On the 32-Character Zodiac Cipher

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Abstract

A possible new approach to the Zodiac Killer's 32-Character Cipher (Z32) is proposed based on the strengths and weaknesses of previous approaches and novel interpretations. This approach does not assume the use of anagrams or similar complex transposition methods; does not assume the identity of a particular Zodiac suspect; and assumes the use of homophonic substitution (as in Z408 and Z340), and simple transposition (as in Z340). Assumptions are clearly defined and tested with sensitivity tests. With Mount Diablo as the pole of a plane polar coordinate system, the instruction "set to Mag. N." is interpreted by setting the hour and minute hands of a watchface to the magnetic declination of the Bay Area circa 1970. Sensitivity tests reveal the exact year and location have little impact on the declination in this case. The hour and minute given by the hands are interpreted as the radial coordinate r and the angular coordinate θ , as in "Radians & # inches along the radians". The hand corresponding to each coordinate is tested, as are 12- and 24-hour interpretations. Impossible or improbable coordinates are excluded leaving one coordinate as a possible solution. This coordinate is explored as the possible plaintext for Z32 using the Z340 transposition method. Further exploration of the proposed method is necessary.

1. Introduction to The Zodiac Ciphers

In the late 1960s, the self-titled 'Zodiac' killed at least five people in the Bay Area of California, and mailed four ciphers to local newspapers (Federal Bureau of Investigation, 2007).

The first ciper, commonly referred to as 'Z408' because of its 408 character length, was delivered in three parts to various newspapers on July 31, 1969. The full ciphertext consists of 24 rows of 17 symbols, with a ciphertext alphabet of 54 distinct symbols. The first 390 characters of Z408 were deciphered on August 8, 1969 by private citizens Donald Gene and Bettye June Harden. The Harden's solution revealed Z408 as a uniliteral substitution cipher with variants, or homophonic substitution cipher. The solution was submitted to the Skaggs Island U.S. Navy Cyptographic Unit (Federal Bureau of Investigation, 1969, p.37) and was soon validated by Vallejo PD (San Francisco Sunday Examiner and Chronicle, 1969) and the Federal Bureau of Investigation (Napa County Sheriff's Department, 1969). The final 18 characters of Z408 remain poorly understood, as the Harden's solution applied to these characters does not produce meaningful plaintext. Unconvincing anagrammatic solutions have been proposed, though it is not clear whether these 18 characters constitute a cipher, or whether they were added as nulls to make the shape of the third part of the cipher consistent with the other two parts, thereby obscuring their order.

The second cipher, similarly referred to as 'Z340', was delivered as a single ciphertext to the San Francisco Chronicle on November 8, 1969. The ciphertext consists of 20 rows of 17 symbols, with a ciphertext alphabet of 64 sym-

bols. This makes Z340 not only shorter than Z408, but more suppressed in its plaintext character frequencies. These factors contributed to its long standing as an unsolved cipher. The cipher was solved in its entirety on December 5, 2020 by cryptologists Sam Blake, Jarl Van Eycke, and David Oranchak (Oranchak, 2020; Blake, 2021). The Blake, Van Eycke, & Oranchak solution revealed Z340 as a multi-layered cipher, with a homophonic substitution layer not unlike Z408, and a transposition layer which divides the cipher into three parts. The solution was validated by the Federal Bureau of Investigation on December 12, 2020 (FBI San Fransisco, 2020).

The third cipher, 'Z13', was delivered to the San Francisco Chronicle on April 20, 1970. The ciphertext consists of a single row of 13 characters, with a ciphertext alphabet of eight characters. Due to its length and lack of any accompanying instructions, no convincing solutions have been proposed for this cipher. The cipher keys of Z408 and Z340 applied to Z13 do not produce meaningful plaintext.

The fourth and final cipher, 'Z32', was delivered to the San Francisco Chronicle on June 26, 1970. The ciphertext consists of two unequal rows of 17 and 15 characters respectively, with a ciphertext alphabet of 29 characters. Although short like Z13, Z32 was accompanied by a Phillips 66® road map of the Bay Area on which Mount Diablo is highlighted with Zodiac's self-branded crosshairs symbol in blue ink. Text alongside the crosshairs reads "is to be set to Mag. N." Text within the June 26 letter refers to an alleged improvised explosive device to be activated by a passing school bus. The June 26 letter states "The map coupled with this code will tell you where the bomb

is set", which may imply that a solution to the map puzzle provides insight into Z32. Because Z13 lacks such a companion puzzle, Z32 is perhaps the next logical Zodiac cipher to approach. A final hint towards the solution to Z32 and the map puzzle comes from a Zodiac letter delivered to the San Francisco Chronicle on July 26, 1970 which states "P.S. The Mt. Diablo Code concerns Radians & # inches along the radians".

