Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence (IJCAI 2015)

Optimal Electric Vehicle Charging Station Placement

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Abstract

Many countries like Singapore are planning to introduce Electric Vehicles (EVs) to replace traditional vehicles to reduce air pollution and improve energy efficiency. The rapid development of EVs calls for efficient deployment of charging stations both for the convenience of EVs and maintaining the efficiency of the road network. Unfortunately, existing work makes unrealistic assumption on EV drivers' charging behaviors and focus on the limited mobility of EVs. This paper studies the Charging Station PLacement (CSPL) problem, and takes into consideration 1) EV drivers' strategic behaviors to minimize their charging cost, and 2) the mutual impact of EV drivers' strategies on the traffic conditions of the road network and service quality of charging stations. We first formulate the CSPL problem as a bilevel optimization problem, which is subsequently converted to a single-level optimization problem by exploiting structures of the EV charging game. Properties of CSPL problem are analyzed and an algorithm called OCEAN is proposed to compute the optimal allocation of charging stations. We further propose a heuristic algorithm OCEAN-C to speed up OCEAN. Experimental results show that the proposed algorithms significantly outperform baseline methods.

1 Introduction

Due to the world's shortage of fossil fuels and the serious environmental pollution from burning them, seeking alternative energy has become a crucial topic of research. Transportation is one of the main consumers of energy and contributors to air pollution. Electric Vehicles (EVs) move pollution away from urban areas and electricity can be efficiently transformed from both traditional fossil fuels and promising renewable energies like solar energy and tidal energy. EVs, as a replacement of traditional internal combustion engine vehicles, provides an environment-friendly solution to modern cities' transportation. A rapid growth of EVs has been seen in recent years along with the rising popularity of the notion of smart cities [Schneider *et al.*, 2008]. This calls for an efficient deployment of relevant supporting facilities, among which charging facility is of top priority. Although EVs can be charged at home, it is time consuming and usually takes 6 to 8 hours, which is at least 12 times the time it takes at charging stations with high voltage [Hess *et al.*, 2012]. The distribution of charging stations determines EV drivers' accessibility to energy sources, and consequently affects the EV flow and traffic conditions in the road network.

There are some existing works studying the CSPL problem. These studies optimize different objectives such as investment cost, maintenance cost [Liu *et al.*, 2013], access cost [Chen *et al.*, 2013], construction cost [Lam *et al.*, 2013; 2014], and coverage of charging stations [Frade *et al.*, 2011; Wang *et al.*, 2010]. A framework based on hitting set problem is used in work of Funke et al. [Funke *et al.*, 2014], which aims to guarantee energy supply in any shortest path. He et al. [He *et al.*, 2013] used a multinomial logit model to compute EVs' charging choices and formulated the problem as a single level optimization. None of these works capture the interrelationship between EV drivers' charging activities and traffic congestion. Additionally, most existing works ignore the selfish behaviors of EV drivers, which affect the traffic condition and queuing time at charging stations and then results in different levels of EV drivers' satisfaction.

This paper studies the Charging Station PLacement (CSPL) problem while considering the mutual impact between allocation of charging stations and EV drivers' charging activities. Queuing time in charging stations is taken into account since long queuing time has been shown to significantly affect the adoption of EVs [Pierre *et al.*, 2011; Hidrue *et al.*, 2011]. More importantly, inspired by the works that model the interactions between driver activities and traffic [Gan *et al.*, 2013; 2015], the influence of charging activities on traffic condition, especially during peak hours when

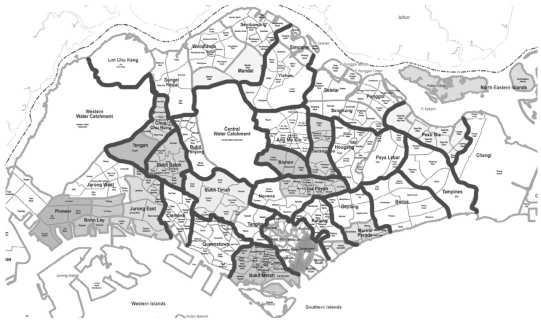


Figure 1: Zonal Map of Singapore

traffic jams are more salient, is considered. Additionally, since EVs are driven by their owners without any centralized control, it is essential to model how EVs' movements are affected by the distribution of charging stations.

