

# An Enhanced Anonymous Password-based Authenticated Key Agreement Scheme with Formal Proof

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## Abstract

With the development of technology, the security of password-based authentication is becoming more and more significant. Recently, Lee et al. proposed an anonymous password-based authenticated key agreement scheme with non-tamper resistant smart card to reduce the computation cost of Wang et al.'s scheme. However, based on analysis, it shows that the scheme can't withstand smart card stolen or lost attack, user impersonation attack and server impersonation attack. Therefore, an enhanced scheme which can resist the attacks mentioned above is presented. By comparing the performance and security with other related schemes, our proposed scheme is more suitable for practical applications.

*Keywords:* Authentication Scheme; BNA Logic; Key Agreement; Network Security; Smart Card

## 1 Introduction

As the internet technology's development, password-based authentication with smart card is significant and widely used for remote system to access to computer network [1, 15]. To enhance the system security and management, research have been focused considerable attention on smart card based password authentication. Since Change and Wu [4] firstly proposed remote user authentication scheme using smart cards in 1993, many other password schemes were present [7, 12, 16, 18]. Traditionally, the smart card is assumed to be tamper-resistant. Namely, an adversary can't obtain the secret information about legal user stored in the smart card. However, recent research has been proved that the secret data stored in the smart card could be extracted by some means, such as monitoring the power consumption [2, 9, 14] or analyzing the leaked information [6, 13]. So such schemes based on the tamper resistance assumption of the smart card are

susceptible to various attacks like impersonation attacks, off-line password guessing attacks, etc.

In 2009, Kim and Chung [8] proposed a remote user authentication scheme which claimed that their scheme is secure. However in 2011, Li et al. [11] pointed out that Kim and Chung's scheme couldn't resist various attacks and further advanced a new remote authentication based on hash function. In their scheme, they suggested that their scheme not only remedy the flaws of Kim and Chung's scheme, but also secure. But in 2012, Wang et al. [17] demonstrated that Li et al.'s scheme is insecure against denial of service attack and off-line password guessing attack under the non-tamper resistance assumption of the smart card. Moreover, their scheme failed to provide user anonymity and forward secrecy. In order to solve the problems mentioned above, Wang et al. presented a robust authentication scheme based on the secure one-way hash function and the well-known discrete logarithm problem. Later, Lee et al. [10] putted forward that Wang et al.'s scheme had high computational overhead. In order to reduce the overhead, they proposed an anonymous authentication scheme with non-tamper resistant smart cards based on password, and proved that their scheme meets all the criteria required for the authenticated key agreement scheme and eliminates security threats. Nevertheless, it indicated that their scheme is prone to smart card stolen or lost attack, user impersonation attack and server impersonation attack base on our analysis. In additional, their scheme can't provide mutual authentication. Then, an enhanced key agreement scheme with non-tamper resistant smart cards is presented. The remainder of the article is sketched as follows. In Section 2, we briefly review Lee et al.'s scheme. Section 3 presents the security analysis of Lee et al.'s scheme. In Section 4, we present an enhanced scheme. The security analysis of the proposed scheme is given in Section 5, and efficiency comparison between our scheme and other related ones is showed in Section 6. Ultimately, in Section 7,

we reach the conclusion.

## 2 Review of Lee et al.'s Scheme

In this section, we will briefly review of Lee et al.'s scheme, which comprises four phases: registration phase, login phase, authentication phase and password change phase. The notations used in this article are described in Table 1.

Table 1: Notation

Notation	Description
$U_i/U_k$	user $i/k$
$S_i$	server $i$
$E$	attacker
$PW_i$	$U_i$ 's password
$ID_i$	$U_i$ 's identity
$x$	secret key generated by $S_i$
$y$	public key generated by $S_i$
$b$	a random number generated by $U_i$
$v$	a random number generated by $U_i$
$w$	a random number generated by $S_i$
$h(\cdot)$	a one-way hash function
$\parallel$	concatenation
$\oplus$	bitwise exclusive-or operation

### 2.1 Registration Phase

$S_i$  generates  $x$  as the server's private key which is only kept secret by himself/herself, and computes  $y = g^x \pmod n$  as its corresponding public key which is stored inside each user's smart card. If a user  $U_i$  wishes to be a legal user of the system so that he/she can utilize resources provided by the server,  $U_i$  should execute the following steps.

- $U_i$  first selects his/her identity  $ID_i$  and password  $PW_i$ . Then,  $U_i$  generates a random number  $b$ , computes  $h(b\parallel PW_i)$  and sends  $\{ID_i, h(b\parallel PW_i)\}$  to  $S_i$ .
- $S_i$  checks the validity of  $ID_i$ . If it is validity,  $S_i$  calculates

$$\begin{aligned}
 C_1 &= h(h(ID_i) \oplus x), \\
 C_2 &= C_1 \oplus h(b\parallel PW_i) \oplus h(ID_i), \\
 C_3 &= h(C_1), \\
 C_4 &= h(b\parallel PW_i) \oplus h(x\parallel y).
 \end{aligned}$$

Then  $S_i$  issues a smart card including  $\{C_2, C_3, C_4, h(\cdot), n, g, y\}$  to  $U_i$  via a secure channel.

- $U_i$  computes  $B = b \oplus ID_i \oplus PW_i$ , and stores  $B$  in the smart card.

