Possibilistic Temporal Reasoning based on Fuzzy Temporal Constraints*

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Abstract

In this paper we propose a propositional temporal language based on fuzzy temporal constraints which turns out to be expressive enough for domains -like many coming from medicine- where knowledge is of propositional nature and an explicit handling of time, imprecision and uncertainty are required. The language is provided with a natural possibilistic semantics to account for the uncertainty issued by the fuzziness of temporal constraints. We also present an inference system based on specific rules dealing with the temporal constraints and a general fuzzy *modus ponens* rule whereby behaviour is shown to be sound. The analysis of the different choices as fuzzy operators leads us to identify the well-known *Lukasiewicz implication* as very appropriate to define the notion of *possibilistic entailment,* an essential element of our inference system.

1 Introduction

Representation and reasoning about time is a major issue to be considered in all those reasoning tasks which take account of a dynamic domain. Most of AI systems incorporating an explicit temporal representation are based in some manner on constraint-reasoning techniques [Allen, 1983; Malik and Binford, 1983; Valdes-Perez, 1986; Vilain and Kautz, 1986; Dean and McDermott, 1987; Knight and Jixin, 1992; Porto and Ribeiro, 1992; Vila, 1994a] [1]. Temporal constraints account for uncertainty in temporal knowledge up to a certain extent. Both qualitative [Allen, 1983; Vilain and Kautz, 1986; van Beek, 1989] and metric temporal constraints [Dechter *et a*l., 1991] represent the set of "equally possible" precise temporal relations between two time units. The larger is this set, the higher is the degree of uncertainty about the temporal relation.

Nonetheless, this is not enough for domains where knowledge about time is highly pervaded with vagueness and uncertainty. Let's illustrate it with an example taken from a medical domain and supplied by an expert in the domain of lumbalgia pathologies. *Brucelosis* is one infectious pathology which may be the origin of serious lumbalgia problems. It has an evolution pattern which can be regarded as a particular instance of the common infectious evolution pattern. It is usually composed of an *Inoculation event* [1], an *Initial Period* [IP], a period of *Ondulating Fever* [OFP] and, finally, it reaches the state of an *Intervertebral Affection* [IA]. We reproduce here the temporal aspects of this knowledge without abstracting the vagueness in terms of which it is obtained from experts. There is some (vague) knowledge about the temporal evolution of *brucelosis* cases:

- The *initial period* usually starts at a time between one and three weeks after the *inoculation event,* although extreme cases range from starting at the very inoculation time up to four weeks after.
- The *initial period* uses to be short: it lasts few days, although in unusual cases it may last up to two weeks at most.
- The *ondulating fever* period always occurs after the *initial period.* It uses to last between 20 and 25 days though other less possible cases range from 4 days as the lowest bound up to 45 days the uppest one.
- Finally, the *intervertebral affection* usually starts between 15 and 20 days after the start of the *ondulating fever* period, having extreme cases where the *intervertebral affection* did not appear after 22 months. The *intervertebral affection* lasts more or less between 3 and 6 months. Extreme cases may range from the shortest case of 40 days and longer periods up to 12 months in cases where it is not properly treated.

Several pieces of work have considered the representation of approximate temporal knowledge. Among them we would stress those based on *possibility theory* [Vitek, 1983; Dutta, 1988; Dubois and Prade, 1989; Kohlas and Monney, 1990; Console *et a*l., 1991; Dubois *et a*l., 1992; Marin *et a*l., 1994; Barro *et a*l., 1994;

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[1] See [Vila, 1994b] for a comparative survey.

Vila and Godo, 1995]. In particular, Barro *et al* [1994] propose an straight forward redefinition of generalization of the notion of metric temporal constraint based on fuzzy sets [Marin *et al.,* 1994].

In this paper we propose an approximate temporal logic based on the embedding of fuzzy temporal constraints into a logical framework. It is provided with an inference system composed of specific inference rules for the fuzzy temporal constraints. For the sake of clarity, we have chosen a simple propositional language. Dealing with fuzzy temporal constraints leads to many-valued interpretations of our formulas, but inference from fuzzy constraints also induces uncertainty as soon as they represent a kind of incomplete information. This induces to extending the whole language to handle both fuzziness and uncertainty. The natural framework where to model fuzziness and uncertainty in a unified way is the possibilistic framework. Therefore this will be the model used througout this paper in accordance with using fuzzy sets for representing the temporal constraints. Uncertainty will be taken into account in the extended language by attaching certainty (necessity-like) degrees to formulas.

