

# Modeling the Basic Meanings of Path Relations

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## Abstract

In the field of spatial reasoning, point-to-point relations have been thoroughly examined, but only little attention has been paid to the modeling of path relations. We propose a computational model that extends the existing referential semantics for point-to-point relations to path relations. On the linguistic side, we present some research on German path prepositions as well as results on their English counterparts. This analysis of path prepositions is used to extract a semantic model for path relations. On the geometric side, we examine the characteristics of trajectories and propose a computational method to find an appropriate path relation for a given situation. Finally, we show how our findings on the linguistic and the geometric sides can be brought together to form a consistent model.

## 1 Introduction

Space plays a central role in human cognition, and has therefore been a research focus in different disciplines like (computational) linguistics [Lakoff, 1987], cognitive sciences [Kosslyn, 1994], psychology [Landau and Jackendoff, 1993], and artificial intelligence [Maaß et al., 1993]. Sophisticated conceptual [Egenhofer, 1991] and computational models [Gapp, 1994] have been developed that made it possible to *compute* the appropriateness of spatial relations in specific situations, thereby providing a better understanding of what is meant by certain spatial expressions. These results paved the way for intelligent systems that are able to analyze and generate natural language descriptions of space [Wahlster et al., 1998].

Within the field of spatial relations, so-called *topological* (e.g. near or at) and *projective* relations (like

right-of or above) have been most thoroughly examined. Both groups are *point-to-point relations* as they establish a spatial relation between two objects of arbitrary shape. (Between is an exception from this rule as it requires at least three objects to be computed correctly [Habel, 1989].)

A different kind of spatial relations are the so-called *path relations* (e.g. along, around, or past). Much less attention has been paid to their conceptualization [Kriiger and Maafi, 1997] and computation than in the case of point-to-point relations. This may be due to the greater complexity: the problem of computing the most appropriate path relation can only be solved if there is a *path* which consists at least of a simple line with a starting point and an ending point. As we will show in section 5, topological, projective, and path relations share nevertheless several (geometric) concepts.

In section 2, we present the basic ideas and concepts we will use throughout the paper. Based on an analysis of the linguistic side of the problem (section 3), we propose a semantic model which is described in section 4. In the following section, we turn to the geometric side, and establish a basic framework for geometric path relations. These two sides are integrated to form a consistent model which then is applied to various examples (section 6). Finally, we summarize our findings and give an outlook on future directions.

## 2 Key Concepts and Related Work

Following [Herskovits, 1986] we distinguish between the *basic meaning* of a spatial relation and its instantiation in a concrete situation: An *object to be localized (LO)* is set, in relation to a *reference object (RO)*. Furthermore, a *frame of reference* has to be established in order to distinguish different spatial relations. Determining the origin and the orientation of the reference frame depends on a multitude of factors such as the point of view of the observer/addressee, or the intrinsic orientation of the RO [Maafi, 1993]. The computation of topological

German preposition	Raw translation	Meaning	Formalization
zu	towards	approach	$D \xrightarrow{D} IMD$
bis	up to	approach	$D \xrightarrow{*} IMD$
in	into	translation from outside to inside	$MD \xrightarrow{M} I$
nach	to	approach	$D \xrightarrow{*} IMD$
an	to	approach until contact	$D \xrightarrow{D} MD$
gegen	against	approach until contact	$D \xrightarrow{M} IM$
von	(away) from	increase of distance	$IMD \xrightarrow{*} D$
aus	out of	translation from inside to outside	$I \xrightarrow{M} MD$
entlang	along	keeping distance (along boundary)	$IMD \xrightarrow{*} IMD$
vorbei	past	approach, increase of distance	$D \xrightarrow{D} D$
durch	through	approach, entering of interior, leaving of interior	$IMD \xrightarrow{I} IMD$
um	around	angular movement	$MD \xrightarrow{MD} MD$
		approach, passing along boundary, increase of distance	$MD \xrightarrow{MD} MD$

Table 1: Several German prepositions used for path description

and projective relations relies on a frame of reference. It is used to extract, the two essential parameters that are needed to evaluate the applicability of point-to-point relations: the distance of the LO from the RO (topological relations), and the angle disparity from a prototypical direction (projective relations).

