A model of political parties

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A MODEL OF POLITICAL PARTIES[1]

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This paper presents a new model of political parties. I assume that the role of parties is to increase the commitment ability of politicians vis-à-vis the voters. Whereas a politician running alone can only offer his ideal policy, the set of policies that a party can commit to is the Pareto set of its members. I show that the commitment mechanisms provided by the institution of parties has no effect when the policy space is unidimensional; the policies parties can induce in equilibrium arise also when politicians are running independently. However, when the policy space is multidimensional, politicians use the vehicle of parties to offer equilibrium policies that they cannot offer in their absence. *Journal of Economic Literature* Classification Numbers: D72, C71.

Key words: Political parties, coalition formation, multidimensional policy space.

1. INTRODUCTION

Political parties are a hallmark of democracies. In every democratic regime, groups establish these institutions. The array of parties and their internal composition has a potential impact on policy outcomes. The purpose of this paper is to analyze whether indeed parties have such an impact. In other words, since it is widely recognized that *institutions matter*, the task of this paper is to assess the significance of the institution of parties, as well as the institutions that parties develop and maintain.

I therefore analyze whether the existence of parties changes the political compromise that is achieved by the different groups in society, compared to a scenario in which parties do not exist and society reaches a political outcome in the absence of this additional institution. To explore this question I offer a new model of endogenous political parties.

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A main feature of political parties is that they are composed of factions; groups of politicians or party members who differ in their ideological views.[3] I assume that the role of parties is to facilitate compromise between these groups. Parties enable compromise by increasing the commitment ability of politicians. A politician, running independently, may wish to offer compromise policies in order to win a larger support. But she is unable to do so credibly; the voters believe that she will implement her own ideological preferences even if she made promises to the contrary (see Besley and Coate [2] and Osborne and Slivinski [16]). Parties, on the other hand, allow politicians to commit to policies that do not coincide with any of their individual preferences, but rather represent a compromise between their ideal policies. Thus, to offer a compromising policy, different groups or factions must join together in one party.

To be more specific, the main assumption of the model is that parties can offer to voters any policy in the Pareto set of their members. Parties cannot commit to offer any policy outside the Pareto set but the party members can find mechanisms (such as bargaining) that allow them to choose policies within their Pareto set. Thus, agreements are enforceable among party members (but not across parties).

Indeed, the mechanisms in which Western European parties reach internal compromise do mimic some form of a weighted average of the ideal policies of their factions; the parties' delegates vote on the policy principals in an annual conference whereas the balance of power between factions, that is, the proportion of votes that each faction receives, is translated into policy recommendations.[4] These policy recommendations are then incorporated into the party's election manifesto. The manifesto is viewed as a "contract" between members and executives, a "contract" by which legislatures and ministers are being committed to the conference decisions. Such commitment is enforced through party discipline, which in itself relies on institutions such as whips and voting by list.[5]

[3]The level of internal cohesiveness may vary across parties. The Japanese Liberal Democrats (LDP) or the Italian Christian Democrats parties correspond to a very factionalist party system whereas the Swedish Social Democrats may be more cohesive. Of course, in some cases, these factions may not reach a compromise. The multiparty system in France may correspond to the non-coincidence of the main cleavages of society (see Duverger [5]).

[4]This institutional feature may also change over time, perhaps as a result of an exogenous shift in the public's preferences. In the 1990's, the British Labor party reduced the maximum proportion of votes that the trade unions could control at the annual conference, to enhance its chances in the general election (see Webb [29]).

[5]It works; recent empirical analysis in the UK and Canada suggests that parties that win election implement the great bulk of their manifesto promises: in the years 1954-1979 for example, the UK governments have fulfilled about 60% of their manifesto pledges while their Canadian counterparts have been successful in implementing an average of 70% of their pledges. See Budge, Hearl and Robertson

1.1 An Outline of the Model

The commitment function of parties is incorporated in the following model. I first assume that politicians are organized into parties. The parties play a platform game, in which each party can choose whether to run and if so, which platform to offer from its feasible set of policies, i.e., its Pareto set. The platform that receives the largest number of votes wins the election and is implemented. I assume that politicians care about policy. I can then characterize the equilibria and consequently the political outcomes given any party structure. Finally, I analyze which party structures and political outcomes are stable. A stable party structure is an array of political parties in which no group of politicians wishes to quit its party and form a smaller one, thus inducing a different equilibrium outcome.[6]

The model differs therefore from the citizen-candidate models, pioneered by Osborne and Slivinski [16] and Besley and Coate [2] in two important ways. First, I introduce parties, an institution that is missing from the citizen-candidate models, and assume that parties can offer to voters more than the set of the ideal policies of its members. Second, I analyze the political outcome in the presence of endogenous parties. To do so, I use a model of coalition formation, which incorporates both cooperative and non-cooperative concepts.[7]

1.2 Summary of Results

The main goal of the analysis is to compare the political outcomes in stable party structures to the political outcomes in the absence of parties. In the absence of the institution of parties, politicians can only run alone and hence offer only their ideal policy. Given that parties allow their members to offer a larger set of policies, the presence of parties changes the feasible set of platforms that politicians can offer. Do parties then have an impact on the political outcome? In other words, does the enlargement of the policy set in the form of political diversity has any effect on the policy that is chosen?

I show that when the policy space is unidimensional, the increased commitment that parties allow for politicians has no effect on the political outcome. Any political outcome in the presence of parties can be achieved when each politician can only run by himself and offer his ideal policy.

Intuitively, in the unidimensional policy space, any party composed of only "left-wing" politicians or only "right-wing" politicians cannot win against the median. But the median can win even if parties do not exist. When parties are comprised of politicians from both sides of the median, i.e., they contain both left-wing and right-wing

[4].

[6]The stability concept is adopted from Ray and Vohra [19].

[7]I discuss the specific stability concept that I analyze in section 1.3.

politicians, they cannot find a compromise that can satisfy all of them relative to the median position. Thus, such a party is not stable, since at least some of its members will prefer the median outcome which can be achieved in the absence of parties.

However, when the policy space has more than one dimension, parties do make a difference. The political outcome in the presence of parties is different than the political outcome in their absence. In the multidimensional policy space, even if a median voter (a Condorcet winner) exists, it is not likely to be in the Pareto set of other groups or politicians. This implies that other politicians can cooperate and offer policies which they - and consequently also a significant part of society - prefer over the Condorcet winner, win the election, and have a true impact on collective decision making.

To illustrate the usefulness of the approach beyond the general results, I analyze a simple example, with two dimensions of conflict. One dimension of conflict is the tax level, where the rich prefer no taxation and the poor prefer full taxation. The second conflict is on how to spend tax revenues. The rich and a segment of the poor prefer to spend it on general public goods, whereas another segment of the poor have special interests and prefers to spend all revenues on a specific, such as a local, public good. The example shows that parties make a difference when the rich join the poor with the special interests. This party suggests a compromising policy, which cannot be offered in the absence of parties; low (but positive) level of taxation and all revenues targeted to special interests. Thus, 'left' and 'right' are endogenously created; the left advocates high taxation with general public spending, whereas the 'right' offers low taxation and hence low but targeted spending. The two-dimensional conflict is converted to a one dimensional conflict between left and right.

The example illustrates that the formation of parties in the multidimensional policy space allows politicians to achieve internal compromises within parties; this is the reason why we may face societies with a *de facto* unidimensional conflict space. It is parties that reduce the conflict in society to a conflict of fewer dimensions.[8]

1.3 Related Literature

Modelling Parties: There are only a handful of papers that model endogenous party formation. In Morelli [15], parties facilitate coordination among voters. Similarly to my analysis, only parties can offer diverse policies, but they can offer any policy and are not restricted by the Pareto set of their members. He then analyzes multi-district elections

[8] This accords with the empirical study of Poole and Rosenthal [18] on roll call voting in the American Congress. They find that voting can be best characterized by a unidimensional model of conflict: "*The political parties, either through the discipline of powerful leaders or through successful trades, function as effective logrollers. Parties thus help to map complex issues (to bundle diverse economic interests) into a low-dimensional space*".

in the unidimensional policy space. In my model there is only one district, but I allow for multidimensional policy spaces. Another related work is by Roemer [23,25] who suggests that parties are composed of different factions that do not differ in ideology, as in my model, but in their political motivation; one faction is an office seeker whereas the other cares only about ideology.

Other papers that analyze endogenous party formation are Feddersen [7] and Baron [1] who view parties as coalitions of voters. In Snyder and Ting [26], parties are brand names, allowing politicians to signal their preferences to voters. In Riviere [21], politicians in one party can share the costs of running.[9] Several papers analyze the role of parties in legislatures. Jackson and Moselle [9] assume that party members in parliament can commit to enforce their internal agreements. In my analysis the commitment mechanism arises in the pre-election stage. In Osborne and Tourky [17], parties arise in legislatures due to economies of scale.

An informal argument that supports regarding parties as commitment devices, compared to its other possible roles, is the existence of interest groups. Interest groups are cheaper and simpler institutions that can fulfil the functions attributed to parties in previous literature. For example, they can raise funds to help a politician elected. Or, a politician can signal her type by receiving support from specific interest groups. Interest groups can also create coordination between voters using public announcements. Thus, if a party evolves instead of an interest group, it is because the party can maintain more complex institutions, which monitor and direct the activities of its members in the legislative and the executive branches of the government. This commitment role cannot be fulfilled by an interest group, who has no direct access to government activities, but by the party members who have control over policy choices.