The paper is organized as follows. In next section, we describe the syntax -which is illustrated by formalizing the example above introduced- and semantics of our basic language. In the third section an extended language and semantics to handle uncertainty is presented. In section 4 we present an inference system and prove its soundness. Finally, we sketch future lines of work.

2 The Basic Temporal Language

We start out from a language where the temporal and the atemporal parts are neatly separated. The atemporal part is simply made over a set of classical crisp propositions. The temporal part is based on fuzzy temporal constraints over a set of temporal propositions. It consists of the introduction of a single predicate FuzzDIST which states a fuzzy temporal constraint between a pair of time points. Whereas in the metric case, a temporal constraint is represented as an interval, now it is represented by a (convex) fuzzy set of time points, inducing a possibility distribution on the set of duration values . Although this approach is very simple in definition, it is highly powerful in expressiveness. Some important constraint based temporal representations, like *point algebra* [Vilain and Kautz, 1986], or the *metric pointwise constraints* [Dechter *et al,* 1991] turn out to be a particular case of it. The link between the temporal propositions and the temporal constraints is performed through the duration-valued functions BEGIN(P) and END(P) that specify the initial and final instants of the period the proposition *p* holds throughout.

Regarding the underlying time structure, and for the sake of simplicity, we take a fix interpretation of the set of duration symbols *VU* as the set of rational numbers *Q*. Accordingly, the set *TVU* of fuzzy durations will be taken as the set of fuzzy subsets of *Q,* i.e. *TVU* = $Q^{[0,1]}$. However, nothing would prevent us to take other particular either *discrete* or *dense* group structures of time.

2.1 Syntax

The sentences of our propositional language $\mathcal{L}$ are built up over propositions and fuzzy temporal constraints. The following symbols are used:

$\mathcal{P}_\infty$, a set of primitive atemporal propositional variables,

$\mathcal{P}_t$, a set of primitive temporal propositional variables, and

t_0, a special time point symbol.

We have two sorts of atomic formulas:

- **Atemporal Propositions:** the elements of $\mathcal{P}_\infty$.
- **Temporal constraints:** consist of a set of propositional variables (indexed by $\mathcal{P}_t \cup \{t_0\} \times \mathcal{P}_t \cup \{t_0\} \times \mathcal{FDU}$) of the form
$$\textsc{FuzzDist}(t, t', \pi),$$
where t and t' are temporal expressions of the form $\textsc{Begin}(p)$, $\textsc{End}(q)$ or t_0, being p and q some temporal propositional variables from $\mathcal{P}_t$, and $\pi \in \mathcal{FDU}$ is a fuzzy duration. The symbol t_0 represents a common reference time point, $\textsc{Begin}(p)$ the initial instant of the time interval where p holds, $\textsc{End}(q)$ the ending instant for q, and π represents a soft constraint on the *duration temporal variable* "$t' - t$".

The well-formed formulas (wffs) of our basic language $\mathcal{L}$ are of the form:

$$B_1 \wedge \ldots B_i \ldots \wedge B_m \Longrightarrow H$$

with $m \geq 0$, where B_i and H are either literals from $\mathcal{P}_\infty$ or temporal constraints.

Like in *definite clauses*, the conjunction of B_i is called the *body* or *antecedent* and H is called the *head* or *conclusion*. We distinguish between two types of formulae according to the form of the *body*:

- **fact**, when $m = 0$ (empty body) and written simply H, and
- **rule**, otherwise.

To illustrate the usage of our language in formalizing domain knowledge let's consider the example in the introductory section. Figure 1 presents a graphical representation of it. Events and properties being part of the temporal evolution description are taken as primitive temporal propositional variables. We shall approximate a soft constraint in FD*U* by a trapezoidal function characterized by four points [2]. From a knowledge adquisition point of view the second and third point determine the interval of those temporal values which are likely whereas the first and fourth points determine the interval out of which the values are absolutely impossible. For example, both the inoculation event and the initial period state are conceptualized as temporal propositional variables and the first statement related to the temporal distance between the inoculation event and the begin of the initial period will be formalized as a fuzzy temporal constraint described by [0, 7, 21, 28] where these values represent days. The *heuristic rule* formalizing the whole piece of knowledge would be as follows:"

[2]Notice that this approximation is only feasible for unimodal fuzzy constraints.

