Heron of Alexandria's Date

NATHAN SIDOLI*

Abstract. This paper is a reassessment of the use of the use of *Dioptra* 35 for the purposes of dating the activity of Heron of Alexandria. It is argued that the text does not contain a carefully recorded eclipse observation that can be attributed to Heron. The most that can be said is that the lunar eclipse of 13 March 62 can probably be taken as a *terminus post quem* for Heron's activity.

Keywords. Heron of Alexandria, *Dioptra* 35, analemma, eclipse observation, astronomical dating, applied historical astronomy, Otto Neugebauer.

This note revisits the question of dating Heron of Alexandria. In particular, I wish to correct an error that I made in a paper that appeared some years ago in this journal and to reassess what we can say about the time in which Heron lived on the basis of the evidence of his *Dioptra* 35.[1]

Until recently, it had generally been accepted that Neugebauer was able to use methods of historical astronomy to settle the vexed question of when Heron lived.[2] Neugebauer (1938–1939, p. 23) argued that Heron was alive during the year 62 CE, based on the claim that the eclipse data in *Dioptra* 35 could be used to date an accurately observed eclipse, which Heron, in fact, personally observed. The argument that Neugebauer advanced depended upon the combined claims that (1) the mathematical methods of *Dioptra* 35 were purely nomographic in nature and that the eclipse of 62 CE, being 'ill suited' to such methods, must have been used as an appeal to the recent memory of his readers,[3] (2) that the data stated in *Dioptra* 35 refers to an eclipse that Heron himself observed, despite the fact that he does not explicitly state this, and (3) that the full lunar eclipse of 62 March 13 is the only one in the range of dates in which Heron might possibly have been active that gives a good fit for the data provided in the text. I previously argued that none of these claims were certain and focused my attention on attempting to undermine claim (3) by arguing that the eclipse of 13 March 62 was not as good a fit with the data as Neugebauer had claimed. As will be explained below, it is now clear that I was mistaken in my attempt to argue against the fit between the eclipse of 13 March 62 and the eclipse data of *Dioptra* 35. Nevertheless, as my study of the *Dioptra* text has shown, claims (1) and (2) are both uncertain and it is highly unlikely that the mathematical methods presented in *Dioptra* 35 were Heron's own invention (Sidoli, 2005). This latter claim is based on a number of considerations such as the fact that the

*School of International Liberal Studies, Waseda University, Tokyo. E-mail: nathan.sidoli@utoronto.ca

CENTAURUS 2011: VOL. 53: PP. 55–61; doi:10.1111/j.1600-0498.2010.00203.x

mathematical astronomy in *Dioptra* 35 is so different from anything we find elsewhere in the *Dioptra* or in the rest of the Heronic corpus, that the mathematical methods set out are so incompletely described from either a nomgraphic or a trigonometric perspective, and the fact that the text contains the summary of a general discussion of cases that are not relevant to the example that Heron gives.[4]

Hence, the entire situation must be reassessed.

In order to fully evaluate the evidence, it may be useful to give a brief overview of *Dioptra* 35 and to quote the passage in which Heron gives the lunar eclipse data. In *Dioptra* 35, Heron sets out a method of using two accurately recorded observations of the same lunar eclipse along with a method of modeling the celestial sphere, known as the *analemma*, to determine the great-arc distance between the observers. As I have argued at some length (Sidoli, 2005), the actual value of the great-arc distance can be determined either nomographically or more precisely, using the techniques of ancient trigonometry applied to the analemma figure. The text begins with a general description of the problem and then shows how a hemisphere can be constructed to model the night sky below one location while an analemma, at the same scale, is used to model various arcs of the celestial sphere at the other location, which are then transferred into the hemisphere. The transmitted text becomes fragmentary midway through the discussion of a case that does not arise in the example Heron gives, but which could arise in other cases. The example that Heron uses is that of Alexandria and Rome. He says,

> Let it be necessary to measure, say, the path between Alexandria and Rome along a line—or rather along a great-circle arc on the earth—if it has been agreed that the circumference of the earth is 252,000 stades—as Eratosthenes, having worked rather more accurately than others, showed in his book entitled *On the Measurement of the Earth.* Now, let the same lunar eclipse have been observed at Alexandria and Rome. If one is found in the records, we will use that, or, if not, it will be possible for us to state our own observations because lunar eclipses occur at five and six month intervals. Now, let the same eclipse be found in the stated regions—in Alexandria in the 5th hour of the night, and the same one in the 3rd hour in Rome—obviously the same night. And let the night—that is, the day circle with respect to which the sun moves on the said night—be 10 days from the vernal equinox in the direction of the winter solstice. (Schöne, 1903, p. 302)

