History-based Interpretation of Finite Element Simulations of Seismic wave Fields

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Abstract

Dynamic objects such as liquids, waves, and flames can easily change their position, shape, and number. Snapshot images produced by finite element simulators show these changes, hut lack an explicit representation of the objects and their causes. For the example of seismic waves, we develop a method for interpreting snapshots which is based on Hayes[7] concept of a history.

1 Introduction

Most work on qualitative reasoning about physical systems is devoted to *technical systems* consisting of a fixed set of components that interact via given connections. Examples given in [Weld and de Kleer, 1990] are electronic circuits, water tanks, and gear systems. In contrast to this, we will consider *natural systems* where objects are *dynamic in position, direction, shape, and number.* The FROB system [Forbus, 1984] simulates springing balls changing their positions and directions, but keeping their shapes. Furthermore, we don't obtain new balls. Flowing liquids [Hayes, 1985a] are different: They easily divide, merge, and change their shapes. In order to capture those interactions between liquids, Hayes developed the concept of a history, i.e. a coherent piece of space-time. Histories provide an adequate means to describe the behaviour of *dynamic objects* such as flames, waves, clusters, clouds, which can all be deformed, divided and merged.

In this paper, we will consider a concrete task requiring history-based reasoning about physical phenomena. We consider the propagation of seismic shock waves in the underground [Lavergne, 1986]. Seismic waves are used by geophysicists to explore the structure of the underground. They are usually launched by an initial vibration on the surface. The resulting spheric shock wave is then propagating downwards as shown in the first snapshot of figure 1. When it hits an interface between two geological layers this causes a reflected and a transmitted wave. The reflected wave returns to the surface and leaves an observable front in the seismograms measured by the geophysicists.

In order to interpret seismograms, the geophysicists incrementally construct a model of the underground based on hypotheses of the histories of the returning waves. Above, we considered a wave that was reflected by the first interface. Further interfaces lead to further direct reflections. Additionally, a seismogram can show multiply reflected fronts, diffractions which are obtained due to corners and many other disturbing fronts. Geophysicists pick out direct reflections using some heuristic approach and use them to construct a model of the underground (based on numerical optimization procedures or further ad-hoc rules).

Newer work on numerical simulators based on finite elements allows a very precise simulation of the wave propagation in complex models of the underground. The snapshot sequence in figure 1 has been produced by such a simulator [Anne and Brae, 1994]. The simulations enable a verification of the geological model. Divergences between observed and simulated seismograms might help to correct the model. To detect them, we have to compare fronts having the same history (e.g. two direct reflections; two diffractions etc.). Unfortunately, numerical simulators based on finite elements do not keep track of the history of waves. They produce a series of images showing the waves, but they lack a representation of the wave objects, their causes, and their histories. When examining a front of a seismogram, we want to know the obstacles and the types of phenomena that produced it.

In this paper, we show how to *interpret* the images produced by the numerical simulator and how to establish a causal relation between seismic events, waves, and obstacles in the underground. Our goal is *to detect Hayes-like histories of waves in snapshot images.* Although the paper is restricted to 2D-models of the underground, its concepts can be generalized to the 3D-case.

The paper is divided into two main sections. Section 2 presents the representation of fields (sec. 2.1), as well as the vocabulary for describing wave histories (sec. 2.2). The interpretation is done in several steps developed in section 3. We first decompose the underground into layers and interfaces (sec. 3.1). Then we show how to detect

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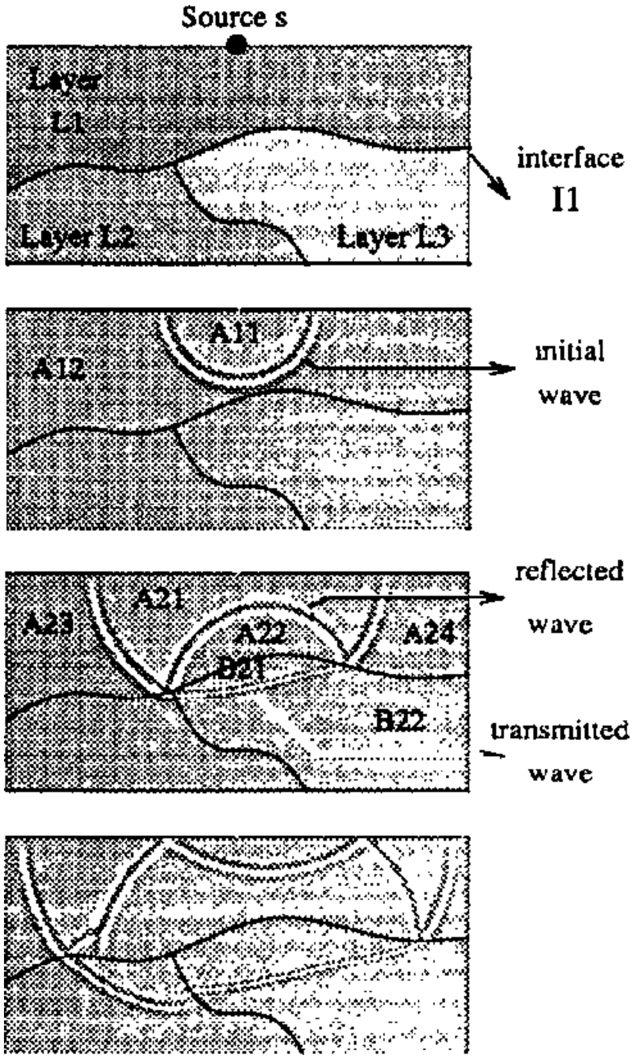