Coalition Formation: Recent models of coalition formation in a non-cooperative setting include Ray and Vohra [20], Yi [30], and Bloch [3]. Departing from the common assumption in cooperative game theory, according to which a coalition's payoff depends only on its members (or size), these papers assume that a coalition's payoff may depend on the whole array of coalitions, i.e., the partition. Thus, instead of a *characteristic function* (see for example Thrall and Lucas [28]), one may use a *partition function* (see for example Ray and Vohra [19]) and analyze which coalitions would form given either a fixed rule of sharing the coalition's value (as in Bloch [3]) or a bargaining game to divide the surplus (as in Ray and Vohra [20]).

In my model, however, the payoff of a coalition (a party) and its members depends not only on which other coalitions form, but also on their actions in equilibrium, due to multiplicity of equilibria. Thus, when I analyze stability or formation of party structures,

[9] I allow for this possibility in section 5.2.

non-cooperative solutions are problematic.[10]

In the paper, I chose to analyze a two-phase process of coalition 'formation' and surplus division inside coalitions. First, I determine the feasible equilibria for any coalition structure. Second, I use a stability notion adopted from Ray and Vohra [19] to analyze stable coalition structures and their equilibria. In this concept, players start from some coalition structure (a partition), and are allowed only to fragment coalitions. Credible threats are deviations to finer partitions which are stable themselves. I refer the reader to the literature discussion in Ray and Vohra [19] which compares their solution concept to others.

In a companion paper, Levy [11], I show how the main result is robust to other stability concepts, cooperative and non-cooperative. Among the non-cooperative games I analyze a membership game (as in Yi [30]). Among the cooperative ones, I analyze Stable Sets, the $\delta-$Core (see Hart and Kurz [8]), and a new concept which I term the 'bi-Core'. The bi-Core allows deviators either to break a coalition into two or to form a new coalition from two existing coalitions.[11]

The rest of the paper is organized as follows. In the next section I present the model. Section 3 discusses two examples which illustrate the main results and the usefulness of the model. The general results are presented in section 4. In section 5, I analyze the robustness of the results; first, by allowing for different incentives for politicians, through campaign costs and benefits from holding office, and second, by using different stability concepts. Finally, I discuss various assumptions and extensions in the concluding section. The appendix contains all proofs.

2. THE MODEL

The set of voters is composed of N (finite) different groups, where each group is characterized by a different preference ordering on the feasible policy set $Q \subseteq R^k$. In particular, I assume that voters who belong to group i, $i \in \{1, 2, ..., n\}$, share single-peaked preferences,[12] represented by a continuous and concave utility function $u_i(q) = u(q, i)$, for $q \in Q$.[13] Thus, each group can be viewed as representing a policy position. I assume a continuum of voters,[14] so that the size of each group can therefore be captured by its

[10] Consider for example bargaining games, in which a player makes an offer to a coalition. His offer must be contingent not only on the other parties that form, but also on what they play, which is not feasible.

[11] Section 5.3 summarizes the results shown in this companion paper.

[12] For a definition of single peakedness in the multidimensional policy space, see Kramer [10].

[13] The results hold also for a weakly concave utility function, with a minor modification (see appendix).

[14] This is a technically convenient assumption which approximates a large number of voters.

measure. For simplicity, I assume throughout the main text that the set of politicians is fixed (although the actual candidates or parties will be endogenously determined) and is composed of a representative from each group. That is, there are N players, with player i having the ideological preferences represented by $u(q,i)$. The results hold also when all voters can participate as politicians and the appendix provides proofs for this more general assumption as well.

The model has two phases of analysis. I first assume that the politicians (henceforth, *the players*) are organized in coalitions/parties. These coalitions choose platforms on which the voters vote sincerely. The platform that receives the highest number of votes is implemented. I then analyze the possible equilibria in the platform game for any coalition structure. The second phase determines which coalition structures are stable.

2.1 The Platform Game

There are N players organized in a coalition structure π which is a partition on the set N. A coalition S is a non-empty subset of players. Denote by Q_S the Pareto set of coalition S :

Definition 1 Q_S is the Pareto set of coalition S, i.e.,
$Q_S = \{q \in Q | \nexists\, q' \in Q \text{ s.t. } \forall i \in S,\ u_i(q') \geq u_i(q) \text{ with at least one strict inequality}\}$.

The platform game has one strategic stage, in which each coalition S chooses, simultaneously, an action. In particular, each coalition S can choose an action $q_S \in Q_S \cup \{\varnothing\}$, where $\varnothing$ means that the coalition offers no policy in the election, i.e., it chooses not to run. Thus, the feasible set of policy platforms differs for each partition and changes the set of possible political outcomes.

This novel assumption introduces the role of parties; if politicians form a party, the party's structure enables them to commit, but only to policies in their Pareto set. The voters believe that any policy in the Pareto set can be enforced. Any policy outside the Pareto set could not be implemented after the election since the party members can renegotiate it.[15] Agreements within coalitions are therefore enforceable. However, agreements across coalitions or the coalition structure itself, are not enforceable.

Denote by $\mathbf{q}(\pi)$ a vector of policy platforms (which may include the null platform $\varnothing$ offered by some coalitions) where each platform is chosen by a coalition $S \subset \pi$. Given the policy vector $\mathbf{q}(\pi)$, election are held and each voter votes sincerely for the policy that she likes most.[16] Voters who are indifferent between several policies, use a fair mixing

[15] An interpretation of this assumption is that parties choose their protocol and not their platform in the pre-election stage. That is, they choose the 'rules of the game' if they win the election. These rules would result in some platform, which the voters anticipate, and which must be ex post efficient due to an efficient ex ante bargaining process with complete information.

[16] In section 6, I discuss the implications of the assumption of sincere voting.

device. If no policy is offered, a default status quo d is implemented. I assume that $u_i(d) = -\infty$, for all i, so at least one coalition chooses to run in equilibrium.[17] Denote by $V_S(\mathbf{q}(\pi))$ the share of votes that a platform q_S receives when $\mathbf{q}(\pi)$ is offered, where $\sum_{q_S \in \mathbf{q}(\pi)} V_S(\mathbf{q}(\pi)) = 1$. The winning policy is determined by plurality rule:

DEFINITION 2 Given $\mathbf{q}(\pi)$, if $\mathbf{q}(\pi) \equiv \varnothing$, d is the election's outcome. Otherwise, let $W(\mathbf{q}(\pi)) = \{q_S | V_S(\mathbf{q}(\pi)) = \max_{q_S} V_S(\mathbf{q}(\pi))\}$. The election's outcome is a fair lottery between the policies in $W(\mathbf{q}(\pi))$. Its expectation is $\frac{1}{\sharp W(\mathbf{q}(\pi))} \sum_{q_S \in W(\mathbf{q}(\pi))} q_S$.

The utility of the players from the platform game is their expected utility from the election's outcome, that is, for all $i \in N$, $U_i(\mathbf{q}(\pi)) = \frac{1}{\sharp W(\mathbf{q}(\pi))} \sum_{q_S \in W(\mathbf{q}(\pi))} u_i(q_S)$ if $\mathbf{q}(\pi) \neq \varnothing$ and $u_i(d)$ if $\mathbf{q}(\pi) \equiv \varnothing$.

Let Δ_S be the set of probability distributions over $Q_S \cup \{\varnothing\}$. A (mixed) strategy for a coalition S is $\delta_S \in \Delta_S$. A set of strategies for each coalition S in the partition π is $\{\delta_S\}_{S \subset \pi}$. Given a mixed strategy, denote the expected utility of player i by $U_i(\{\delta_S\}_{S \subset \pi})$. Let δ_{-S} denote the strategies taken by all coalitions but S.

DEFINITION 3 An equilibrium in the platform game is a collection $\{\boldsymbol{\delta}_S\}_{S \subset \pi} \equiv \boldsymbol{\delta}(\pi)$ such that for all S there exists no $\delta'_S \in \Delta_S$, $\delta'_S \neq \boldsymbol{\delta}_S$ that satisfies $U_i(\delta'_S, \boldsymbol{\delta}_{-S}) \geq U_i(\boldsymbol{\delta}_S, \boldsymbol{\delta}_{-S})$ for all $i \in S$, with at least one strict inequality.

In equilibrium, I assume that any player within a coalition has veto power, i.e., a switch to a different policy demands consent of all party members.[18] The equilibrium concept itself, implicitly, reflects the trade-off facing a politician who joins a party. When in a party, the politician may be able to attract more voters since she can offer a larger set of platforms. But, these policies force her to compromise. Moreover, she also has to give up control rights over decision making since other party members may block desirable deviations. The first proposition assures the existence of equilibria.

PROPOSITION 1 *For all $\pi \in \Pi$, there exists an equilibrium $\boldsymbol{\delta}(\pi)$.*

I focus on pure-strategy equilibria. Among them, I focus on 'partisan' equilibria, whenever they exist. A 'partisan' equilibrium is an equilibrium in which all party members vote for their party's platform, if it offers one (party members are not restricted in their votes if their party is not offering a platform).[19] As a tie-breaking-rule, I assume that if all coalition members are indifferent between running and not running, the coalition chooses not to run.

[17] Osborne and Slivinski [16] also make this assumption.

[18] The equilibrium concept is similar to the one used in Roemer [23].

[19] Focusing on 'partisan' equilibria serves to eliminate equilibria with four or more parties running in election. Also, the non partisan equilibria have an undesirable feature; party members object to deviations to platforms which are closer to their ideal policies.

The analysis of the platform game fixes the possible utilities for a player from each partition. In particular, each partition is characterized by an equilibrium (or a set of equilibria) and the utility from such an equilibrium, $U_i(\boldsymbol{\delta}(\pi))$, is the utility that player i may accrue from a partition π. When party members are not satisfied with a partition, they may be able to induce other party structures and de-stabilize the current one. This is what we explore in the second phase of the analysis.