2. Previously Proposed Solutions to Z32

A number of solutions to Z32 have been proposed:

2.1. Gareth Penn and 'Radian Theory'

In November 1981, Gareth Penn published an article titled 'Portrait of the Artist as a Mass Murderer' in California Magazine (Penn, 1981). In this article, Penn proposed a series of links between the June 26, 1970 letter and the locations of Zodiac crime scenes. Penn's method interprets the numbers 0, 3, 6, and 9 around Mount Diablo on the Phillips 66[®] road map as representing a watchface, which is a reasonable assumption and may be supported by the fact that Zodiac Watches[®] are a brand of wrist watch which similarly feature a crosshairs symbol.

The next steps taken by Penn are more nebulous. Rather than centering the watchface on Mount Diablo, Penn places Mount Diablo and Lake Berryessa (the location of Zodiac's September 27, 1969 attack on Cecelia Shepard and Bryan Hartnell) on the circumference of a circle such that they correspond to the times on a watchface 7:04 and 12:00 respectively. Blue Rock Springs Park (the location of Zodiac's July 4, 1969 attack on Darlene Ferrin and Mike Mageau) lies within this circle. Penn alleged that the radial line drawn from the center of this circle through Blue Rock Springs Park meets the circumference at 09:00, and that the angle between this line, and the radial line which meets the circumference at Mount Diablo, is equal to 58°, which is almost one radian (1 rad $\approx 57.29578^{\circ}$). However, commenters have noted that Penn's grid positioning was imperfect, and that the real angle is larger than one radian (Butterfield, 2007).

Penn's hypothesis does not have predictive power, as further Zodiac crime scenes do not follow a pattern of one radian separation. Furthermore, this method does not provide details as to the encryption method or plaintext of Z32.

2.2. Henrik Sundberg and David Thelin's Solution

In January 2014, engineers Henrik Sundberg and David Thelin proposed their solution to Z32 (Sundberg and Thelin, 2014). Like Penn, Sundberg and Thelin interpret the numbers 0, 3, 6, and 9 around the Mount Diablo crosshairs as the numbers around a watchface. Using Zodiac's "Mag. N." instruction, they suggest that the 24 hour bezel featured on some watches must be rotated clockwise by the magnetic declination (that is the angle between magnetic

north and geographic north), which is taken as 15°. Next, Sundberg and Thelin guess (based on which letters in the English language look closest to the ciphertext symbols) that Z32 contains the chemical equation $C_3H_3 + OCTANE$ (C₈H₁₈). Adding up the number of atoms of each element gives 11 carbon atoms and 21 hydrogen atoms. Because the 12 hour time 11:00 falls below the 24 hour time 21:00 on the rotated bezel, and because there are 11 carbon atoms and 21 hydrogen atoms in $C_3H_3 + C_8H_{18}$, Sundberg and Thelin take this as proof that the radial line from Mount Diablo through 11:00 gives the direction of the bomb location. Sundberg and Thelin also believe that Z32 contains the chemical HClO₃ as well as 'E' for East and 'W' for West. Again by adding up numbers of atoms and subtracting one from the other, Sundberg and Thelin suggest that the number of inches required is one inch. But rather than taking this distance along the 11:00 direction, they take this to mean one inch West by arguing the use of vectors. Finally, they propose that the location of the bomb is at the intersection between the radial 11:00 line and the one-inch West line.

Sundberg and Thelin add additional steps to reach their solution for which there is no precedent in previous Zodiac ciphers. Indeed, chemical equations did not appear in Z408 and Z340. In fairness, neither did map puzzles. But Sundberg and Thelin fundamentally interpret Z32 not as a true homophonic substitution cipher but as a kind of shorthand notation or half-cipher half-plaintext which is not consistent with the two solved Zodiac ciphers. Also, somewhat arbitrary choices are made, such as the decision to add the number of atoms rather than subtracting them, multiplying them, or performing any other operation.

2.3. Richard Grinell's Solutions

In September 2018, Richard Grinell published an article titled 'The 32 Symbol Code- Hidden in Plain Sight' on his website zodiacciphers.com (Grinell, 2018). In this article, Grinell suggests that because the Zodiac's bomb threats were mailed to the San Francisco Chronicle, the bomb location must lay within San Francisco proper, and not the surrounding areas. Based on Zodiac's message "Radians & # inches along the radians", Grinell seeks an angle ('Radians') and distance ('inches') from Mount Diablo on the map corresponding to San Francisco.

