### Gen-ichiro Kinoshita

Department of Electrical Engineering, Chuo University Bunkyo-ku, Tokyo, Japan

### Shuhei Aida

University of Electro-Communications Chofu-shi, Tokyo, Japan and Masahiro Mori

Tokyo Institute of Technology Meguro-ku, Tokyo, Japan

### Abstract

This paper describes an artificial tactile sense with multi-elements that not only takes the pressure information without crushing an object in the hand, but also discriminates one pattern from another on the surfaces being in touch therewith.

The eigenvalue of pattern of stress distribution on the surface of the receptor elements is derived from the lielmholtz equation. The element spacing of receptor is designed by using the relative error of the eigenvalue of the sampled pattern as the design specification.

A simple experiment classifying cylinders, trigonal prism and square pillar was performed by the artificial hand with multi-elements. As a result, a high percentage of correct answer *is* obtained,

# i Introduction

Some hand-eye systems and manipulators have already been constructed. These systems accept the information for satisfying the request of task sufficiently, which is given by a basic behavioral pattern of the system or relationship between an object and a tip of finger. The receptors of the artificial tactile sense for giving the contact information with an object are a number of micro switches, strain gauge and conductive rubber. These elements are used for detecting the contact and the pressure information without crushing the object in the hand. 1

T.B.Sheridan has developed a prototype touch sensor having on its surface a deformable mirror which distorts a regular grid pattern. The human operator observing the reflected image through a closed circuit television can infer the point by point pattern of normal forces on the gripping surface of the manipulator hand.

The authors have proposed the tactile model, whose elements convert the stress distribution to<sub>2</sub> the potential distribution, and the VARISHOLD method for classifying its distribution. This paper describes the twodimensional pattern for each threshold and the element spacing of the artificial tactile sense.

2. A prototype of the artificial tactile sense

As the grasped object deforms the surface of the artificial tactile sense, its stress distribution is given by the output of the tactile receptors. A schematic diagram of these receptors is shown in Fig.I. The receptor elements or input are arranged at the node of regular net on the plane in the elastic material of the artificial tactile sense . Inner potential I in the receptors is compared with the threshold 9.

In case of the artificial tactile sense being arranged close to each other, these inner potential of the receptors may be considered to be the potential distribution V(x,y,z). The threshold is also considered to be the threshold distribution. Moreover, if each threshold 6. of receptors has the same value, the threshold distribution P( $O_{i}.$ ) in the artificial tactile sense may be regarded  $\it Is$  being a plane.

Then, comparing the inner potential with the threshold distribution, an obvious geometrical relation is obtained among them as shown in the next section.

# 5. Dupin's indicatrix of the potential distribution

For the threshold distribution P{ B. ) corresponding to the hyperplane of the potential distribution V(x,y, z), the relationship between P{ 0. ) and V{ x,y, z ) is described by the differential geometrical concept.

Let us define a point of contact p between P( 6. ) and V(x,y,z). We define the second ?undamental differential form at a point of contact p on the threshold distribution. The second fundamental quantity is derived from its near surface of V(x,y,z) at a point of contact p. It is clear that to get the seccond fundamental quantity corresponds to discriminating the pattern which is proposed by the threshold distribution P( 0. - A0<sub>1</sub> ) as shown in Fig.2.

Let us suppose the surface of the object is convex or concave within the region of surface of the tactile sense and V{ x,y,z ) is described by the following parameter representation,

$$V = V(u,v) \qquad (1)$$

The second fundamental differential form of V(u,v) at a point of contact p can be expressed as the following equation.

$$II = Ldu^2 + Mdudy + Ndy^2$$
(2)

where L = V<sub>UU</sub>. e, M = V<sub>UV</sub> . e and N = V<sub>VV</sub> . e,

$$V = \frac{3}{2} V$$
, e is a normal vector.

L, M and N represent the second fundamental quantity of the parameter representation u and v. Then, we have the following theorem from the shape of potential distribution for the second fundamental quantity L, M and N.

### Theorem

I) At a point of contact p of the threshold distribution PC 9, ), the points of V(u,v) being near to the point  $p_Q$  are on one side of the threshold distribution PC 6 ). If the point of contact p is the elliptic point, the following equation is established.

$$L > 0, LN - M^2 > 0$$
 (3)

2) At a point of contact p of the threshold distribution P(9), the points of V(u,v) being near to the point p are on both side of the threshold distribution P(0?). If the point of contact p is the hyperbolic point, the following equation is established.

$$L < 0$$
,  $LN - M^2 < 0$  [4)

3} If a point of contact p is the parabolic point and satisfies the condition 1). The following equation is established.

$$LN - M^2 = 0$$
 C5)

It is clear from the theorem that the surface shape of the object at a point of contact p is identified approximately by discriminating the two-dimensionally shaded pattern as shown in Fig.2.

