METHOD FOR THE BEST SOLUTIONS SEARCH IN MULTIOBJECTIVE DECISION PROBLEMS

V.M. Ozernoi and M.G. Gaft
Institute of Control Sciences
b1 Profsoyuznaya, Moscow

Abstract

This paper describes a method for constructing a decision rule providing the isolation of the most preferable solutions in multiobjective decision problems. The method is based on the iterative procedure for constructing decision rules. Criteria of meaning-fulness and consistency of received information are formulated. The results are given of applying the method to solution of the most preferable alternatives search problems concerning the mineshaft layout.

Introduction

Decision theory which belongs to the area of artificial intelligence is quickly developing. Decision making is closely related to problem solving, though these are not identical processes. Problem solving implies the search of means to achieve the clearly visible goal which nevertheless cannot be achieved directly [1]; the decision process in general implies the search of all possible means of achieving the Bet objective, preference comparison among them and choice of the best one.

The most important specific of decision-making problems is the presence of a decision-maker who has to act in the situation which is characterized by lack of information about the environment, about the possible outcomes of the decisions and about the values of some outcomes. It is not only that objective functions (optimal criteria) in these problems are not stated but also their existence is not indisputable.

The need to solve complex practical problems and the interest to scientific problems accompaning them led to quick development of decision theory. It is very important that decision theory postulates nonexistence of a decision that could be considered the best one in any absolute sence, but only for a given decision maker in connection with the set objective. Thus the two main problems of the decision theory are as follows:

1) to substitute man in repeating routine decision problems (programmable problems);

2) to help man in complex and uncertain situations to choose decisions consistent with his preference judgments using formalized models and methods which allow avoiding mistakes in long and complex chains of logical arguments (unprogrammable problems).

Decision theory must provide the basis for development of models and methods whereby all the information on the problem, including the decision maker's preference judgements is used in order to decide which of the alternative courses of actions is the best. When decision problems are formalized difficult questions such as analysis of complex and uncertain situations become conceptually equivalent to simple problems that can be solved by "common sense".

Presently it is universally acknowledged that in most important real life decision problems the solution quality cannot be estimated by one scalar -valued performance criterion. Here arises the problem of estimating and comparing alternatives taking into consideration a great number of criteria (multicriteria or multiobjective decision problems[2j. Themain difficulties in developing formalized decision methods which could be applied to a wide range of practical problems are caused by the fact that multiobjective decision problems are ill-structured [3], and in order to formalize them successfully one must consider a lot of factors such as sociological, psychological, measurement-theoretic, etc. Disregard of these factors at the stage of formalizing the process of decision-making shows up as distrust on the part of the decision-makers of the results of formalized methods which are thus downgraded.

When stating and solving multiobjective problems it is essential to consider a great number of meaningful circumstancies and ideas which cannot be easily given a strict mathematical motivation. Another difficulty is in what criteria must be considered in a specific problem, whether all feasible solutions are considered, etc. These questions must be solved by all means and all of them are outside the mathematical formulation of the problem. At the same time working out the methods of choosing most preferable alternatives is impossible without using mathematical methods which are applied to formalized models rather than to real problems. Thus in model approach to solving certain multiobjective problems meaningful considerations which cannot be strictly formalized have to co-exist with formalized methods and to be used in combination with them. It is only in interaction that they can lead to useful results.

The development and use of a model it possible to get ordered sets of feasible solutions, provided that the sets are consistent with the model used. At the same time it must be stressed that there is no "right" or "objective" model for any specific multiobjective problem. In connection with the fact that in each problem there is a great number of factors which may be taken into or left out of consideration, there always exists the possibility to present the same situation by different models. One can find whether these models are fit or not only using them in real situations.

The process of developing multicriteria models in decision problems may be broken into the following series of stages: 1) goal formulation and problem type identification: 2) working out a feasible solutions set; 3) working out a criteria set; 4) criteria scale development; 5) mapping of feasible solutions set in the set of vector-valued estimates; 6) identification of a decision-maker's preference judgments; 7) the decision rule construction. The specifics of the first six stages of constructing multicriteria models and means of their implementation are described in [4]. We shall discuss in more detail the questions connected with constructing a decision rule.