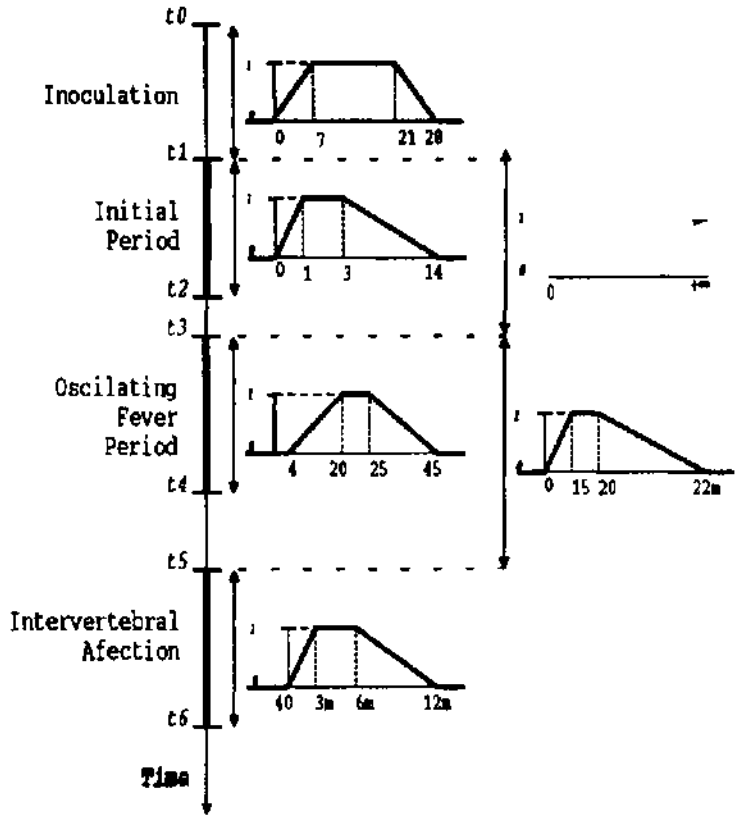

Figure 1: Brucelosis Evolution Pattern.

$\text{FuzzDist}(\text{Begin}(I), \text{Begin}(IP), [0, 7, 21, 27])$
$\wedge$
$\text{FuzzDist}(\text{Begin}(IP), \text{End}(IP), [0, 1, 3, 15])$
$\wedge$
$\text{FuzzDist}(\text{Begin}(OFP), \text{End}(OFP), [4, 20, 25, 45])$
$\wedge$
$\text{FuzzDist}(\text{Begin}(IP), \text{Begin}(OFP), [0, 0, +\infty, +\infty])$
$\wedge$
$\text{FuzzDist}(\text{Begin}(IA), \text{End}(IA), [40, 3m, 6m, 12m])$
$\wedge$
$\text{FuzzDist}(\text{Begin}(OFP), \text{Begin}(IA), [0, 15, 20, 22m])$
$\Longrightarrow$ Brucelosis

where m stands for *month* (*Xm* can be taken as a short hand of the value $X * 30$).

2.2 Semantics

The semantics of our propositional language $\mathcal{L}$ involves, for each model, first the assignment of intervals of time points to the temporal propositional variables, and second the interpretation in terms of truth-values of the formulas of the language. Atemporal propositional variables can be directly assigned to either 1 *(True)* or 0 *(False)*, while temporal constraints expressions are assigned truth-values of [0,1) via the fuzzy duration function they contain [3]. As for compound formulas, we have chosen to interpret conjunction by the *min* function and implication by the Godel's many-valued implication function (see Definition 2). The choice of *min* for conjunction is the usual one in fuzzy logics and Gödel's implication is its corresponding residuum. In doing so, we assure a correct behaviour of the implication with respect to a suitable notion of logical consequence (see Proposition 1, next Section) and later on with respect to modus ponens (see Proposition 5, Section 4).

Definition 1 *An $\mathcal{L}$-model is a pair $\omega = \langle \omega_t, \omega_\infty \rangle$, where*

$\omega_t : \mathcal{P}_t \rightarrow \text{Pairs}(\mathcal{DU})$, where $\text{Pairs}(\mathcal{DU})$ is the set of ordered pairs (x, y) from $\mathcal{DU}$ such that $x \leq y$, and
$\omega_\infty : \mathcal{P}_\infty \rightarrow \{0, 1\}$

Definition 2 *Let Ω denote the set of $\mathcal{L}$-models. The satisfaction relation $\models$ is a mapping*

$$\models : \Omega \times \mathcal{L} \rightarrow [0, 1]$$

defined as:

$\models (\omega, p) = \omega_\infty(p)$, *if* $p \in \mathcal{P}_\infty$
$\models (\omega, \neg p) = 1 - \omega_\infty(p)$, *if* $p \in \mathcal{P}_\infty$
$\models (\omega, \text{FuzzDist}(t, t', \pi)) = \pi(M_\omega(t') - M_\omega(t))$,
where M_ω is the interpretation function of temporal terms defined as
$M_\omega(\text{Begin}(p)) = t_1$, *if* $\omega_t(p) = (t_1, t_2)$
$M_\omega(\text{End}(p)) = t_2$, *if* $\omega_t(p) = (t_1, t_2)$
$M_\omega(t_0) = 0$
$\models (\omega, B \Rightarrow H) = \mathcal{I}_G(\omega(B), \models (\omega, H))$, *where $\mathcal{I}_G$ is the Gödel's many-valued implication function:*

$$\mathcal{I}_G(x, y) = \begin{cases} 1 & x \leq y \\ y & x > y \end{cases}$$

and, if $B = B_1 \wedge \ldots \wedge B_n$ then $\omega(B) = min(\models (\omega, B_1), \ldots, \models (\omega, B_n))$.

From now on, and for the sake of a simpler notation, we will write $\omega(\phi)$ for $\models (\omega, \phi)$.

2.3 Fuzzy Temporal Constraint Inference Rules

Now, we are interested in presenting a set of inference rules that deal exclusively with temporal constraints and that capture the completion operations performed in temporal constraint networks. Actually, they are the extension for fuzzy constraints of those proposed in [Vila and Escalada-Imaz, 1994] that, together with modus ponens, were proved to be complete for crisp constraints. We will show their soundness w.r.t. the following notion of logical consequence in accordance to the well-known Zadeh's *entailment principle* in *fuzzy logic* [Zadeh, 1979].

Definition 3 *A well-formed formula ϕ is valid, written $\models \phi$, iff $\omega(\phi) = 1$ for all $\omega \in \Omega$.*

Definition 4 *A well-formed formula ϕ is a logical consequence of a set of other well-formed formulas $\phi_1, ..., \phi_n$, written $\{\phi_1, ..., \phi_n\} \models \phi$ iff for any $\omega \in \Omega$, $min(\omega(\phi_1), ...\omega(\phi_n)) \leq \omega(\phi)$.*

An immediate consequence of this definition of logical consequence is the following proposition which is a kind of restricted deduction theorem.

[3] Without gain of complexity we could allow the atemporal propositions to be fuzzy as well and thus to have a more general language but, for the sake of clarity we prefer to only allow fuzziness in the temporal expressions, which is the focus of the paper.

Proposition 1 *Let $\phi_1, ..., \phi_n$ and ϕ be facts. Then it holds*

$$\{\phi_1, ..., \phi_n\} \models \phi \quad \textit{iff} \quad \models \phi_1 \wedge ... \wedge \phi_n \Longrightarrow \phi.$$

The definition of the inference rules make use of the following two special fuzzy duration sets:

Null duration function,

$$\pi_0(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Universal duration function,

$$\pi_U \equiv 1$$

and of the following three operations on the set of fuzzy durations:

Symmetrization: $\pi^-(x) = \pi(-x)$

Conjunction: $(\pi \sqcap \pi')(x) = min\{\pi(x), \pi'(x)\}$

Composition: $(\pi \oplus \pi')(x) = \sup_{x=y+z} \{min\{\pi(y), \pi'(z)\}\}$

Next we present the inference rules.

$$\frac{}{\text{FUZZDIST}(t, t, \pi_0)} \{\text{Reflexivity}\}$$

$$\frac{}{\text{FUZZDIST}(t_1, t_2, \pi_U)} \{\text{Universal constraint}\}$$

$$\frac{\text{FUZZDIST}(t_1, t_2, \pi_{d_{12}})}{\text{FUZZDIST}(t_2, t_1, \pi^-_{d_{12}})} \{\text{Symmetry}\}$$

$$\frac{\text{FUZZDIST}(t_1, t_2, \pi_{d_{12}}), \text{FUZZDIST}(t_2, t_3, \pi_{d_{23}})}{\text{FUZZDIST}(t_1, t_3, \pi_{d_{12}} \oplus \pi_{d_{23}})} \{\text{Transitivity}\}$$

$$\frac{\text{FUZZDIST}(t_1, t_2, \pi_{d_{12}}), \text{FUZZDIST}(t_1, t_2, \pi'_{d_{12}})}{\text{FUZZDIST}(t_1, t_2, \pi_{d_{12}} \sqcap \pi'_{d_{12}})} \{\text{Intersection}\}$$

$$\frac{\text{FUZZDIST}(t_1, t_2, \pi_{d_{12}})}{\text{FUZZDIST}(t_1, t_2, \pi'_{d_{12}}), \text{ if } \pi_{d_{12}} \leq \pi'_{d_{12}}} \{\text{Inclusion}\}$$