Concerning the angle, Grinell shows that San Francisco lies at an approximate clockwise angle of 4 rad from the vertical axis of the Mount Diablo crosshairs. Like Sundberg and Thelin, Grinell interprets the Zodiac's instruction "is to be set to Mag. N." by taking the magnetic declination at Mount Diablo in 1970 (as 17° compared to Sundberg and Thelin's 15°), and simply adding this to 4 rad ($\approx 229.2^{\circ}$) to give an angle of 246.2°. Concerning the distance, Grinell similarly shows that San Francisco is approximately five inches across the map in a straight line from Mount Diablo.

With this, Grinell suggests the plaintext of Z32 reads 'RADIANS AND 5 INCHES ALONG THE RADIANS'.

A simple logic check can be made by determining whether the recurring characters in Z32 encipher the same plaintext character each time. Z32 features three recurring characters: 'C' (in the 1st and 26th positions), ' \triangle ' (in the 2nd and 32nd positions), and 'O' (in the 6th and 14th positions). In Grinell's 2018 solution, 'C' appropriately enciphers the plaintext 'R' in both occurrences, but ' \triangle ' enciphers the plaintext 'A' in the first occurrence and 'S' in the second.

In June 2020, Grinell published a follow-up article which revised this solution to 'ESTIMATE FOUR RADIANS AND FIVE INCHES', in which 'C' enciphers 'E', '△' enciphers 'S', and 'O' enciphers 'A' in all occurrences (Grinell, 2020).

Circumstantial evidence is provided to support these conclusions. Grinell argues that the angle 246.2° is also featured in a Zodiac doodle from the July 26, 1970 letter. In this doodle, the Zodiac crosshairs symbol is drawn within which the text "SFPD=0" is written. Next to the symbol, the text "=13" is also written. Zodiac implies that the San Francisco Police Department (SFPD) have a score of zero (since the Zodiac remains uncaught), and that Zodiac themself has a score of 13 (perhaps alluding to 13 victims, though only five are confirmed). The zero in this doodle is particularly bold, as if Zodiac went over it many times with a pen. Grinell suggests that this zero is actually a marker, and that the clockwise angle between the vertical axis of the crosshairs doodle and the zero marker also equals 246.2°. Grinell also identifies the Ingleside Police Station as being close to the (5 inch, 4 rad) coordinate, and that the road nearby this police station very crudely resembles a Zodiac sketch of the bomb plot.

Uncertainties remain with Grinell's solution. First, the San Francisco Chronicle was available outside of San Francisco in the wider Bay Area, therefore the idea that mailing bomb threats to this newspaper precludes the location being anywhere other than San Francisco is unfounded. Indeed, other Zodiac attacks lay outside of the city. The Chronicle may have been selected by Zodiac for its large circulation, and because this paper had published the Zodiac's letters the previous year. This makes the 246.2° angle highly uncertain.

Although Grinell argues that the same angle appears in the July 26, 1970 letter, this similarity is very approximate as the doodle is rough. Second, the choice of reference direction, as well as the reference axis, from which the 4 rad is taken are selected arbitrarily. It is equally likely that the Zodiac intended for the angle to be taken from the horizontal axis of the Mount Diablo crosshairs and not the vertical axis, or that the Zodiac intended the angle to be taken anticlockwise (which is more common in plane polar coordinate systems) rather than clockwise. Grinell argues that the numbers 0, 3, 6, and 9 around the Mount Diablo crosshairs suggest a clockwise direction, but other meanings for these numbers are possible. Finally, there are hundreds of police stations, schools, newspaper offices, and other relevant locations within San Francisco and the Bay

Area which Zodiac could have equally targeted. Virtually any spot on the Phillips 66[®] road map is in the vicinity of one or more of these locations. Therefore, Grinell's coordinate being close to one such location is not convincing evidence.

2.4. Faycal Ziraoui's Solution

In January 2021, Faycal Ziraoui published a forum post titled 'Detailed solution to Z13 and Z32' to zodiackiller.net (Ziraoui, 2021). In this article, Ziraoui proposes solutions to both Z13 and Z32 using somewhat similar methodology, though here only the Z32 solution is considered.

Ziraoui applies the Blake, Van Eycke, & Oranchak Z340 key to Z32, which does not immediately provide meaningful plaintext and represents only a partial key since Z32 features two cipher symbols (each appearing once) not present in Z340. These two undeciphered characters are disregarded from the remaining analyses in Ziraoui's solution. Like Grinell, Ziraoui next seeks to find two numbers within Z32 to act as coordinates, but Ziraoui interprets the Zodiac's 'Mag. N.' instruction as referring to geomagnetic coordinates instead of plane polar coordinates.