### 2.2 Login Phase

When  $U_i$  logs in the system, he/she can perform the next steps.

- $U_i$  inserts his/her smart card into a card reader and enters the identity  $ID_i$ , password  $PW_i$ . The smart card  $SC$  computes  $b' = B \oplus ID_i \oplus PW_i$ ,  $C'_1 = C_2 \oplus h(b' \parallel PW_i) \oplus h(ID_i)$ ,  $C'_3 = h(C'_1)$ , and compares  $C'_3$  with  $C_3$  stored in the smart card. Only if the equation holds,  $SC$  performs the following steps.
- $SC$  generates a random number  $v$  and computes  $V = g^v \pmod n$ ,  $h(x\parallel y) = c_4 \oplus h(b\parallel PW_i)$ ,  $CID_i = h(ID_i) \oplus h(V \parallel h(x\parallel y))$ ,  $M_1 = h(CID_i \parallel V \parallel C_1)$ . Then,  $U_i$  sends login request message  $\{CID_i, V, M_1\}$  to  $S_i$ .

### 2.3 Authentication Phase

$U_i$  and  $S_i$  achieve mutual authentication as follows.

- Upon receiving the login message  $\{CID_i, V, M_1\}$ ,  $S_i$  computes  $h(x\parallel y)$ ,  $h(ID_i) = CID_i \oplus h(V \parallel h(x\parallel y))$ ,  $C'_1 = h(h(ID_i) \oplus x)$ ,  $M'_1 = h(CID_i \parallel V \parallel C'_1)$ , and checks whether  $M'_1$  equals to the received  $M_1$ . If they are not equal, the session is terminated. Otherwise,  $S_i$  selects a random number  $w$  and computes  $W = g^w \pmod n$ ,  $SK = V^w \pmod n$ ,  $M_2 = h(SK \parallel W \parallel C'_1)$ . Then,  $S_i$  sends  $\{M_2, W\}$  to  $U_i$ .
- $SC$  receives the message and computes the session key  $SK' = W^v \pmod n$ . And,  $SC$  verifies  $M_2$  with the computed value of  $h(SK' \parallel W \parallel C_1)$ . If the verification holds,  $SC$  computes  $M_3 = h(M_2 \parallel C_1 \parallel SK')$  and send  $\{M_3\}$  to  $S_i$ .
- Upon receiving  $\{M_3\}$ ,  $S_i$  computes  $M'_3 = h(M_2 \parallel C_1 \parallel SK')$  and checks whether the equation  $M'_3 = M_3$  holds. If it holds,  $S_i$  and  $U_i$  finish mutual authentication, and share a common session key  $SK = g^{vw} \pmod n$ . Otherwise, the session is terminated.

### 2.4 Password Change Phase

Assume that  $SC$  has the ability to detect the login failure trials. If the failure times exceed a given number,  $SC$  will be soon locked to prevent from guessing password attack.

- $U_i$  inserts the smart card into a card reader and inputs identity  $ID_i$ , password  $PW_i$  and a new password  $PW_i^{new}$ .
- $SC$  calculates  $b' = B \oplus ID_i \oplus PW_i$ ,  $C'_1 = C_2 \oplus h(b' \parallel PW_i \oplus h(ID_i))$ ,  $C'_3 = h(C'_1)$  and verifies whether  $C'_3 = C_3$ . If they are the same,  $SC$  accepts the change request. Otherwise, the session is terminated.
- $SC$  computes  $B^{new} = b \oplus ID_i \oplus PW_i^{new}$ ,  $C_2^{new} = C'_1 \oplus h(b' \parallel h(b' \parallel PW_i^{new})) \oplus h(ID_i)$ ,  $C_4^{new} = C_4 \oplus h(b' \parallel PW_i) \oplus h(b' \parallel PW_i^{new})$ . Finally,  $SC$  replace  $C_2, C_4, B$  with  $C_2^{new}, C_4^{new}, B^{new}$  in the smart card.

### 3 Security Analysis of Lee et al.'s Scheme

In Lee et al.'s scheme, they claim that their scheme can resist some attacks, containing off-line password guessing attack, user impersonation attack, server masquerading attack, and so on. By analysis and study, we find that the scheme fails to resist the attacks mentioned above. The details are as follows.

#### 3.1 Smart Card Stolen or Loss Attack

Assume that  $U_i$ 's smart card was stolen by a legal but malicious user  $U_k$ , and  $U_k$  had monitored the login request message  $\{CID_i, V, M_1\}$  which was sent to  $S$  by  $U_i$ .

A legal but malicious user  $U_k$  acquires  $\{C_2^*, C_3^*, C_4^*, h(\cdot), n, g, B^*\}$  from his/her own smart card and computes  $b^* = B^* \oplus ID_k \oplus PW_k$ ,  $h(x||y) = C_4^* \oplus h(b^*||PW_k)$ . And the value of  $h(x||y)$  is not changed for every user. Then  $U_k$  can obtain  $h(ID_i)$  and  $C_1$  by computing  $h(ID_i) = CID_i \oplus h(V||h(x||y))$ ,  $C_1 = C_2 \oplus h(b||PW_i) \oplus h(ID_i) = C_2 \oplus C_4 \oplus h(x||y) \oplus h(ID_i)$  where  $C_2, C_4$  is extracted from  $U_i$ 's smart card. Then,  $U_k$  can continue guesses the identity as follows.