2.2 Stable Coalition Structures and Platforms

We are looking for stable partitions and their respective equilibria, i.e., $(\pi, \boldsymbol{\delta}(\pi))$. Note that $\boldsymbol{\delta}(\pi)$ has to be defined as an *equilibrium* in the platform game given π and not just a strategy vector. Partitions may be stable for one equilibrium but not for others. The analysis of stability pins down therefore not only the stable party structures, but also the stable political outcomes. An unstable partition is one that is not supported by any equilibrium in the platform game.

I use a stability concept derived by Ray and Vohra [19]. In this solution concept, players start from some coalition structure (a partition), and are only allowed to break parties to smaller ones. The deviations can be unilateral or multi-lateral (i.e., several players can deviate together). The deviators take into account future deviations, both by members of their own coalitions and by members of other coalitions. Credible threats are deviations to finer partitions which are stable themselves and therefore the nature of the definition is recursive. Stable $(\pi, \boldsymbol{\delta}(\pi))$ are immune to such deviations.

Consider a partition π and let $R(\pi)$ denote all coalition structures that are refinements of π. A coalition $\tilde{\pi} \in R(\pi)$ is *induced* from π if $\tilde{\pi}$ is formed by breaking a coalition in π into two. We say that a coalition S is a *deviator* if it can induce $\tilde{\pi} \in R(\pi)$ from π. Obviously, S is a sub-coalition of a coalition in π.

Denote the finest coalition structure, in which $|S| = 1$ for all S, by π^0. There are no deviations allowed from π^0 and therefore $(\pi^0, \boldsymbol{\delta}(\pi^0))$ is defined as stable, $\forall \boldsymbol{\delta}(\pi^0)$.

Recursively, suppose that for some π, all stable coalitions with their respective equilibria were defined for all $\pi' \in R(\pi)$, i.e., for all structures which are finer than π.

Definition 4 $(\pi, \boldsymbol{\delta}(\pi))$ is *sequentially blocked* by $(\pi', \boldsymbol{\delta}(\pi'))$ for some $\pi' \in R(\pi)$ if there exists a sequence $\{(\pi^1, \boldsymbol{\delta}(\pi^1)), (\pi^2, \boldsymbol{\delta}(\pi^2)), ..., (\pi^m, \boldsymbol{\delta}(\pi^m))\}$ such that:

1. $(\pi^1, \boldsymbol{\delta}(\pi^1)) = (\pi, \boldsymbol{\delta}(\pi))$, $(\pi^m, \boldsymbol{\delta}(\pi^m)) = (\pi', \boldsymbol{\delta}(\pi'))$ and for every $j = 2, ..., m$, there is a deviator S^j that induces π^j from π^{j-1}.
2. $(\pi', \boldsymbol{\delta}(\pi'))$ is stable.
3. $(\pi^j, \boldsymbol{\delta}(\pi^j))$ is not stable for any $\boldsymbol{\delta}(\pi^j)$ and $1 < j < m$.
4. $U_i(\boldsymbol{\delta}(\pi')) > U_i(\boldsymbol{\delta}(\pi^{j-1}))$ for all $j = 2, ..., m$, and $i \in S^j$.

Intuitively, to block a coalition, a leading deviator suggests a particular sequence in which the other deviators will move and at each step, the final outcome justifies the next step. In no step in the way the process can stop since it does not encounter any stable partitions.

Then, *$(\pi, \boldsymbol{\delta}(\pi))$ is stable if there is no $(\pi', \boldsymbol{\delta}(\pi'))$ for $\pi' \in R(\pi)$ that sequentially blocks $(\pi, \boldsymbol{\delta}(\pi))$.* This completes the recursive definition.

A party structure and a political outcome are therefore stable if no player (or a group of players) wishes to induce a finer partition, which is stable in itself. This stability concept captures threats of players to break up their party in order to enhance their desired platform. It can also be viewed as short-term deviations, in which it is easier to break agreements than to form agreements with new partners. I later show that the main result is robust to other stability concepts as well.

We can now turn to the characterization of endogenous parties and their effect on the political outcome. First, I do so by analyzing two simple examples.

3. EXAMPLES

Consider 3 players, a, b and c. These players are representatives of the 3 groups in society. The measure of each group, as its share of the population, is such that no group has a majority. The groups/politicians have the following preferences: $u_i(x, y) = -\alpha(i_x - x)^2 - (1-\alpha)(i_y - y)^2$, for $i \in \{a, b, c\}$ and $\alpha \in [0, 1]$. The players would like therefore to minimize the weighted distance of the implemented policy from their ideal policy, (i_x, i_y).

3.1 One Dimension of Conflict

Let $\alpha = 1$, $(a_x, a_y) = (0, 0)$, $(b_x, b_y) = (1, 0)$ and $(c_x, c_y) = (2, 0)$. Restrict the policy space to $x \in [0, 2]$. Note that b is the median voter in this example, depicted in figure 1.

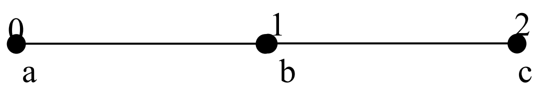

Figure 1: unidimensional policy space

We first find the equilibria of the platform game for each partition on the set of politicians, $\{a, b, c\}$. In the partition $a|b|c$, b running alone (and hence winning) is generically the unique equilibrium. When the partition is $ab|c$, ab can offer anything in $(0, 1]$ and win the election. Similarly, in the partition $a|bc$, the unique form of equilibrium is bc running unopposed and offering anything in $[1, 2)$. These last two coalition structures illustrate the costs of being in a party from the point of view of b. When in a party with either a or c, he does not necessarily offer his ideal point as opposed to the partition in which he runs by himself.

If the partition is $ac|b$, the equilibria are that either ac or b run, offering 1 which is the ideal point of b. To see that these are the only possible equilibria, consider ac running with a platform that differs from $x = 1$. In this case, b can join the race and win the election. Such an equilibrium cannot be sustained. Finally, the grand coalition abc can offer anything in the feasible policy set. Figure 2 describes the society and possible equilibrium outcomes in the platform game for each partition.

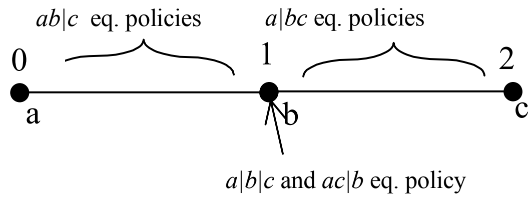

Figure 2: equilibria in the unidimensional example

We can now analyze which partitions and equilibria are stable. The partition $a|b|c$ in which b runs alone is stable by definition (this is therefore also the outcome in the absence of parties). The partition $ab|c$, has $x \in (0, 1]$ as its equilibrium outcome. This partition is stable only if $x = 1$. Otherwise, b prefers to break it. Similarly, $a|bc$ is stable only if bc offers $x = 1$.

The partition $ac|b$ yields the median voter's outcome in any of its equilibria, when either b runs or ac run and offer $x = 1$. It is therefore stable because if a or c break their coalition, they cannot increase their payoff. Finally, we consider the grand coalition abc. If the outcome in abc is biased away from the median voter, then b should break it. It then results in $ac|b$ which has $b's$ ideal policy as a stable equilibrium outcome.

The implication of this example is that coalition structures are stable only if they yield the median voter's outcome. Thus, in this example, parties are 'neutral'. The same political outcome arises both in the presence of parties and in the absence of parties, that is, in the partition in which each politician can only run by himself. In other words, the commitment ability provided by parties to politicians, which increases the set of possible policies that can be offered to voters, does not eventually allow them to increase and change the set of political outcomes in equilibrium.

3.2 Two Dimensions of Conflict

Let $0 < \alpha < 1$ and $(a_x, a_y) = (0, 0)$, $(b_x, b_y) = (1, 0)$, $(c_x, c_y) = (1, 1)$. Restrict the policy set to be the triangle formed from the ideal points. Note that c (a) prefers $b's$ ideal point to that of a (c). Thus, b is still a 'median voter'. Let $\alpha \leq .5$ which implies that b weakly prefers $a's$ ideal point to $c's$. Figure 3 describes the society.

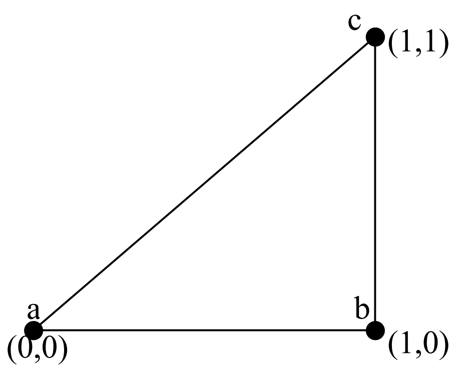

Figure 3: a two-dimensional example

This structure could be interpreted in the context of an economic environment with scarce resources. Groups b and c may represent low-income voters, while group a is composed of high-income voters. Thus, the $x - axis$ denotes the level of tax t, from 0 to 1, which represents a transfer from the rich to the poor. The tax revenues can be spent on public goods, which all voters can enjoy, or on a special local good, which *only* group c can enjoy. These local goods can be for example specific schooling needs, such as ethnic or religious. The $y - axis$ represents therefore the level of tax revenues targeted to the special interests of group c. When $y = 0$, all tax revenues, if exist, are spent on general public goods. The higher we go up the $y - axis$, the more is spent on group c and less on general public goods.

What are the equilibria for each partition? Consider the partition $a|b|c$. Generically, b running unopposed and winning the election is the unique pure-strategy equilibrium. In other words, b is the Condorcet winner, although a and c may share some policies which they prefer to $b's$ ideal point. Nevertheless, none of their representatives can beat b since each can offer only his ideal point (if $a(c)$ runs against b, the voters of $c(a)$ still vote for b).