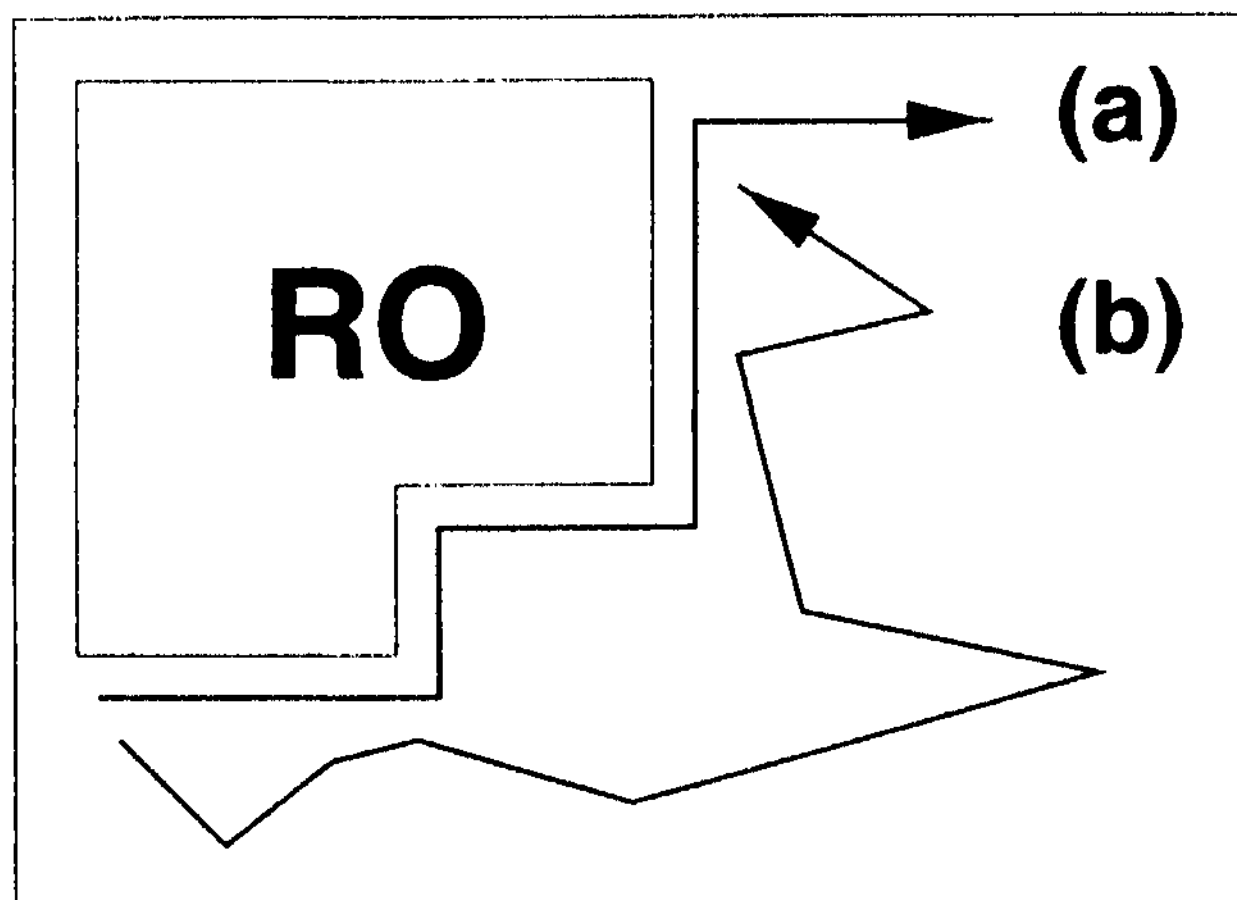


Figure 1: Two trajectories

[Gapp, 1997] proposed a model for spatial relations that is based on a three level *referential semantics*. On the lowest level, only purely visual information is available. This information is abstracted on the semantic level to a geometrical representation which the referential semantic itself relies on. *Idealized meanings* of spatial relations are compared to the actual situation taking into account contextual factors that are modeled on the *conceptual layer*. A *degree of applicability (DA)* is computed that rates how well a relation corresponds to the

prototypical meaning, on a scale from 'zero' (not applicable) to 'one' (fully applicable). For a detailed description of the factors and algorithms used to determine the DA of point-to-point relations, refer to [Gapp, 1997].