- 1) Guess an identity  $ID_i'$ .
- 2) compute  $h(ID_i')$  and compare it with the values of  $CID_i \oplus h(V||h(x||y))$ . If they are not equal, go back to 1). Otherwise,  $U_k$  finds the user  $U_i$ 's identity  $ID_i$ .

After acquiring the user  $U_i$ 's identity  $ID_i$ ,  $U_k$  can go on continuing guess user's password.

- 1) Guess a password  $PW_i'$ .
- 2) Compute  $b' = B \oplus ID_i \oplus PW_i'$ ,  $C_4' = h(b'||PW_i') \oplus h(x||y)$ , where  $B$  is extracted from  $U_i$ 's smart card and  $h(x||y)$  can be obtained by Step 1. Then  $U_k$  verifies  $C_4' \stackrel{?}{=} C_4$ . If it holds,  $U_k$  finds the correct password  $PW_i$ .

#### 3.2 User Impersonation Attack

From Section 3.1, we know that a legal but malicious user  $U_k$  can obtain  $h(x||y)$ ,  $h(ID_i)$ ,  $C_1$ . Then he/she can forge the login request message  $\{CID_i, V, M_1\}$  to disguise the user  $U_i$ .

- 1)  $U_k$  generates a random number  $v^*$  and computes  $V^* = g^{v^*} \bmod n$ ,  $CID_i^* = h(ID_i) \oplus h(V^*||h(x||y))$ ,  $M_1^* = h(CID_i^* || V^* || C_1)$ . Then,  $U_k$  sends  $\{CID_i^*, V^*, M_1^*\}$  to  $S_i$ .
- 2)  $S_i$  computes  $h(x||y)$ ,  $h(ID_i) = CID_i^* \oplus h(V^*||h(x||y))$ ,  $C_1' = h(h(ID_i) \oplus x)$ ,  $M_1' = h(CID_i^*||V^*||C_1')$ , and checks whether  $M_1'$  equals to the received  $M_1^*$ . If they are equal, then  $S_i$  selects a random number  $w^*$  and computes  $W^* = g^{w^*} \bmod n$ ,  $SK^* = (V^*)^{w^*} \bmod n$ ,

$M_2^* = h(SK^*||W^*||C_1')$ . Then,  $S_i$  sends  $\{M_2^*, W^*\}$  to  $U_i$ .

- 3)  $U_k$  computes the session key  $SK' = (W^*)^{v^*} \bmod n$ ,  $M_3^* = h(M_2^*||C_1||SK^*)$  and send  $\{M_3^*\}$  to  $S_i$ .
- 4)  $S_i$  computes  $M_3' = h(M_2^*||C_1'||SK^*)$  and checks whether the equation  $M_3 = M_3^*$  holds. As  $SK^* = (V^*)^{w^*} \bmod n = (g^{v^*})^{w^*} \bmod n = (g^{w^*})^{v^*} \bmod n = (W^*)^{v^*} \bmod n = SK$ ,  $M_3' = h(M_2^*||C_1'||SK^*) = h(M_2^*||C_1||SK') = M_3^*$ .  $S_i$  authenticates  $U_k$  as  $U_i$ .

#### 3.3 Server Impersonation Attack

A legal but malicious user  $U_k$  acquires  $h(x||y), h(ID_i), C_1$  by the method mentioned in Section 3.1, then  $U_k$  can impersonate server  $S_i$  to communicate with  $U_i$ .

- 1) When  $U_i$  sends the login request message  $\{CID_i, V, M_1\}$  to  $S_i$ ,  $U_k$  eavesdrops the message, selects a random number  $w^*$  and computes  $W^* = g^{w^*} \bmod n$ ,  $SK^* = V^{w^*} \bmod n$ ,  $M_2^* = h(SK^*||W^*||C_1)$ . Then,  $S_i$  sends  $\{M_2^*, W^*\}$  to  $U_i$ .
- 2) When  $U_i$  receives the message, the smart card computes the session key  $SK' = (W^*)^v \bmod n$ .  $SK' = (W^*)^v \bmod n = (g^{w^*})^v \bmod n = (g^v)^{w^*} \bmod n = (V)^{w^*} \bmod n = SK^*$ , so  $M_2^* = h(SK'||W^*||C_1)$ . Then,  $SC$  computes  $M_3 = h(M_2||C_1||SK')$  and send  $\{M_3\}$  to  $S_i$ .

Thus,  $U_k$  is authenticated as the legitimate server by the user  $U_i$ .

## 4 Our Proposed Scheme

In this section, we propose a new scheme based on Lee et al.'s scheme, which can resist the attacks mentioned in Section 3. It composes four phase: registration phase, login phase, authentication phase and password change phase. The detail description of each phase are shown below.

#### 4.1 Registration Phase

$S_i$  generates  $x$  as the server's private key which is only kept secret by himself/herself, and computes  $y = g^x \bmod n$  as its corresponding public key which is stored inside each user's smart card. A user  $U_i$  must register to be a legal user of the system, before utilizing resources provided by the server.

- $U_i$  first selects his/her identity  $ID_i$  and password  $PW_i$ . Then,  $U_i$  generates a random number  $b$ , computes  $RPW_i = h(b||PW_i)$  and sends  $\{ID_i, RPW_i\}$  to  $S_i$ .