Construction of Decision Rules

A number of various decision rules have been proposed, and each of them has some shortcomings which considerable limit the sphere of its possible application [5-9] . Multicriteria problem analysis drives one to the conclusion that the construction of a generally applicable decision rule appears to be absolutely impossible. The following fact may account for it. Depending on the decision-makers goals his preference judgments and the set of assumptions used in the model, different decision rules may be constructed that naturally lead to different ordered sets of feasible solutions [4].. The impossibility of constructing a generally applicable decision rule requires development of a method for constructing decision rules leading to the needed result in every specific situation.

Difficulties of finding out preference judgments of the decision-maker are responsible for the iterative nature of the decision rule construction procedure involving stage-wise acquisition of information about decision-maker's preference judgments [8] . Information received at each stage must be used to construct an intermediate decision rule on the basis of which decisions are ordered. Procedure of constructing the decision rule leading to the problem solution must be organized in such a way that the received sequence of intermediate decision rules has the following characteristics: the initial (weakest) one is based on the simplest and most evident assumptions; the following rules follow from the previous ones because we make extra assumptions that do not contradict the earlier ones and receive extra information. If an intermediate decision rule leads to the desired result it is accepted as the final and this is the end of the procedure,

Basic Principles

In spite of the fact that decision rules may be constructed in different ways depending *on* the problems at hand, one may formulate principal requirements that every iterative procedure of such a kind must meet:

a) Additional information on preferences, received by the researcher from the decision-maker at the i+1-th step of the procedure must allow establishment of the preference at least between two vector-valued estimates that cannot be compared at the i-th step (meaningfulness criterion of additional information^

b) The preference between any two vector-valued estimates received at i-th step must not change during the subsequent steps (consistency criterion of adopted assumptions and addittional information) .

When comparing vector-valued estimates, the use of decision rule defines uniquely a binary relation (quasi-order in the general case) on the set Y *of* vector-valued estimates and, thus, on the set of feasible solutions.

Major Relationships

Let the expression $\langle R^i, Y \rangle$ denote preference-indifference relation received when comparing vector-valued estimates from Y on the basis of the i-th decision rule. The relation $\langle R^i, Y \rangle$ allows to define relations of strict preference $\langle P^i, Y \rangle$ equivalence $\langle I^i, Y \rangle$, incomparability $\langle N^i, Y \rangle$ in the following way [9]:

$$P^i = \{(x,y) \in Y \times Y / (x,y) \in R^i, (y,x) \notin R^i\};$$

$$I^i = \{(x,y) \in Y \times Y / (x,y) \in R^i, (y,x) \in R^i\};$$

$$N^i = \{(x,y) \in Y \times Y / (x,y) \notin R^i, (y,x) \notin R^i\}$$

The requirements to iterative procedures formulated earlier for constructing decision rules may be formally written down in the following way:

$$R^i \subset R^{i+1}; \qquad (1)$$

$$P^i \subseteq P^{i+1}. \qquad (2)$$

Strictness of inclusion (1) follows from the first requirement. Nonstrictness of inclusion (2) follows from the possibility to find strict preference between vector-valued estimates incomparable at the i-th step as well as to establish their equivalence using the information received at the i+1-th step.

<u>Definition 1</u>. Relations $\langle R^i, Y \rangle$, $\langle R^{i+1}, Y \rangle$, satisfying expressions (1) and (2) will be called inserted.

<u>Assertion 1</u>. If relations $\langle R^i, Y \rangle$, $\langle R^{i+1}, Y \rangle$ are inserted, the following expressions are valid: $I^i \subseteq I^{i+1}$,

$$N^i \supset N^{i+1}, \quad P^i \cup N^i \supseteq P^{i+1}, \quad I^i \cup N^i \supseteq I^{i+1}.$$

At the same time at least one of the two last inclusions is strict.