When the partition structure is $a|bc$, the Pareto set of b and c is $(1, y)$ for $y \in [0, 1]$. Since $(0, 0) \succ_b (1, 1)$ and $(0, 0) \prec_b (1, 0)$, there exists y' such that $(0, 0) \sim_b (1, y')$. Thus, in any equilibrium, bc offers $(1, y)$ for $y < y' = \sqrt{\frac{\alpha}{1-\alpha}}$, and wins the election since against such a policy a cannot enter and win $b's$ votes. The structure $ab|c$ can be similarly analyzed: the coalition ab wins the election with any policy $(x, 0)$, for $x \in [0, 1]$.

Consider now the partition $ac|b$. The Pareto set of a and c is the diagonal (x, x) for $x \in [0, 1]$. The policies $(x, x) \succ_{a,c} (1, 0)$ can win the election against b. Any other policy offered by ac will 'induce' b to run and win. Thus, an equilibrium must consist of ac running, and offering (x, x) for $x \in (1 - \sqrt{1 - \alpha}, \sqrt{\alpha})$. The analysis of the partition $ac|b$ shows the power of the commitment assumption: a and c can gain higher utility when in the same coalition than when separated. Figure 4 illustrates this equilibrium:

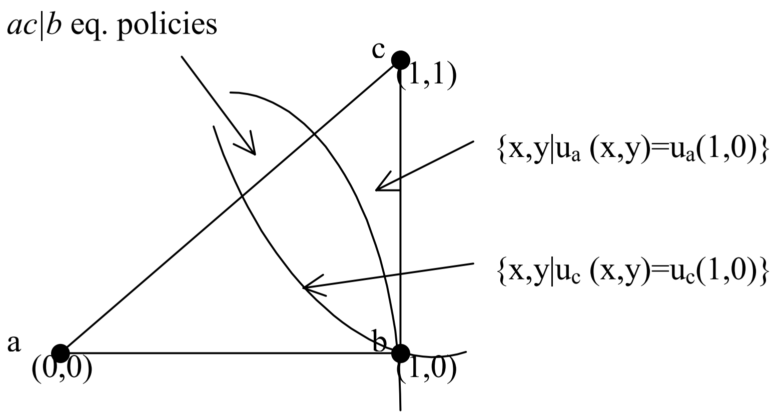

Figure 4: $ac|b$ equilibria

Finally, the coalition abc can offer anything in the Pareto set of the whole polity.

Let us analyze which party structures and equilibria are stable in this environment. By definition, $a|b|c$ is stable. On the other hand, the structures $ab|c$ or $bc|a$ can be stable only if ab and bc respectively, offer $b's$ ideal policy, $(1,0)$. Otherwise, b has an incentive to deviate from each, and win the election alone in $a|b|c$.

What about the structure $ac|b$? Given the threat of b winning the election in the partition $a|b|c$, both a and c prefer to cooperate and run for election with the platform (x,x) for $x \in (1-\sqrt{1-\alpha}, \sqrt{\alpha})$. Any of these platforms is better for both of them than b. The structure $ac|b$ is therefore stable and yields an outcome which is different from the outcome in $a|b|c$.

Now consider abc. If a or c break it by themselves, then the resulting outcome is $b's$ ideal policy, $(1,0)$. Thus, a stable abc must create a political outcome that Pareto dominates b for both a and c. If ac break it together, the outcome is an equilibrium of $ac|b$. Since a and c can induce outcomes in their Pareto frontier if they deviate together, abc must implement one of the platforms that ac does. But because b can break it as well to $ac|b$, abc must provide him with his preferred position in the set of feasible policies (x,x) for $x \in (1-\sqrt{1-\alpha}, \sqrt{\alpha})$. We can therefore pin down exactly the policy which is offered by abc.

The result in this example is therefore as follows. When politicians can only run alone, b wins the elections. The political outcome is $(1,0)$. However, when we allow for parties, the political outcome can also be (x,x) for $x \in (1-\sqrt{1-\alpha}, \sqrt{\alpha})$. Parties, therefore, make a difference. When b wins, it represents a compromise between a and c but not an ideal one. It is a compromise that satisfies each group only on one dimension. When a and c win together, the outcome represents a compromise on *both* dimensions.

According to our economic interpretation, this result means that special interests (c) and the rich group (a) join together in a coalition. They will offer a relatively low level of tax whose revenues will be directed to the special interests instead of to the whole population at large. This policy outcome not only illustrates the impact of parties, but may also explain why the poor do not expropriate the rich. The poor constitute a majority of the voters, and a maximum level of tax would be imposed in the absence of parties. However, the poor are divided on how to spend taxes. Against the threat of a maximum tax level, the rich find it in their interest to compromise with a segment of the poor on how to spend public revenues.[20]

Moreover, the identity of the groups who join together in one party, re-shapes the map of political conflicts and re-defines their dimensions. Instead of a conflict along the x dimension, and a conflict along the y dimension, the conflict in society becomes along one of the diagonals. For example, if the population at position b is large enough, there may be equilibrium in which two platforms are offered to voters in equilibrium; a stable party structure has both ac and b running and each winning with probability $\frac{1}{2}$. The coalition ac offers (x, x) for $x \in (1 - \sqrt{1-\alpha}, \sqrt{\alpha})$ whereas b offers $(1, 0)$. The dimension of conflict is the function $\frac{x}{x-1}t - \frac{x}{x-1}$ for $x \in (1 - \sqrt{1-\alpha}, \sqrt{\alpha})$ and $t \in [x, 1]$. Figure 5 depicts the prevailing conflict in society in the presence of parties.

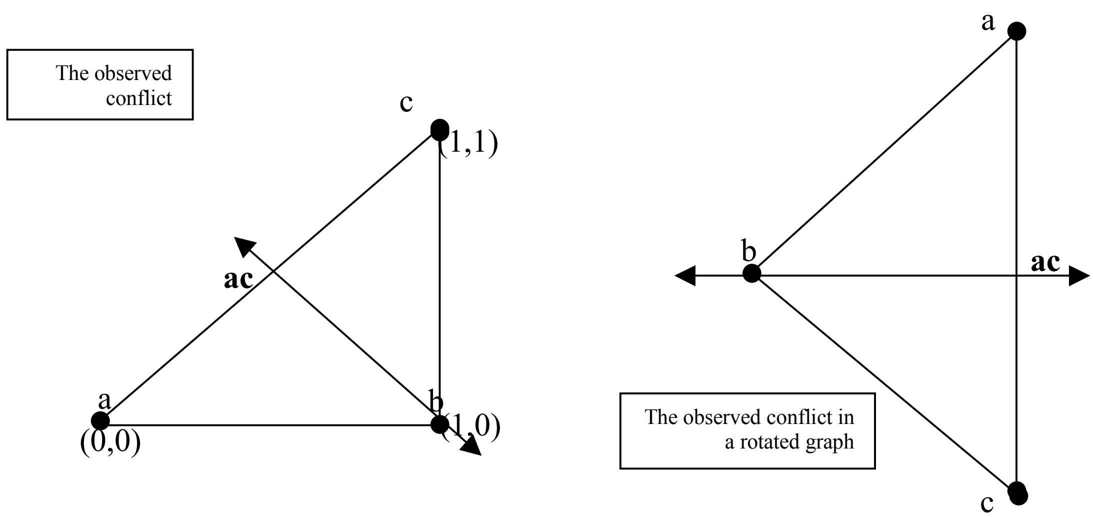

Figure 5: endogenous Left (b) and Right (ac)

Assuming that c represents clerical special interests we would now define the conflict in society as a conflict between a party of big government and secular education ('left-wing'), and a party of small government and subsidized religious education ('right-wing').[21] The conflict would not only be re-defined, but it would also be reduced from

[20]Roemer [24] also uses a two-dimensional analysis to explain why the poor do not expropriate the rich. In his analysis though, parties are exogenous, and their structure is given. That is, the left wing party represents the poor and the anti-clerical, whereas the right wing party represents the rich and the pro-clerical. In the present model, the form of representation, i.e., who joins who in a party, is endogenous.

[21]This example may describe India's right-wing Hindu Bharatiya Janata Party (BJP), a party which

two dimensions of conflict, to a unidimensional conflict. Observations of societies in which only one dimension of conflict is apparent may be misleading. The underlying preferences may form two dimensions of conflict, but the mitigating power of parties reduces them to one observable dimension.

Remark 1 The ability of parties to form may depend on the importance voters place on each of the different dimensions (often termed "salience"), which is captured in the above example by the parameter α.[22] An interesting case is the party system in Belgium. Until the end of the 1960's, the two main parties, Catholic and Liberal, were composed of different linguistic groups, the Flanders and the Walloons. Each party reached an internal compromise on the language conflict, while the external conflict between parties focused on the religious dimension. But, the increasing salience of the linguistic cleavage in the 1960's had made it impossible for the Flanders and the Walloons factions to cooperate within one party. From that point onwards, the conflict in the Belgian society is on the linguistic dimension as well.[23]

Remark 2 The results of the two examples hold if we allow all voters to run for office (instead of one representative for each group). In particular, parties make a difference only in the two dimensional example, in which case, a party of representatives from a and c is established. I use the more restrictive assumption in the examples for clarity of exposition but the appendix proves the general results, to be presented now, also for the case in which all voters can be politicians.

4. General Results

In the examples illustrated in the previous section, parties make a difference only when the policy space is multidimensional. The main results generalize this insight, but we need few more definitions before introducing it:

Definition 5 Let k^* be the dimensionality of a Euclidean space V that is spanned by the N ideal policies in society. Then, k^* denotes the dimensionality of conflict in society.

brought together Hindu fundamentalists from the poorer sections who wanted targeted provision of public goods and wealthy candidates who wanted low taxes and reduction in the role of the state.