Table 2: The proposed scheme of registration phase

$U_i$	$S_i$
$RPW_i = h(b  PW_i)$	$ID_i, RPW_i$
$B = b \oplus ID_i \oplus PW_i$ stores $B$ in the smart card	Server's public key $y = g^x \text{mod} n$ . checks the validity of $ID_i$ generates a random number $d$ $C_1 = h(ID_i  x), C_2 = C_1 \oplus RPW_i$ $C_3 = h(C_1  d), C_4 = h(C_1  RPW_i) \oplus d,$ $D = g^d \text{mod } n, C_5 = h(C_1 \oplus ID_i) \oplus h(x  y  D).$
	$\xleftarrow{\text{smart card}}$

- $S_i$  checks the validity of  $ID_i$ . If it is validity,  $S_i$  generates a random number  $d$  for user  $U_i$ . Then  $S_i$  performs the following computations.  $C_1 = h(ID_i||x)$ ,  $C_2 = C_1 \oplus RPW_i$ ,  $C_3 = h(C_1||d)$ ,  $C_4 = h(C_1||RPW_i) \oplus d$ ,  $D = g^d \text{mod } n$ ,  $C_5 = h(C_1 \oplus ID_i) \oplus h(x||y||D)$ . Then  $S_i$  sends a smart card including  $\{C_2, C_3, C_4, C_5, h(\cdot), n, g, y\}$  to  $U_i$  via a secure channel.
- $U_i$  computes  $B = b \oplus ID_i \oplus PW_i$ , and stores  $B$  in the smart card.

## 4.2 Login Phase

When  $U_i$  logs in the system, he/she can perform the next steps.

- $U_i$  inserts his/her smart card into a card reader and enters the identity  $ID_i$ , password  $PW_i$ . The smart card  $SC$  computes  $b = B \oplus ID_i \oplus PW_i$ ,  $RPW_i = h(b||PW_i)$ ,  $C_1 = C_2 \oplus RPW_i$ ,  $d = C_4 \oplus h(C_1||RPW_i)$ ,  $C_3 = h(C_1||d)$ , and compares  $C_3$  with  $C_3$  stored in the smart card. Only if the equation holds,  $SC$  performs the following steps.
- $SC$  generates a random number  $v$  and computes  $V = g^v \text{mod } n$ ,  $D = g^d \text{mod } n$ ,  $h(x||y||D) = C_5 \oplus h(C_1||ID_i)$ ,  $CID_i = ID_i \oplus h(V||h(x||y||D))$ ,  $F_1 = RPW_i \oplus h(C_1||ID_i)$ ,  $F_2 = C_4 \oplus h(V||C_1) \oplus h(x||y||D)$ ,  $M_1 = h(ID_i||RPW_i||V||C_1||d)$ . Then,  $U_i$  sends login request message  $\{CID_i, V, D, F_1, F_2, M_1\}$  to  $S_i$ .

## 4.3 Authentication Phase

$U_i$  and  $S_i$  achieve mutual authentication as follows.

- Upon receiving the login message  $\{CID_i, V, D, F_1, F_2, M_1\}$ ,  $S_i$  computes  $h(x||y||D)$ ,  $ID_i = CID_i \oplus h(V||h(x||y||D))$ ,  $C_1 = h(ID_i||x)$ ,  $RPW_i = F_1 \oplus h(C_1||ID_i)$ ,  $C_4 = F_2 \oplus h(V||C_1) \oplus h(x||y||D)$ ,  $d = C_4 \oplus h(C_1 \oplus RPW_i)$ ,  $M_1^* = h(ID_i||RPW_i||V||C_1||d)$ , and checks whether  $M_1^*$  equals to the received  $M_1$ . If they are not equal, the session is terminated. Otherwise,  $S_i$  selects a random number  $w$  and computes  $W = g^w \text{mod } n$ ,  $SK = V^w \text{mod } n$ ,  $M_2 =$

$h(SK||W||C_1||RPW_i||d)$ . Then,  $S_i$  sends  $\{M_2, W\}$  to  $U_i$ .

- $SC$  receives the message and computes the session key  $SK' = W^v \text{mod } n$ . And,  $SC$  verifies  $M_2$  with the computed value of  $h(SK'||W||C_1||RPW_i||d)$ . If the verification holds,  $SC$  computes  $M_3 = h(M_2||C_1||SK'||d)$  and send  $\{M_3\}$  to  $S_i$ .
- Upon receiving  $\{M_3\}$ ,  $S_i$  computes  $M_3^* = h(M_2||C_1||SK||d)$  and checks whether the equation  $M_3^* = M_3$  holds. If it holds,  $S_i$  and  $U_i$  finish mutual authentication, and share a common session key  $SK = g^{vw} \text{mod } n$ . Otherwise, the session is terminated.

## 4.4 Password Change Phase

Assume that  $SC$  has the ability to detect the login failure trials. If the failure times exceed a given number,  $SC$  will be soon locked to prevent from guessing password attack.