Therefore, as the thick elastic rubber coated on the receptor elements plays an important role for giving the pattern based upon the threshold change A8, we have developed a prototype artificial tactile sense consisting of the simple touch sensitive on-off switches, which is sufficient to discriminate the pattern of the surface shape of object.

### 4. Design method for spacing the receptor elements

The surface of the object touching with the artificial tactile sense has some pattern of shape. Then, the pattern of shape is discriminated as a two-dimensional pattern which is received by the receptors as shown in Fig.2. The two-dimensional pattern is regarded as a circle, square or its modification. It is desirable that the pattern can be discriminated precisely by as few receptors as possible. The problem of spacing and sampling of the two-dimensional patterns have not been subjected to such extensive investigations. G.C.Cheng and R.S. Ledley discussed the error analysis, that is, the normalized mean-square error etc. for the two-dimensional digitized picture of a chromosome image on a photomicrograph.

In this section, we discuss a design method for spacing the receptor elements by using the concept of the eigenvalue satisfying the Helmholtz equation.

# 4.1 The eigenvalue of a pattern

The pattern described here is the two-dimensional pattern at the receptor output which deals with the predetermined threshold and the output pattern of receptors, that is, on-off switches, etc. being arranged at the node of regular net under the thick elastic rubber. Therefore, it is important that a given pattern of objective surface on the artificial tactile sense would be truly represented to a certain extent by the digitized pattern.

The pattern has the eigenvalue satisfying the Helmholtz equation. By using the eigenvalue, we describe the method of spacing the receptor elements.

At first, the eigenvalue of pattern are discussed. (f) Let us assume that the elements of the subset  $\Omega$  (f)  $\beta_{r=1}$ , 2, 3, ..., m, of pattern set  $\Omega$  are denoted by  $\chi(\beta_{r}, \beta_{r=1}, 2)$  $\chi^{(\mathfrak{b})}, \quad \mathfrak{f} = 1, 2, \dots, m.$  Moreover, these subsets  $\Omega^{(\mathfrak{f})}, \quad \mathfrak{f} = 1, 2, \dots, m,$  correspond to category  $J^{(\mathfrak{f})},$ β = 1,2, .. , m.

Area of pattern X  $\in \Omega$  and unit area X  $\in \Omega$  are denoted by  $\Gamma(X_{\alpha})$  and  $\Gamma(X_{\alpha}) = 1$ , respectively. Suppose that the receptor elements under the thick elastic rubber are arranged regularly with spacing h on the x-y plane as shown in Fig.3.

The eigenvalues of pattern  $\chi_{\lambda}$  are defined by the values  $\lambda^*$  of following Helmholtz equation,

$$\Delta \Phi + \lambda * \Phi = 0 \quad \text{in } \mathbf{R}_{\alpha} \quad \{\mathbf{6}\}$$

$$\Phi = 0$$
 on  $C_{\alpha}$  C7)

where R and C are the connected domain and its boundary of pattern X , respectively.

As the artificial tactile senses are arranged by a finite number of receptor elements, the output of receptor elements expresses only the sampling value of digitized pattern. To solve the above equation using the digitized values of pattern is to use the finite difference method for Eqs. (6) and (7). As the eigenvalues of pattern depend on the area, the pattern is normalized by the following,

$$\Gamma(X_{n}) = \gamma \Gamma(X_{n})$$
 (8)

where  $\gamma$  is a parameter of the area of pattern.

We use the pattern X of  $\Gamma(X) = 1$  in the following discussion. Laplacian

$$\Delta \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}$$
 (9)

is expressed by the following difference equation,

$$\Delta \Phi(\mathbf{i},\mathbf{j}) = \frac{1}{h^2} \{ \Phi(\mathbf{i}+\mathbf{1},\mathbf{j}) + \Phi(\mathbf{i},\mathbf{j}+\mathbf{1}) + \Phi(\mathbf{i},\mathbf{j}+\mathbf{1}) + \Phi(\mathbf{i},\mathbf{j}-\mathbf{1}) - 4\Phi(\mathbf{i},\mathbf{j}) \}$$
(10)

Then, the Heimholtz equation is given by the following,

$$\left[-\frac{1}{b^2}\mathbf{A} + \lambda \mathbf{E}\right]\mathbf{\beta} = 0 \quad (11)$$

where A, 🛔 and E are a matrix of difference oparator, the vetor of function  $\Phi$  and a unit matrix, respectively. As 1 0,

> $\frac{1}{b^2} = \frac{1}{b^2} A_{\rm L} + \lambda E_{\rm L} = 0$ (12)

Then, we have

$$\lambda = \frac{1}{h^2} \Lambda \qquad (13)$$

where A is an eigenvalue of A.