Proposition 2 *The above inference rules are sound w.r.t. the previous semantics.*

Proof: See [Godo and Vila, 1995]. □

3 The Possibilistic Temporal Language

To fully exploit the use of fuzzy expressions for temporal durations in the language, it seems very natural to also allow for partial degrees of matching between fuzzy expressions. As a matter of example, consider the following piece of knowledge: the duration of the *Ondulating Fever Period* of a patient has been between 2 and 3 weeks, but it is known that in any case it has been not less than 17 days and not more than 32. This knowledge can be represented by the proposition FUZZDIST(BEGIN(OFP),END(OFP),π), being the trapezoidal possibility distribution corresponding to the parameters [17, 22, 27, 32] and presented graphically in figure 2.

On the other hand, the example rule in Section 2.1, codifying the Brucelosis evolution pattern, has FuzzDIST(BEGIN(OFF), END(OFP), [4, 20,25,45]) as

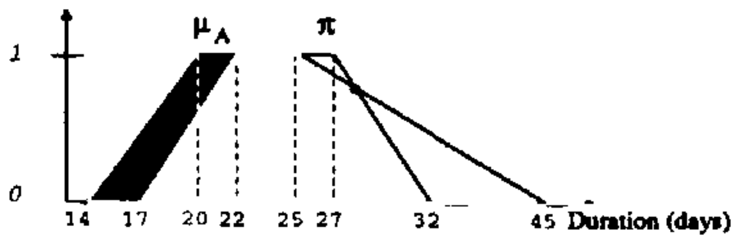

Figure 2: Partial matching of fuzzy constraints.

one of conditions, being the membership function A of the fuzzy duration represented by the tuple [4, 20, 25, 45] also shown in the figure 2. Of course $\Pi < \mu_A$, and therefore FUZZDIST(BEGIN(OFP),END(OFP),Π) does not entail FUZZDIST(BEGIN(OFP), END(OFP),μ_A), in the sense of Definition 4, but one would say that it nearly entails it. In such a context, if we want to use the above mentioned rule to conclude about the possibility for that patient of having *Brucelosis,* it makes sense to think of a way to measure at what extent π is included in μ_A *AN*d use this measure as a certainty degree with which that condition of the rule is satisfied [4]. Therefore, in reasoning with fuzzy constraints one is led to deal with partial degrees of certainty, mainly of propositions involving fuzzy duration constraints but also non-fuzzy propositions. On the other hand, many AI domains, being the medical one a good example, require the management of uncertainty from a knowledge representation language. *Possibility theory* (Dubois and Prade, 1988] offers a unified framework where to model both uncertainty and fuzziness. We would like to stress here again that the kind of uncertainty the possibilistic model deals with comes from the use of imprecise knowledge modelled by a fuzzy set, and differs from other kinds of uncertainty, like probability, which are of different nature. We present below an extension of the language decribed in the previous section where lower bounds of a necessity-like degree are attached to formulas, with a semantics based on ideas in [Dubois and Prade, 1990; 1992] and extending the Dubois, Lang and Prade's Possibilistic Logic semantics [Dubois *et a*l., 1994] for crisp propositions.

Let us make more precise the above claim. Temporal constraint inference rules provide the tightest constraints between durations of events entailed by a given set of temporal facts. Such constraints can be used as inputs in heuristic rules that may help in turn to obtain additional temporal facts. Therefore, when trying to apply heuristic rules, we are interested in certainty qualifying the conditions of such rules given for granted the constraints provided by the temporal facts. Next subsection is devoted to discuss how such certainty evaluation can be performed.

[4] Such certainty degrees should not be confused with the truth degrees arising from the many-valued approach introduced in the previous section to evaluate fuzzy temporal expressions. There, L-models evaluate the truth degree of formulas in a purely functional way. This is not the case with the certainty degrees we propose in the possibilistic temporal extension.