[22] For simplicity, I assumed that α is equal among all voters. This is not necessary for the results.

[23] See Lijphart [12]. Similarly, the US Democratic coalition broke down in the 1860's when Abraham Lincoln, then a Republican candidate, had raised the salience of the conflict about slavery. This prevented cooperation between the northern (anti-slavery) and southern democrats (the slave-owners) in spite of their shared economic interests. This explanation follows Riker [22].

The second definition states what it means for parties to be effective. Recall that an equilibrium $\boldsymbol{\delta}(\pi)$ generates a political outcome which is a fair lottery over policy platforms in the winning set, the set of policies that receive the highest number of votes.

Definition 6 Parties are *effective* if there exists a stable party structure $(\pi, \boldsymbol{\delta}(\pi))$, such that the political outcome of the equilibrium $\boldsymbol{\delta}(\pi)$ differs from the political outcome of any equilibrium $\boldsymbol{\delta}(\pi^0)$ in π^0, where π^0 is the partition in which each politician can only run by himself.

When we analyze the model, the set of all stable political outcomes is the set of all outcomes of $\boldsymbol{\delta}(\pi)$ such that $(\pi, \boldsymbol{\delta}(\pi))$ is stable. Among which are also the outcomes generated by $\boldsymbol{\delta}(\pi^0)$ since π^0 is stable. I compare this set to the equilibria in the absence of parties, i.e., the equilibria generated only by π^0. Parties are therefore effective if when we allow for parties, the set of the political outcomes is larger, i.e., they can induce an outcome which cannot arise in their absence. The next result characterizes whether parties are effective when the policy space is unidimensional.

PROPOSITION 2 *Parties are not effective when* $k^* = 1$; *any stable party structure can only generate political outcomes that exist also in the absence of parties. In particular, the only stable outcomes are the median voter's ideal policy, with certainty or in expectations.*

Thus, although parties allow politicians to increase their commitment ability, as it turns out, commitment cannot buy them much when the conflict in society is only unidimensional. It is easy to see how parties that contain members from both sides of the median cannot be effective. In the unidimensional policy space, the median voter represents a natural compromise. If no parties exist, the median voter's ideal point is a possible political outcome. But no other ideological compromise can then satisfy coalition members from both sides of the median. If the policy outcome is biased to the left, a right-wing member will break the coalition under the prediction that the resulting stable outcome will be the median.

The key element of the proof, which is Lemma 1 in the appendix, is to show that when parties are only one-sided, that is, each party is composed of members from the same side relative to the median, either left-wing or right-wing members, all possible equilibria also exist in the absence of parties (in the partition π^0). In particular, these equilibria yield the median's policy, either with certainty or in expectations.

To see the intuition for this lemma, consider equilibria with one or two platforms. When only one platform is offered in equilibrium, it must be the median voter's platform. Otherwise, the median can run against any other platform and win. When two platforms are offered, one by a left-wing party and one by a right-wing party, then each of them must receive half of the votes. Otherwise, the loser should not run for election. To receive half of the votes, their platforms must be positioned symmetrically around the median's ideal policy. Thus, each party must offer its most moderate position; otherwise it can deviate closer to the median, win alone and increase the utility for all its members (who all belong to the same side relative to the median). These platforms have to coincide therefore with the position of the most moderate member in each party. But this equilibrium can then potentially exist in the absence of parties since these moderate members can offer their ideal policy, and indeed have an incentive to do so in equilibrium.[24] The appendix extends this argument for equilibria with more platforms.

Thus, the entire set of equilibria that exists when parties are one-sided, exists also in the absence of parties, whereas the equilibria that exist when some parties are two-sided, cannot be stable if they yield an outcome which is biased from the median.

To conclude, with only one major social cleavage, parties do not have any major effect on the political outcome, because politicians do not master sufficient tools for compromise. Adding another dimension of conflict provides politicians with such a tool. As we have seen in the examples, parties may be effective in the multidimensional policy space and their presence influences the political outcome. The next result is set to characterize sufficient conditions for parties to be effective when $k^* > 1$. In particular, it illustrates that in contrast to the unidimensional policy space, parties can change the set of political outcomes even if a natural compromise such as the median voter exists when politicians can only run alone:

PROPOSITION 3 *Parties can be effective when $k^*>1$ even if a median voter (a Condorcet winner) exists in the absence of parties. In particular, a sufficient condition for parties to be effective when the Condorcet winner is a unique equilibrium in π^0 and preferences are Euclidean,[25] is that $k^* = N - 1$, where N is the number of ideal policies (groups) in society.*

The conditions that Proposition 3 creates in π^0 are similar to the unidimensional policy space; there exists one natural political outcome - a Condorcet winner/median voter. In the multi-dimensional policy space, however, this outcome is not necessarily the political result when we allow for the establishment of parties. It is important to note

[24] We also have to show that this equilibrium indeed exists in the absence of parties. It may be that in the absence of parties, there is a candidate in between the two positions that can upset the equilibrium by running and attracting enough votes, whereas in the presence of parties, this candidate belongs to a party that offers no platform. The proof shows that when all parties are one-sided, if there is a politician who can upset the equilibrium in the absence of parties, his party members would also prefer him to do so in the presence of parties.

[25] Euclidean preferences imply that the utility of a player is monotonically decreasing with the euclidean distance from her ideal policy.

that a Condorcet winner is relatively likely to exist in the citizen-candidate environment in π^0, in which a finite number of politicians can offer only their ideal policy.[26]

The Proposition implies that the relation between the number of political dimensions and the number of different groups can determine whether the formation of parties has an impact or not. It is easy to understand the intuition when we look again at the unidimensional case. Consider 3 groups (less than 3 groups is a trivial case) lined on the $x - axis$. The middle group, which is also the median voter, must belong to the Pareto set (the convex hull) of the other groups. The two extremes cannot cooperate against it. Now add another dimension of conflict by increasing slightly the position of the middle group on the $y - axis$. The middle group no longer belongs to the Pareto set of the other groups and the other groups can cooperate against it.

Similarly, when we have N groups, if the dimensionality of the conflict space is $N - 1$, then no group is in the Pareto set of all the others. This renders it possible for a subset of players to cooperate against the Condorcet winner, effectively changing the political outcome in their favor.

Remark 3 The result in Proposition 3 characterizes conditions for parties to be effective in the multidimensional policy space while stressing their role of offering policies which otherwise cannot be offered. The reader may be led to think that parties may matter only because they convexify the policy space. However, the following example shows that it is not the case. In this example, a party offers in (a stable) equilibrium the ideal policy of one of its members. Although this politician is able to offer his ideal policy without parties, there is no such equilibrium in the absence of parties.

Consider a society with 5 groups, represented by the politicians $\{a, b, c, d, e\}$. Group 'sizes' as their shares in the population are $\{.4, \varepsilon, \varepsilon, .3 - \varepsilon, .3 - \varepsilon\}$ respectively, for a small ε. The ideal point of a is $(0, -y)$ for $y \geq 0$, the ideal policy of groups b and c is $(0, 3)$ for both, whereas that of d and e is $(-1, 2)$ and $(1, 2)$ respectively. Preferences are Euclidean; b, c and a are indifferent between d and e, group a prefers d and e to b and c whereas d and e prefer b or c to a.

It is then easy to verify that in the absence of parties there are only mixed strategy equilibria. In all these equilibria, politician a enters the race with probability 1, and wins with a strictly positive probability. The utility of all players but a decreases then in an unbounded way with y, the distance of a from the rest of society. For sufficiently large values of y, however, parties make a difference: the party structure $a|bd|ce$ is stable and in the equilibrium, the party bd offers the ideal policy of b and wins.[27]

[26] This is in contrast to models which allow any policy to be an election candidate (see for example Mckelvey [13]).

[27] It is an equilibrium for this partition because a cannot win against the policy b, and c blocks any

Parties are therefore effective even without convexifying the policy space, i.e., even when they offer a position which can be offered in their absence. The reason is that the strategic environment, namely, the feasible set of actions and deviations for each player, is different in the presence of parties. In particular, politicians in the same party can commit not to run against each other. This is an additional aspect of the commitment role of parties.

5. ROBUSTNESS

5.1 Robustness to Winning-Motivated Politicians

The main result has illustrated that politicians need more than one dimension of policy in order to facilitate compromise. Another tool which may enable compromise is some form of transferable utility. If politicians are office-motivated and receive utility from winning per se, they may prefer to join a party even in the unidimensional policy space. Assume therefore that the utility of the players from the platform game is $U_i(\mathbf{q}(\pi)) = E(u_i(\mathbf{q}(\pi)) + \theta I(\mathbf{q}(\pi)))$, where $I(\mathbf{q}(\pi)) = 1$ if i is a member of a winning coalition and 0 otherwise, for $\theta > 0$. I also assume that given the same equilibrium, politicians prefer to be in smaller coalitions than in larger ones[28], since it is better to share benefits from office with fewer politicians. I then find:

PROPOSITION 4 *For small enough values of θ, parties are not effective when $k^*=1$ but may still be effective when $k^* > 1$.*

The main result is therefore robust to small changes in the incentives of politicians. Intuitively, when there are positive benefits from holding office, it may reduce politicians' incentive to cooperate in parties since these may force them to share benefits from holding office with other party members. Consider for example an equilibrium with two platforms offered by a left-wing and a right-wing party. Then, as explained in the intuition for Proposition 2, these platforms must be the ideal policies of the most moderate members of the parties and the equilibrium must also exist in the absence of parties. But, because these moderate politicians can run by themselves and are not in need of the commitment mechanism provided by parties, they actually prefer to dissolve their parties and enjoy the ego rents by themselves.