When the spacing h + 0, namely, the spacing of receptor elements is greatly reduced, the condition of convergence is proved by Wasow . S.Sengoku and T.I Shiketa <sup>5</sup> give the following equation which can be modified to some extent by comparing both the analytical value and approximate value with Jacobi method.

$$\overline{\lambda}_{\delta} = \frac{\lambda_{\delta}}{1 - \frac{0.08\lambda}{5}\delta - \cdots}$$
(14)

where  $\lambda_{\delta}$  is the  $\delta$ -th eigenvalue, and  $\bar{s} = 1 / h^2$ .

# 4.2 Design method of spacing the receptor elements

The minimum value of the eigenvalues for given pattern in category  $\beta$  is denoted by  $\lambda \begin{pmatrix} r \\ 0 \end{pmatrix}$ , that is,

$$\lambda_{0}^{(\beta)} = \min\{\lambda_{1}^{(\beta)}, \lambda_{2}^{(\beta)}, \dots, \lambda_{\delta}^{(\beta)}\} \quad (15)$$

where  $\delta$  is dimension of matrix A ( the number of inner points of R ) The eigenvalues of digitized pattern being given by

the receptor elements in regularly arranged net of the

artificial tactile sense present the approximate value of the eigenvalue  $\lambda^*$  or the modified  $\lambda$  in Eq. [14]. In the case where  $\sigma$  is a relative error of the eigenvalue of pattern and digitzed pattern,  $\lambda_0^{(p)}$  can be written as in Eq. (16) be written as in Eq.( 16 ).

$$\lambda \left( \frac{\beta}{\delta} \right) \left( 1 - \sigma^{\left( \beta \right)} \right) \lambda_{0}^{*\left( \beta \right)}$$
(16)

Then, the spacing of the receptor elements is decided uniquely for the given relative error  $\sigma(f)$ , when the spacing h is a small value. The design method is written as follows. For the design specifications, the spacing of receptor elements  $h_{p}^{(P)}$ ,  $\beta = 1, 2, \ldots, m$ , are decided for each category, we make choice of minimun spacing as follows.

$$h = \min \{h^{(1)}, h^{(2)}, \dots, h^{(m)}\}$$
 (17)

Therefore, the spacing of the receptor elements is given by h. If the pattern of each category has a reducibility  $\gamma$  (F)  $\beta = 1, 2, ..., m$ , the spacing is given as follows.

$$h = \min\{ h^{(1)} / \sqrt{\gamma^{(1)}}, h^{(2)} / \sqrt{\gamma^{(2)}}, \dots, h^{(m)} / \sqrt{\gamma^{(m)}} \}$$
(18)

# 4.3 Example

The minimum eigenvalue for the circle and square patterns is given by use of Jacobi method as shown in Fig.4. The pattern in Fig.3 is a circle which has 9 inner points and h = 0.282. In this pattern, the matrix A and eigenvalue A can be written as

A =	$ \left\{\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	0 -1 4 0 0 -1 0 0 0	$     \begin{array}{r}       -1 \\       0 \\       4 \\       -1 \\       0 \\       -1 \\       0 \\       0 \\       0     \end{array} $	0 -1 -1 4 -1 0 -1 0	0 -1 -1 -1 4 0 -1	0 0 -1 0 4 -1 0	0 0 0 -1 0 -1 4 -1	0 0 0 0 -1 0 -1 4	λ ≠		(19)	ן
-----	--	---	---	--	-------------------------------------	-----------------------------------	---	--	-----	--	------	---

The minimum eigenvalug as shown in Eq.(13) is  $\lambda_{0} = \Lambda / h^{2}$ = 14.7233

We make a mark with the asterisk in Fig.4 as the modified eigenvalue  $\lambda_{\rm c}$  using Eq.( 14 ). The numerals in Fig.4 show the number of equally divided parts of a diameter or side of the pattern. The relative error  $\sigma$  (F) in Fig.4 is given by Eq.(16). As is evident from Fig.4, the relative error o of the square pattern is smaller than one of its corresponding circular pattern. This means that the output pattern of receptors closely follows the given pattern ( input pattern ) as a side of square pattern corresponds to the node in a row of regular net for arranging the receptor elements. The eigenvalue of circular pattern does not change through the rotation of the pattern around its center, but the eigenvalue of square pattern changes as shown in Fig.5. It shows the eigenvalues for spacing h =0.1 43 and 0,200. The eigenvalue for changing the center of pattern is shown in Figs. 6 and 7. Moreover, for a specific value of the design parameter  $\sigma$ , the spacing h is given by the curve in Fig.4. For example, we can determine the spacing h = 0.14 for  $\sigma = 0.05$ .