After applying the Z340 key, Ziraoui interprets the resulting text as a sort of A1Z26 cipher in reverse. That is, Ziraoui assumes each letter represents a number such that A=1, B=2, ..., Z=26. Applying this second key, and dropping the digit in the tens column (e.g., Z=26 becomes Z=6), leaves a string of numbers 30 characters in length. Ziraoui decides that the 21st to 30th characters in this string, '4560958719' represent the geomagnetic coordinates required (i.e., 45.609°N, 58.719°W). As circumstantial evidence, Ziraoui points out that these coordinates lie close to both South Tahoe Middle School, a possible target for the Zodiac's school bus bomb, and the home of Lawrence Kane, a notable Zodiac Killer suspect.

Next, the remaining numbers (besides the geomagnetic coordinate numbers) are replaced back again with their original Z340 key decipherments (e.g., 6=Z again). Ziraoui notes that the words 'FIND', 'PABOR', 'DAY', 'NORT' and 'WEST' appear anagrammatized among these letters (the exact transposition method Ziraoui used to arrive at these is not made clear). Finally, by considering 'NORT' as a misspelling of 'NORTH' and 'PABOR' as a misspelling of 'LABOR', Ziraoui proposes that the Z32 plaintext reads 'LABOR DAY FIND 45.609 NORTH 58.719 WEST'.

Despite attracting widespread media attention [including articles in the San Francisco Chronicle (2021) and New York Times (2021)], there are uncertainties surrounding this solution. For one, Z408 and Z340 represented homophonic substitution ciphers, with an additional transposition layer in Z340. Like Sundberg and Thelin's solution, Ziraoui's solution goes beyond homophonic substitution and transposition by adding an additional cipher layer (A1Z26 variant). Although it is not impossible that the Zodiac added an additional layer of cryptographic security to their cipher, there is no precedent for this step in their

previous works. Additionally, Z340 remained unsolved at the time that Zodiac published Z32. Zodiac taunted this fact in their April 20, 1970 letter: 'By the way have you cracked the last cipher I sent you?' Thus, there would have been no need for Zodiac to alter the previous encipherment method at this time. It is reasonable to assume that Z32 features homophonic substitution, possibly with transposition, but not necessarily additional steps.

Furthermore, some of Ziraoui's steps seem arbitrary. For example, the A1Z26 variant (swapping letters for numbers and dropping the digit in the tens column), could easily be replaced for example by reversing the number assignment (A=26, B=25, ..., Z=1), or by adding together the digits until one remains instead of dropping the tens. The possibilities for manipulating numbers are countless. Another questionable decision is to designate the 21st to 30th characters as the geomagnetic coordinates, when other characters could just as easily represent coordinates. The transposition method is also unclear. In Z340, the transposition method was relatively simple and systematic; read across the characters following the moves of the knight piece in chess, and reverse some portions of the text. By comparison, Ziraoui's transposition step which is described as 'search[ing] for a solution of a Rubik's Cube by brute force', is far more complicated and, without further details, appears somewhat forced.

In addition, the two Z32 symbols not included in the Z340 key are simply ignored in the Ziraoui solution. It is true that in Z408, the final 18 characters are probably null characters, which sets a precedent for nulls in Zodiac's other ciphers. However, these appeared at the end of Z408, whereas the two 'nulls' in Ziraoui's solution appear in the middle.

Finally, Ziraoui's solution makes little use of the Phillips 66^{\circledR} road map Zodiac provided. By opting for geomagnetic coordinates as opposed to plane polar coordinates, Ziraoui does not incorporate 'Radians' and 'inches' into the solution, nor the numbers 0, 3, 6, and 9 and Mount Diablo crosshairs. While these all could be red herrings, it's not clear why Zodiac would provide these, especially because the "Radians & # inches along the radians" message was provided after the Z32 ciphertext, as if Zodiac was disappointed the cipher had not been solved. Other criticisms have been published by Oranchak (2021a).

2.5. The Case Breakers' Solution

In October 2021, a group of former FBI agents, private investigators, and consultants known as 'The Case Breakers' claimed to have solved Z13 and Z32, re-solved Z340 (implying that a second message was hidden within this cipher beyond the accepted solution), solved other secret messages allegedly included in Zodiac's letters, and discovered the identity of the Zodiac (The Case Breakers, 2021).

Leaving these other claims aside, Jennifer Bucholtz of the Case Breakers described the decryption methods as follows: "All you do is take any of the taunting letters (and accompany envelope) he sent to police, remove the letters of his full name, and re-arrange the remaining letters/numbers into the "real" message" (Bucholtz, 2021).

While vague, this description is enough to raise concerns because an gram-based solutions are notoriously able to produce whichever plaintext the cryptologist wishes to uncover. This makes verification next to impossible; astronomically many solutions may all be derived from this method with no reason to prefer any one solution besides personal preference, e.g., when the solution appears to confirm the identity of a suspect the investigator preferred a priori.