The data that Heron uses is 10 days before the vernal equinox, in the third hour of the night at Rome and the fifth hour at Alexandria. As his language makes clear, he is using these two cities as an example to demonstrate a general method and the data for this example was either taken from some published source, or derived from a new observation. Another, unstated possibility is that, simply for the sake of an example, the data was generated from some mathematical model that could be used to predict lunar eclipses. Although this last possibility may seem remote, it should be pointed out that the time difference that Heron states between Alexandria and Rome is 2:00 hours, whereas they are actually only about 1:10 hours apart, so that it is clear that at least one of these data sets does not correspond to an accurately observed eclipse.[5]

Although I had previously argued that the data given by Heron is not a good fit with Neugebauer's proposed dating of 13 March 62, this comparison was based on using published tables for the times of the equinoxes that I carelessly did not notice were dated according to the Gregorian calendar, even for periods prior to the institution of that calendar. After Franz Krojer, in private communication, kindly alerted me to this confusion, I revisited the question, using the eclipse calculations given by Liu and Fiala (1992) and directly calculating the times of the equinoxes for lunar eclipses that took place 9–12 days before the vernal equinox between the years 200 BCE to around 300 CE.[6]

While there were a fair number of umbral lunar eclipses that were 9–12 days before the equinox for this period.[7] Only two took place at times that could reasonably fit the data in *Dioptra* 35. The eclipse of 12/13 March -3 took place 10 days before the equinox of 23 March -3, which took place at around 1:35 in Alexandria, but the start times of this eclipse were around 00:31 (13 March) in Rome and 1:41 in Alexandria (Liu and Fiala, 1992, p. 89). Since these times would probably have been recorded as during the seventh hour at Rome and the eighth hour at Alexandria, this is not a very good fit with the data given in *Dioptra* 35.

The eclipse of 13 March 62, however, started around 22:50 in Alexandria, which, in fact, might have been recorded as *during* the fifth hour, and at 21:40 in Rome, which should more accurately have been recorded as *during* the fourth hour; while the vernal equinox that year occurred at around 22:00 on the night of 22 March. If the time of the equinox was observed using an instrument like the equinox rings that Hipparchus and Ptolemy, quoting and discussing him, say were set up in Alexandria then it will not have been observed until the sunrise of 23 March.[8] Another possibility is that the date of the equinox was taken from a calendrical type device, like a parapegma.[9] In either case, the lunar eclipse could have been recorded as 10 days prior to the equinox and we are left with only some uncertainties concerning the stated times of the observations. These uncertainties arise both because Heron does not state whether the times he gives in *Dioptra* 35 are the beginning or the middle of the eclipse and also from the vagueness of the time expressions themselves. The text literally reads,

> Now, let it be that the same eclipse is found at the stated latitudes–at Alexandria, hour (ὥρας) 5 of the night, while at Rome, the same one, hour 3 of the night...[10]

It is not certain from this if the times indicated are at beginning of the hours or during those hours. Nevertheless, if we take 'hour' as the genitive of time-within-which, it is possible to read the text as asserting that the beginning of the eclipse was seen *during* the third and fifth hours, where the first hour is the period after sunset lasting 1/12 of the nighttime.[11] At this time of year, the sun will have crossed the ideal horizon at roughly 18:00 local time, so depending on how the observers defined the start and end of nighttime, the first hour of the night would have been the period from roughly 18:00 to roughly 19:00, and so on through the night. Since the beginning of the eclipse would, in theory, have been visible in Alexandria from about 22:50 on, even if the observer did not notice this immediately, it might well have been recorded as having begun just before the start of the sixth hour, or sometime during the fifth hour. In Rome, however, the eclipse would, in theory, have been visible from 21:40 on, so that even if the observer noticed the very start of the eclipse this should have been recorded as during the fourth hour, not at the start of, or during the third hour, as we read in the text. Hence, while the date and time for Alexandria could have been the result of a careful observation, the time for Rome appears to have been a calculation based on an assumed time difference between the two cities. Nevertheless, as Neubebauer asserted, the eclipse of 13 March 62 is a fairly good fit for Alexandria and certainly the best fit of all the lunar eclipses between 200 BCE to 300 CE.

It should be stressed, however, that although the data given in *Dioptra* 35 for the eclipse as seen from Alexandria could have been *based* on an accurately recorded eclipse observation, this data does not itself constitute an accurate observation record. Simply stating that some unspecified phase of an eclipse took place at some time during a certain hour does not meet the standards of precision that we find elsewhere in the ancient technical literature. Moreover, as Heron certainly knew, the method of finding the great-arc distance between two locations set out in *Dioptra* 35 depends on the strict accuracy of the two observations. Hence, the data sets given in the text must be taken as examples for the sake of a general demonstration of the method, not as a viable pair of observations for accurately calculating the distance between Alexandria and Rome.