The first contribution of this paper is a realistic CSPL model that considers EV drivers' strategic behaviors to minimize their charging cost. Besides, traffic condition and queuing time in charging stations, which play a vital role in the EV diffusion, are also considered in our model. The CSPL problem is formulated as a bilevel optimization problem, where the upper-level objective is the social cost to be minimized by the government who acts as market regulators to decide the placement of charging stations; and the sub-level is a charging game played by EV drivers, which falls into the class of congestion games as we show in this paper. The second contribution of our work is that through exploiting the structure of the charging game's Nash Equilibrium, we successfully obtain an equivalent single level optimization problem and propose algorithm OCEAN to compute the optimal solution. Unfortunately OCEAN cannot scale up due to the integer variables and large searching space. Therefore, we propose an effective heuristic algorithm OCEAN-C to speed up the computation and solve realistic CSPL problems. The third contribution of this paper is that we conduct extensive experiments based on the Singapore scenario. Experimental results show that our algorithms significantly outperform some baseline methods.

2 Charging Station Placement in Singapore

While Singapore enjoys a relatively good quality of air, it is important to be mindful that land transport is one of the major contributors to air pollution. In Singapore, land transport contributes about 20% of the total carbon emission, and 75% of air pollution is attributed to motorised traffic [LTA, 2014]. Singapore government is planning on introducing EVs to replace traditional vehicles. Since 2011, the authorities have been studying the feasibility of EVs on Singapore roads. According to the electric-car manufacturer BYD Asia-Pacific, Singapore has the "best potential" to implement EVs because of its advanced power grid system [Woo, 2014].

To adopt EVs islandwide, the government needs to foot the bill for constructing the necessary infrastructure. However, simply considering the financial perspective is insufficient. Without efficient planning of charging stations, the introduction of EVs can result in serious traffic congestion. Unlike most urban metropolises in other countries, Singapore has a very small territory of $718.3km^2$ (Figure 1), with a maximum east west and north south distance of 42 km and 23 km respectively. Thus, the limited range of EVs (between 160 and 320 km [EVMileage, 2014]) is not an issue in Singapore. However, Singapore has a large population, resulting in a large population of vehicles. In 2014, the total number of motor vehicles in Singapore has grown to more than $970,000$. This implies that rather than limited range, the queuing time of EVs is likely to have a much larger impact on traffic.

Minimizing traffic congestion is our first consideration while planning the location of charging stations. EV drivers' charging activities influence traffic congestion. In turn, EV drivers make decisions to choose charging destinations considering traffic congestion. Besides this, we also study how queuing time in charging stations affects EV drivers' charging decisions; this is because long queuing time translates to larger space requirements to accommodate queuing EVs and also in frustrated EV drivers. By exploiting EV drivers' selfish charging behaviors, we model the interactions among the allocation of charging stations, EVs' strategic charging activities, traffic congestion and queuing time in charging stations, and propose OCEAN and OCEAN-C to efficiently compute the optimal solution.

3 Charge Station Placement Problem

The goal of CSPL is to optimize the charging station placement in a set of interconnected zones so that the social cost (defined in Section 3.3) is minimized. In this section, we begin with defining the topology of the studied area, and the costs of EVs. Then we propose a congestion-game-based interpretation of the CSPL problem, and formulate the CSPL problem as a bilevel optimization program.

3.1 Zones

The region to be analyzed is divided into n zones $\mathcal{N} = \{1, 2, ..., n\}$. We use a matrix $A = \{a_{ij}\}_{n\times n}$ to represent the adjacency relationship between different zones, with $a_{ij} = 1$ (or $a_{ij} = 0$) representing zones i and j as adjacent (or not). A pair of zones are adjacent if they share a geographical border and there is a road between them. A zone is defined to be adjacent to itself, i.e., $a_{ii} = 1$. The distances between different pairs of zones are denoted as a matrix $D = \{d_{ij}\}_{n\times n}$, where d_{ij} is the average length of trips of EV drivers that reside in zone i and charge in zone j, estimated by the distance between the centers of the pair of zones. For all $i \in \mathcal{N}$, d_{ii} is set as the radius of zone i. We divide Singapore into a number of zones according to the conventional partitioning method (Figure 1).

3.2 The EV Model

In each zone i, there is an estimated number γ_i of resident EVs who charge at charging stations instead of at home. Let x_i denote the number of chargers in zone i, and y_i denote the number of EVs that charge in zone i everyday. When an EV needs to charge, the driver chooses to charge at an adjacent zone that minimizes her cost for charging. The assumption is realistic in the sense that each zone represents a relatively large area and an EV may not be willing to drive too far to charge. We will relax this assumption and allow EVs to charge at nonadjacent zones in Section 5.2. We assume that electricity prices are the same in stations, so that EVs consider only the time cost, which is a combination of the travel time and the queuing time.