3. A New Approach to Z32

The previous solution attempts described in Section 2 have strengths and weaknesses. Arguable strengths of these solutions include use of Zodiac's hints (the map, the watchface numbers, "Mag. N.", and "Radians" and "inches"), as well as cryptographic techniques previously used by Zodiac; homophonic substitution and simple transposition. Arguable weaknesses of these solutions are techniques for which there is no precedent in Z408 and Z340, e.g., anagrams or complex transposition, references to chemistry or suspect names, and untested assumptions.

The contribution of the present study is a new approach toward Z32 based on the above analysis of previous solutions. This new method is predicated on the following assumptions:

- 1. "Radians & # inches along the radians" in the July 26, 1970 letter refers to the use of a plane polar coordinate system with "# inches" being the radial coordinate r > 0, and "Radians" being the the angular coordinate $\theta \in [0, 2\pi)$.
- 2. The Phillips 66^{\circledR} road map represents a plane polar coordinate system with Mount Diablo as the pole. Two numbers specifying the coordinate (r, θ) are required to identify a location of interest on the map using this system.
- 3. The numbers 0, 3, 6, and 9 around Mount Diablo refer to hours on a watchface, and "is to be set to Mag. N." instructs the reader to set the hands of a watchface (centered on Mount Diablo) to the direction of magnetic north.
- 4. By setting the hands to magnetic north, the 'time' on the watchface provides two numbers (the hour and minute) which correspond to r and θ .
- 5. The 32 character cipher may then represent an enciphered variation of the coordinates (r, θ) , or perhaps an enciphered variation of the name of the location on the map which (r, θ) identifies. Z32 may therefore not be intended as a standalone cipher, but a check for solutions to the map puzzle.

Assumptions 1 and 2 are perhaps the least speculative; they represent the most literal interpretation of Zodiac's July 26, 1970 letter. Indeed, Grinell and others have previously proposed the plane polar coordinate interpretation.

Assumption 3 is more contentious. While the solutions of Penn, as well as Sundberg and Thelin, made use of the 0, 3, 6, and 9 numbers, other solutions disregarded this feature of the map. Crucially, Penn did not center the watch on Mount Diablo, and Sundberg and Thelin referred to the bezel of a watch rather than the hands. Because the numbers are written around Mount Diablo, and because not all watches or clocks feature bezels, Assumption 3 is perhaps less speculative than those made by others. An argument for the hands interpretation of this study is that it readily provides a means of identifying two numbers which are further assumed to represent (r, θ) , whereas other interpretations do not. The Zodiac's use of the word "set" in the context of the numbers of a watchface perhaps makes more sense under this assumption: 'To set a watch' is a common phrase in the English language.

Assumptions 4 and 5 are somewhat natural conclusions if Assumption 3 is true. Indeed, other purposes for setting the hands of a watchface to magnetic north besides providing numbers for coordinates aren't clear. Assumption 5 in particular may explain why Z32 is so short. Based on Zodiac's previous ciphers, Z408 and Z340, it is clear that Zodiac had acquired at least an amateur understanding of classical cryptography, having used homophonic substitution and transposition methods which may be learned from a textbook. Most cryptography textbooks (and classes) teach the simple concept that a shorter cipher is generally more difficult to solve than a longer cipher with the same encryption method. Zodiac then may well have been aware that Z13 and Z32 were too short to solve as standalone ciphers (hence Zodiac's confidence to possibly have enciphered their name within Z13, though this is by no means certain). Zodiac could therefore have intended Z32 to be a companion puzzle, rather than a cipher to be solved unaided.

A further complication is that even if all five assumptions are correct, they may not lead to a singular solution. Some of these assumptions have multiple possible approaches:

(a) Assumption 3 requires a value for the magnetic declination, but this varies across the surface of the Earth, and also across time (Livermore et al., 2020). It is possible to calculate the magnetic declination at Mount Diablo on June 26, 1970 (the date of the Z32 letter), but different sources provide different latitude/longitude for Mount Diablo, and Zodiac may have used outdated declinations (or even declinations corresponding to different locations besides Mount Diablo). Indeed, the solutions given by Grinell and by Sundberg and Thelin used different magnetic declinations. Thus, a range of likely values for the magnetic declination must be explored.