3.1 Certainty evaluation of fuzzy constraints

In technical terms, for a given duration variable *X* on D*U,* we aim at finding the certainty evaluation of a fuzzy proposition *XisA* (the condition of a rule), being *A* a fuzzy subset of D*U,* knowing that the values of *X* are restricted by a possibility distribution (the constraint induced by the temporal fact base). Dubois and Prade [1992] have discussed this issue and they propose to use the following measure:

$$\mathcal{E}(\mu_A|\pi) = \inf_{u \in \mathcal{DU}} \mathcal{I}_G^R(\pi(u), \mu_A(u))$$

where $\mathcal{I}_G^R$ is the reciprocal of the Godel's implication, i.e.:

$$\mathcal{I}_G^R(x, y) = \mathcal{I}_G(1 - y, 1 - x)$$

and *HA* is the membership function of the fuzzy set *A.* It is remarkable to notice that $\mathcal{E}(\mu_A|\pi)$ — 1 iff $\mu_A \geq \pi$, and that $\mathcal{E}(\mu_A|\pi)$ reduces, when A is non-fuzzy, to $N_\pi(A) = inf\{1 - \pi(u) | u \notin A\}$, the necessity measure of *A* based on TT and used in Possibilistic Logic . In fact, Dubois and Prade [1990,1992] discuss the inverse problem, that is, which possibility distribution corresponds to the semantical interpretation of the qualified proposition *"(XisA)* is a — *certain",* and to which evaluation of *A* is identified. This is also of much interest since it will allow us to use uncertain constraints derived from a set of heuristic rules as inputs in the temporal fact base. An interesting line of argumentation leads them to represent the above qualified proposition by the following family of inequalities

$$\pi(u) \leq max(\mu_A(u), 1 - \alpha), \forall u \in \mathcal{DU}$$

which, as expected, turns to be equivalent to

$$\mathcal{E}(\mu_A|\pi) \geq \alpha.$$

However, the certainty degree $\mathcal{E}(\mu_A|\pi)$ provides not very natural results in very common situations. In particular, the existence of only one element *u* in the domain for which *n(u)* — 1 and $\mu_A(u) < 1$ causes the certainty degree $\mathcal{E}(\mu_A|\pi)$ to be 0, independently whether the value $\mu_A(u)$ is close to 0 or close to 1. For instance, this is the case depicted in Figure 2 where an easy computation shows that $\mathcal{E}(\mu_A|\pi) = 0$ while π is very close to entail *A.*

This counter-intuitive behaviour has led us to look for an alternative definition of the certainty degree. If one wants to keep the property that $\mathcal{E}(\mu_A|\pi)$ = 1 iff *TT* < n_A, one is forced to stay either with residuated many-valued implications [5] or with their reciprocals. Residuated implications, in general, share the problem that the resulting certainty degree does not collapse to the necessity degree in the non-fuzzy case (actually it becomes a trivial {0,1}-valued measure), and thus the resulting semantics is not an extension of that of Possibilistic Logic. On the other hand, the reciprocal implications, in general again, share the above mentioned problem of the Godel's reciprocal implication. However, among these two families of implication functions, there is one exception (up to isomorphisms), the well-known Lukasiewicz implication

$$\mathcal{I}_L(x, y) = min(1, 1 - x + y),$$

that avoids the above problems. Namely, defining

$$E(\mu_A|\pi) = \inf_{u \in \mathcal{DU}} \mathcal{I}_L(\pi(u), \mu_A(u))$$

we keep most of the interesting properties of the previous definition while solving the main problem with it. Now the interpretation of *"given* π*, (X is A) is (at least)* α*-certain"* as $E(\mu_A|\pi) \geq \alpha$ is semantically equivalent to

$$\pi(u) \leq \mathcal{I}_L(\alpha, \mu_A(u)), \forall u \in \mathcal{DU}$$

This representation can be provided with practically the same argumentation used in [Dubois and Prade, 1992] to justify their proposal, only a slight modification in one step is needed. The agreement of this proposal with the original one in the non-fuzzy case is easy to establish by noticing that $\mathcal{I}_L(\alpha, \mu_A(u)) = max(1 - \alpha, \mu_A(u))$i when *A* is non-fuzzy.

3.2 Possibilistic Semantics

Now, we are prepared to define our *Possibilistic Temporal Language* and show that captures the above requirements.

Definition 5 *The set of possibilistic temporal formulas is the set* $\mathcal{L}_\mathcal{T} = \{\phi[\alpha] | \phi \in \mathcal{L} \text{ and } \alpha \in [0, 1]\}$.

Definition 6 *A* Possibilistic Temporal model II *is a possibility distribution over the set* Q *of C-models,* II : $\Omega \rightarrow [0, 1]$ [6]

Definition 7 (Possibilistic Entailment) *A possibilistic temporal model* Π *satisfies a* $wff\ \phi$ *with a certainty degree* α*, written* $\Pi \models_\mathcal{T} \phi[\alpha]$, *iff*

$$E(\phi|\Pi) = \inf_{\omega \in \Omega} \mathcal{I}_L(\Pi(\omega), \mu_\phi(\omega)) \geq \alpha$$

where we have identified the $\mathcal{L}$ *-formula* ϕ *with its corresponding fuzzy subset of the set of* $\mathcal{L}$ *-models* Ω *defined as* $\mu_\phi(\omega) = \omega(\phi)$*. The notion of logical consequence is the natural one, i.e. a possibilistic temporal formula G is a logical consequence of a set of possibilistic temporal formulas* F1,...,Fn, *written* $\{F_1, ..., F_n\} \models_\mathcal{T} G$, *iff for any possibilistic model* Π, $\Pi \models_\mathcal{T} F_1, ..., \Pi \models_\mathcal{T} F_n$ *imply* $\Pi \models_\mathcal{T} G$.