With this as background, we can see that there are a fair number of possibilities for what has happened. For example, (1a) The eclipse of 13 March 62 might have been carefully observed from Alexandria, by Heron, and from Rome, by another astronomer, who then both proceeded to record the times of the eclipse fairly crudely. (1b) Careful observations may have been made in both cities and the data of these observations then loosely recorded in the source from which Heron drew the mathematical methods of *Dioptra* 35. (2a) A causal observation of the eclipse might have been made, by Heron in Alexandria, from which the time of the eclipse in Rome was determined by calculation. (2b) The author of Heron's source might have seen the eclipse in one of the cities and calculated the time in the other. Some mathematical theory might have been used to calculate the times of both eclipses, either (3a) by Heron or (3b) by his source. Finally, the eclipse of 13 March 62 may have been chosen (4a) by Heron or (4b) his source, not because it was a carefully recorded observation, but because, at that time, it was an eclipse which was well known to have been seen in both cities. In this case, the eclipse itself may have occurred before the time of either Heron, or his source.

Since the time difference between Rome and Alexandria is about 1:10 equinoctial hours whereas the stated time difference in the *Dioptra* 35 data is 2:00 hours, with a fairly large difference of 0:50 minutes, and since the data for Alexandria is passably accurate whereas that for Rome is not, this may be an indication that the eclipse was actually observed in Alexandria and then calculated for Rome. Nevertheless, the conditions of ancient observational practice and the vagueness of the text with regard to the temporal data are enough to raise some doubts. Even if the data were calculated, however, the best explanation for why this particular eclipse was chosen may still be that it had actually been seen in both cities by the contemporaries of Heron, or his source, so that for the purpose of dating Heron, or his source, it probably does not make much difference whether the eclipse was observed or calculated. Finally, in *Geography* I 4, Ptolemy makes it clear that, much to his disappointment, there were almost no carefully recorded simultaneous eclipse observations and mentions only one - the famous eclipse seen at Arbēla and Carthage, which was famous, not because it was carefully recorded, but because it took place eleven days before Alexander defeated Darius III (Berggren and Jones, 2000, p. 63). Hence, there is a real possibility that the reason the eclipse of 13 March 62 was used is that it was, at the time, a sort of canonical example. Certainly, in Heron's text it is used *as* an example.

Hence, although the evidence of *Dioptra* 35 might, indeed, be used to argue that Heron was active in 62 CE, at the time of the recorded eclipse, all that we can say with certainty is that this date can be taken as a *terminus post quem* for his activity. Since the mathematical methods developed in the text are derived from some other source, it comes down to whether we read Heron as working through a new data set or taking the reader through a previously worked example. For the purposes of deciding this question, however, Heron's statement that if an observation 'is found in the records, we will use that, or, if not, it will be possible for us to state our own observations' is not of much help, since it does not, in fact, state which option he chose. Thus, while it is highly unlikely that Heron would have used data for an eclipse prediction sometime in the future of his own activity as an example, there is a real possibility that he took such data from the same source in which he found the mathematical methods set out in *Dioptra* 35.

Acknowledgments

I would like to thank Alexander Jones and Hans Michael Schellenberg for their comments on a previous version of this paper.

NOTES

1. See Sidoli (2005, pp. 250–253) for my previous remarks on this issue. Franz Krojer was kind enough to point out an error in this account in a personal correspondence and referred me to his published discussion of the situation (Krojer, 2009, pp. 32–38).
2. In fact, a date around the middle of the first century had already been put forward based on historical and philological arguments, for example by Schmidt (1903, vol. 1, p. xix) and Drachmann (1932, p. 125). Many scholars, however, have taken Neugebauer's arguments as definitive, as can be seen, for example, by consulting standard reference works such as the *Dictionary of Scientific Biography* and *The Encyclopedia of Ancient Natural Scientists* (Gillispie, 1970–1980, pp. 6, 310; Keyser and Irby-Massie, 2008, p. 384).