Path relations differ from their topological and projective counterparts in two ways. On one side, the LO is expected to be path-like: either its shape has to be path-like, or it can be abstracted to a path-like shape. In some cases, this also holds for the RO. (In the computation of the applicability of point-to-point relations, the shape of the LO is of lesser importance.) On the other side, the computation of path relations cannot be reduced to a simple two point problem. Figure 1 illustrates this fact: Trajectory (a) is certainly a better match for a relation *alongi* (describing a path that follows the form of the RO) than is trajectory (b). But there is no *single* point on either trajectory that can be used to determine the applicability of this relation. (While one may argue about the meaning of "along", *alongi* might actually capture a meaning facet.)

### 3 Analysis of German Path Prepositions

From a computational perspective, one of the main problems in understanding natural language is its inherent ambiguity. This is also true for path prepositions: The German path preposition<sup>1</sup> "zu" (roughly "to", "towards"), for example, is used to describe trajectories that lead towards the reference object. The description "der Weg zu dem Park" ("the way to the park") can mean different things. It is not obvious where the tra-

<sup>1</sup>We use the term '(path) preposition' although, from a strictly linguistic perspective, not all of them are prepositions per se.

jectory starts, nor where it ends: Does it end outside the park, just at the border, or inside? The verbalization process is affected by ambiguity, too: To describe a path that starts within the park and leads outside of it, one could, for example, use "aus" (approx. "out of") or "von" (approx. "from").

Table 1 lists some of the most commonly used German path prepositions. We tried to express the basic meanings of the main uses in natural language and - at a finer level of detail - in a more formal syntax. It should be stated that the table does not contain a complete description of all possible meanings of path prepositions but only *meaning facets*. The formal syntax is based on a subset of Egenhofers semantics for topological relations [Egenhofer, 1991]. "I" stands for *containment* of the LO within the HO (inside), "M" for *contact* of the boundaries of LO and RO (meet), and "D" for the *disjoints* of them (disjoint). So, a formula like

$$\mathbf{IMD} \xrightarrow{M} \mathbf{D}$$

is to be read as

"The corresponding relation describes a path that starts either within the border, or on the border, or outside of the RO. On its way, the contact relation becomes true at least once, and it ends outside of the RO."

We use bold style for the main usage, plain style for possible uses, and italics for unlikely (but still possible) uses of a relation. An asterisk (\*) above the arrow indicates that no special relation has to be fulfilled during transition. Although the table was created with German prepositions in mind, preliminary research on the corresponding English prepositions indicates that a basic set of concepts exists across different languages. Exploratory studies in French and Japanese support this hypothesis, and justify the search for a language independent conceptual model.

## 4 Basic Semantic Concepts

A first look at the meaning column in Tab. 1 reveals that the majority of the prepositions describes an approach towards an object. Only a few words are available to express an increase of distance (e.g. "von", "aus"). (This is not surprising since one usually follows a path with the target in mind.) Furthermore, there is a concept of angular movement (such as in "um" and "entlang"). So, the two essential parameters (distance and angle) needed for the evaluation of point-to-point relations play an important role in the conceptualization of path relations, too.

A second observation is that the meaning component of some path prepositions (e.g. "in") contains a simple point-to-point relation. This relation is applied to either one of the endpoints of the trajectory (in the case of "in": to the ending point), or to the entire trajectory (e.g. "past"). If we take out that element, we are left with a *simple path relation* that is only related to the *path* but not to a single point.

Based on these observations, we can establish a semantics for path relations which relies on simple path relations. We propose the following five simple path relations as building blocks for more complex ones. They can be combined with each other, and/or with point-to-point relations to form higher order path relations.