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deviation of the party *ce*. It is also stable because if each of the party members break their party they can only induce mixed strategy equilibria in which a wins with a strictly positive probability. These are worse for them, when a is located far enough.

[28] Condition 4 in definition 4 should therefore be restated:

4'. Either $U_i(\boldsymbol{\delta}(\pi')) > U_i(\boldsymbol{\delta}(\pi^{j-1}))$ for all $i \in S^j$ or $\boldsymbol{\delta}(\pi') = \boldsymbol{\delta}(\pi^{j-1})$ for all $j = 2, ..., m$.

If we employ this condition to the previous analysis as well, all the results hold.

5.2 Robustness to Campaign Costs

Assume now that there are costs γ for offering a platform. If a politician runs by herself, then she has to incur γ. However, a party can divide the campaign costs γ among its members. Moreover, the party can choose to divide it as it wishes, that is, some members may pay more or less than others. Thus, whenever a party S chooses to run for election with a platform $q_S \in Q_S$, it also chooses a cost structure $\{\gamma_i\}_{i \in S}$, such that $\sum_{i \in S} \gamma_i = \gamma$. The utility of a player i is therefore $E(u_i(\mathbf{q}(\pi)) - \gamma_i r_i(\mathbf{q}(\pi)),$ where $r_i(\mathbf{q}(\pi))$ is the probability that i runs (alone or within a coalition). Although this assumption should increase the attractiveness of joining a party, I find that the results of the model are still robust when costs are small enough:

PROPOSITION 5 *For small enough values of γ, parties are not effective when $k^*=1$ but may still be effective when $k^* > 1$.*

Intuitively, costs for running can create two effects. Although parties may be a useful vehicle for sharing the burden of costs, a free riding effect is also created. That is, players would prefer others to incur the costs of running. To see why the second effect dominates, consider once more the two-platform equilibrium example, offered by a right-wing party and a left-wing party. These platforms are equidistant from the median. But then, all party members, from either side of the median, prefer that the median would run in their stead; in terms of policy it increases their utility when they are risk averse and in terms of the costs it lets this burden fall on someone else's shoulders. Thus, it is profitable for them to break away from their party and induce a partition in which the median runs uncontested.

5.3 Robustness to Other Stability Concepts

In order to check and understand the role of the stability concept in the derivation of the main result, I have analyzed the game using other stability concepts. Among the cooperative notions, I have examined the Core, Stable Sets and a notion which I denote by bi-Core. The bi-Core stability concept restricts the deviations of players by allowing them either to break a party into two parties, or to form a new party from two existing parties. Among the non-cooperative notions, I have examined a membership game in which each player announces which coalition he would like to join. Details and definitions of this concepts are provided in an accompanying working paper, Levy [11]). The result of the robustness analysis is reported in the next proposition.

PROPOSITION 6 *When $k^* = 1$, parties are not effective under the Core, the bi-Core, Stable Sets and the membership game. When $k^* > 1$, the Core may yield empty predictions. However, parties may be effective under the bi-Core, Stable Sets and the membership game.*

The implications of this result are as follows. First, concerning the unidimensional policy space, the neutrality of parties is robust. In the multidimensional policy space, solutions that allow for too many deviations can create chaos and lead to instability. However, if deviations are restricted, or, if we analyze non-cooperative games, then stable structures exist and affect policies.

6. DISCUSSION

I conclude by discussing some of the assumptions of the model. In the model, I have assumed that voters are sincere, mainly for simplicity and as a useful benchmark. From an empirical point of view, whether voters vote strategically or not is an open question. Note however that in equilibrium with more than one platform, since platforms tie and a single vote can change the outcome, voters vote sincerely even if they reason strategically, as Feddersen [6] shows.[29] In the citizen-candidate model, under a unidimensional policy space, strategic voting can increase the set of equilibria (see Besley and Coate [2]).[30] Thus, assuming sincere voting in my model refines the set of equilibria in the unidimensional policy space. In the multidimensional policy space, however, sincere voting equilibria cannot always be supported by strategic voting strategies. Nevertheless, this has no effect on the results of this paper.

In the model, I have not assumed anything about how decisions are reached among party members and how specific platforms are chosen. Supposedly, there exists some explicit bargaining process or mechanism that selects a platform, a process which I have treated as a black box. Interpreted differently, the results of the paper can be viewed as an analysis of the different intra-party bargaining mechanisms. Since parties can be stable only when they offer some specific platforms, only bargaining protocols which yield these platforms can allow parties to form. Hence, we can interpret the model as predicting which intra-party bargaining mechanisms will survive.

Finally, I assume that parties have positions on all dimensions and hence I rule out the possibility of a single-issue party. Although some parties may have been established as single-issue parties, most of them evolve into multi-issue parties due to the nature of political decision making. The praxis of ruling a country demands decisions on all

[29] Morelli [15] also shows that there exists a substitution effect between strategic voters and strategic candidates or politicians. Whenever the politicians are strategic, the voters do not need to be.

[30] For example, with strategic voting, equilibria with many candidates may hold. When voters are sincere, an extreme candidate will drop from the race allowing the candidate closest to him to win. With strategic voting, candidates are reluctant to leave the race, because voters may 'threat' to switch their votes not to the closest candidate but to a candidate with an extreme position.

dimensions. Moreover, voters expect politicians to take decisions on all issues and hence form beliefs about how parties will act when facing these other issues.

Appendix

Proof of Proposition 1. The strategy set of any coalition is convex, nonempty and compact. The utility of each player is continuous and quasi-concave. For each coalition define the following set of "better than" by:

$P_S(\delta) = \{\delta'_S \in \Delta_S | U_i(\delta'_S, \delta_{-S}) > U_i(\delta_S, \delta_{-S})$ for all $i \in S\}$

This is an incomplete preference ordering but it is convex and its graph is open so there exists δ_S which is a best response of S for δ_{-S}, for any S and thus an equilibrium exists.[31] ■

For the next proofs, we use the following notation.

A partition π^k denotes a partition from which k deviations are needed to reach π^0, where deviations allow each sub-coalition to break a party into two, as defined in the text.

$O(\delta(\pi)) = (q_1, p_1; q_2, p_2; ..; q_i, p_i)$ denotes the political outcome of the vector of strategies $\delta(\pi)$. In particular, it describes a lottery in which outcome q_i is winning with probability p_i, whereas $EO(\delta(\pi))$ denotes the expected political outcome. For brevity, whenever there exists q_i such that $p_i = 1$, let $O(\delta(\pi)) = q_i$.

Finally, when the policy space is unidimensional, the notation $i < (>) j$ implies that i is to the left (right) of j, whereas m denotes the ideal policy of the median voter.

Proof of Proposition 2. The proof below holds both for the case presented in the paper in which there is one player on behalf of any policy preferences, and for the case in which all citizens can be players.

I prove first the following, important, Lemma.

Lemma 1 *When $k^* = 1$:*

(i) in π^0, there exists an equilibrium in which m runs and wins, and there may exist equilibria in which two platforms are offered, where each platform is equidistant from m. No other equilibria exist.

(ii) If for all $S \subset \pi$, for any $i,j \in S$, either $i>j> m$ or $i<j< m$, then any equilibrium in which one, two, or three parties/candidates participate, must be an equilibrium in π^0 as well. Moreover, it is an equilibrium for any $\pi' \in R(\pi)$. Any equilibrium with four or more parties running for election, cannot be a partisan equilibrium.

(iii) If $\exists S \subset \pi$, with $i, j \in S$ and $i \leq m < j$ or $i < m \leq j$, then there exist partisan equilibria with an outcome that differs from m.

[31] The proof follows Ray and Vohra [19] and uses the result of Shafer and Sonnenschein [27].

(iv) For all π, there exists an equilibrium in which m wins the election.

Proof: (i) Consider π^0 and equilibria with one platform offered. If only one platform is offered, it cannot be offered by more than one politician, since then one can withdraw without affecting the outcome. But then, if in equilibrium only one platform is offered, this platform must be m. If not, m can deviate and run against it.

If in equilibrium two platforms are offered, i and j, then since no loser runs in equilibrium, each must win with equal probability which implies that they are equidistant from m. Also, this equilibrium exists only if there is no k, $i < k < j$, that can win the election if she enters the race (obviously, $k \geq j$ ($k \leq i$) does not enter since then she will induce a less favorable outcome - i (j) winning).

Consider now equilibria with three or more platforms. Assume for example that 3 platforms are offered, i, k and j, with $i < k < j$. Assume that $i \leq EO(\boldsymbol{\delta}(\pi^0)) \leq k$. In this case, a politician that offers j is better off dropping from the race. She either induces the same outcome or the outcomes k or j which she prefers. Similar argument holds when $k \leq EO(\boldsymbol{\delta}(\pi^0)) \leq j$. Analogously, when more than 3 platforms are offered, there is always an extremist who rather deviate and not offer a platform.

(ii) In this part we consider only π in which for all $S \subset \pi$, for any $i, j \in S$, either $i > j > m$ or $i < j < m$.

Step 1: characterization of equilibria with one platform:

Assume that one platform is offered in equilibrium. By the tie-breaking rule, it is offered by one party only. But this platform must be m, because otherwise, m, who is not a party member, can run against it and win. By part (i), this equilibrium exists in π^0 as well.