- (b) Assumption 4 takes the hour and minute given by the arms of a watchface centered on Mount Diablo set to magnetic north. However, it is not clear whether a 12 hour or 24 hour interpretation of the time should be taken (if the hour hand is at 1, this may represent either 01 or 13). Both interpretations must be explored.
- (c) Assumption 4 maps the minute and hour to polar coordinates, but it is not clear whether the minute corresponds to r and the hour to θ or vice versa. Thus, both must be tried.
- (d) To identify a location on the map using the coordinates (r, θ), a reference direction must be taken for the angle θ. Typically, the reference direction is taken to be anticlockwise from the polar axis (or positive x-axis of Cartesian coordinates centered on the pole), but other reference directions and axes are possible.
- (e) Assumption 5 interprets the plaintext of Z32 to contain either (r, θ) or the name of the location corresponding to this coordinate on the map. Of course, many possible plaintexts may be constructed which encipher any given location or coordinate. Thus, care must be taken not to 'force' a given 'solution' strongly, e.g. by using anagrams.

Uncertainties (a)-(e) require combinations of possible approaches to be taken, which may provide multiple solutions to the cipher. Nonsensical options such as coordinates which lay outside of the map boundaries and coordinates which do not satisfy the conditions r > 0 and $\theta \in [0, 2\pi)$ may be ruled out to narrow the possibilities. The three Z32 ciphertext characters which appear more than once in the cipher, 'C', ' \triangle ', and 'O', also provide some weak restrictions on solutions. Some of the above possibilities may have little impact on the solution. For example, different possible magnetic declinations may only vary the watchface hands by a minute or so. Other options may lead to very disparate solutions (or branches of solutions), for example the 12 or 24 hour interpretation. By exploring a range of reasonable possibilities, the sensitivity of Assumptions 1-5 are tested. Sensitivity tests were not performed in any of the previously discussed solutions.

To summarise these approaches, Table 1 compares the features of previous solutions to the present study.

Table 1: Potential Strengths and Weaknesses of Z32 Solutions

	Solution	Potential Strengths							Potential Weaknesses			
,	Solution	Uses Homophonic Substitution With or Without Simple Transp.	Uses Map	Uses Watchface Numbers	Uses "Mag. N."	Uses "Radians" and "inches"	Sensitivity Tests	Uses Anagrams or Complex Transp.	Uses Extra Encryption Steps E.g. Chemistry, A1Z26, Character Removal Rules	Uses A Priori Suspect	Assumes Particular Location	
	Penn	No	Yes	Yes	No	Yes	No	No	No	No	No	
	Sundberg & Thelin	No	Yes	Yes	Yes	Yes	No	No	Yes	No	No	
	Grinell	Yes	Yes	Yes	Yes	Yes	No	No	No	No	Yes	
ĺ	Ziraoui	Yes	Yes	No	Yes	No	No	Yes	Yes	No	No	
ĺ	The Case Breakers	Unclear	Unclear	Unclear	Unclear	Unclear	Unclear	Yes	Yes	Yes	Unclear	
	Foxon (This Study)	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	

4. Results

4.1. Magnetic Declination

First, a value for the magnetic declination was obtained. This was calculated with the National Oceanic and Atmospheric Administration's (NOAA) magnetic declination calculator (NOAA, 2022a) with the International Association of Geomagnetism and Aeronomy's 13th Generation International Geomagnetic Reference Field model, which provides magnetic declination estimates from the year 1590 to 2024 (International Association of Geomagnetism and Aeronomy, 2019). This calculator requires the geographic coordinates (obtained from NOAA (2022b-e)) of the reference point, as well as a date in time.

For the main result, Mount Diablo is taken as the reference point and the date of the Z32 letter as the time.

Three sensitivity tests were performed. First, the coordinates of the reference point depend on the geodetic datum. NOAA provide the geographic coordinates for Mount Diablo for numerous geodetic datum. Datum such as the North American Datum of 1983 (NAD 83) were not available at the time the letter was written (1970), therefore only the US Standard Datum (USSD; 1901) and North American Datum of 1927 (NAD 27) were considered. Second, three other locations on the Phillips 66[®] road map of the Bay Area were tested: San Francisco, Vallejo, and San Jose. These approximately represent the northern-most, southern-most, eastern-most, and western-most locations on the road map, allowing for possible variation in declination across the map. Third, declinations were estimated both for June 26, 1970 (the Z32 letter date) and for January 1, 1950 (selected somewhat arbitrarily) to assess the impact of time on the declination (Zodiac may have used an outdated declination estimate).

The declination results are shown in Table 2. Across the locations, datum, and dates tested, the declination only varied from 16.3° to 17.7°. Thus, assuming Zodiac used an estimate for the magnetic declination somewhere in the bay area with some pre-1970 datum for some time between 1950 and 1970, the estimate was likely between 16° and 18°. This addresses uncertainty (a) in Section 3.