3. Neugebauer made his case in Neugebauer (1938–1939, p. 23) and again in Neugebauer (1975, p. 846). Nomography was the mathematical field of using graphical techniques to perform analog calculations that flourished from the end of the 19th century until the rise of the electronic computer (Funkhouser, 1937, pp. 367–374). My usage of the adjective *nomographic* follows Neugebauer (1975, p. 852). By *nomographic techniques*, I mean some tradition of using techniques of ancient geometry to produce line segments or arc lengths on an instrument in such a way that they could then be measured by an analog measuring tool. The nomographic aspects of *Dioptra* 35 have been set out by Rome (1923), Neugebauer (1938–1939) and Sidoli (2005).
4. For a full treatment of these issues see Sidoli (2005).
5. Indeed, as we will see below, neither of these data sets can correspond to an *accurately* recorded observation. The time difference of 1:10 is in equinoctial hours whereas the times of observation stated by Heron should be in seasonal hours. Nevertheless, since the day in question is only 10 days before the vernal equinox, the difference is negligible. Notice that the times in this paper are given either in ancient seasonal hours, or in equinoctial hours, which is true local time, not local time as defined by the current system of time zones.
6. Since Heron mentions Archimedes and is in turn mentioned by Pappus of Alexandria, we can take the possible span of his dates as from around 200 BCE to 300 CE. The range of possible eclipse dates is taken because we cannot be certain how the equinox was determined, whether by observation or by consulting a calendrical type instrument, such as a parapegma, nor can we be sure, in any case, what level of accuracy would have been obtained.
7. Steele (2000, p. 10) has pointed out that there are 'no firmly dated observations of penumbral eclipses from pre-telescopic times'.
8. Although Ptolemy disparages the accuracy of equinox rings, both his comments and the Hipparchus text that he quotes, make it clear that these instruments were in use in Alexandria (Toomer, 1984, pp. 133–134).
9. See Lehoux (2007, pp. 70–97) for a discussion of ways in which a parapegma could be used in conjunction with a local calendar, which would make it possible to date an equinox.
10. ἔστω οὖν ερὑημένη ἐν τοῖς εἰρμένοις κλίμασιν αὕτη <ἡ> ἔκλειψισ, ἐν Ἀλεξανδρείᾳ μὲν νυκτὸς ὥρας ε, ἐν Ῥώμη δὲἡ αὐτὴ νυκτὸς ὥρας γ (Schöne, 1903, p. 302).
11. For a night 10 days before the equinox, 1/12 of the nighttime would be about equal to one equinoctial hour.

REFERENCES

Berggren, J. L. and Jones, A. (2000) *Ptolemy's geography: an annotated translation of the theoretical chapters* (Princeton: Princeton University Press).

Drachmann, A. G. (1932) *Ancient oil mills and presses* Det kongelige Danske Videnskabernes Selskab, Archaeologisk-kunsthistoriske Meddelelser, 1 (Copenhagen: Levin & Munksgaard).

Funkhouser, H. G. (1937) Historical development of the graphical representation of statistical data, *Osiris*, 3, 269–404.

Gillispie, C. C. (1970–1980) *Dictionary of scientific biography* (New York: Charles Scribner's Sons).

Keyser, P. T. and Irby-Massie, G. L. (2008) *The encyclopedia of ancient natural scientists* (London, New York: Routledge).

Krojer, F. (2009) *Astronomie der Spätantike, die Null und Aryabhata* (München: Differenz-Verlag).

Lehoux, D. (2007) *Astronomy, weather, and calendars* (Cambridge: Cambridge University Press).

Liu, B.-L. and Fiala, A. D. (1992) *Canon of lunar eclipses 1500 B.C. to A.D. 3000* (Richmond: Willmann-Bell).

Neugebauer, O. (1938–1939) Über eine Methode zur Distanzbestimmung Alexandria-Rom bei Heron I & II, *Det Kongelige Danske Videnskabernes Selskab*, 26 (2 & 7), 3–26, 3–11.

Neugebauer, O. (1975) *A history of ancient mathematical astronomy* (New York: Springer).

Rome, A. (1923) Le problème de la distance entre deux villes dans la *Dioptra* de Héron, *Annales de la Société Scientifique de Bruxelles*, 43, 234–258.

Schmidt, W. (1903) *Heronis Alexandrini: Pnumatica, Heronis Alexandrini opera quae supersunt omnia*, Vol. 1 (Leipzig: Teubner).

Schöne, H. (1903) *Heron von Alexandria: Vermessungslehre und Dioptra, Heronis Alexandrini opera quae supersunt omnia*, Vol. 3 (Leipzig: Teubner).

Sidoli, N. (2005) Heron's *Dioptra* 35 and analemma methods: an astronomical determination of the distance between two cities, *Centaurus*, 47, 236–258.

Steele, J. (2000) *Observations and predictions of eclipse times by early astronomers* (Dordrecht: Kluwer Academic Publishers).

Toomer, G. J. (1984) *Ptolemy's almagest* [Reprinted by Princeton University Press, Princeton, 1998] (London: Duckworth).