Travel time. Travel time depends on the distance and traffic condition (i.e., congestion level) on the road, which can be denoted with Eq. 1, where λ is a constant and α_{ij} is the congestion level of the road from zone i to zone j [Boarnet *et al.*, 1998].

$$f_{ij} = \lambda d_{ij} \alpha_{ij}, \tag{1}$$

When there are more than one road directly leading from zone i to zone j, we consider the average traffic condition, road capacity and distance. Following transportation science research [Banner and Orda, 2007; Bertini, 2006; Sweet, 2011; Wang *et al.*, 2013], the congestion level depends on the traffic on the road, and is defined as Eq. 2, where α^0_{ij} is the normal traffic congestion caused by driving activities with any other objectives except for charging which can be estimated by the ratio of traffic flow on the road to the capacity of the road.

$$\alpha_{ij} = \alpha^0_{ij} + k_{ij} \frac{y_{ij}}{\tau}, \tag{2}$$

$k_{ij} \frac{y_{ij}}{\tau}$ represents the congestion caused by EVs' charging activities. The parameter k_{ij} is in inverse proportion to the road capacity, y_{ij} represents the charging flow from zone i to zone j, and $\frac{1}{\tau}$ is the fraction of EVs that charge during peak hours. Specifically, α_{ii} is the congestion level within zone i, which is a function of the average congestion level of the main roads in zone i. Note that we particularly focus on studying the peak hour period because the worst case traffic congestion usually occurs during peak hours, while to some EVs the demand for charging during peak hours is inevitable, e.g., EVs may run out of electricity while their owners have to use them immediately.

Queuing time. We consider charging activities' influence on traffic and charging stations' queuing time during peak hours, such that the worst case is optimized. Since we assume that 1 in every τ EVs would charge at charging stations during peak hours, the number of EVs that arrive in zone i during peak hours is $\frac{y_i}{\tau}$. The estimated queuing time is defined to be directly proportional to the number of EVs charging during the peak hours. The queuing time is then

$$g_i = \frac{y_i}{\mu \tau x_i}, \tag{3}$$

where μ is the serving rate of chargers, i.e., the number of EVs that a charger can serve within a unit period.

3.3 A Congestion-Game-Based Interpretation

According to the definitions in Section 3.2, the cost associated with a road or a charging station is determined only by the number of EVs using this facility when background traffic and charger numbers in the zones are decided. We can thus treat zones and roads as congestible elements, and formulate the CSPL as a congestion game [Nisan *et al.*, 2007] consisting of the following components:

- Two sets of congestible elements are the zones $\mathcal{N} = \{1, ..., n\}$ and the roads $\mathcal{R} = \{\langle i, j\rangle | i, j \in \mathcal{N}, a_{ij} = 1\}$, where $\langle i, j\rangle$ represents a road leading from zone i to adjacent zone j. The costs defined in Eqs. 1 and 3 are respectively taken as delay functions for the congestible elements $i \in \mathcal{N}$ and $\langle i, j\rangle \in \mathcal{R}$. We denote them as $g_i(\cdot)$ and $f_{ij}(\cdot)$, which take the numbers of EVs choosing the elements as variables.
- The EVs residing in one zone are one type of players and are identically treated, such that they have the same strategy space and adopt the same strategy in the equilibrium. Specifically, each EV plays a mixed strategy, which is a probability distribution over a set of pure strategies. Each pure strategy for EVs in zone i is to choose zone i or an adjacent zone j to charge and use corresponding road from i to j. We use $\mathbf{p}_i = \{p_{ij}\}$ to denote the mixed strategy. Assume that EVs only charge in adjacent zones[1], $a_{ij} = 0 \Rightarrow p_{ij} = 0, \forall i, j$. Therefore, the congestion of elements in $\mathcal{R}$ and $\mathcal{N}$ are respectively

$$y_{ij} = \gamma_i p_{ij}, \tag{4}$$

$$y_j = \sum_{i \in \mathcal{N}} y_{ij}. \tag{5}$$

Furthermore, let $\mathbf{P} = \langle \mathbf{p}_i \rangle$ denote the strategy profile for all players and $\mathcal{A}_i = \{j | a_{ij} = 1\}$. The cost of each type i of players is a function of $\mathbf{P}$, defined as

$$C_i(\mathbf{P}) = \sum_{j \in \mathcal{A}_i} p_{ij}(g_j(y_j) + f_{ij}(y_{ij})). \tag{6}$$