- decrease-distance: the ending point of the trajectory is closer to the RO than the starting point.
- increase-distance: the starting point of the trajectory is closer to the RO than the ending point.
- maintain-distance: the distance of every point on the trajectory from the RO is the same.
- change-angle: the starting and the ending point form an angle with the RO.
- maintain-angle: the starting and the ending point show no angular disparity in relation to the RO.

Since the direction of the angular disparity cannot be expressed easily using path prepositions (at least as far as German, French, and English are concerned), it makes sense to have just one relation expressing undirected change. This is not true in the case of distance, where we consequently differentiate approach and an increase of distance.

## 5 Geometric path relations

As far as the geometric side of the computation of path relations is concerned, we first want to define clearly the object, to be accomplished:

Given an LO in path-like shape (or abstraction) represented by a trajectory, find the path relation which describes best the relation of the LO to an arbitrary reference object (and compute a corresponding degree of applicability).

A trajectory is defined by  $n$  points ( $p_1 \dots p_n$ ,  $P_N$ ). The endpoints  $p_1$  and  $P_N$  denoting beginning and end of the trajectory are defined either by an explicit direction or by the order of the computational analysis itself. This description of the trajectory is provided by the conceptual layer mentioned in section 2. Its construction (e.g. by choosing a specific idealization or by indicating the starting point) is beyond the scope of this paper.

We rely on the following assumptions:

1. The LO is represented as a trajectory. This corresponds to the fact that a path preposition can hardly be applied to an object which is not path-like shaped at all (e.g. "the ball along the wall").
2. We want to describe the trajectory as a whole. Though it might be preferable to subdivide the trajectory into parts (which analyzed one by one could possibly be associated more evidently with *several* path relations), the entire trajectory has to be analyzed in order to detect possible subdivisions. Therefore, it is appropriate to generate first an overall description using a *single* relation.