- $U_i$  inserts the smart card into a card reader and inputs identity  $ID_i$ , password  $PW_i$  and a new password  $PW_i^{new}$ .
- $SC$  calculates  $b = B \oplus ID_i \oplus PW_i$ ,  $RPW_i = h(b||PW_i)$ ,  $C_1 = C_2 \oplus RPW_i$ ,  $d = C_4 \oplus h(C_1||RPW_i)$ ,  $C_3 = h(C_1||d)$  and verifies whether  $C_3 = C_3$ . If they are the same,  $SC$  accepts the request. Otherwise, the session is terminated.
- $SC$  computes  $B^{new} = b \oplus ID_i \oplus PW_i^{new}$ ,  $RPW_i^{new} = h(b||PW_i^{new})$ ,  $C_2^{new} = C_1 \oplus RPW_i^{new}$ ,  $C_4^{new} = d \oplus h(C_1||RPW_i^{new})$ . Finally,  $SC$  replace  $C_2, C_4, B$  with  $C_2^{new}, C_4^{new}, B^{new}$  in the smart card.

## 5 Security Analysis

The proposed scheme advanced Lee et als scheme and can resist the attacks analyzed above. The details are described in the following content.

Table 3: The proposed scheme of the login and authentication phase

$U_i$	$S_i$
inputs $ID_i, PW_i$ computes $b = B \oplus ID_i \oplus PW_i$ , $RPW_i = h(b  PW_i), C_1 = C_2 \oplus RPW_i$ , $d = C_4 \oplus h(C_1  RPW_i)$ , $C'_3 = h(C_1  d)$ , verifies $C'_3 \stackrel{?}{=} C_3$ . selects a random number $v$ , computes $V = g^v \text{mod } n$ , $D = g^d \text{mod } n$ , $h(x  y  D) = C_5 \oplus h(C_1 \oplus ID_i)$ , $CID_i = ID_i \oplus h(V  h(x  y  D))$ , $F_1 = RPW_i \oplus h(C_1  ID_i)$ , $F_2 = C_4 \oplus h(V  C_1) \oplus h(x  y  D)$ $M_1 = h(ID_i  RPW_i  V  C_1  d)$	$CID_i, V, D, F_1, F_2, M_1 \xrightarrow{\hspace{2cm}}$
$SK' = W^v \text{ mod } n$ , verifies $M_2 \stackrel{?}{=} h(SK'    W    C_1    RPW_i    d)$ , computes $M_3 = h(M_2    C_1    SK'    d)$ .	computes $h(x  y  D)$ , $ID_i = CID_i \oplus h(V  h(x  y  D))$ , $C_1 = h(ID_i  x)$ , $RPW_i = F_1 \oplus h(C_1  ID_i)$ , $C_4 = F_2 \oplus h(V  C_1) \oplus h(x  y  D)$ , $d = C_4 \oplus h(C_1 \oplus RPW_i)$ , $M_1^* = h(ID_i  RPW_i  V  C_1  d)$ , checks $M_1^* \stackrel{?}{=} M_1$ . selects a random number $w$ , computes $W = g^w \text{ mod } n$ , $SK = V^w \text{ mod } n$ , $M_2 = h(SK    W    C_1    RPW_i    d)$ .
	$M_2, W \xleftarrow{\hspace{2cm}}$
	$M_3 \xrightarrow{\hspace{2cm}}$
	computes $M_3^* = h(M_2    C_1    SK    d)$ checks $M_3^* \stackrel{?}{=} M_3$

### 5.1 Analysis the Proposed Scheme with BNA Logic

We analyzes out proposed scheme with BNA logic [3] in this section. The main notations of the BNA logic are shown in Table 4. Note that symbols  $P$  and  $Q$  stands for principals,  $X$  and  $Y$  range over statement, and  $K$  represent encryption keys.

Table 4: Notations of BNA logic

Notation	Meaning
$P \models X$	$P$ believes that $X$ is true.
$P \triangleleft X$	$P$ once received a message including $X$ .
$P \sim X$	$P$ once said $X$ .
$P \Rightarrow X$	$P$ has jurisdiction over $X$ .
$\#(X)$	$X$ is fresh.
$(X, Y)_K$	$X$ and $Y$ are hashed with the key $K$ .
$\{X, Y\}_K$	$X$ and $Y$ are encrypted with the key $K$ .
$P \xleftrightarrow{K} Q$	$P$ communicates with $Q$ by a shared key $K$ .

1) Idealization forms

$$\begin{aligned}
 U_i: & (ID_i, V)_{U_i \xleftrightarrow{h(x||y||D)} S_i}, & V, & & U_i \xleftrightarrow{d} S_i, \\
 & (RPW_i, ID_i)_{U_i \xleftrightarrow{h(ID_i||x)} S_i}, \\
 & (C_4, V, U_i)_{U_i \xleftrightarrow{h(x||y||D)} S_i}, & S_i)_{U_i \xleftrightarrow{h(ID_i||x)} S_i},
 \end{aligned}$$

$$\begin{aligned}
 & (ID_i, RPW_i, V, U_i) \xleftrightarrow{d} S_i)_{U_i \xleftrightarrow{h(ID_i||x)} S_i}, \\
 & ((U_i)_{U_i \xleftrightarrow{SK} S_i}, S_i, W, RPW_i, U_i) \xleftrightarrow{d} S_i, \\
 & S_i)_{U_i \xleftrightarrow{h(ID_i||x)} S_i}, U_i \xleftrightarrow{SK} S_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftrightarrow{h(ID_i||x)} S_i} \\
 S_i: & (U_i)_{U_i \xleftrightarrow{SK} S_i}, S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftrightarrow{h(ID_i||x)} S_i}, W
 \end{aligned}$$