3.4 Bilevel Optimization Formulation

In the above defined congestion game, EVs want to minimize their charging cost. We adopt the Nash equilibrium as the solution concept, in which all EVs are assumed to be aware of the strategies of other EVs, and no EV has the incentive to deviate to other mixed strategies. Formally, we have

$$\mathbf{p}_i \in \arg\min_{\mathbf{p}'_i} C_i(\mathbf{P}_{-i}, \mathbf{p}'_i), \forall i \in \mathcal{N}.$$

The government authority is able to induce different equilibria through allocating charging stations in the zones (as the number of chargers affects EVs' cost in Eq. 3). Given a fixed amount B of budget, the government's goal is to decide the optimal allocation of charging stations, so that the social cost in equilibrium is minimized. Specifically, we consider the overall benefits, and use the sum of costs of all EVs as the social cost, thus

$$C(\mathbf{P}) = \sum_{i \in \mathcal{N}} C_i(\mathbf{P}). \tag{7}$$

Our framework can be easily extended to optimize other social cost functions.

Let $\mathbf{P}_{-i}$ denote the strategy profile of EVs except type i EVs. Therefore, the CSPL problem is formulated as the fol-

[1]This can also be assumed as a set of en-route zones.

lowing bilevel program.

$$\textbf{P1:} \quad \min_{\mathbf{x},\mathbf{P}} C(\mathbf{P}), \tag{8}$$

$$\text{s.t.} \quad \mathbf{p}_i \in \arg\min_{\mathbf{p}'_i} C_i(\mathbf{P}_{-i}, \mathbf{p}'_i), \forall i \in \mathcal{N}, \tag{9}$$

$$\sum_{i\in\mathcal{N}} x_i \le B, x_i \in \mathbb{N}, \tag{10}$$

$$\sum_{j\in\mathcal{A}_i} p_{ij} = 1, \forall i \in \mathcal{N} \tag{11}$$

$$p_{ij} = 0, \forall i \in \mathcal{N}, \forall j \notin \mathcal{A}_i, \tag{12}$$

$$p_{ij} \ge 0, \forall i, j \in \mathcal{N}. \tag{13}$$

Note that we compute the best equilibrium with respect to the social cost, in case there exist multiple equilibria leading to different social costs. In practice, the government may incentivize EV drivers to choose the equilibrium with best social cost by providing small incentives (e.g., slightly decrease the charging price at a station). Such ideas have been widely used, such as tie-breaking in security games [Tambe *et al.*, 2014].

4 Solve the CSPL Problem

In this section, we present our methods for solving the CSPL problem, which is a bilevel problem with a congestion game as the sub-problem. The bilevel problem has an upper-lever non-linear objective and multiple non-linear sub-level optimization objectives, which makes it complex and intractable with existing solvers. Therefore, we need to search for efficient approaches to compute the mixed strategy Nash equilibria of the congestion game, as well as the optimal solution for the bilevel CSPL problem. We begin with reformulating the problem by analyzing conditions of strategy deviation.

4.1 Deviation of Strategies

Given a strategy profile $\mathbf{P}$, when type i EVs change the charging strategy, we denote the strategy change as an n-dimensional vector $\Delta\mathbf{p} = (\Delta_1, ..., \Delta_n)$, such that

$$\sum_{j\in\mathcal{N}} \Delta_j = 0, \tag{14}$$

$$-p_{ij} \le \Delta_j \le 1 - p_{ij}, \forall j \in \mathcal{A}_i. \tag{15}$$

When type i players change their strategy from $\mathbf{p_i}$ to $\mathbf{p_i}' = \mathbf{p_i} + \Delta\mathbf{p}$, recall that y_{ij} denotes the charging flow from zone i to zone j and y_j denotes the number of EVs that charge in zone j, we have $y'_{ij} = y_{ij} + \gamma_i\Delta_j$, $y'_j = y_j + \gamma_i\Delta_j$, and the change in type i EVs' cost can be formulated as:

$$\begin{aligned}\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) &= C_i(\mathbf{P}_{-i}, \mathbf{p_i}') - C_i(\mathbf{P}) \\ &= \sum_{j\in\mathcal{A}_i} [p_{ij}(\lambda d_{ij} k_{ij} \frac{\gamma_i\Delta_j}{\tau} + \frac{\gamma_i\Delta_j}{\mu\tau x_j}) \\ &\quad + \Delta_j(\lambda d_{ij}\alpha_{ij} + \lambda d_{ij} k_{ij} \frac{\gamma_i\Delta_j}{\tau} + \frac{y_j}{\mu\tau x_j} + \frac{\gamma_i\Delta_j}{\mu\tau x_j})] \\ &= \sum_{j\in\mathcal{A}_i} [(\frac{p_{ij}\gamma_i}{\tau}(\lambda d_{ij} k_{ij} + \frac{1}{\mu x_j}) + \lambda d_{ij}\alpha_{ij} + \frac{y_j}{\mu\tau x_j})\Delta_j \\ &\quad + (\lambda d_{ij} k_{ij} \frac{\gamma_i}{\tau} + \frac{\gamma_i}{\mu\tau x_j})\Delta_j^2].\end{aligned} \tag{16}$$

For the ease of description, we rewrite it as

$$\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = \sum_{j\in\mathcal{A}_i} (\xi_{ij}\Delta_j + \eta_{ij}\Delta_j^2). \tag{17}$$

In a Nash equilibrium, no player has the incentive to deviate, we therefore can reformulate the CSPL problem in **P1** as

$$\textbf{P2:} \quad \min_{\mathbf{x},\mathbf{P}} C(\mathbf{P}), \tag{18}$$

$$\text{s.t.} \quad \Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \ge 0, \forall i \in \mathcal{N}, \forall \Delta\mathbf{p}, \tag{19}$$

10–13.

Here Eq. 19 replaces Eq. 9 as a new criterion for Nash equilibrium. However, the above reformulation still involves two levels of optimization, as $\Delta\mathbf{p}$ in Eq. 19 is continuous. To further resolve the difficulty, we propose a *simple deviation approach*, and reduce the program to a single level optimization problem as described next.

4.2 Simple Deviation Approach

We define a special type of deviation called *simple deviation*. As we will show below, one property of the CSPL problem is that if any simple deviation cannot help reduce the player's cost, neither can any other (more complex) deviation. Therefore, the equilibrium criterion can be simplified by focusing on only simple deviations.

Definition 1 (simple deviation). *A simple deviation of type i player is a strategy change, where only the probabilities of a pair of pure strategies are changed (one increases and the other decreases by the same amount), while the probabilities of all the other pure strategies remain unchanged. A simple deviation is denoted as a tuple $\langle l, h, \delta\rangle$ with $\delta > 0$, which corresponds to a deviation vector $\Delta\mathbf{p}$, such that $\Delta_l = -\delta$, $\Delta_h = \delta$, and $\Delta_j = 0, \forall j \notin \{l, h\}$.*

Lemma 1. *Given a strategy profile $\mathbf{P}$ with $p_{il} > 0$, type i player cannot reduce her cost through a simple deviation from pure strategy l to h (i.e., reduce p_{il} and increase p_{ih}), if and only if $\xi_{ih} \ge \xi_{il}$.*

Proof. Given a simple deviation $\langle l, h, \delta\rangle$, the change in type i player's cost is

$$\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = (\eta_{il} + \eta_{ih})\delta^2 + (\xi_{ih} - \xi_{il})\delta, \tag{20}$$

which is a quadratic function of δ. Therefore, player i cannot reduce her cost through a simple deviation from pure strategy l to h, if and only if $\Delta C_i \ge 0, \forall\delta \in [0, p_{il}]$. According to Eq. 16, we have $\eta_{il} + \eta_{ih} > 0$, so that $\Delta C_i \ge 0$ holds for all $\delta \in [0, p_{il}]$ if and only if $\xi_{ih} - \xi_{il} \ge 0$. This is because 1) if $\xi_{ih} - \xi_{il} \ge 0$, $\Delta C_i \ge 0$ for all $\delta \ge 0$; 2) if $\Delta C_i \ge 0$ holds for $\delta \in [0, p_{il}]$, the derivative of ΔC_i at $\delta = 0$ should be non-negative (otherwise, $\Delta C_i < 0$ in a right neighbourhood of 0 as ΔC_i is continuous with respect to δ), so that $\frac{\mathrm{d}\Delta C_i}{\mathrm{d}\delta}(0) = \xi_{ih} - \xi_{il} \ge 0$. ☐