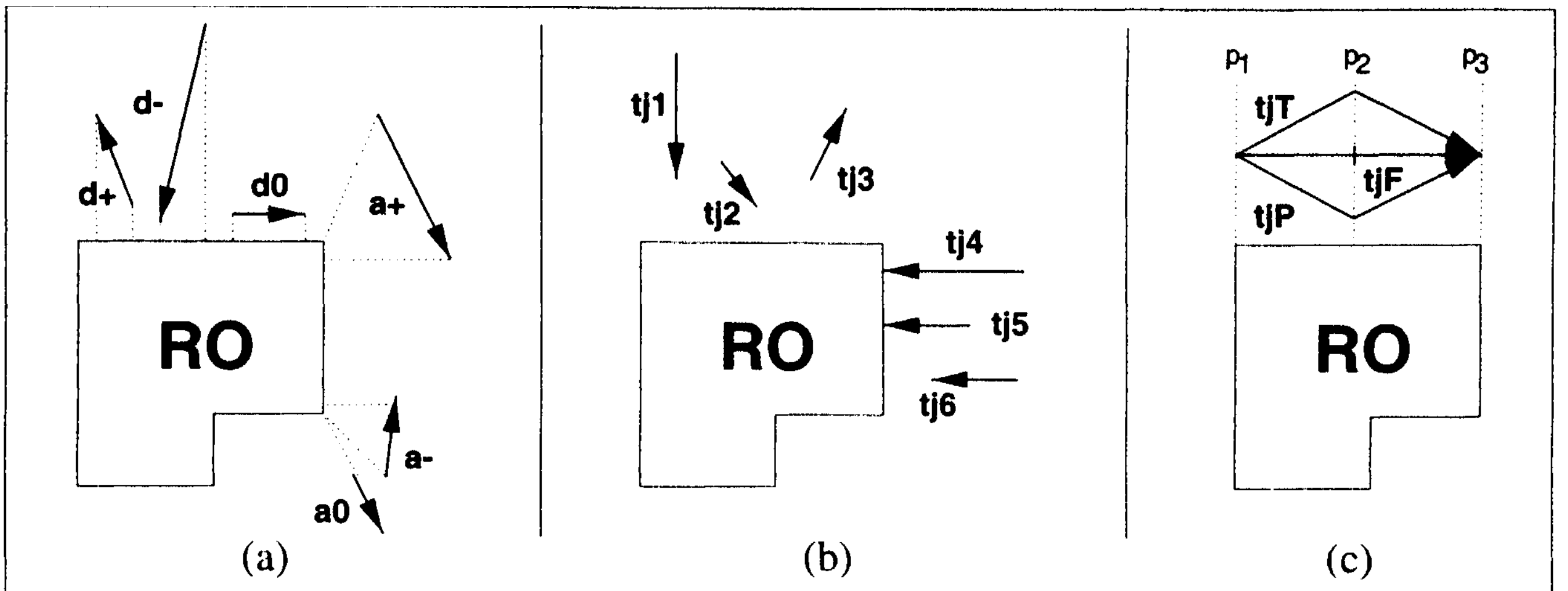


Figure 2: Trajectories: (a) changes (b) qualities (c) curvatures

3. We want to describe the *course* of a trajectory. In this case, point-to-point relations are not sufficient and path relations are needed.
4. We want to extend the computational model of static relations to path relations. Accordingly, the DAs have to be comparable. This will be ensured by using the same essential parameters distance and angle -, identically calculated frame of reference, reference points (the nearest points between LO and RO), and intermediate results of the computation of point-to-point relations.
5. We focus on path relations that correspond to path prepositions. As the geometric model is linked to linguistic concepts via the reference semantics, path *prepositions* have to be kept in mind while exploring geometric path *relations*.

### 5.1 Two-point-trajectories

The most basic trajectory consists of exactly two distinct points and is called *two-point-trajectory*. On one hand, more complex trajectories - *n-point-trajectories* can easily be constructed by concatenating  $n-1$  two-point-trajectories. On the other, even the most complex trajectory can be split into a unique series of two-point-trajectories. This implies that the first, step to analyze path relations is to study the relations between two-point-trajectories and the reference object.

Any trajectory can be localized exactly using the concepts of distance and angle. This corresponds to assumption four and to the observations made in section 4. In order to describe the course of a two-point-trajectory as a whole (see assumption three) using distance and angle, the changes of these essential parameters between  $p_1$  and  $p_2$  have to be analyzed: Distance and/or angle can either be increased, maintained or decreased as depicted

in Fig. 2a. This distinction - similar to the one made in the previous section - enables us to define *basic path relations* (see Tab. 2; ew: clockwise, ccw: counter-cw).

Change	Distance	Angle
Increasing	depart $d+$	turn-cw $a+$
Decreasing	approach $d-$	turn-ccw $a-$
None	follow $d0$	no-turn $a0$

Table 2: Basic path relations (see Fig. 2a)

Figure 2b shows that we can even sort trajectories according to the degree of being an optimal representant of a path relation  $tj1$  is closer to the ideal meaning of approach than is  $tj2$ . Furthermore, we can *compare different* relations: The quality of the relation depart represented by  $tj3$  lies inbetween the qualities of  $tj1$  and  $tj2$ . To describe the differences between  $tj4$  to  $tj6$  other factors than course, e.g. the distance to the RO, have to be taken into account. Nevertheless, these trajectories are all optimal representants of the relation approach since we assumed that path relations depend on their course only. Consequently, the degree of applicability can be expressed as the difference in the distances ( $\Delta d$ ) of  $p_1$  and  $p_2$  with respect to the RO, divided by the length of the trajectory. As the computation of each of these distances corresponds to the one used for point-to-point relations, the resulting DAs are comparable. Table 3 illustrates the computation of the DAs for the basic path relations defined above.

### 5.2 N-point-trajectories

Actually, in most cases trajectories representing abstractions of real world objects consist of more than two points. In order to extend the use of basic path relations

Relation	Measure ( $M_{cssParam}$ )	DA
depart	$M_{dist} = \frac{\Delta d_{RO}(p_2, p_1)}{ p_1 p_2 }$	$M_{dist}$
approach		$-M_{dist}$
follow		$1 -  M_{dist} $
turn-cw	$M_{angle} = \frac{\Delta \angle_{RO}(p_2, p_1)}{2\pi}$	$M_{angle}$
turn-ccw		$-M_{angle}$
no-turn		$1 -  M_{angle} $

Table 3: Calculation of degrees of applicability

to n-point-trajectories it suffices to build the weighted average over all parts of the trajectory.