2) Security goals

- G1  $S_i \models U_i \models U_i \xleftrightarrow{SK} S_i$
- G2  $S_i \models U_i \xleftrightarrow{SK} S_i$
- G3  $U_i \models S_i \models U_i \xleftrightarrow{SK} S_i$
- G4  $U_i \models U_i \xleftrightarrow{SK} S_i$

3) Initiative assumption

- A1  $U_i \models U_i \xleftrightarrow{h(ID_i||x)} S_i$
- A2  $S_i \models U_i \xleftrightarrow{h(ID_i||x)} S_i$
- A3  $U_i \models U_i \xleftrightarrow{d} S_i$
- A4  $S_i \models U_i \xleftrightarrow{d} S_i$
- A5  $U_i \models U_i \xleftrightarrow{h(x||y||D)} S_i$
- A6  $S_i \models U_i \xleftrightarrow{h(x||y||D)} S_i$
- A7  $S_i \models U_i \Rightarrow U_i \xleftrightarrow{SK} S_i$
- A8  $U_i \models S_i \Rightarrow U_i \xleftrightarrow{SK} S_i$

Table 5: BNA logical postulates

Rule	Formula	Meaning
Message-meaning rule	$\frac{P \equiv P \xleftarrow{K} Q, P \triangleleft \{X\}_K}{P \equiv Q \sim X}$	If $P$ believes that $K$ is the secret key shared by $P$ with $Q$ , and $P$ sees $X$ encrypted with $K$ , then $P$ believes that $Q$ once said $X$ .
Nonce-verification rule	$\frac{P \equiv \#(X), P \equiv Q \sim X}{P \equiv Q \equiv X}$	If $P$ believes that $X$ is fresh and $Q$ once said $X$ , then $P$ believes that $Q$ believes $X$ .
Freshness-conjunction rule	$\frac{P \equiv \#(X)}{P \equiv \#(X, Y)}$	If $P$ believes that $X$ is fresh, then $P$ believes that $(X, Y)$ is fresh.
Jurisdiction rule	$\frac{P \equiv Q \Rightarrow X, P \equiv Q \equiv X}{P \equiv X}$	If $P$ believes that $Q$ controls $X$ and $P$ believes $Q$ believes $X$ , then $P$ believes $X$ .

#### 4) Scheme analysis

The main analysis of our proposed scheme is described as follows: Since  $S_i \triangleleft ((U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftarrow{h(ID_i||x)} S_i}, U_i \xleftarrow{SK} S_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftarrow{h(ID_i||x)} S_i}$  and  $S_i |\equiv U_i \xleftarrow{h(ID_i||x)} S_i$ , we can know

$$S_i |\equiv U_i |\sim ((U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftarrow{h(ID_i||x)} S_i}, U_i \xleftarrow{SK} S_i, U_i \xleftrightarrow{d} S_i) \quad (1)$$

based on message-meaning rule.

According to freshness-conjunction rule and  $S_i |\equiv \#(W)$ , we can derive

$$S_i |\equiv \#((U_i \xleftarrow{SK} S_i), W, RPW_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftarrow{h(ID_i||x)} S_i}, U_i \xleftarrow{SK} S_i, U_i \xleftrightarrow{d} S_i). \quad (2)$$

On the basis of Equations (1), (2) and nonce-verification rule, the following can be derived

$$S_i |\equiv U_i |\equiv ((U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftarrow{h(ID_i||x)} S_i}, U_i \xleftarrow{SK} S_i, U_i \xleftrightarrow{d} S_i). \quad (3)$$

The G1  $S_i |\equiv U_i |\equiv U_i \xleftarrow{SK} S_i$  will be deduced from Equation (3).

Based on A7, G1 and jurisdiction rule, we can derive G2  $S_i |\equiv U_i \xleftarrow{SK} S_i$ .

Since  $U_i \triangleleft (U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i)_{U_i \xleftarrow{h(ID_i||x)} S_i}$  and  $U_i |\equiv U_i \xleftarrow{h(ID_i||x)} S_i$ , we can know

$$U_i |\equiv S_i |\sim (U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i) \quad (4)$$

based on message-meaning rule.

If  $M_3 = h(M_2||C_1||SK'd)$ ,  $U_i |\equiv \#(W)$ . According to freshness-conjunction rule, we can derive

$$U_i |\equiv \#(U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i). \quad (5)$$

On the basis of Equations (4), (5) and nonce-verification rule, the following can be derived

$$U_i |\equiv S_i |\equiv (U_i \xleftarrow{SK} S_i, W, RPW_i, U_i \xleftrightarrow{d} S_i). \quad (6)$$

The G3  $U_i |\equiv S_i |\equiv U_i \xleftarrow{SK} S_i$  will be deduced from Equation (6).

Based on A8, G3 and jurisdiction rule, we can derive G4  $U_i |\equiv U_i \xleftarrow{SK} S_i$ .