Lemma 2. *If a player cannot reduce her cost by any simple deviation, then she can neither reduce her cost by any strategy deviation.*

Proof. We first show that any strategy deviation of a player can be decomposed into a set of simple deviations. Actually, given a strategy deviation vector $\Delta\mathbf{p} = (\Delta_1, ..., \Delta_n)$, we can define two sets $\mathcal{L} = \{i | i \in \mathcal{N}, \Delta_i < 0\}$ and $\mathcal{H} = \{i | i \in \mathcal{N}, \Delta_i > 0\}$, and then implement deviation $\Delta\mathbf{p}$ by simply deviating from each $l \in \mathcal{L}$ to each $h \in \mathcal{H}$ by an amount $\delta_{hl} = |\Delta_l| \cdot \frac{\Delta_h}{\sum_{i \in \mathcal{H}} \Delta_i}$.

Therefore, the cost change of player i can also be decomposed and it is always larger than the sum of cost changes caused by the set of simple deviations defined above, i.e.,

$$\begin{aligned}
\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) &= \sum_{j \in \mathcal{A}_i} (\xi_{ij}\Delta_j + \eta_{ij}\Delta_j^2) \\
&= \sum_{l \in \mathcal{L}} (\xi_{il}(-\sum_{h \in \mathcal{H}} \delta_{hl}) + \eta_{il}(-\sum_{h \in \mathcal{H}} \delta_{hl})^2) + \\
&\quad \sum_{h \in \mathcal{H}} (\xi_{ih}(\sum_{l \in \mathcal{L}} \delta_{hl}) + \eta_{ih}(\sum_{l \in \mathcal{L}} \delta_{hl})^2) \\
&\geq \sum_{l \in \mathcal{L}} (\xi_{il}(-\sum_{h \in \mathcal{H}} \delta_{hl}) + \eta_{il}(\sum_{h \in \mathcal{H}} \delta_{hl}^2)) + \\
&\quad \sum_{h \in \mathcal{H}} (\xi_{ih}(\sum_{l \in \mathcal{L}} \delta_{hl}) + \eta_{ih}(\sum_{l \in \mathcal{L}} \delta_{hl}^2)) \\
&= \sum_{l \in \mathcal{L}} \sum_{h \in \mathcal{H}} (\eta_{il} + \eta_{ih})\delta_{hl}^2 + (\xi_{ih} - \xi_{il})\delta_{hl}
\end{aligned}$$

According to Lemma 1, when no simple deviation can reduce the player's cost, we have $\xi_{ih} \geq \xi_{il}$ for all $l \in \mathcal{L}$ and $h \in \mathcal{H}$, and it follows that the last expression is non-negative, so $\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \geq 0$. Since $\Delta\mathbf{p}$ and i are arbitrary, thus no player can reduce her cost by any strategy deviation. □

Proposition 3. *A strategy profile* $\mathbf{P}$ *forms a Nash equilibrium if and only if* $\xi_{ih} \geq \xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{il} > 0$.

Proof. The proof follows directly from Lemmas 1, 2 and the converse direction of Lemma 2, i.e., $\xi_{il} \leq \xi_{ih}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{ij} > 0 \Leftrightarrow$ no player can reduce her cost through a simple deviation $\Leftrightarrow$ no player can reduce her cost through any deviation $\Leftrightarrow$ Nash equilibrium. Note that the converse direction of Lemma 2 holds because simple deviation is a special case of general deviation. □

According to Proposition 3, we propose a substitute approach OCEAN (Optimizing eleCtric vEhicle chArging statioN placement) to compute the optimal solution of CSPL problem **P2**. The key idea of OCEAN is that we replace the infinite number of constraints specified by Eq. 19 with a finite number of constraints based on Proposition 3. OCEAN can be formulated as follows.

$$\textbf{P3:} \quad \min_{\mathbf{x}, \mathbf{P}} C(\mathbf{P}), \tag{21}$$

$$\text{s.t.} \quad \xi_{ih} \geq \xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{il} > 0, \tag{22}$$

10–13.

The above program is a single-level non-linear optimization problem and can be handled by a standard non-linear optimization solver.