4.2. Setting the Hands to Magnetic North

If the degrees minutes declinations are interpreted with no further steps as the plane polar coordinates, e.g., 16° 53' E as (16,53) or perhaps (53,16), this is problematic because although these satisfy the convention r > 0, they do not satisfy the convention $\theta \in [0,2\pi)$. This also ignores the numbers 0,3,6, and 9 around Mount Diablo. Thus, the decimal degrees declinations are found and used to set the hands of the watchface to magnetic north.

One hour corresponds to a clockwise rotation of $\frac{360^{\circ}}{12} = 30^{\circ}$, and one minute corresponds to a clockwise rotation of $\frac{360^{\circ}}{60} = 6^{\circ}$. Thus, the hour $= \lfloor \frac{\text{declination}}{30^{\circ}} \rfloor$ and the minute $= \lfloor \frac{\text{declination}}{6^{\circ}} \rceil$.

Assuming the hands are to be rotated clockwise from the vertical position by the magnetic declination, both the lower and upper bounds for the magnetic declination (16° and 18°) give a time of 12:03 or 00:03 (uncertainty (b) in Section 3).

These times each provide two numbers which may be interpreted as the plane polar coordinates (0,3), (3,0), (12,3), or (3,12) [uncertainty (c) in Section 3].

The coordinates (0,3) do not satisfy the convention r > 0, and the coordinates (3,12) do not satisfy the convention $\theta \in [0,2\pi)$.

Table 2: Magnetic Declination Results for the Bay Area Circa 1970

Location	Datum	Coordinates	Date	Magnetic Declination			
Location	Datum	Coordinates	Date	Degrees Minutes	Decimal Degrees		
	USSD	37° 52' 55.48200" N	January 1, 1950	17° 34' E	17.6°		
Mount Diablo	USSD	121° 54′ 48.35500″ W	June 26, 1970	16° 53' E	16.9°		
Mount Diablo	NAD 27	37° 52' 54.38700" N	January 1, 1950	17° 34' E	17.6°		
		121° 54′ 47.10700″ W	June 26, 1970	16° 53' E	16.9°		
	USSD	N/A	N/A	N/A	N/A		
Hunter West 1,	USSD	IV/A	N/A	N/A	N/A		
San Francisco	NAD 27	37° 43′ 42.17100″ N	January 1, 1950	17° 34' E	17.6°		
		122° 21' 39.78600" W	June 26, 1970	16° 53' E	16.9°		
	USSD	N/A	N/A	N/A	N/A		
Long Pond 2,	USSD	IV/A	N/A	N/A	N/A		
Vallejo	NAD 27	38° 07' 38.41026" N	January 1, 1950	17° 44' E	17.7°		
		122° 19' 29.07967" W	June 26, 1970	17° 20' E	17.3°		
	USSD	N/A	N/A	N/A	N/A		
American Reset,	USSD	IV/A	N/A	N/A	N/A		
San Jose	NAD 27	37° 17' 15.84200" N	January 1, 1950	17° 18' E	17.3°		
		121° 51' 53.42800" W	June 26, 1970	16° 19' E	16.3°		

The point on the map corresponding to the coordinates (3,0) taken from the pole at Mount Diablo where $\theta=0$ rad and r=3 inches lays approximately on Morgan Territory Road, which is semirural and falls so close to the Zodiac crosshairs on the map that it seems unlikely that this is the intended location. Finally, this leaves the coordinates (12,3). It is probably just coincidence that the number 12 also appears in the June 26, 1970 letter: "Zodiac-12 SFPD-0".

4.3. Z32 Plaintext and Transposition

If Z32 encrypts the plane polar coordinates (12, 3) identified above, then possible solutions for the plaintext of Z32 include 'THREE RADIANS AND TWELVE INCHES ALONG' and 'TWELVE INCHES ALONG THE THREE RADIANS'. These are each 32 characters long, and are consistent with the phrasing of Zodiac's July 26, 1970 letter: "Radians & # inches along the radians". However, these solutions are not immediately consistent with the positioning of the recurring characters in the original Z32 ciphertext. Therefore, if one of these is correct, then it would require a transposition layer in Z32 as in Z340. This possibility is explored in the following, though it should be noted that the author does not suggest that this is necessarily the true solution to Z32.

Based on the relatively simple and systematic transposition rules of Z340, it is unlikely that Zodiac used complex transposition or anagrams, as some have postulated. Two transposition rules were used in Z340: 1) read across the characters following the moves of the knight piece in chess; and 2) reverse some portions of the text.

Oranchak (2021b) discusses one way to replicate the transposition method Zodiac used in Z340. The first part of the Z340 plaintext can be rewritten into a grid with nine rows and 17 columns. Next, each character (from left to right, top to bottom) from the first grid is taken sequentially and placed into a second nine by 17 grid, populating the columns first. The final positions are obtained by shifting the second row to the right by two positions, the third row by four positions, the fourth row by six positions, etcetera (characters shifted out of the grid loop back on the opposite side of the same row). This can also be thought of as splitting the grid diagonally into two triangles with 17 characters in the first row of the upper triangle, 15 characters in the second row, etcetera, and then flipping the triangles.