## 5.2 Informal Security Analysis

### 5.2.1 User Anonymity

1) A legal but malicious user  $U_k$  acquires  $\{C_2^*, C_3^*, C_4^*, C_5^*, h(\cdot), n, g, y, B^*\}$  from his/her own smart card and computes  $b^* = B^* \oplus ID_k \oplus PW_k$ ,  $RPW_k = h(b^*||PW_k)$ ,  $C_1^* = C_2^* \oplus RPW_k$ ,  $d^* = C_4^* \oplus h(C_1^*||RPW_k)$ ,  $D^* = g^{d^*} \bmod n$ ,  $h(x||y||D^*) = C_5^* \oplus h(C_1 \oplus ID_k)$ .  $U_k$  can't obtain any common values for every legal user.

2) Even If  $U_k$  obtain  $\{C_2, C_3, C_4, C_5, h(\cdot), n, g, Y, B\}$  from  $U_i$ 's smart card, he/she impossible to get  $d$  without knowing  $C_1$ ,  $RPW_i$ , or  $h(x||y||D)$  without the values of  $C_1$ ,  $ID_i$ .

3) In unsecure channels,  $U_k$  intercepts the message  $\{CID_i, V, D, F_1, F_2, M_1\}$ , and tries to trace the user  $U_i$ . But the user  $U_i$  communicates with  $S_i$  by  $CID_i$  instead of his/ her own identity  $ID_i$ . It is infeasible to derive  $ID_i$  without knowing  $h(x||y||D)$ . On the other hand, it is hard to get the random number  $d$  from  $D = g^d \bmod n$  due to discrete logarithm problem.

Consequently, any legal but malicious user cannot obtain some useful values concerning with user  $U_i$ .

### 5.2.2 Offline Password Guessing Attack

- 1) Form the analysis of Section 5.2.1, we know that any legal but malicious user  $U_k$  cannot the common value  $h(x||y)$  for all legal users.
- 2) If  $U_k$  acquires  $\{C_2, C_3, C_4, C_5, h(\cdot), n, g, y, B\}$  from  $U_i$ 's smart card, he/she has to guess the user  $U_i$ 's identity  $ID_i$  and password  $PW_i$  correctly at the same time to compute  $b = B \oplus ID_k \oplus PW_k$ . As we all known, it is difficult to guess the two parameters chosen freely by the user at the same time in polynomial time. And the proposed scheme can provide user anonymity by the above analysis. Furthermore, the adversary needs to know the server's private key  $x$  to compute  $C_1 = h(ID_i||x)$ ,  $RPW_i = h(b||PW_i)$ . Then, he/she could get right password by comparing  $C_1 \oplus RPW_i$  with  $C_2$ .
- 3) Assume  $U_k$  intercepts the message  $\{CID_i, V, D, F_1, F_2, M_1\}$  which  $U_i$  once sent to  $S_i$ . However  $U_k$  does not have the knowledge of  $b$ ,  $C_1$  and  $ID_i$ , the verification of the computed  $F_1 = RPW_i \oplus h(C_1||ID_i)$  will fail.

### 5.2.3 Stolen Verifier Attack

The server  $S_i$  does not store any sensitive verification information corresponding to users in its database in our proposed scheme. Therefore even if any adversary accesses the server's database, he/she is impossible to gain any verification information related to registered users. So, the proposed scheme can withstand stolen verifier attack.

### 5.2.4 Insider Attack

Assume that the privileged user gets  $ID_i$ ,  $RPW_i$  when a legal user  $U_i$  registers to the system  $S_i$ . However, the privileged couldn't extract  $PW_i$  from  $RPW_i$  due to one-way property of hash function. At the same time,  $PW_i$  is protected by random number  $b$ , and the privileged user is not able to guess the right password. Thus, the proposed scheme can resist insider attack.

### 5.2.5 Replay Attack

Suppose that an adversary  $E$  eavesdrops the login request message and tries to perform replay attack in future. Upon receiving  $\{CID_i, V, D, F_1, F_2, M_1\}$  from  $E$ , the server  $S_i$  verifies  $M_1 \stackrel{?}{=} h(ID_i||RPW_i||V||C_1||d)$ . The message has not been changed by  $E$ , so  $S_i$  selects a random number  $w^*$  and computes  $W^* = g^{w^*} \bmod n$ ,  $SK^* = V^{w^*} \bmod n$ ,  $M_2^* = h(SK^*||W^*||C_1||RPW_i||d)$ . Then,  $S_i$  sends  $\{M_2^*, W^*\}$  to the adversary  $E$ . It is indispensable for the adversary  $E$  to reply  $\{M_3\}$  to  $S_i$ , where  $M_3 = h(M_2||C_1||SK||d)$ . Because  $E$  not only couldn't compute  $SK$  without random number  $v$ , but also couldn't get  $C_1$  and  $d$ . Thus, the server cannot authenticate  $E$ . Namely, the scheme is secure against replay attack.

### 5.2.6 User Impersonation Attack

If an adversary  $E$  wants to pretend  $U_i$  to communicate with  $S_i$ , he/she must forge the login request message  $\{CID_i, V, D, F_1, F_2, M_1\}$ . Then, he/she selects a random number  $v^*$  and computes  $V^* = g^{v^*} \bmod n$ ,  $U^* = Y^{v^*}$ . Unfortunately,  $E$  couldn't compute  $CID_i^*$  without the user  $U_i$ 's identity  $ID_i$ , server's private key  $x$ . Meanwhile,  $U_k$  requires to compute  $F_1^* = RPW_i \oplus h(C_1||ID_i)$ ,  $F_2^* = C_4 \oplus h(V^*||C_1) \oplus h(x||y||D)$ , which is not possible, since  $E$  does not know  $ID_i$ ,  $RPW_i$ ,  $x$ . That is, the proposed scheme is able to against the user spoofing attack.