4.3 Speeding Up OCEAN

OCEAN has a large number of non-linear constraints, as well as mixed integer variables, which make it unscalable. Therefore, we propose a heuristic algorithm OCEAN-C (OCEAN with Continuous variables) in Algorithm 1 to compute the optimal solution in two steps. Firstly, we relax $\mathbf{x}$ to be continuous variables and solve the optimal solution $\mathbf{x}^*$ of **P3**. Since the number of chargers in $\mathbf{x}^*$ are not integers, we round $\mathbf{x}^*$ to $\hat{\mathbf{x}}$, and compute the optimal solution of CSPL problem with $\mathbf{x}$ set as $\hat{\mathbf{x}}$, the result of which is the output of OCEAN-C. With $\mathbf{x}$ determined, the single level CSPL problem's runtime sharply decreases.

Algorithm 1: OCEAN-C

1 Relax $\mathbf{x}$ to be continuous;
2 Solve optimal solution $\mathbf{x}^*$ of **P3**;
3 $\hat{\mathbf{x}} \leftarrow$ rounded $\mathbf{x}^*$;
4 Compute the optimal solution Obj of **P3** with $\mathbf{x}$ set as $\hat{\mathbf{x}}$;
5 **return** $Obj, \hat{\mathbf{x}}$;

5 Experimental Evaluation

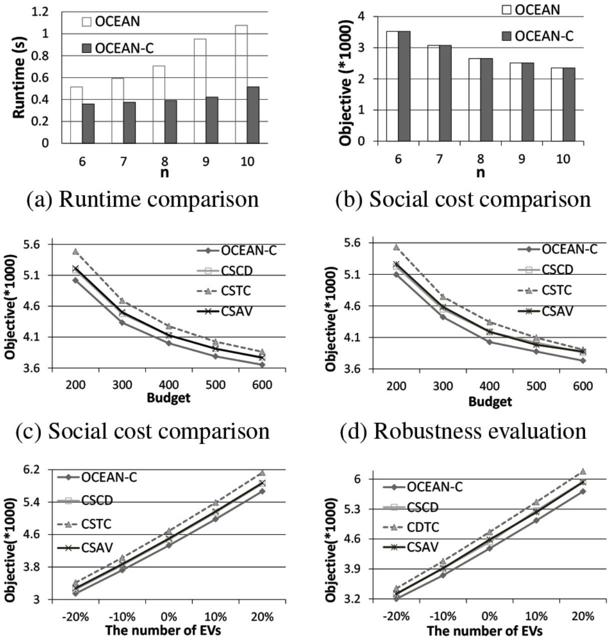
In this section, we run experiments on the real data set from Singapore to evaluate our approach. To compare multiple methods, all experiments were run on the same data set using a $3.4GHz$ Intel processor with $16GB$ of RAM, employing KNITRO (version 8.0.0) for nonlinear programs. The results were averaged over 20 trials.

5.1 Data Set and Baseline Methods

According to the statistics in the official websites of Singapore Land Transport Authority (LTA) and Singapore Department of Statistics (DOS), the population of all motor vehicles in Singapore is $969,910$ in year 2012. We divide Singapore into 23 zones according to the conventional partition method as shown in Figure 1 and the accessible graphical and residential distribution data. We then assume the number of vehicles proportional to the number of residents in each zone. The proportion of EVs among vehicles is set to 5% and the proportion of EVs that charge in charging station rather than at home is 10%. The distance between adjacent zones is estimated using the distance measure tool in Google Maps; the normal congestion α_{ij}^0 during peak hours is estimated by the ratio of travel time during peak hours and the distance between zones i and j. We assume that each road between two zones has the same capacity, i.e., $k_{ij} = 0.01$ for all pairs i and j. Serving rate of chargers is set as $\mu = 6$ and the proportion of EVs that charge during peak hours is set as $\frac{1}{\tau} = \frac{1}{10}$. The linear coefficient λ is fixed at 0.2. Unless otherwise mentioned, the above parameters are fixed in all our experiments. Since OCEAN has scalability issues, we combine some small zones to generate data of different n (ranging from 6 to 10) to compare the performance of runtime and solution quality between OCEAN and OCEAN-C.

We compare our approach with three baseline methods:

- Baseline 1, CSCD assigns charging stations in consideration of charging demand in each zone. The number of



(a) Runtime comparison (b) Social cost comparison

(c) Social cost comparison (d) Robustness evaluation

(e) Social cost comparison (f) Robustness evaluation

Figure 2: Compare OCEAN-C with OCEAN and baselines

charging stations in each zone is set proportional to the number of EVs in each zone, i.e., $x_i \propto \gamma_i$.