An interesting feature of this transposition method is that the 17 character row followed by the 15 character row is exactly the structure of Z32. Incidentally, the third row of the triangle has 13 characters like Z13, but this may simply be coincidence.

Taking the Z32 ciphertext (Figure 1(A)) and shifting the bottom (15-character) row two places to the right (Figure 1(B)) resembles the structure of the first two rows of the upper triangle of the grid described above. Next, starting from the first character in the upper left corner, the characters of Z32 may be transposed by following the same knight piece move as in Z340 (Figure 1(C)). Finally, the plaintext 'TWELVE INCHES ALONG THE THREE RADIANS' fits within Z32 albeit with two misspellings: 'TWELVE' \rightarrow 'TWELVT' or 'EWELVE', and 'RADIANS' \rightarrow 'RNDIANS' or 'RADIAAS' (Figure 1(D)).

Figure 1: Z32 Ciphertext (in ASCII Characters) and Possible Plaintext $\,$

- (A) C 1 J I 2 O 3 4 A M 5 6 7 O R T G X 8 F D V 9 ! H C E L ? P W 1

Of course, other transposition methods and plaintexts are possible, and should and must be explored. For example, Z32 could instead encrypt a location (road or building name) on the map correspond to the coordinates (12,3), or even other coordinates.

4.4. Map Locations

The point on the map corresponding to the coordinates (12,3) taken from the pole at Mount Diablo where $\theta=3$ rad and r=12 inches depends on the chosen reference axis and direction of rotation.

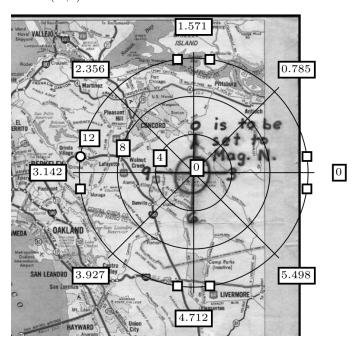
In plane polar coordinate systems, the reference axis is typically the polar axis (pointing east), and the reference direction is anticlockwise. With these references, the coordinates (12,3) lay approximately on Highway 24 between Orinda Village and Lafayette (white circle in Figure 2). Circumstantially, and not intended as proof of the present study, this coordinate is close to Bentley Upper School, Lafayette. As noted previously there are very many schools and other notable locations across the Bay Area, so it is not impressive that these coordinates lay close to one.

However, Zodiac may have taken other reference axes and directions (uncertainty (d) in Section 3). By trying all possible combinations of reference axes and clockwise/anticlockwise directions, alternative locations for the coordinates (12, 3) are shown by the white squares in Figure 2.

Evidently, the negative x-axis is not a likely reference axis because coordinates taken from this axis fall off the map to the east. Similarly, the negative y-axis is unlikely because coordinates taken from this axis fall into Suisun Bay and Grizzly Island Wildlife Area- not likely targets for a bomb threat. However, the coordinate (12,3) when taken clockwise from the positive x-axis falls in Lost Valley

which is somewhat close to a number of schools. Similarly, the positive y-axis as the reference axis yields locations in Livermore and Pleasanton, which are populous urban areas off Interstate 580. Any of these may be targets for Zodiac's bomb plot.

Figure 2: Phillips 66[®] Road Nap of the Bay Area with the Coordinates (12, 3) Illustrated for all Reference Axes and Directions



5. Discussion

In the present study, various proposed solutions to the Zodiac Killer's 32-character cipher are discussed with their strengths and weaknesses outlined. Based on these approaches, a new approach is presented. Assumptions are clearly defined and sensitivity tests are performed to explore possible variations in obtained 'solutions'. A single coordinate is obtained, but this coordinate corresponds to various map locations depending on the choice of reference axis and direction. Additionally, one possible plaintext is presented using homophonic substitution and a simple transposition method, though the plaintext contains tvpos, and is not particularly convincing. The motivation for this transposition method, which is the same as Zodiac used in the Z340 cipher, is a similarity between a method for the Z340 transposition step described by Oranchak, and the structure of Z32. But other possible transposition methods (if used at all), as well as alternative plaintext, cannot be ruled out.

This study is preliminary and exploratory, but perhaps represents the most systematic and rigorous assessment of Z32 to date. Future analyses may seek to find simple substitution methods which fit variations of the (12, 3) coor-

dinates (or corresponding map locations) without typos, and which may offer a more 'convincing' solution.

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