq 44/65/77$ for KATAN32/48/64 respectively is added due to the limitation of computing resource. The data complexity N of the linear hull attack is set by $2 * ALH^{-1}$. Suppose the length of the guessed-key is l_k , thus the time complexity is $N * 2^{l_k}$. The complexity is summarized in Table 1.

Table 4. The involved subkey bits in the attack process

version	round of guessed-key for k_a
KATAN32	13, 9, 8, 6, 4, 3, 2, 1, 0, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130
KATAN48	7, 4, 3, 2, 1, 0, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119,
KATAN64	5, 4, 3, 2, 1, 0, 89, 90, 91, 92, 93
version	round of guessed-key for k_b
KATAN32	10, 7, 5, 4, 3, 2, 1, 0, 111, 113, 115, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130
KATAN48	10, 6, 3, 2, 1, 0, 113, 116, 117, 118
KATAN64	5, 4, 2, 1, 0, 89, 91, 92, 93
version	non-guessed key
KATAN32	$k_a^5, k_a^{16}, k_a^{107}, k_a^{112}, k_a^{117}, k_b^8, k_b^{13}, k_b^{17}, k_b^{105}, k_b^{116}$
KATAN48	$k_a^{10}, k_a^{14}, k_a^{106}, k_b^5, k_b^8, k_b^{110}, k_b^{114}, k_b^{119}$
KATAN64	$k_a^5, k_a^8, k_a^9, k_a^{88}, k_a^{90}, k_a^{91}, k_a^{93}, k_b^2, k_b^4, k_b^9, k_b^{88}, k_b^{89}, k_b^{92}$

CONCLUSION

We first propose a third-party linear cryptanalysis on KATAN in this paper. What is more, we first take the dependence of the S-box into the analysis for KATAN. At the same time, we evaluate the security analysis on KATAN32 in the

Table 5. The involved bits of the registers for KATAN32

Round	bits of register L_1	bits of register L_2
0	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
1	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18
2	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18
3	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18
4	0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18
5	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 18
6	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 16, 17
7	1, 2, 3, 4, 5, 6, 7, 10, 11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 17, 18
8	0, 2, 3, 4, 5, 6, 7, 8, 11, 12	1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 16, 18
9	0, 1, 3, 4, 5, 7, 8, 12	0, 2, 3, 4, 6, 7, 8, 9, 10, 11, 14, 17
10	1, 2, 4, 5, 6, 9	0, 1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15, 18
11	0, 2, 3, 5, 6, 7, 10	16, 1, 2, 4, 5, 6, 8, 10, 12
12	1, 3, 4, 6, 7, 8, 11	17, 2, 3, 5, 6, 7, 9, 11, 13
13	2, 4, 5, 7, 8, 9, 12	18, 3, 4, 6, 7, 8, 10, 12, 14
14	0, 10, 3, 5, 6	0, 4, 5, 7, 9, 15
15	1, 11, 4, 6, 7	16, 1, 5, 6, 8, 10
16	8, 2, 12, 5, 7	17, 2, 6, 7, 9, 11
17	8, 3	0, 18, 3, 7, 8, 10, 12
18	0, 9, 4	1
19	1, 10, 5	2
20	2, 11, 6	3
21	3, 12, 7	4
105	10	18
106	0, 11	8, 9, 11, 4, 13
107	1, 12	9, 10, 12, 5, 14
108	8, 9, 2, 6	0, 6, 10, 11, 13, 15
109	9, 10, 3, 7	16, 1, 7, 11, 12, 14
110	8, 10, 11, 4	17, 2, 8, 12, 13, 15
111	9, 11, 12, 5	16, 18, 3, 9, 13, 14
112	0, 6, 8, 9, 10, 12	0, 4, 8, 9, 10, 11, 13, 14, 15, 17
113	1, 4, 6, 7, 8, 9, 10, 11	0, 1, 5, 9, 10, 11, 12, 14, 15, 16, 18
114	0, 2, 5, 7, 8, 9, 10, 11, 12	1, 2, 4, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17
115	1, 3, 4, 6, 8, 9, 10, 11, 12	0, 2, 3, 5, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18
116	0, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18
117	0, 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18
118	0, 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17
119	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
120	0, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
121	0, 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
122	0, 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
123	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
124	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
125	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
126	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
127	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
128	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
129	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
130	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
131	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Table 6. The involved bits of the registers for KATAN48