However, as shown in Fig. 2c, there are differences in the course of n-point-trajectories we are still not able to describe using the basic path relations defined above: A distinction of the curvature of n-point-trajectories has to be made. Three types can be distinguished:

- $tjP$  first approaches the RO and then departs: this curvature corresponds to the path relation past.
- $tjT$  first departs from the RO and then approaches it again: this curvature corresponds to trip.<sup>2</sup>
- $tjF$  keeps the distance over the whole course, no curvature exists: this equals follow.

There are two possible ways to cope with curvatures: either by finding a suitable segmentation (which contradicts assumption two), or by analyzing the essential parameter distance more thoroughly. A curvature can only exist if a trajectory has at least three points: In this case, the difference of  $\Delta d_{RO}(p_2, p_1)$  and  $\Delta d_{RO}(p_3, p_2)$  is either positive (trip) or negative (past). The degree of the curvature is measured as follows:

$$\text{sign}(\Delta d_{RO}(p_2, p_1) - \Delta d_{RO}(p_3, p_2)) \left( 1 - \frac{|p_1 p_3|}{|p_1 p_2| + |p_2 p_3|} \right)$$

A n-point-trajectory's curvature can easily be expressed by the weighted curvatures of the inner points.

## 6 Results

Table 4 integrates the semantic aspects of German path prepositions developed in section 3 and path relations constructed according to section 5. They may be combined with each other, or with point-to-point relations. (Path relations refer to the entire trajectory while point-to-point relations refer to the trajectory point(s) mentioned explicitly in brackets.)

However, there is no 1:1-correspondence between a given preposition and a specific realization. Since a single preposition can be used to describe different situations, and a single situation can be described using different prepositions, there is a n:m-relation between language and geometry. To overcome the vagueness of language, contextual factors can be taken into account. Additionally, vagueness (precision) can be modeled explicitly as proposed in [Kray, 1998..

<sup>2</sup>There seems to be no corresponding path preposition neither in German nor in English.