Assertion 1 is needed to prove whether the information received from the decision-maker in practical situations is noncontradictory.

Let the expression $A \subseteq Y$ denote the subset of vector-valued estimates corresponding to feasible solutions.

Definition 2. The subset

$$A^0_{R^i} = \{x \in A / \neg(\exists y \in A)[y P^i x]\}$$

will be called the set of the most preferable estimates from A with respect to preference-indifference relation $\langle R^i, Y \rangle$.

Assertion 2. If the relations $\langle R^i, Y\rangle, \langle R^{i+1}, Y\rangle$ are inserted the expression $A^0_{R^i} \supseteq A^0_{R^{i+1}}$ is valid.

Therefore the insertion of relations $\langle R^i, Y\rangle, \langle R^{i+1}, Y\rangle$ provides the possibility of reducing at the i+1-th step the most preferable estimates subset received at the i-th step. It is sufficient for a strict inclusion $A^0_{R^i} \supset A^0_{R^{i+1}}$ to establish, at the i+1-th step, preferences between incomparable estimates from $A^0_{R^i}$.

Information on Decision Maker's Preferences

When organizing a decision rule generating procedure it is essential to define and substantiate a sequence in which different kinds of information on the decision-maker's preferences are received. Thus it becomes necessary to make a classification of such information. Reference [8] classifies the information in terms of its relation with one or several criteria. This approach makes it possible to distinguish three types of information:

1) Information on the effect of estimates changes over one criterion scale on the decisions value (utility) in general.

2) Information on the effect of estimates changes over one criterion scale in comparison with estimates changes over another criterion scale on the decisions value in general.

3) Information on the effect of estimates changes over the scales of criteria belonging to one group in comparison with the estimates changes over the scales of criteria belonging to another group on decisions value in general; at the same time at least one of the compared groups must contain more than one scale.

Information of each type is calssified with due regard to the degree of its effect on elimination of vector-valued estimates incomparability. The sef of all possible combinations of the resultant ordered information classes may be presented as a flowchart which determines the sequence of obtaining different types information on decision-maker's preference judgments. The classification suggested in [8] allows finding a decision rule for each type of information presented by the flowchart.

Application

The suggested procedure of constructing the decision rule was applied to developing the method of isolating the needed amount of the most preferable versions of a mineshaft layout [10]. When designing coal mines the number of feasible layout versions may be as high as tens of thousands. At the same time the mineshaft layout is such a complex plant that the number of its versions cannot be estimated by one scalar-valued performance criterion. On the basis of the detailed design examination which consumes much time no more than 20-30 versions may be studied. Thus the selection of the best design version through a detailed design examination is to a great extent determined by the versions under study.

The developed method for most preferable solutions search makes it possible at the early stages of designing:

- to work out all feasible layout versions for a wide range of mineral-geological factors on the basis of morphological analysis;
- to estimate all versions with respect to 25 nonanalytical technological and economic criteria;
- to compare all feasible versions and isolate from them the desired number of most preferable ones.

When developing the method the decision rule was constructed in three stages. At the first stage only information on different estimate preferences over each criterion scale was used; at the second stage extra information was received in terms of some criteria on the preference of estimates change over the scale of one of them from the best estimate to some other in comparison with similar estimates change over the scale of another criterion; at the third stage information on utility changes relations corresponding to estimates changes both over one criterion scale and over some different criteria scales was received.

The method of search for the most preferable decisions was repeatedly applied to designing mines for certain coal deposits. In one problem the initial set contained 7704 feasible versions of the layout; from these 21 versions were chosen as most preferable. This subset contained layouts that had been disregarded earlier as well as those that had been usually included in the number of the best ones.

At the same time some versions which were traditionally used in designing were not included in the number of most preferable ones. The subsequent research and design examination fully confirmed the wisdom of that selection.

Analysis of the results of the methodology application to certain problems solution shows that it allows considerable saving of time and cost of design work and improvement of the quality of the decisions made. The methodology allows substitution of man in choosing the most preferable system versions at early stages of design.

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