Figure 1: Snapshots of seismic waves.

wave fronts in a single snapshot (sec. 3.2). Tracking a front from one snapshot to the other is considered in section 3.3. In section 3.4, we discuss how to detect new objects and their causes.

2 Multiple representations

2.1 Fields

In order to describe complex phenomena (e.g. liquids, waves, flames etc.), physicists use parameter fields. A *field* is the distribution of a physical parameter in the given space. For shock waves, we consider a velocity field, giving the velocity of a wave at a certain point, and the field of the amplitudes of the waves (i.e. the snapshots in figure 1.). A *physical law* captures a relationship between parameter fields, which is valid at each point. In general, such a law is a differential equation (e.g. the wave equation). Its solution describes the temporal development of a field. Some of the fields such as the velocity field are static (stationary), whereas the amplitude field is changing in time (non-stationary). We restrict our discussion to a single static and a single dynamic field.

A well-suited technique for simulating changes of complex and arbitrary fields is the *funite element method.* A numerical simulator based on this technique is supplied

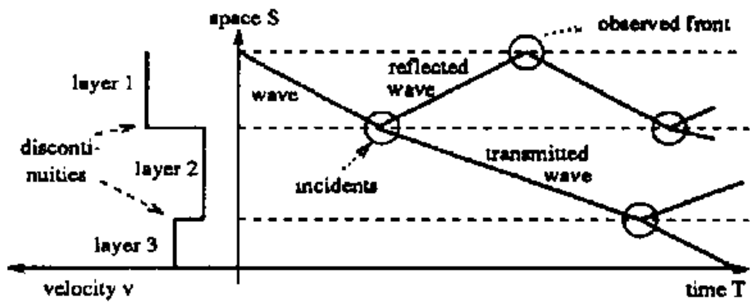

Figure 2: Dividing seismic histories into waves and incidents

with the initial parameter fields and then produces a series of snapshots showing the fields at selected instances of time. The finite element method can handle complex shapes because it uses a kind of an *analogical representation of fields:*

- It is based on a *grid* *(P,N)* where V is a set of selected points and $N \subset V \times V$ is a symmetric neighbourhood relation.
- It describes the spatial distribution of a parameter using a mapping $f : V \longrightarrow \mathbb{R}$ of the points to the real numbers.
- It is specified extensionally (e.g. a matrix of floating point, numbers.)

Seismic simulators normally use regular grids obtained by rows and columns. They are characterized by a starting point $s := (s_1, s_2)$, a unit distance Δ, the number n of columns and the number m of rows. The set of points is then given by

$$\mathcal{P} := \begin{array}{l}\{(s_1 + i * \Delta, s_2 + j * \Delta) \mid \\ i = 0, \ldots, n-1, \; j = 0, \ldots, m-1\}\end{array} \quad (1)$$

Two points are neighbours if they have successive positions in the same row or column.

$$\mathcal{N} := \{(p, q) \in \mathcal{P} \times \mathcal{P} \mid |p - q| = \Delta\} \quad (2)$$

2.2 Histories of dynamic objects

Fields don't represent objects explicitly. They just show certain patterns of activity that are reproduced in the next instants. For example, figure 1 shows wave fronts that are propagating, hitting interfaces, and generating new waves. In order to describe these phenomena, we need an ontology for dynamic objects in fields.

Our discussion is based on a given (continuous) space S, for example the two-dimensional space defined by $\mathbb{R}^2$, and a linear (continuous) time defined by $T := \mathbb{R}$. Dynamic objects such as waves evolve in time and occupy a region at each time t. This region is a subset of $S \times \{t\}$. If we consider different time points the occupied region of an object can change. We require that these changes are local. If we put the regions of an object at different times together, we obtain a subset of $S \times T$. This subset must be a 'connected piece of space-time', i.e. a *history* as defined in [Hayes, 1985b].