Round	bits of register L_1
0	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
1	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18
2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17
3	1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 17, 18
4	0, 1, 3, 5, 6, 9, 11, 12, 14, 17
5	2, 3, 5, 8, 11, 14
6	16, 1, 4, 5, 7, 10, 13
7	0, 1, 3, 6, 7, 9, 12, 15, 18
8	2, 3, 5, 8, 11, 14
9	16, 1, 4, 5, 7, 10, 13
10	18, 3, 6, 7, 9, 12, 15
11	8, 0, 11, 5
12	10, 7, 2, 13
13	4, 15, 12, 9
14	17, 11, 6, 14
15	-
16	-
106	0, 9, 18
107	17, 2, 11, 14, 9
108	16, 8, 11, 4, 13
109	18, 13, 10, 6, 15
110	17, 8, 9, 12, 14, 15
111	1, 7, 8, 10, 11, 13, 14, 16, 17
112	3, 8, 9, 10, 12, 13, 15, 16, 18
113	5, 8, 9, 10, 11, 12, 14, 15, 17, 18
114	0, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17
115	0, 2, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18
116	2, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
117	0, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
118	0, 1, 2, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
119	1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
120	1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
Round	bits of register L_2
0	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 27, 28
1	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 26, 28
2	0, 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 28
3	0, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 24, 27, 28
4	0, 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 20, 23, 24, 26
5	0, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 22, 25, 26, 28
6	2, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 24, 27, 28
7	4, 6, 8, 11, 12, 13, 14, 17, 19, 21, 26
8	1, 6, 8, 10, 13, 14, 15, 16, 19, 21, 23, 28
9	16, 18, 3, 25, 10, 12
10	18, 20, 5, 27, 12, 14
11	1
12	3
13	5
14	7
15	0, 9
16	2, 11
106	20
107	1, 22
108	0, 24, 3
109	2, 26, 5
110	1, 4, 28, 7
111	0, 17, 3, 21, 6, 23, 8, 9, 15
112	0, 2, 5, 8, 10, 11, 17, 19, 23, 25
113	1, 2, 4, 7, 10, 12, 13, 19, 21, 25, 27
114	0, 1, 3, 4, 6, 7, 9, 12, 14, 15, 16, 20, 21, 22, 23, 27
115	0, 2, 3, 5, 6, 7, 8, 9, 11, 14, 16, 17, 18, 20, 22, 23, 24, 25
116	1, 2, 4, 5, 7, 8, 9, 10, 11, 13, 16, 18, 19, 20, 22, 24, 25, 26, 27
117	0, 1, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 24, 26, 27, 28
118	0, 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 26, 28
119	0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28
120	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28