The possibilistic entailment [=7- in £7- is related to the entailment relation [= of the basic language *C* (without uncertainty) as follows.

Proposition 3 *Let* $\phi_1, ..., \phi_n$ *and* ϕ *well-formed formulas of C. Then it holds that* $\{\phi_1, ..., \phi_n\} \models \phi$ *iff* $\{\phi_1[1], ..., \phi_n[1]\} \models_\mathcal{T} \phi[1]$

[5] Residuated many-valued implications are binary operations in [0,1] defined as $I(x, y) = Sup\{c \in [0, 1] | x \otimes c \leq y\}$, where $\otimes$ stands for a t-norm, i.e. a a binary operation in [0,1] which is associative, commutative, non-decreasing in each variable, with 1 as neutral element and 0 as absorbent element.

[6] These possibilistic models differ from those of Possibilistic Logic in that the possibility distributions are defined on [0, l]-valued ^-models, rather than on {0, l}-valued L-models.

This proposition shows that the possibilistic entailment $\models_T$ actually extends $\models$ in a natural way, as it could be expected. In particular, the set of inference rules for fuzzy constraints presented in section 2.3 are then also sound w.r.t. for $\models_T$ once the fuzzy temporal constraints appearing in those rules are attached the certainty value 1.

A *possibilistic temporal knowledge base* is a pair *KB* = *(FB, HB)* of a set of weighted facts *TB* and a set of weighted rules R*B*. The temporal fact base will be represented as a network of fuzzy temporal constraints.

4 Inference

In this section we supply the set of inference rules which compose the deductive system of our logic. The separation between the non-temporal and the temporal part also holds for them: we distinguish between rules specific for temporal propositions and rules applied to arbitrary well-formed forms in the *KB*.

Since temporal inference rules deal exclusively with temporal constraints which certainty degree is 1, for the sake of completeness, additional inference rules are needed to state the *degree of fulfillment* for a temporal proposition and viceversa.

Fuzzy Constraint Inference Rules. As already mentioned, the *Reflexivity, Universal Constraint, Symmetry, Transitivity, Intersection* and *Inclusion* inference rules, with the certainty value [1] attached to premises and conclusions, are sound rules w.r.t. to the possibilistic semantics, and they capture constraint network procedures.

Constraint Certainty Inference Rules. The following inference rules show how uncertainty influences fuzzy temporal constraints, and thus how they provide a kind of bridge between knowledge from the temporal constraint network and knowledge from a heuristic rule set. In other words they provide a way to infer certain fuzzy constraints from uncertain ones, and viceversa.

$$\frac{\textsc{FuzzDist}(t_1,t_2,\pi_{d_{12}})[1]}{\textsc{FuzzDist}(t_1,t_2,\pi'_{d_{12}})[\alpha],\ \text{where } \alpha=E(\pi_{d_{12}}|\pi'_{d_{12}})}\ \{R1\}$$

And the converse inference rule is defined as follows:

$$\frac{\textsc{FuzzDist}(t_1,t_2,\pi_{d_{12}})[\alpha]}{\textsc{FuzzDist}(t_1,t_2,\pi'_{d_{12}})[1],\ \text{where } \pi'_{d_{12}}=\mathcal{I}_L(\alpha,\pi_{d_{12}})}\ \{R2\}$$

Taking back the example of the beginning of Section 3 and applying the *Rl* inference rule, from FUZZDIST(BEGIN(OFP), END(OFP), [4, 20,25, 45]), with certainty 1, we can derive the fuzzy constraint FuzzDIST(BEGiN(OFP), END(OFP), [17, 22, 27, 32]), with certainty $E(_A\pi)$ = 0.9 (see Figure 2). This certainty value could be used after to conclude *Brucelosis* from the rule when applying the modus ponens inference rule introduced below.