The region occupied by an object can change in a continuous or discontinuous way. For example, the initial

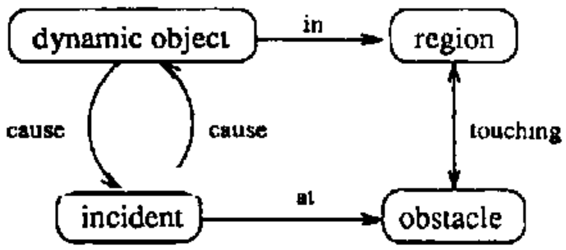

Figure 3: Ontological concepts for dynamic objects

wave front in figure 1 is split into two parts when hitting the interface. Changes are caused by the dynamic and the static field. In the case of seismic waves, discontinuities caused by the static field can be characterized precisely: If the velocities of a zone are changing continuously, a (convex) wave propagating through this zone will change continuously. Discontinuities in the velocity field however cause discontinuities in wave propagation.

In order to keep track of causes for discontinuous changes, we divide the static field into regions and obstacles. A *region* is a (maximal) coherent subset of S that does not contain discontinuities in the static field. The regions of the seismic velocity field are also called geological layers. An *obstacle* separates one, two, or several regions. It is a (maximal) coherent subset of S of discontinuity points in the static field. Its dimension is inferior to that of *S*. The geological model of figure 1 is composed of three 2D-regions of constant velocities, which are separated by three ID-obstacles called *interfaces.* The interfaces are linked by a corner which is a OD-obstacle. Regions, interfaces, and corners constitute a place vocabulary in the sense of [Hayes, 1985b; Forbus, 1994].

Thus, we divided the static field into regions where motion of dynamic objects is continuous and into obstacles which disturb motion in a discontinuous way. We now use this topological structure to divide the global histories into episodes of dynamic objects and incident events linking these episodes. We require that a dynamic object is contained into a single region. If it reaches an interface then the continuation of its history on the other side of the interface is considered to be a new object, namely the transmission of the incident object. A *dynamic object* is an episode of the global history that is contained in the static history of a certain region.

An *incident* is the event when a dynamic objects hits an obstacle. It is in fact the intersection of the history of the dynamic object with the history of the obstacle. The incident is caused by the incident object and causes itself new objects in the regions surrounding the obstacle. The global history is branching at the incidents as illustrated in figure 2. An incident is the start of the histories of the waves it causes and it links them with the history of the incident wave

Thus, we have structured histories in the dynamic fields into dynamic objects, incidents, and their causal relation. In fact, we have adapted the basic concepts of naive physics [Hayes, 1985b] to physical fields and now have a vocabulary for interpreting the simulated fields.

3 Interpreting snapshots

3.1 Detecting static histories

First we show bow to decompose the static velocity field into geological layers and interfaces. Thus, we obtain the regions where to look for waves and the obstacles where to look for incidents.

Let *{V,N)* be a grid and / : *V* — R be a field. We define regions as follows: let *C* C N be a symmetric criteria that specifies whether two neighbour points belong to the same region. We consider the reflexive transitive closure of *C* and denote it by *C.* *C** is the smallest superset of *C* that is reflexive and transitive. Since *C* is symmetric, *C* is an equivalence relation. The regions are obtained as the equivalence classes of C* The *C*-region of a point *p E V* is defined as the equivalence class containing ;>:

$$R_C(p) := \bar{p}^{C^*} \tag{3}$$

To define regions in the velocity field *v* of seismic waves, we link two neighbour points if there is no discontinuity between them. Since grids have a fixed resolution, we use a threshold *i* to operationalize this criteria. The velocity difference of two points must be smaller than E*:*

$$V := \{(p,q) \in \mathcal{N} \mid |v(p) - v(q)| < \epsilon\} \tag{4}$$

The *geological layer* of point *p* is then the equivalence class *Fiv{p).*

Next we define interfaces separating two C-regions *R* and R_2 An interface is just a set of neighbourhood links *(p,q)* € N that do not satisfy the given criteria *C* and that connect a point in *R* with a point in *R2'.*

$$I_C(R_1, R_2) := ((R_1 \times R_2) \cap \mathcal{N}) - C \tag{5}$$

The set I(R_1,R_2) is called C-interface between R_1 R_2 iff *1*(R_1, R_2) is not empty. The geological layers are separated by *V-interfaces.* For the sake of brevity, we neither discuss corners, nor the case that the interface between two regions is interrupted by a third region.

3.2 Detecting objects in a snapshot

In the next sections, we consider a sequence of snapshots $a_1, a_2, a_3, \ldots$. showing the amplitude field at increasing time points t_l, t_2, t_3 ■ ■ ■■ We proceed in three steps in order to detect histories of wave objects. First, we identify wave objects in a single snapshot. Then, we link the possible interpretations of succeeding snapshots. After that, we show how to detect histories of new objects caused by incidents at interfaces.