In this case, it should be considered that the position of pattern is normalized under the design of spacing, as the pattern moving on the arranged node changes its eigenvalue.

#### Pattern classification of the grasped 5. object by the artificial hand

Pattern classification of the grasped object is carried out by the artificial hand with the artificial tactile senses. An important function of artificial tactile sense is to classify the surface shape of the grasped object by the artificial hand, This section describes a prototype of the artificial tactile sense which is designed according to Sec.4.2 and an experiment of the pattern classification of the grasped object by the artificial hand.

# 5.1 The artificial tactile sense with multi-elements

It is necessary that the artificial tactile sense is small-sized on a finger tip of the artificial hand and consists of multi-elements feeling a small finger tip pressure for classifying the grasped object. In this section, we describe a prototype of the artificial tactile sense developed under the above condition. One of the trial artificial tactile senses arranges the 20 elements in 5 rows by 4 columns as shown in Fig.8. The whole size is 20mm in width, 25mm in length and 5mm in thickness, and an element is 3min in diameter, and the spacing is h = 4mm. Therefore, in case of the circular pattern, the relative error is  $\sigma = 0.26$ .

As shown in Fig.8, the elements are similar to onoff switches and a point of contact is closed by applied pressure of about 20-40 g/cm".

# 5.2 Pattern classification of the grasped object by the artificial hand with learning mechanisum

The artificial hand with the artificial tactile sense described above is controlled sequentially by the following steps( Fig.9).

- (1 J The object is set up.
- (2) The thumb and the index finger grasp the object.
- f 3) The middle, the third and the little fingers grasp sequentially.

C 4 ) The fingers grasp the object firmly. The output pattern of the artificial tactile sense are given by the point of contact with the grasped object. By a feature extraction of the output pattern, its output is given to the input of a trainable pattern classifier as shown in Fig.10.

The grasped object is a trigonal prism (90mm on a side 1, a square pillar (75mm on a side ) or a column ( 80mm in diameter J, The 16 output patterns are used in the learning process as shown in Fig.II. We have tried a pattern classification by using the above adjusted weights of the trainable pattern classifier and gotten the result of Table 1,

#### 6. Conclusion

In this paper, we have proposed a design method for the receptor elements of the artificial tactile sense. This method is an application of the eigenvalue satisfying the lleJmholtz equation. We think this method will have a wide application.

Pattern classification of an objective shape has been done by using the artificial hand with the artificial tactile sense which consists of multi-elements. A high percentage of correct classifications have been obtained.

### References

- 1) J.B.Sheridan, W.R.Ferrell : Human Control of Remote Computer-manipulators, International Joint Conference on Artificial Intelligence, 483(1965)
- 2) S.Aida and G.Kinoshita : A Pattern Recognition by Time-varying Threshold Method, International Joint Conference on Artificial Intelligence, 417(1969)
- 3) G.Kinoshita, S.Aida and M.Mori : Pattern Recognition by an Artificial Tactile Sense, Second International Joint Conference on Artificial Intelligence,376 (1971)
- 4) Hilderbrand : Finite-difference Equations and Simulations. Prentice Hall Inc.(1968)
- G.E.Forsyth, K.R.Wasow : Finite Difference Methods for PartialDifferential Equations, John Wiley & Sons (1967)

- 6) S.Sengoku and T.Ishiketa : Tree of Eigenvalue of Two-dimensional Figure and its application, Information Processing, 12, 1(1971)
- 7) G.C.Cheng and R.S.Ledley : A Theory of Picture Digi-tization and Applications, Pictorial Pattern Recognition ( G.C.Cheng, R.S.Ledley, O.K.Pollock and A, Rosenfeld eds.) Thompson 329-352 (1968)



Fig.1 A schematic diagram of the tactile receptor





Fig.2 Dupin's indicatrix



Fig.3 Pattern and receptor elements





Λ







Fig. 8 The artificial tactile sense with multielements





Fig.9 The artificial hand and the output pattern of the artificial tactile sense.



Fig.10 A block diagram of trainable pattern classifier



Fig.11 Learning process

Table 1 The percentage of correct answers

Object	trigonal prism(90mm on a side)	square pillar(75 mm on a sid	column (80,100mm e)in dia~ meter]
Correct answer(%)	78.6	85.7	100.0