Proposition 4 *The above two inference rules R1 and R2 are sound.*

Proof: See [Godo and Vila, 1995]. □

General Inference Rule. General inference is performed through a single inference rule which is a sort of *possibilistic modus ponens* (PMP) and extends the one for necessity-valued formulas in Possibilistic Logic [Dubois *et al.*, 1994]:

$$\frac{B_1 \wedge \ldots \wedge B_m \overset{[\alpha]}{\Longrightarrow} H,\quad B_1[\alpha_1], \ldots, B_m[\alpha_m]}{H[\alpha'],\ \text{where } \alpha' = min(\alpha, \alpha_1, \ldots, \alpha_n)}\ \{PMP\}$$

Proposition 5 (soundness) *The above possibilistic modus ponens inference rule PMP is sound*

Proof: For simplicity, we take $m = 1$. Thus we will show that $H[min(\beta,\alpha)]$ is a logical consequence of $\{(B \Longrightarrow H)[\alpha], B[\beta]\}$. Suppose then $E(B \Longrightarrow H|\Pi) \geq \alpha$ and $E(B|\Pi) \geq \beta$. This implies, for all $\omega \in \Omega$, that

$\mathcal{I}_L(\Pi(\omega), \mathcal{I}_G(\omega(B), \omega(H)) \geq \alpha$ and

$\mathcal{I}_L(\Pi(\omega), \omega(B)) \geq \beta$.

Therefore, since $\mathcal{I}_L$ is non-decreasing in the second variable, we have

$min(\alpha,\beta) \leq \mathcal{I}_L(\Pi(\omega), min(\mathcal{I}_G(\omega(B), \omega(H)), \omega(B)))$,

and since the inequality $min(x, \mathcal{I}_G(x,y)) \leq y$ always holds, we also have $min(\alpha,\beta) \leq \mathcal{I}_L(\Pi(\omega), \omega(H))$. Finally, taking infimum w.r.t. ω, we get

$min(\alpha,\beta) \leq inf_{\omega\in\Omega}\mathcal{I}_L(\Pi(\omega), \omega(H)) = E(H|\Pi)$

which ends the proof. □

It is interesting to observe that in the above proof, concerning the certainty degrees, we only make use of the monotonicity of $\mathcal{I}_L$, and so, the *possibilistic modus ponens* rule would be valid also for other definitions of $E(.|.)$ using other implications, in particular for the reciprocal of Gödel's implication –used in [Dubois *et al.*, 1994] to define an extended resolution-like inference rule–, largely discussed in a previous section.

5 Concluding Remarks

We have presented a propositional temporal language based on fuzzy temporal constraints, and able to deal also with uncertainty within the possibilistic framework. Although this is a very restricted language, it turns out to be expressive enough for a large set of applications in the medical domain and, eventually, in other domains where knowledge is of propositional nature, yet explicit account of temporallity and uncertainty are required. This language is provided with:

1. A formal semantics based on possibilistic models to account for the uncertainty issued by the fuzziness of our temporal constraints.
2. A sound inference system composed of a set of fuzzy temporal constraint inference rules, a possibilistic modus ponens and a pair of constraint certainty inference rules. An overall picture of the way this inference system can operate is graphically presented in figure 3.

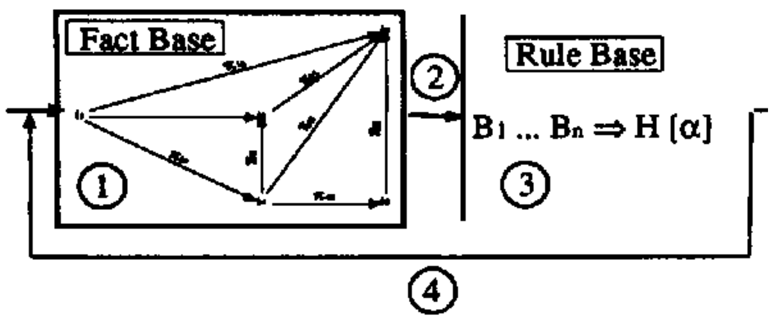

Figure 3: Steps of the uncertain fuzzy temporal Inference: 1. fuzzy temporal constraint inference, 2. constraint certainty evaluation (RI), 3. possibilistic modus ponens, and 4. constraint certainty update (R2).

We are currently studying the completeness of our logic. Previous results on the non-fuzzy case [Vila and Escalada-Imaz, 1994] seem to be a guarantee on the way to proving it. We are also working in developing correct and efficient deductive algorithms to make our language operational.

The approach we have presented here allows for further work on two main lines. First the extension to first-order Horn clauses, incorporating relations on the Fuzzy Duration functions, and second, to involve more general types of constraint networks which will be a matter of study for fuzzy networks as well.

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