Wave fronts as shown in figure 1 consist of a small number of oscillations. In a snapshot, they appear as thin regions of negative or positive amplitudes, which can clearly be distinguished from the background having zero amplitude. To capture this phenomena formally, we divide the set *V* of points into three classes: *positive, negative,* and *zero* ones. Since there are small distortions in the simulated field, we use a $\delta > 0$ to define the zero class. Let a,- be the amplitude field of the *i-th* snapshot:

$$\begin{array}{lcl} \mathcal{P}_i^+ & := & \{p \in \mathcal{P} \mid a_i(p) > \delta\} \\ \mathcal{P}_i^0 & := & \{p \in \mathcal{P} \mid -\delta \leq a_i(p) \leq \delta\} \\ \mathcal{P}_i^- & := & \{p \in \mathcal{P} \mid a_i(p) < -\delta\} \end{array} \tag{6}$$

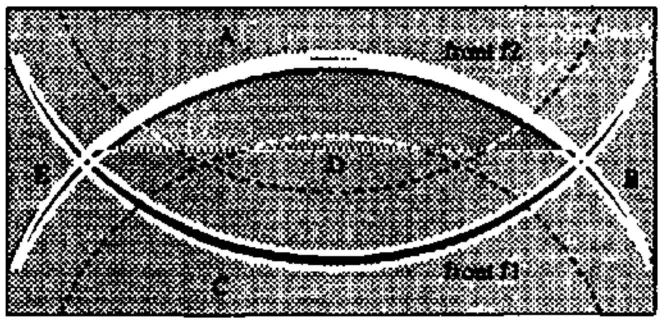

Figure 4: Two crossing fronts

Coherent regions of either positive, negative, or zero points are obtained by linking two neighbour points of the same class. Since we want to reconstruct wave fronts and each wave front is limited to a single geological layer we cut the regions in the amplitude field at the geological interfaces by using the criteria V.

$$A_i := V \cap \begin{array}{c}\{(p,q) \in \mathcal{N} \mid \exists c \in \{-,+,0\} : \\ p \in \mathcal{P}_i^c, q \in \mathcal{P}_i^c\}\end{array} \qquad (7)$$

This criteria A_i gives rise to A_i-regions and A_i-interfaces decomposing the geological layers at time t_i into positive, negative, and zero regions.

Characterizing wave fronts by their positive and negative regions is problematic because we can obtain intersections between several fronts (cf. figure 4). Due to this, the regions of several waves can merge. A more elegant and robust characterization is obtained by the zero regions that are enclosed by a wave. If a wave front f in the layer L is enclosing a zero region A then we obtain the front as the interface between the regions A and $L-A$. In case of interruptions, a front encloses several zero regions. Front f_1 in figure 4 encloses the zero regions A, D, whereas front f_2 encloses the regions D, C. f_1 also encloses the gap between the fronts A and D which is caused by f_2.

In order to close this gap, we will enlarge the zero regions of a layer L until the complete layer is covered. A single growing step adds the neighbouring layer points to a region $X \subseteq \mathcal{P}$:

$$grow(X) := X \cup \{p \in \mathcal{P} \mid \exists x \in X : (x,p) \in \mathcal{N} \cap V\} \qquad (8)$$

We iterate this step until the layer L is completely covered. Let $\mathcal{Z}_i(L)$ be the set of the zero regions contained in the layer L at time t_i. The *i-th cover factor* of L is the smallest number k s.t.

$$L \subseteq \bigcup_{Z \in \mathcal{Z}_i(L)} grow^k(Z) \qquad (9)$$

If Z is a zero region in layer L at time t_i and k is the cover factor of L at this time, then the *enlargement* of Z is defined as

$$\hat{Z} := grow^k(Z) \qquad (10)$$

The enlargement enable us to define a wave front using the zero regions enclosed by it. Suppose that w is a wave in layer L and that the front of w at time t_i encloses[1] the zero regions $Z_1, \ldots, Z_m$ of L. Then

1. $\mathcal{Z}_i(w) := \{Z_1, \ldots, Z_m\}$ is called ***i-the characterization of w***.
2. $from_i(w) := \hat{Z}_1 \cup \ldots \cup \hat{Z}_m$ is called the ***i-th enclosed region of w***.
3. $to_i(w) := L - from_i(w)$ is called the ***i-th propagation region of w***.

The interface between $from_i(w)$ and $to_i(w)$ represents the wave front. For example, the front f_1 is obtained as the interface between $\hat{A} \cup \hat{D}$ and $\hat{B} \cup \hat{C} \cup \hat{E}$, whereas f_2 is the interface between $\hat{C} \cup \hat{D}$ and $\hat{B} \cup \hat{A} \cup \hat{E}$.

If $from_i(w)$ is empty then the wave w has not yet appeared in snapshot i. If $to_i(w)$ is empty w has gone.

3.3 Tracking existing histories

Dynamic objects are steadily changing form and position. The time steps between two succeeding snapshots are usually too large to track these changes locally (cf. figure 1). In this section, we show how to track the history of an object even in presence of gaps.