Table 7. The involved bits of the registers L_1 for KATAN64

Round	bits of register L_1
0	0, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 18, 19, 22, 23
1	0, 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 21, 22
2	0, 2, 3, 4, 5, 6, 9, 10, 11, 13, 14, 15, 18, 19, 20, 24
3	0, 1, 3, 5, 6, 7, 8, 9, 12, 13, 14, 16, 17, 18, 21, 22
4	3, 4, 6, 8, 9, 10, 11, 12, 15, 17, 19, 20, 21, 24
5	2, 6, 7, 9, 11, 13, 15, 18, 20, 22, 24
6	1, 18, 5, 9, 10, 14
7	17, 4, 21, 8, 12, 13
8	16, 2, 20, 7, 24, 11, 15
9	10, 19, 5, 14, 23
10	8, 17, 2
11	11, 20, 5
12	8, 14, 23
88	8, 24, 18, 12, 15
89	2, 21, 23, 11, 18, 14, 15
90	0, 1, 5, 13, 14, 17, 18, 21, 22, 24
91	3, 4, 8, 12, 14, 16, 17, 18, 20, 21, 23, 24
92	1, 2, 6, 7, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24
93	0, 1, 4, 5, 9, 10, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24
94	0, 2, 3, 4, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24
Round	bits of register L_2
0	0, 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 35, 36, 37, 38
1	0, 1, 3, 4, 5, 7, 8, 9, 10, 12, 14, 15, 16, 17, 19, 20, 21, 23, 24, 25, 28, 29, 31, 33, 34, 36, 38
2	0, 3, 4, 6, 7, 8, 10, 11, 12, 13, 15, 17, 18, 19, 20, 23, 24, 27, 28, 31, 32, 36, 37
3	2, 3, 6, 7, 9, 11, 13, 14, 15, 18, 35, 20, 21, 22, 26, 30, 31
4	0, 5, 6, 9, 10, 12, 14, 16, 17, 18, 21, 23, 24, 25, 29, 33, 34, 38
5	32, 2, 3, 37, 8, 9, 13, 15, 19, 20, 21, 24, 26, 27
6	0, 2, 35, 5, 6, 11, 12, 16, 18, 22, 24, 29, 30
7	32, 33, 3, 5, 38, 8, 9, 14, 15, 19, 21, 25, 27
8	18, 35, 22, 6, 8, 11, 30
9	33, 2, 38, 9, 11, 14, 21, 25
10	1, 5, 14
11	8, 17, 4
12	11, 20, 7
88	2, 5, 38, 11, 34, 14, 18, 20, 23, 29
89	32, 2, 36, 37, 8, 12, 14, 17, 21, 23, 24, 26, 28, 5
90	1, 34, 35, 5, 8, 10, 11, 15, 16, 17, 20, 22, 23, 24, 26, 27, 29, 31
91	0, 32, 34, 4, 37, 38, 8, 11, 13, 14, 18, 19, 20, 23, 2, 25, 26, 27, 29, 30
92	1, 2, 3, 5, 7, 11, 12, 14, 16, 17, 21, 22, 23, 24, 26, 27, 28, 29, 30, 32, 33, 35, 36, 37
93	0, 1, 2, 4, 5, 6, 8, 10, 11, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38
94	0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38

case of taking the dependence of the S-box into consideration. Furthermore, the 131/120-round attack mounted by our linear hull on KATAN32/48 respectively is the best single-key known-plaintext attack for KATAN.

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APPENDIX

Table 8. The 84-round linear characteristic for KATAN32

Round	input masks of register L_1	input masks of register L_2
0	100010001000	00000000000010000
1	000000000000	000000000000100001
2	000000000000	0000000000001000010
3	000000000000	000000000010000100
4	000000000000	000000000100001000
5	000000000000	000000001000010000
6	000000000000	0000000010000100000
7	000000000000	0000000100001000000
8	000000000000	0000001000010000000
9	000000000000	0000010000100000000
10	000000000000	0000100001000000000
11	000000000000	0001000010000000000
12	000000000000	0010000100000000000
13	000000000000	0100001000000000000
14	000000000000	1000010000000000000
15	000000000001	0000100000100000000
16	000000000010	0001000001000000000
17	000000000100	0010000010000000000
18	0000000001000	0100000100000000000
19	0000000010000	1000001000000000000
20	0000000100001	0000010000100000000
21	0000001000010	0000100001000000000
22	0000010000100	0001000010000000000
23	0000100001000	0010000100000000000
24	0001000010000	0100001000000000000
25	0010000100000	1000010000000000000
26	0100001000001	0000100000100000000
27	1000010000010	0001000001000000000
28	0000000000100	0010000010000000001
29	0000000001000	0100000100000000010
30	0000000010000	10000010000000000100
31	0000000100001	000000000100001000
32	0000001000010	0000000001000010000
33	0000010000100	0000000010000100000
34	0000100001000	0000000100001000000
35	0001000010000	0000001000010000000
36	0010000100000	0000010000100000000
37	0100001000000	0000100001000000000
38	1000010000000	0001000010000000000
39	0000000000000	0010000100000000001
40	0000000000000	0100001000000000010
41	0000000000000	10000100000000000100
42	0000000000001	0000100000100001000
43	0000000000010	0001000001000010000
44	0000000000100	0010000010000100000
45	0000000001000	0100000100001000000
46	0000000010000	1000001000010000000
47	0000000100001	0000000000000000000
48	0000001000010	0000000000000000000
49	0000010000100	0000000000000000000
50	0000100001000	0000000000000000000
51	0001000010000	0000000000000000000
52	0010000100000	0000000000000000000
53	0100001000000	0000000000000000000
54	1000010000000	0000000000000000000
55	0000000000000	0000000000000000001
56	0000000000000	0000000000000000010
57	0000000000000	00000000000000000100
58	0000000000000	00000000000000001000
59	0000000000000	000000000000000010000
60	0000000000000	0000000000000000100000
61	0000000000000	000000000000000001000000
62	0000000000000	00000000000000000010000000
63	0000000000000	000000000000000000000000
64	0000000000000	00000000010000000000
65	0000000000000	00000000100000000000
66	0000000000000	00000001000000000000
67	0000000000000	00000010000000000000
68	0000000000000	00000100000000000000
69	0000000000000	00001000000000000000
70	0000000000000	00010000000000000000
71	0000000000000	00100000000000000000
72	0000000000000	01000000000000000000
73	0000000000000	10000000000000000000
74	0000000000001	0000000001000000000
75	0000000000010	0000000001000000000
76	0000000000100	0000000010000000000
77	0000000001000	0000000100000000000
78	0000000010000	0000001000000000000
79	0000000100000	0000010000000000000
80	0000001000000	0000100000000000000
81	0000010000000	0001000000000000000
82	0000100000000	0010000000000000000
83	0001000000000	0100000000000000000
84	0010000000000	1000000000000000000