Step 2: characterization of equilibria with two different platforms, each offered by one party:

In equilibria with two distinct platforms, each should win with probability .5. Therefore, they must be equidistant from m. Denote these platforms by R and L, with $L < m < R$, where $u_m(R) = u_m(L)$ and $\frac{1}{2}L + \frac{1}{2}R = m$. Consider, for example, the party S that offers L. Let $i = \max\{j | j \in S\}$ and assume that $i > L$. However, a deviation to the platform $i > L$ is both feasible for S and increases the utility for all its members, since $u_j(i) > u_j(m) \geq \frac{1}{2}u_j(L) + \frac{1}{2}u_j(R)$ for all $j < i < m$. Thus, in equilibrium, $L = \max\{j | j \in S\}$ and similarly, R is the ideal policy of the most moderate member of the party that offers this platform.

The above argument shows that any two platform equilibrium in π can potentially exist in π^0. The politicians with the ideal points L and R may participate in such an equilibrium and will not deviate given each other's strategies. The only obstacle for such an equilibrium to exist in π^0, as part (i) demonstrates, is an individual k, $L < k < R$, that can beat both whereas in π, $k \in S'$ for some $S' \subset \pi$ that does not run in equilibrium.[32] Without loss of generality, let $m < k < R$. For all $i \in S'$, $i > m$ by the construction of π. However, $u_i(k) > u_i(m) \geq \frac{1}{2}u_i(R) + \frac{1}{2}u_i(L)$ for all $i \in S'$ such that $i > k$, and $u_i(k) > u_i(R) > \frac{1}{2}u_i(R)+\frac{1}{2}u_i(L)$ for all $i \in S'$ such that $m < i < k$ since $u_m(R) = u_m(L)$. Hence, all members in S' rather deviate and offer k, which implies that the conjectured equilibrium cannot exist in π, a contradiction.

Moreover, note that any such equilibrium exists in π' for $\pi' \in R(\pi)$. The players that offer a platform do not wish to deviate and can hence block their coalitions from deviating. Players who do not run have a weakly smaller set of feasible platforms to deviate to in $\pi' \in R(\pi)$ compared to π. Also, if a player had wished to deviate and run in $\pi' \in R(\pi)$ he would have done so in π as well since all his coalition members, which belong to the same side of the median, would support this deviation as shown above.

Step 3: the impossibility of equilibria with two different platforms, where at least one platform is offered by more than one party:

Consider an equilibrium in which the platforms L and R are offered. It must be that $L < m < R$. Otherwise, m can run and win. Assume wlog that $u_m(R) \geq u_m(L)$. If R is offered only by one party, then R wins for sure and will win even if a party that offers L deviates and does not run. By the indifference condition, a party that offers L is therefore not best responding. Hence, in equilibrium, R is offered by more than one party. Let $q_S = R$. If whenever S offers $\varnothing$ the equilibrium outcome does not change, then a deviation to $\varnothing$ is profitable, a contradiction. Hence, whenever S offers $\varnothing$ it must be that R wins with a greater probability. But then $q_S = R \rightarrow \exists i \in S$ such that $u_i(L) > u_i(R)$. This implies $i < m < R \leq j$ for some $j \in S$, which contradicts the construction of π.

Step 4: the impossibility of equilibria with three platforms, each offered by one party:

Consider an equilibrium with three different platforms, i, k and j, for which $i < k < j$ where each is offered by one party. In such an equilibrium, m votes for k. Otherwise, if m votes for j for example then j wins and one of the other parties can drop from the race without affecting the outcome. Assume, wlog, that $i \leq EO(i,k,j) \leq k$. Consider the party S with $q_S = j$ (individuals cannot offer j by part (i)). It has to be that $q_S = \varnothing$ will affect the outcome and let k win with a greater probability. If the above is an equilibrium, then $\exists h \in S$ that votes for i. Otherwise, if h votes for k, $u_h(k) > \max\{u_h(i), u_h(j)\}$ by revealed preferences and hence she prefers k to any lottery of i, j and k, and if h votes for j, then $u_h(j) > u_h(k) \geq u_h(EO(i,k,j)) \geq u_h(i,k,j)$. Since $q_S = j$, $\exists f \in S$, $f \geq j > m$. But if $h \in S$ votes for i whereas m votes for k, this implies $h < m < f$, a

[32]It cannot be that S' offers L or R because this contradicts the claim that each party offers the position of its most moderate member.

contradiction to the construction of π.

Step 5: the impossibility of a partisan equilibria with three or more platforms, where some may be offered by more than one party:

Consider an equilibrium $\boldsymbol{\delta}(\pi)$ with k platforms, indexed $\{1, 2, ..., k-1, k\}$. In such an equilibrium, it cannot be that individuals offer the extreme platforms, by part (i). Assume that the expected outcome is to the left of the platform $k-1$. Consider party S that offers k. If instead, S chooses $\varnothing$, then either no change occurs or more weight is placed on either $k-1$ or on k (which may be offered by another party). If this is an equilibrium, then $\exists i \in S$ that objects to such a deviation. However, if i votes for k, then by revealed preferences, $u_i(k) > u_i(k-1) > u_i(EO(\boldsymbol{\delta}(\pi))) \geq u_i(\boldsymbol{\delta}(\pi))$, which implies that she would not object to such a deviation. Thus, $\exists i \in S$, i does not vote for k, implying that the above is not a partisan equilibrium because $k \in Q_S \rightarrow \exists j \in S$, $j \geq k$ and j votes for k. If the expected outcome is to the right of $k-1$, we can repeat the same exercise with the party that offers platform 1.

(iii) consider for example π^1 with the party $\{i, m\}$, $i < m$, that offers $m - x \in [i, m]$. If there is no $j \in (m - x, m + x]$, this is an equilibrium.

(iv) trivial.$\square$

We are now ready to prove the Proposition. The proof goes by induction.

Consider π^1, i.e., partitions in which there exists only one party, with two members. Assume that this party has members i and j with $i \leq m < j$ or $i < m \leq j$. But $(\pi^0, \boldsymbol{\delta}(\pi^0))$ such that $O(\boldsymbol{\delta}(\pi^0)) = m$ sequentially blocks $(\pi^1, \boldsymbol{\delta}(\pi^1))$ for any $\{\boldsymbol{\delta}(\pi^1) | O(\boldsymbol{\delta}(\pi^1)) \neq m\}$ since either i or j will break the party. Consider now a one-sided party which is right-wing, without loss of generality. That is, $i > j > m$. By Lemma 1, any one platform equilibrium must be m, and therefore $(\pi^1, \boldsymbol{\delta}(\pi^1))$ such that $O(\boldsymbol{\delta}(\pi^1)) = m$ is stable. Any two platform equilibria yields $EO(\boldsymbol{\delta}(\pi^0)) = m$ so that $(\pi^0, \boldsymbol{\delta}(\pi^0))$ with $O(\boldsymbol{\delta}(\pi^0)) = m$ sequentially blocks it (in any case, all such equilibria exists in π^0 as well). No equilibria with a biased outcome may exist. We therefore showed that the equilibrium in which m wins stabilizes any π^1 whereas no other equilibrium is stable.

Assume that the Proposition is true for $0 < l < k$. In particular, that $(\pi^l, \boldsymbol{\delta}(\pi^l))$ is stable if and only if $O(\boldsymbol{\delta}(\pi^l)) = m$. Consider π^k. Since m is an equilibrium, it is stable since no stable outcome provides any higher utility for any $i \in N$. Consider now other equilibria. If $\exists S \subset \pi^k$ with $i \leq m < j$ or $i < m \leq j$, then $(\pi^k, \boldsymbol{\delta}(\pi^k))$ for any $\{\boldsymbol{\delta}(\pi^k) | O(\boldsymbol{\delta}(\pi^k)) \neq m\}$ cannot be stable. In particular, $(\pi^{k-1}, \boldsymbol{\delta}(\pi^{k-1}))$ with $O(\boldsymbol{\delta}(\pi^{k-1}))) = m$ sequentially blocks it. If, on the other hand, for all $S \subset \pi$, and $i, j \in S$ either $i > j > m$ or $i < j < m$, then by Lemma 1, the only feasible equilibrium besides m yields $EO(\boldsymbol{\delta}(\pi^k)) = m$ which can be blocked by $(\pi^{k-1}, \boldsymbol{\delta}(\pi^{k-1}))$ with $O(\boldsymbol{\delta}(\pi^{k-1}))) =$

m.■[33]

Proof of Proposition 3. We will now prove that $k^* = N-1$ is a sufficient condition for parties to be effective when preferences are Euclidean and a Condorcet winner is a unique equilibrium in π, using two Lemmatas.

Let $Q_S(q) = \{q'|u_i(q') > u_i(q) \text{ for all } i \in S\}$. Note that $Q_S(q) \neq \varnothing \longleftrightarrow q \notin Q_S$ and that $q \notin Q_S(q)$ for any S.

LEMMA 2. Consider the equilibria of π^0, the set $\boldsymbol{\Delta}(\pi^0)$ with $\boldsymbol{\delta}(\pi^0) \in \boldsymbol{\Delta}(\pi^0)$. If there exists an i such that for all $\boldsymbol{\delta}(\pi^0) \in \boldsymbol{\Delta}(\pi^0)$, $\cap_{\boldsymbol{\delta}(\pi^0)\in\boldsymbol{\Delta}(\pi^0)} Q_{N\backslash i}(EO(\boldsymbol{\delta}(\pi^0))) \cap Q_{N\backslash i}(i) \neq \varnothing$, then parties are effective.

Proof: Assume the lemma is wrong and that each stable party structure can at most reproduce the same equilibrium outcomes as π^0. Consider the partition π for which $N\backslash i \in \pi$ and $i \in \pi$. That is, there exists one party composed of all players but (group) i. The $N\backslash i$ coalition can offer a platform $q^* \in \cap_{\boldsymbol{\delta}(\pi^0)\in\boldsymbol{\Delta}(\pi^0)} Q_{N\backslash i}(EO(\boldsymbol{\delta}(\pi^0))) \cap Q_{N\backslash i}(i)$. This platform will win the election because all voters but i prefer q^* to i, which is the only other possible election candidate.