For a given set $\mathcal{W}$ of waves and the set $\mathcal{Z}$ of zero regions of snapshot $i+1$, the task is to identify the $i+1$-th characterizations of the waves in $\mathcal{W}$ by using their i-th characterizations. We require that the new characterizations satisfy some additional criteria:

1. The new fronts should be in the propagation regions of the old fronts.
2. Each observed frontier between two zero regions should be explained by a wave.
3. The new fronts should be as close as possible to the old fronts.

The first constraint is based on the hypothesis that all points are propagating to the same side of the front. It implies that the enclosed region of a wave w is growing from one instant to the other, which is expressed by $from_i(w) \subseteq from_{i+1}(w)$. If a zero region $Z \in \mathcal{Z}$ of snapshot $i+1$ is overlapping with $from_i(w)$ it is also overlapping with $from_{i+1}(w)$ and therefore an element of the $i+1$-th characterization of w:

$$\text{if} \quad Z \cap from_i(w) \neq \emptyset \quad \text{then} \quad Z \in \mathcal{Z}_{i+1}(w) \qquad (11)$$

The second constraint requires the detection of frontiers between two zero regions in a snapshot. In figure 4, D has a frontier with A and C, but not with B and E. If we let grow all regions then the borders of D will overlap with that of A, B, C, E. However, the overlap with B and E is also covered by the borders of A and C. This leads to the following definition: There is a ***frontier*** between two zero regions X and Y in $\mathcal{Z}$ iff

$$(grow(\hat{X}) \cap grow(\hat{Y})) - \bigcup_{Z \in \mathcal{Z}-\{X,Y\}} grow(\hat{Z}) \neq \emptyset \qquad (12)$$

A frontier between X and Y is ***explained*** by a wave w iff w is enclosing exactly one of these zero regions, i.e. $\mid \{X,Y\} \cap \mathcal{Z}_{i+1}(w) \mid = 1$.

[1]Our characterization is based on the assumption that wave fronts always enclose a region. If a front ends inside a layer we can further divide the zero regions by lengthening this end.

These two constraints already reduce the number of possible interpretations. Consider figure 4 showing two waves w_1 and w_2 at two snapshots 1 and 2. Their wave fronts in snapshot 1 are indicated by dotted lines. The 1-st enclosed region of wave w_1 overlaps with the zero regions A and D, whereas the 1-st enclosed region of wave w_2 overlaps with C and D. Due to our first constraint, we get:

$$\{A, D\} \subseteq \mathcal{Z}_2(w_1) \qquad \{C, D\} \subseteq \mathcal{Z}_2(w_2) \tag{13}$$

In order to explain the frontier between C and D, the region C cannot be an element of $\mathcal{Z}_2(w_1)$. Similarly, A cannot be an element of $\mathcal{Z}_2(w_2)$. What about B and E? Their frontiers to the regions A and C are explained if they are not enclosed by wave w_1 and w_2. However, the frontiers of B are also explained if B is enclosed by w_1 and w_2. A similar argument holds for E. Thus, there are four characterizations explaining all frontiers:

$$\begin{array}{lll} 1. & \{A, D\} = \mathcal{Z}_2(w_1) & \{C, D\} = \mathcal{Z}_2(w_2) \\ 2. & \{A, D, E\} = \mathcal{Z}_2(w_1) & \{C, D, E\} = \mathcal{Z}_2(w_2) \\ 3. & \{A, D, B\} = \mathcal{Z}_2(w_1) & \{C, D, B\} = \mathcal{Z}_2(w_2) \\ 4. & \{A, D, B, E\} = \mathcal{Z}_2(w_1) & \{C, D, B, E\} = \mathcal{Z}_2(w_2) \end{array} \tag{14}$$

In order to reduce these ambiguities, we require that wave fronts are as close as possible to the old fronts. This can be achieved by minimizing the characterizations of waves. Let $\mathcal{Z}^1_{i+1} : \mathcal{W} \to 2^{\mathcal{Z}}$ and $\mathcal{Z}^2_{i+1} : \mathcal{W} \to 2^{\mathcal{Z}}$ be two characterizations. $\mathcal{Z}^1_{i+1}$ is *preferred* to $\mathcal{Z}^2_{i+1}$ iff

$$\mathcal{Z}^1_{i+1}(w) \subseteq \mathcal{Z}^2_{i+1}(w) \tag{15}$$

for each $w \in \mathcal{W}$. We don't accept a characterization $\mathcal{Z}'_{i+1}$ if there exists a characterization $\mathcal{Z}^*_{i+1}$ that is preferred and different to $\mathcal{Z}'_{i+1}$ and that satisfies the two constraints introduced above.

This eliminates the characterizations 2, 3, 4 of our example. Hence, the wave w_1 is characterized by $\{A, D\}$ and the wave w_2 is characterized by $\{C, D\}$. Our three principles are sufficient to track histories if the gaps between two succeeding fronts of a wave are not too large.