### 5.2.7 Server Impersonation Attack

If an adversary  $E$  eavesdrops the login request message  $\{CID_i, V, F_3, F_4, M_1\}$  from user  $U_i$ , he/she performs the following steps to act as the legal server  $S_i$ .  $E$  must compute  $M_2 = h(SK||W||C_1||RPW_i||d)$  to respond the login request message. Even If  $U_k$  selects a random number  $w^*$  and computes  $W^* = g^{w^*}$ ,  $SK^* = V^{w^*} \bmod n$ , he/she cannot forge  $M_2$  without  $RPW_i$ ,  $C_1$ ,  $d$ . From the above analysis, our proposed scheme could resist server impersonation attack.

### 5.2.8 Mutual Authentication

- 1) In the proposed scheme,  $S_i$  authenticates  $U_i$  by checking the validity of equation  $M_3 \stackrel{?}{=} h(M_2||C_1||SK||d)$ . We have demonstrated that the proposed scheme can provide user anonymity and off-line password guessing attack. If an adversary replays the former login request message  $\{CID_i, V, D, F_1, F_2, M_1\}$  sent to  $S_i$  by  $U_i$ , he/she would fail according to the analysis of section 5.2.5. On the other hand, suppose the adversary forge the login request message to cheat the server, we will find that it is impossible by the analysis of Section 5.2.6.
- 2) On the contrary, the legal user  $U_i$  authenticates  $S_i$  by comparing  $M_2$  with the computed value  $h(SK||W||C_1||RPW_i||d)$ . Based on the analysis of Section 5.2.7, no one can act as legal user to deceive the server.

Therefore, the proposed scheme can provide mutual authentication.

### 5.2.9 Forward Secrecy

In the improved scheme, the user  $U_i$  and the server  $S_i$  establish the same session key  $SK = W^v \bmod n = V^w \bmod n = g^{vw} \bmod n$ . Due to discrete logarithm problem (DLP), no one is able to compute the previously established session keys without knowing  $v$ ,  $w$ . As a result, the proposed scheme provides perfect forward secrecy.

Table 6: Performance comparison

	Total of login and authentication phase	Time
[17]	$6T_e + 11T_h$	13.7303
[10]	$4T_e + 13T_h$	9.1575
[5]	$6T_e + 5T_h$	13.7261
[20]	$8T_e + 7T_h$	18.3017
[21]	$6T_e + 8T_h$	13.7282
Ours	$5T_e + 17T_h$	11.4474

## 6 Performance Analysis

In this section, we will show efficiency and functionality comparison among our proposed scheme and other related schemes. According to Wu et al.'s report [19], the time of executing one modular exponentiation is 2.2871ms, while the computation time of a one-way hash function is 0.0007ms. For the convenience, we define the following notations used in this section.

- $T_h$ : time for executing a one-way hash function.
- $T_e$ : time for executing exponential operation.
- $T_{\oplus}$ : time for executing XOR operation.

Compared with  $T_e$  and  $T_h$ , the time of executing XOR operation can be neglected. Usually, a legal user only needs to perform once registration operation, but login and authentication phase are carried out more times in a short time. So we display the comparison of the computational cost in login and authentication phase among these schemes in Table 6. In Table 7, we show security comparison between our proposed scheme and other related ones.

From the comparison of Table 6 and Table 7, we can conclude that the performance of our scheme has better efficiency than other related schemes. Taking all into account, the proposed scheme is more suitable for practical applications.

## 7 Conclusions

In this paper, we review an anonymous password-based authenticated key agreement scheme with non-tamper resistant smart cards which is proposed by Lee et al. to reduce time cost under the condition of safety. However, Lee et al.'s scheme is vulnerable to smart card stolen or lost attack, user impersonation attack, server impersonation attack and cannot provide mutual authentication. To overcome the weakness mentioned above, an improved scheme is proposed. Finally, we demonstrate that our scheme is more secure and applicable to practice by comparing the performance and efficiency of our scheme with other related ones.

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Table 7: Security comparison

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
[17]	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
[10]		✓	✓	✓			✓	✓		✓		✓
[5]	✓	✓		✓			✓			✓		
[20]	✓	✓	✓				✓	✓		✓	✓	✓
[21]	✓						✓			✓		✓
Ours	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

C1 No verifier table.

C2 Password can be chosen freely.

C3 The password cannot be derived by the privileged administrator of the server.

C4 The security of the scheme is not based on the tamper resistance assumption of the smart card.

C5 Resistance to known attacks, such as offline password guessing attack, replay attack, parallel session attack, denial of service attack, stolen verifier attack, user/server impersonation attack.

C6 The password cannot be broken by guessing attack even if the smart card is lost/stolen and compromised.

C7 Establish a common session key.

C8 The scheme is not prone to the problems of clock synchronization and time-delay

C9 the user can change the password locally without any interaction with the authentication server.

C10 Mutual authentication

C11 User anonymity.

C12 Forward secrecy.

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