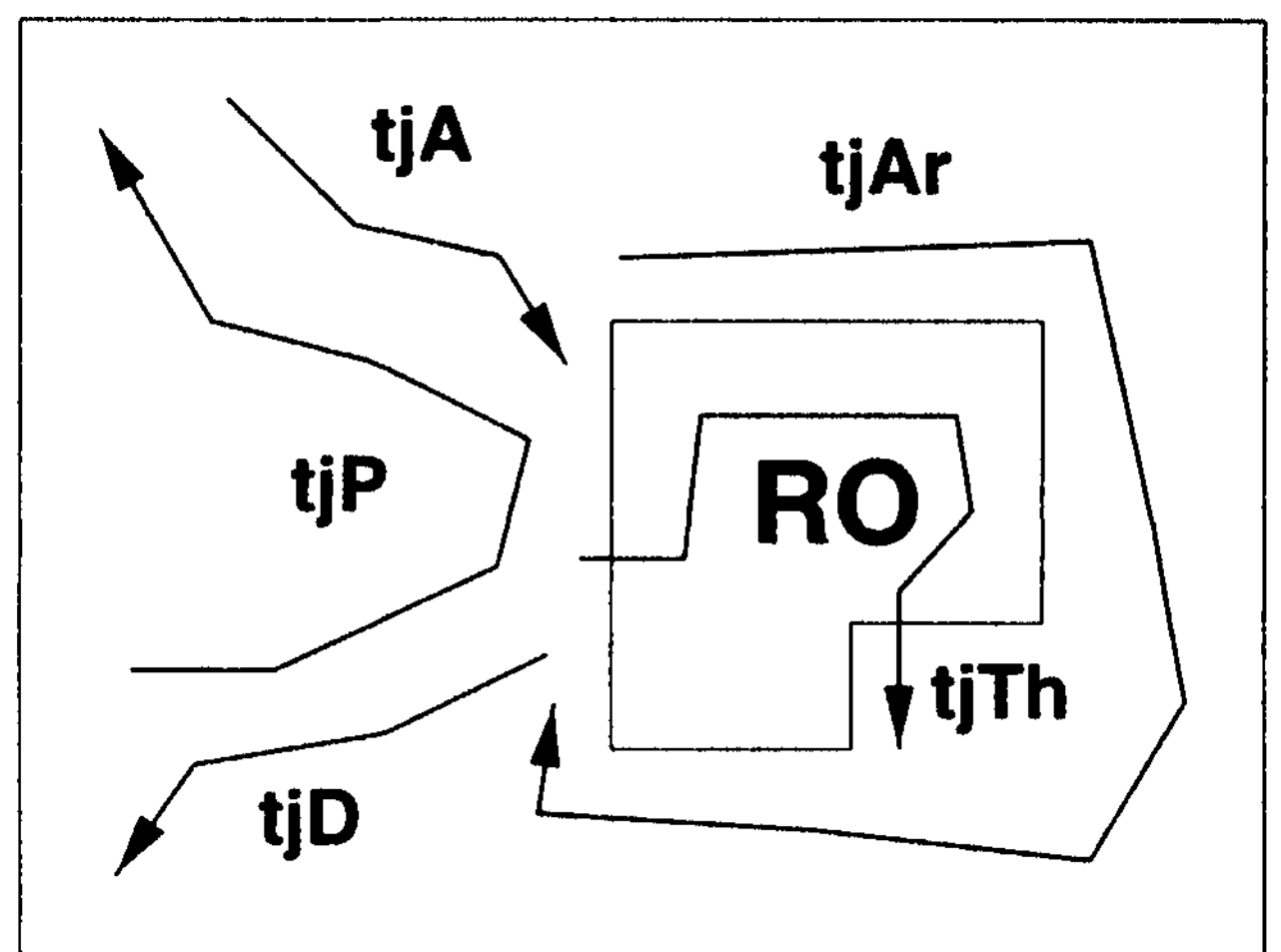


Figure 3: Exemplary trajectories

Table 5 shows exemplary results for the trajectories in Fig. 1 and 3. As expected, the DAs for 1(a) and 1(b) are significantly different. The examples in Fig. 3 also yield reasonable results.

Fig.	Tj.	Relation	DA	Preposition
1	(a)	follow	0.76	“entlang”
1	(b)	follow	0.59	“entlang”
3	tjA	approach	0.68	“nach”, “zu”
3	tjD	depart	0.68	“von”
3	tjTh	thru	0.71	“durch”
3	tjAr	turn-cw	0.63	“um”
3	tjAr	past	0.57	“vorbei”

Table 5: Exemplary results

## 7 Conclusion

The connection between visual and verbal space is an important issue in the development of natural language systems that are concerned with spatial information. In this paper, we presented an analysis of German path prepositions, and used the results to deduce a basic semantics for path relations. Extending the model for spatial relations, we showed how those findings can be integrated with geometric path relations.

Currently, our results are being integrated in a localization agent [Wahlster *et al.*, 1998] and in a mobile tourist [Deep Map, 1999] guide. In addition, their potential for anytime behavior (inferrability with increasing quality over time) [Dean and Boddy, 1988] is being investigated. In the future, we plan to develop segmentation algorithms that are based on the methods proposed in this paper. A subdivision of a complex trajectory may improve the quality of path descriptions due to the finer granularity. Furthermore, we intend to evaluate the meaning facets of path prepositions empirically.



German preposition	Raw translation	Formalization	Path relation (basic or combined)
zu	towards	$D \xrightarrow{D} IMD$	approach
bis	up to	$D \xrightarrow{*} IMD$	approach
in	into	$MD \xrightarrow{M} I$	approach $\wedge$ in( $p_N$ )
nach	to	$D \xrightarrow{*} IMD$	approach $\wedge$ in( $p_N$ )
an	to	$D \xrightarrow{D} MD$	approach $\wedge$ contact( $p_N$ )
gegen	against	$D \xrightarrow{M} IM$	approach $\wedge$ contact( $p_N$ )
von	away from	$IMD \xrightarrow{*} D$	depart
aus	out of	$I \xrightarrow{M} MD$	in( $p_1$ ) $\wedge$ depart
entlang	along	$IMD \xrightarrow{*} IMD$	follow
vorbei	past	$D \xrightarrow{D} D$	past
durch	through	$IMD \xrightarrow{I} IMD$	past $\wedge \exists i : \text{in}(p_i)$
um	around	$MD \xrightarrow{MD} MD$	turn-cw $\vee$ turn-ccw
		$MD \xrightarrow{MD} MD$	past $\wedge$ (turn-cw $\vee$ turn-ccw)