3.4 Detecting new histories

Intersections between different histories can be the start of new histories. In the case of seismic waves, the intersection of two wave histories is without interaction, whereas the intersection between a wave history and the static history of an obstacle bears new wave histories in the regions surrounding the obstacle. In this section, we show how to detect incidents and their resulting waves. For the sake of shortness, we consider only incidents at interfaces causing reflections and transmissions.

First, we discuss how to detect and characterize incidents to interfaces. Let w be a wave in layer L_1 and I be the (coherent) interface $I_V(L_1, L_2)$ between L_1 and a neighbouring layer L_2. The event of an incident of w to I is denoted by $e(w, I)$.

To detect an incident $e(w, I)$ in snapshot i, we are checking whether w hits I in this snapshot. A wave hits an interface iff the region behind it, as well as the region in front of it are touching this interface. A region $X \subseteq \mathcal{P}$ touches an interface $I \subseteq \mathcal{N}$ if some points of X are linked to other points by I. In this case, the following set of edges is non-empty:

$$touch(X, I) := \{(p, q) \in I \mid p \in X \text{ or } q \in X\} \tag{16}$$

We can now characterize incidents by the points in $to_i(w)$ and $from_i(w)$ that are touching I:

$$\begin{array}{rcl} from_i(e(w, I)) & := & touch(from_i(w), I) \\ to_i(e(w, I)) & := & touch(to_i(w), I) \end{array} \tag{17}$$

The set $from_i(e(w, I))$ is called the i-th *enclosed interface* of the incident $e(w, I)$, whereas $to_i(e(w, I))$ is called the i-th *propagation interface* of the incident $e(w, I)$.

If the i-th enclosed interface is empty then the incident $e(w, I)$ has not yet occurred in snapshot i. If the i-th propagation interface is empty then the incident is completed in snapshot i. If neither the i-th propagation interface, nor the i-th enclosed interface are empty then the incident *occurs* in snapshot i.

If an incident $e := e(w, I)$ occurs in a snapshot i then it causes a reflected wave $r(e)$ in the layer L_1 of the incident wave and a transmitted wave $t(e)$ in the layer L_2 on the other side of the interface. These waves exist if and only if the incident occurs:

$$\begin{array}{lll} from_i(e(w, I)) \neq \emptyset & \text{iff} & from_i(r(e(w, I))) \neq \emptyset \\ from_i(e(w, I)) \neq \emptyset & \text{iff} & from_i(t(e(w, I))) \neq \emptyset \end{array} \tag{18}$$

The enclosed region of the reflected wave is touching the interface I exactly at the enclosed interface of the incident. Furthermore, the enclosed region of the reflected wave is included in the enclosed region of the incident wave:

$$\begin{array}{rcl} touch(from_i(r(e(w, I))), I) & = & from_i(e(w, I)) \\ from_i(r(e(w, I))) & \subseteq & from_i(w) \end{array} \tag{19}$$

The transmitted wave is touching the interface I at least at the enclosed interface of the incident:

$$touch(from_i(t(e(w, I))), I) \supseteq from_i(e(w, I)) \tag{20}$$

We don't get the inverse inclusion because the transmitted wave can be faster than the incident wave.

We are thus able to detect reflections and transmissions of existing waves, but we have not yet discussed how to detect the initial waves. Initial waves are obtained around a given source point $s \in \mathcal{P}$. Let L be the layer containing s. The initial wave w_0 encloses a zero region containing this source point provided the region does not cover the complete layer L:

$$\text{if} \quad Z \in \mathcal{Z}_i(L),\ s \in Z,\ Z \neq L \quad \text{then} \quad Z \in \mathcal{Z}_i(w_0) \tag{21}$$

As an example, we interpret the first and second snapshots of figure 1. Snapshot 1 decomposes layer L_1 into two zero regions A_{11}, A_{12} that are separated by a frontier. The region A_{11} contains the source point s. Hence, A_{11} is enclosed by the initial wave w_0. w_0 cannot enclose A_{12} because otherwise the frontier between A_{11} and A_{12} is not explained. Therefore, $\mathcal{Z}_1(w_0) = \{A_{11}\}$.

In the second snapshot, layer L_1 is divided into the zero regions $A_{21}, A_{22}, A_{23}, A_{24}$. The region A_{21} has frontiers with A_{22}, A_{23}, A_{24}. The old enclosed region of w_0 is overlapping with A_{21} and A_{22}. Hence:

$$\{A_{21}, A_{22}\} \subseteq \mathcal{Z}_2(w_0) \tag{22}$$

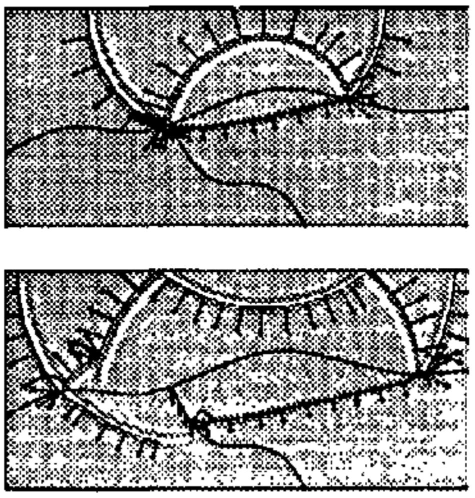

Figure 5: Wave fronts detected by the Sismonaute using rays

Region A_{22} is enclosed by w_0 and touching the interface I_1 between the layers L_1 and L_3. Therefore, the incident $e(w_0, I_1)$ occurs in snapshot 2 and the waves $r(e(w_0, I_1))$ and $t(e(w_0, I_1))$ exist in this snapshot. The reflected wave is in layer L_1 and its enclosed region is touching the interface I_1 at the enclosed interface of the incident. Therefore, the reflected wave contains the region A_{22}:

$$\{A_{22}\} \subseteq \mathcal{Z}_2(r(e(w_0, I_1))) \tag{23}$$

Layer L_3 is divided into the zero regions B_{21} and B_{22} which are separated by a frontier. B_{21} is touching the interface I_1 at the part that is touched by A_{22}. Since the transmitted wave is touching I_1 at the enclosed interface of the incident we get:

$$\{B_{21}\} \subseteq \mathcal{Z}_2(t(e(w_0, I_1))) \tag{24}$$

In order to explain all observed frontiers, the waves cannot enclose more regions as deduced above. Therefore, we get the following characterizations of waves:

$$\begin{array}{lcl} \mathcal{Z}_2(w_0) & = & \{A_{21}, A_{22}\} \\ \mathcal{Z}_2(r(e(w_0, I_1))) & = & \{A_{22}\} \\ \mathcal{Z}_2(t(e(w_0, I_1))) & = & \{B_{21}\} \end{array} \tag{25}$$

This example shows that it is possible to interpret snapshots by a qualitative analysis of zero regions and their neighbourhood relationships. This method can be extended to other kinds of waves such as diffracted waves. Problems are encountered if 1. fronts end in a region without enclosing it and 2. two fronts of a different origin are linked without showing an indication where this link can be found. In order to treat these problems, we need additional physical knowledge that cannot been extracted from the images. In [Junker, 1994], a wave front has been characterized by a sequence of rays called *polyray* (cf. figure 5). Polyrays provide the additional knowledge, but are difficult to manage when traversing curved interfaces. A compromise could be the use of two auxiliary rays marking the left and right ends of wave fronts to meet the problems of the qualitative interpretation method.

4 Related work

Recent work demonstrates the power of augmenting numerical simulation with qualitative notions. [Forbus and Falkenhainer, 1990] define the notion of *self-explanatory simulations* where the simulator itself is able to explain its behavior:

> a self-explanatory simulation integrates qualitative and numerical models to produce accurate predictions and causal explanations of the behavior of continuous physical systems.

They illustrate this definition with the SIMGEN program on physical systems simulated by ordinary differential equations.

Other examples of programs mixing quantitative simulation with qualitative notions can be found in the AI literature : the most famous being Q3 [Kuipers and Berleant, 1988], POINCARE [Sacks, 1991], the Kineticist's Workbench [Eisenberg, 1991], and others [Yip, 1987], [Zhao, 1991]. [Forbus, 1991] addressed an extension of qualitative reasoning to spatial information. In this work, Forbus advocates that, in order to be able to reason about, spatially distributed system, one needs to mix two representations which he calls a *metric diagram* (the quantitative part), and a *place vocabulary* (the symbolic part). The metric diagram is used for calculation whereas the place vocabulary is used for describing the system's behavior at a more abstract level, and for guiding the numeric computations which take place on the metric diagram. Moreover, these two representations are intertwined so that there is a correspondence between the places identified by the place vocabulary, and the quantities manipulated in the analog representation. In a more recent paper [Forbus, 1994], he proposes six challenge problems for spatial reasoning, the fourth one being :

> develop a system which can, given a sequence of weather maps for a region, provide a consistent qualitative explanation of the atmospheric behavior during that period ...

The problem we have addressed is very similar: Given a sequence of 2D snapshots of seismic amplitudes within the underground, our method provides a consistent qualitative explanation of the propagation of acoustic waves during that period. This has been achieved by effectively integrating several representations, namely a metric diagram (i.e. *fields)* used for simulation and a place vocabulary (i.e. *objects)* describing the geological structures. [junker, 1994] additionally experimented with a physical representation based on rays.

Research in qualitative and model-based reasoning has focused since its beginning on systems that could be simulated by ordinary differential equations (ODEs). Numerical simulators using differential equations can be divided into two classes: Those using scalar variables and those using field variables. Scalar variables describe different quantitative properties of a system and are not distributed over a space. Good examples of this class of systems are simple physical devices, chemical processes, chemical kinetics, global socio-economical models, or econometric models. In contrast to this, field variables are distributed over a space, often related to the real world in one, two or three dimensions. Within this category, we can distinguish between fields of scalar variables and fields of vector variables. Different simulation techniques are used for approaching this kind of problems. Finite difference and finite elements are the conventional tools used by applied mathematicians for the simulation of field variables. Examples are fluid dynamics, geophysics or mechanics. Other approaches for field variables are naive physics and cellular automata, the basis for a number of ALife experiments such as Conway's game of life.