- Baseline 2, CSTC, is a dynamic method that assigns charging stations based on the traffic conditions. Here x_i is set in inverse proportion to the weighted sum of normal congestion of the zone and all the roads that lead to the zone: $x_i \propto \sum_{j \in \mathcal{A}_i} 1/(\alpha_{ji}^0 d_{ji})$.
- Baseline 3, CSAV assigns charging stations averagely.

The baseline methods first allocate charging stations to each zone, then compute the equilibrium charging strategies according to program in **P3** with determined $\mathbf{x}$.

5.2 Performance Evaluation

OCEAN-C VS. OCEAN We compare the performance of OCEAN and OCEAN-C regarding to runtime and social cost with different n (ranging from 6 to 10) generated from combining some zones. Budget is set as 300. Figures 2a and 2b show that when n increases, i.e., the number of variables and constraints increases, OCEAN-C's runtime is shorter and also increases slower than OCEAN. Meanwhile, OCEAN-C results in the same optimal social cost and solutions as OCEAN, thus OCEAN-C is an efficient substitution.

OCEAN-C VS. Baseline Methods The optimal social cost of OCEAN-C in comparison with baselines are shown in Figure 2c. When $n = 23$ and budget increases, the social cost also decreases accordingly, but OCEAN-C yields the lowest social cost. We also compare their performances when the number of EVs in the region decreases/increases by different proportions. As Figure 2e shows, the social cost of all the approaches increase proportionally regarding the total charging demand (i.e., the number of EVs) and OCEAN-C always performs best.

Robustness Evaluation We also test the robustness of OCEAN-C considering that some EV drivers may use non-equilibrium charging strategy (due to lack of knowledge). We assume that their charging strategies vary by 10%, i.e., $p_{ij} \pm 10\%$. Figures 2d and 2f depict the social cost of OCEAN-C and the baseline methods when $n = 23$ with varying budget and charging demand. It shows that OCEAN-C outperforms the baseline methods considering EV drivers' decision deviation.

EVs Charge in Remote Zones Previously, we assumed that EVs charge in adjacent zones. Here we relax this assumption by allowing EVs in zone i to charge at two-stop remote zones, which are neighbors of zone i's adjacent zones. After relaxation, the charging strategy and social cost change slightly (less than 0.001 while the original social cost is more than 4000). Thus we conclude that our assumption of charing at adjacent zones is reasonable.

Road Capacity Improvement When the government authority wants to invest to improve the road capacity to lower the social cost, our approach can provide meaningful suggestion. Assume that the investment of improving capacity of road $\langle i, j \rangle$ results in both normal congestion α_{ij}^0 and parameter k_{ij} decreasing by 20%. Taking zone 8 as an example, based on the data set of Singapore with $n = 23$ and budget 300, we get the result as in Table 1 (row "$-$" representing the result before investment), which indicates that the social cost can be decreased in different levels when investing on different roads, and the respective charging strategy $p_{8,j}$. Thus to invest on the roads inside zone 8 is the best choice. Meanwhile, investment on roads $\langle 8, 2 \rangle$ and $\langle 8, 7 \rangle$ is meaningless since these two roads are too far and not used by EVs in zone 8 for charging.

j	C	$p_{8,2}$	$p_{8,7}$	$p_{8,8}$	$p_{8,9}$
–	4332.73	0	0	0.5842	0.4158
2	4332.73	0	0	0.5842	0.4158
7	4332.73	0	0	0.5842	0.4158
8	**4301.25**	0	0	**0.7081**	0.2919
9	4328.68	0	0	0.4416	**0.5584**

Table 1: Social cost C and charging strategy $p_{8,j}$ when capacity of road $\langle 8, j \rangle$ is improved.

6 Conclusion

The key contributions of this paper include: (1) a realistic model for the CSPL problem in cities like Singapore considering the interactions among charging station placement, EV drivers' charging activities, traffic congestion and queuing time: (2) an equivalent single level CSPL optimization problem of the bilevel CSPL optimization problem obtained through exploiting the structure of the charging game; (3) an effective heuristic approach that can speed up the mixed integer CSPL problem with a large amount of non-linear constraints; (4) experiments results based on real data from Singapore, which show that our approach solves an effective allocation of charging stations and outperforms baselines.

7 Acknowledgements

This work is supported by Singapore MOE AcRF Tier 1 grant RG33/13. This research is supported by the National Research Foundation, Prime Minister's Office, Singapore under its IDM Futures Funding Initiative and administered by the Interactive and Digital Media Programme Office.

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