Table 4: German path prepositions with their corresponding path relations

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## References

- [Dean and Boddy, 1988] T. Dean and M. Boddy. An Analysis of Time-Dependent Planning. In *Proc. of AAAI-88*, pages 49-54, St. Paul, MN, 1988.
- [Deep Map, 1999] Deep Map. Intelligent next-generation Geo-Information Systems. <http://www.em1.org/englisch/prjekte/deepmap/deepmap.html>
- [Egenhofer, 1991] M. J. Egenhofer. Reasoning about Binary Topological Relations. In O. Gunther and H.-J. Schek, editors, *Advances in Spatial Databases*, pages 144-160. Springer, Berlin, Heidelberg, 1991.
- [Gapp, 1994] K.-P. Gapp. Basic Meanings of Spatial Relations: Computation and Evaluation in 3D Space. In *Proc. of AAAI-91*, Seattle, WA, 1991.
- [Gapp, 1997] K.-P. Gapp. *Objektlokalisierung: Ein System zur sprachlichen Raumbeschreibung*. Studien zur Kognitionswissenschaft. Deutscher Universitätsverlag, Wiesbaden, 1997.
- [Habel, 1989] C. Habel. Zwischen-Bericht. In C. Habel, M. Herweg, and K. Rehkemper, editors, *Raumkonzepte in Verstehensprozessen: Interdisziplinäre Beiträge zu Sprache and Raum*, pages 37-69. Niemeyer, Tübingen, 1989.
- [Herskovits, 1986] A. Herskovits. *Language and Spatial Cognition. An Interdisciplinary Study of the Prepositions in English*. Cambridge University Press, Cambridge, London, 1986.
- [Kosslyn, 1994] S. M. Kosslyn. *Image and Brain*. MIT Press, Cambridge, MA, 1994.
- [Kray, 1998] C. Kray. Ressourcenadaptierende Verfahren zur Präzisionsbewertung von Lokalisationsausdrücken und zur Generierung von linguistischen Hecken. Memo 66, Universität des Saarlandes, Sonderforschungsbereich (SFB) 378, 1998.
- [Krieger and MaaB, 1997] A. Krieger and W. MaaB. Towards a Computational Semantics of Path Relations. In *Workshop Language and Space, AAAI'97*, pages 101-109, Providence, RI, 1997.
- [Lakoff, 1987] G. Lakoff. *Women, Fire, and Dangerous Things. What Categories Reveal about the Mind*. Chicago University Press, Chicago, 1987.
- [Landau and Jackendoff, 1993] B. Landau and R. Jackendoff. "What" and "Where" in Spatial Language and Spatial Cognition. *Behavioral and Brain Sciences*, 16:217-265, 1993.
- [MaaB et al., 1993] W. MaaB, P. Wazinski, and G. Herzog. V1TRA GUIDE: Multimodal Route Descriptions for Computer Assisted Vehicle Navigation. In *Proc. of the Sixth Int. Conf. on Industrial and Engineering Applications of Artificial Intelligence and Expert Systems IEA/AIE-93*, pages 144-147, Edinburgh, Scotland, 1993.
- [MaaB, 1993] W. Maati. A Cognitive Model for the Process of Multimodal, Incremental Route Description. In *Proc. of the European Conference on Spatial Information Theory*. Springer, Berlin, Heidelberg, 1993.
- [Wahlster et al., 1998] W. Wahlster, A. Blocher, J. Bans, E. Stopp, and H. Speiser. Ressourcenadaptierende Objektlokalisierung: Sprachliche Raumbeschreibung unter Zeitdruck. *Kognitionswissenschaft, Sonderheft zum Sonderforschungsbereich (SEE) 378*, 1998.