5 Conclusion

We developed a method for interpreting snapshot images produced by finite element, simulators for seismic wave propagation. As a result, the regions in the images are linked with Hayes-like histories of waves:

1. In order to detect wave fronts in a snapshot, we characterized them by the zero regions in the background they are enclosing. The first, snapshot contains a single front enclosing the zero region that contains the source point.
2. Symbolic constraints are posed on the zero regions to track a given wave from one snapshot to the other and to detect new waves. We obtain new waves when wave histories intersect with the static histories of obstacles.

A first prototype of a snapshot interpreter which is called SISMONAUTE [Junker, 1994] has been implemented using the ILOC tools LELISP, AIDA, and SMECI. This experience enabled us to find the crucial concepts for characterizing waves and for describing histories, as well as symbolic constraints, which enables the use of constraint programming tools to find globally consistent interpretations.

As a future perspective, the interpretation method could be adapted to other kinds of numerical simulations (e.g. that of flame fronts in simulations of combustions).

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References

[Anne and Brae, 1994] Laurent Anne and Jean Brac. 3D acoustic modelling software - B-version - Documentation and validation. Technical report, Institut Franc.ais du Petrole, Rueil-Malmaison, France, 1994,

[Eisenberg, 1991] M. Eisenberg. The kineticist's workbench : Combining symbolic and numerical methods in the simulation of chemical reaction mechanisms. Technical Report 1306, MIT Artificial Intelligence Laboratory, 1991.

[Forbus and Falkenhainer, 1990] K.D. Forbus and B. Falkenhainer. Self-explanatory simulations: An integration of qualitative and quantitative knowledge. In *Proceedings of the Eighth National Conference on Artificial Intelligence (AAAI-90),* pages 380-387. AAA! Press, 1990.

[Forbus, 1984] K. D. Forbus. Qualitative process theory. *Artificial Intelligence,* 24:85-168, 1984.

[Forbus, 1991] K.D. Forbus. Qualitative spatial reasoning. *Artificial Intelligence,* 51(3), 1991.

[Forbus, 1994] K. D. Forbus. Qualitative spatial reasoning: Framework and frontiers, report available by internet, Northwestern University, Evanston, IL, 1994.

[Fox, 1989] M. Fox. Knowledge-based simulation. In N. Nielsen L. Widman, K. Loparo, editor, *Artificial Intelligence, Simulation and Modeling.* Wiley Interscience, 1989.

[Hayes, 1985a] P.J. Hayes. Naive physics i: Ontology for liquids. In J, Hobbs and B. Moore, editors, *Formal Theories of the Commonscnse World,* pages 71-89. Ablex Publishing Corporation, 1985.

[Hayes, 1985b] P.J. Hayes. The second naive physics manifesto. In J. Hobbs and B. Moore, editors, *Formal Theories of the Commonscnse World,* pages 1-36. Ablex Publishing Corporation, 1985.

[Junker, 1994] U. Junker. Model-based reasoning meets 2I)-simulalions: Detecting physical wave phenomena in a series of snapshots. In *IA 94,* Paris, 1994. EC2.

[Kuipers and Berleant, 1988] B.J. Kuipers and D. Berleant Using incomplete quantitative knowledge in qualitative reasoning. In *Proceedings of the Seventh National Conference on Artificial Intelligence (AAAI 88),* pages 324-329. Morgan Kaufmann, San Mateo, CA, 1988.

[Lavergue, 1986] M. Lavergne. *Seismic methods.* Editions Technip et Institut Francais du Petrole, Paris, 198G.

[Sacks, 1991] E.P. Sacks. Automatic analysis of one-paraineter planar ordinary differential equations by intelligent numerical simulation. *Artificial Intelligence,* 48(1), 1991.

[Weld and de Kleer, 1990] D. S. Weld and J. de Kleer, editors. *Readings in Qualitative Reasoning About Physical Systems.* Morgan Kaufmann Publishers, San Mateo, CA, 1990,

[Yip, 1987] K.M. Yip. Extracting qualitative dynamics from numerical experiments. In *Proceedings of the Sixth National Conference on Artificial Intelligence (AAAI-87),* pages 665-671. Morgan Kaufmann, Los Altos, CA,, 1987.

[Zhao, 1991] F. Zhao. Extracting and representing qualitative behaviors of complex systems in phase spaces. In *Proceedings of the 18th International Conference on Artificial Intelligence,* pages 1144-1149. Morgan Kaufmann, Los Altos, CA,, 1991.