Table 9. The 43-round linear characteristic for KATAN48

Round	input masks of register L_1	input masks of register L_2
0	000000001000001001	000001000000001000000000000000
1	000000010000010010	000100000000100000000000000000
2	000001000001001000	010000000100000000000000000000
3	0001000001001000001	000000000000000000000000000000
4	0100000100100000100	000000000000000000000000000000
5	0000000010010010000	0000000000000000000000000000001
6	0000001001001000000	00000000000000000000000000000100
7	0000100100100000000	0000000000000000000000000000010000
8	0010010010000000000	000000000000000000000000000001000000
9	1001001000000000000	0000000000000000000000000100000000
10	0000000000000000000	0000000000000000000100000000010
11	0000000000000000000	0000000000000000010000000001000
12	0000000000000000000	0000000000000001000000000100000
13	0000000000000000000	000000000000100000000010000000
14	0000000000000000000	00000000001000000001000000000
15	0000000000000000000	00000000100000000100000000000
16	0000000000000000000	00000010000000010000000000000
17	0000000000000000000	00001000000001000000000000000
18	0000000000000000000	00100000000100000000000000000
19	0000000000000000000	10000000010000000000000000000
20	0000000000000000010	00000000000000000000000000000
21	000000000000000001000	00000000000000000000000000000
22	0000000000000100000	00000000000000000000000000000
23	0000000000010000000	00000000000000000000000000000
24	0000000001000000000	00000000000000000000000000000
25	0000000100000000000	00000000000000000000000000000
26	0000010000000000000	00000000000000000000000000000
27	0001000000000000000	00000000000000000000000000000
28	0100000000000000000	00000000000000000000000000000
29	0000010000010000000	000000000000000000000000000001
30	0001000001000000000	0000000000000000000000000000100
31	0100000100000000000	000000000000000000000000000010000
32	0000000000000000000	00000000000000000000000001000001
33	0000000000000000000	0000000000000000000000000100000100
34	0000000000000000000	000000000000000000000000010000010000
35	0000000000000000000	0000000000000000000000000100000100000
36	0000000000000000000	0000000000000000000000000100000100000000
37	0000000000000000000	0000000000000100000100000000000
38	0000000000000000000	0000000000100000100000000000000
39	0000000000000000000	0000000010000010000000000000000
40	0000000000000000000	0000001000001000000000000000000
41	0000000000000000000	0000100000100000000000000000000
42	0000000000000000000	0010000010000000000000000000000
43	0000000000000000000	1000001000000000000000000000000