For all coalition members, q^* dominates any equilibrium in any partition $\pi' \in R(\pi)$ because these partitions yield only the outcomes in π^0. Thus, no member breaks $N\backslash i$, rendering it stable with a policy outcome that differs from the ones in π^0, a contradiction.□

Then, obviously, if the equilibrium $\boldsymbol{\delta}(\pi^0)$ is unique, $O(\boldsymbol{\delta}(\pi^0)) = i$ for some player i, and $Q_{N\backslash i}(i) \neq \varnothing$, then stable parties exist and parties are effective. To complete the proof of the Proposition, it is left to show that if $k^* = N-1$, then $Q_{N\backslash i}(i) \neq \varnothing$.

LEMMA 3. If $k^* = N-1$, then $i \notin Q_{N\backslash i}$.

Proof: When preferences are Euclidean, then the Pareto set of a coalition identifies with the convex hull of the member's ideal points.[34] The convex hull of the N players is a simplex combining all the ideal points, with sub-simplices that are the lines connecting each two members. Clearly, $k^* \leq N-1$. Assume, to the contrary, that $i \in Q_{N\backslash i}$. By convexity, $Q_N = Q_{N\backslash i}$. Let k^*_H denote the minimal number of vectors that spans a space V such that $Q_h \in V$. Hence, $k^*_N = k^*_{N\backslash i} \leq N-2 < N-1$, a contradiction.■

Proof of Proposition 4. Consider $k^* = 1$. Note that parts (i), (ii), and (iii) of Lemma 1 hold for a small enough θ. We will prove the proposition by induction on the number of deviations that it takes to reach π^0.

Consider π^1. If $m \in S$, for S such that $|S| > 1$, then $(\pi^1, \boldsymbol{\delta}(\pi^1))$ is not stable for

[33] When players are risk neutral, then Lemma 1 holds as well. If we assume that players prefer to be in smaller coalitions than in larger ones, given everything else equal, then the qualitative result of Proposition 2 holds; parties are not stable and hence cannot be effective for $k^* = 1$. Proposition 3 holds without qualifications when we assume risk neutrality.

[34] See Miller [14].

any $\boldsymbol{\delta}(\pi^1)$ because $(\pi^0, \boldsymbol{\delta}(\pi^0))$ with $O(\boldsymbol{\delta}(\pi^0)) = m$ sequentially blocks it when m breaks his party. $O(\boldsymbol{\delta}(\pi^0)) = m$ is the best possible outcome for m who then can get both her ideal policy and θ. If on the other hand $\exists i, j \in S$, with $i < m < j$, then $(\pi^1, \boldsymbol{\delta}(\pi^1))$ can be stable only if $O(\boldsymbol{\delta}(\pi^1)) \to m$ and S participates in election. Otherwise, if for example $O(\boldsymbol{\delta}(\pi^1)) - m > \varepsilon$ for a small ε, i rather break the coalition and even give up θ to reach $(\pi^0, \boldsymbol{\delta}(\pi^0))$ with $O(\boldsymbol{\delta}(\pi^0)) = m$.[35]

Consider π^1 such that $\exists i, j \in S$ with $i > j > m$. In a two platform equilibria, if S participates in election, it offers j and this equilibrium exists in π^0 as well. Then, it cannot be stable because j rather break the party and play the same equilibrium in π^0 in which she can win $\frac{\theta}{2}$ by herself. If S does not participate in election, still, any equilibrium must exist in π^0 as well. Since politicians prefer to be in smaller coalitions than in larger ones, the partition cannot be stable since any equilibrium is sequentially blocked by the same equilibrium in π^0 or by the equilibrium in which $O(\boldsymbol{\delta}(\pi)) = m$.

Assume that the Proposition is true for $l < k$. In particular, assume that if for all $S \subset \pi$, for any $i, j \in S$, either $i > j > m$ or $i < j < m$, then $(\pi, \boldsymbol{\delta}(\pi))$ is not stable for any $\boldsymbol{\delta}(\pi)$ whereas otherwise, $(\pi, \boldsymbol{\delta}(\pi))$ is stable only if $O(\boldsymbol{\delta}(\pi)) \to m$.

Consider π^k. If for all $S \subset \pi^k$, for any $i, j \in S$, either $i > j > m$ or $i < j < m$, then $(\pi^k, \boldsymbol{\delta}(\pi^k))$ is not stable for any $\boldsymbol{\delta}(\pi^k)$. To see this, note that any equilibrium exists in π^0 by Lemma 1. Moreover, any equilibrium exists also in π^l for any $\pi^l \in R(\pi^k)$. Then, if a coalition breaks to $\pi^l \in R(\pi^k)$, there exists $(\pi^0, \boldsymbol{\delta}(\pi^0))$ that sequentially blocks it, which is the same $(\pi^0, \boldsymbol{\delta}(\pi^0))$ that sequentially blocks $(\pi^l, \boldsymbol{\delta}(\pi^l))$ for $\boldsymbol{\delta}(\pi^k) = \boldsymbol{\delta}(\pi^l) = \boldsymbol{\delta}(\pi^0)$. The initial deviation of players from π^k to π^l is profitable since at the end result, in π^0, they are in smaller coalitions and can win θ by themselves.

If, on the other hand, there exists $S \subset \pi^k$, with $i, j \in S$, and either $i < m \leq j$ or $i \leq m < j$, then $(\pi^k, \boldsymbol{\delta}(\pi^k))$ with $O(\boldsymbol{\delta}(\pi)) \nrightarrow m$ is not stable. Consider for example $O(\boldsymbol{\delta}(\pi)) - m > \varepsilon$ for a small ε. Then i has an incentive to break π^k to $\pi^{k-1} \in R(\pi^k)$. Either $(\pi^{k-1}, \boldsymbol{\delta}(\pi^{k-1}))$ is stable, which implies that $O(\boldsymbol{\delta}(\pi^{k-1})) \to m$, or it is not stable. If it is not stable, then there exists $(\pi^l, \boldsymbol{\delta}(\pi^l))$ which sequentially blocks $(\pi^{k-1}, \boldsymbol{\delta}(\pi^{k-1}))$ with $O(\boldsymbol{\delta}(\pi^l)) \to m$. However, in this case, $(\pi^l, \boldsymbol{\delta}(\pi^l))$ also sequentially blocks $(\pi^k, \boldsymbol{\delta}(\pi^k))$. On the other hand, $(\pi^k, \boldsymbol{\delta}(\pi^k))$ with $O(\boldsymbol{\delta}(\pi)) \to m$ may be stable.

For $k^* > 1$, we can use the two dimensional example with 3 players which holds for all $\theta > 0$.■

Proof of Proposition 5 *(sketch):* When there are campaign costs, then the following are the equilibria of the model. First, in π^0, the only one platform equilibria are

[35] Note that a party S with $i < m < j$ cannot participate in a two platform equilibrium, in which R and L are offered for $R - m = m - L$. The reason is that if S offers L for example, then it should deviate and offer $m \in Q_S$. This provides S with θ instead of $\frac{\theta}{2}$ and utility from m instead of $E(m)$.

in the interval $[m - x(\gamma), m + x(\gamma)]$ where $x(\gamma) \to 0$ when $\gamma \to 0$. Note that the binding constraint is set by m himself; that is, if costs are too low, m runs against another platform and wins, so a one platform equilibria cannot have a platform which is too far from m. The set of two platform equilibria is also the same for small enough campaign costs as for $\gamma = 0$. Since there are no three or more platform equilibria without costs, moreover, no such equilibrium exists with costs.

Consider now coalition structures with one sided parties, that is, with $i > j > m$ or with $i < j < m$ for all $i, j \in S$ for any S. The one platform equilibria are as above (since the constraint on $x(\gamma)$ is that m does not run and upset the equilibrium), whereas the two platform equilibria are such that each party offers the position of its most moderate member. Otherwise it can keep the same cost structure, deviate towards the median, win, and increase the utility of all its members. This equilibrium has to exist also in π^0 when the costs are small enough, due to the same reasons as in Lemma 1. Similarly to Lemma 1, other equilibria cannot exist with one sided coalitions.

We will now prove the Proposition by induction. Consider first π^1. Assume that the coalition is one-sided, that is, with $i > j > m$ or with $i < j < m$. Then, the only stable equilibrium is the one in which m runs, since otherwise party members break their coalition. If they run in a two platform equilibrium the equilibrium in which m runs in π^0 dominates in terms of costs and ideology. If the party is two-sided, such that $i \leq m \leq j$, consider an equilibrium which, without loss of generality, is biased from m to the right. Then i is better off breaking it and letting m run in π^0, both in terms of costs and ideology. Even if the members i and j are such that $i < m < j$ and they offer m, they would still rather break the party and let m run by himself and incur the costs. Thus, for a two-sided coalition to be stable, m has to be a member of the coalition, it has to offer m, and m has to incur all costs.

Assume that the Proposition is true for all $l < k$, and in particular that all stable equilibrium outcome converge to m, and now consider π^k. Assume first that all the coalitions are one-sided. If the equilibria in π^k are one platform equilibria, then the outcome converges to m. If it is a two platform equilibrium, each player who belongs to a coalition prefers an equilibrium in which m runs, both in terms of costs and ideology. Thus, the only stable equilibria have m winning. If there exist two-sided coalitions, then similarly, if the outcome is biased from the median to the right, any left wing member will prefer to break this structure to reach an equilibrium in which m is winning, an equilibrium which exists for any π.

For the case of $k^* > 1$, we can use again the example of the two dimensional society to derive effective parties.■

Proof of Proposition 6. See Levy [11].■

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