near the surface in semimetals (antimony) is the radical difference between the reflection of the electrons (specular) and the holes (diffuse) from the sample surface.[13]

The presence of band bending near the surface causes formula (7) to underestimate the dimensions of surface roughnesses. In this case formula (7) must be modified:

$$q(\theta)=1-P(\theta)\alpha\sin\theta, \tag{8}$$

where $P(\theta)$ is the probability that the electron will "reach" the boundary. Owing to tunneling, the function $P(\theta)$ cannot be represented in the form $P(\theta)=0$ at $\theta<\theta_c$ and $P(\theta)=1$ at $\theta_c\leqslant\theta\leqslant\pi/2$. The roughnesses of the investigated surfaces, estimated on the basis of (8), exceed by one order the estimates obtained with formula (7), assuming that (8) is valid for arbitrary θ and that $P(\theta)\approx 1$ at large θ.

The diffuse reflection of normally incident electron from the binary plane and specular reflection from the trigonal plane may not be due to a difference between the scattering mechanisms, but only to the different dimensions of the surface roughnesses. It is known, for example, that different crystallographic planes can have different macroscopic roughnesses.

[1)]In the calculations of $U(H)$ the contact dimensions were assumed to be the same and the Fermi surface was assumed to be cylindrical.

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Nonlinear effects during the motion of vortices in superconductors

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The order parameter depends strongly on the coordinates in the mixed state in superconductors. A kinetic equation is derived in this case which describes the excitation-energy distribution function. This function varies strongly in a comparatively weak electric field E. At low temperatures, the effective electron temperature is proportional to $E^{2/5}$. If this temperature exceeds the energy gap, most of the energy of the stationary electric field should transform into energy of almost-monochromatic phonons. The boundary condition for the diffusion equation is obtained in the case of high impurity concentrations.

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INTRODUCTION

The energy relaxation time in metals $\tau_\varepsilon\sim\Theta_D^2/T^3$ is large at low temperatures. A large change in the electron energy distribution function therefore takes place in comparatively weak electric fields. In a normal metal, such a change has no effect on the conductivity. In a superconductor, the current density and the value of the energy gap are strongly dependent on the shape of the distribution function; therefore, departures from Ohm's law set in rather rapidly during motion of vortices. The significant change in the electron distribution function during motion of vortices can be observed from the spectrum of the emitted phonons. At sufficiently low temperatures and not too weak electric fields, almost all the energy of the electric field should transform into energy of monochromatic phonons with a frequency equal to 2Δ. In this case, the electron excitations are produced by the electric field at the center of the vortex, are accelerated to energy Δ, and then leave

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the region of the vortex and undergo annihilation with emission of a phonon. The usual kinetic equation describing the coordinate and momentum distributions of the excitations is applicable only in those cases in which both the path length and the size of the investigated region are large in comparison with v/Δ. At least one of these conditions is not satisfied in the vortex state. Therefore, the momentum and coordinate of an excitation do not have any meaning, but the excitation energy distribution function does have a meaning. The kinetic equation for this function is obtained below.

1. KINETIC EQUATION FOR TWO DISTRIBUTION FUNCTIONS

In obtaining the kinetic equation for the distribution functions, we use the Keldysh method.[1] In this method, the Gor'kov equations for the Green's function have the form[2]

$$\left\{i\hat{\tau}_z\frac{\partial}{\partial t}+\frac{1}{2m}\partial^2+\check{\Delta}-e\varphi+\mu-\hat{\Sigma}\right\}\hat{G}(\mathbf{r},\mathbf{r}',t,t')=\delta(t-t')\delta(\mathbf{r}-\mathbf{r}'), \tag{1}$$

where $A(\mathbf{r},t)$ is the vector potential, φ is the scalar potential, $\partial=\partial/\partial\mathbf{r}-ie\mathbf{A}\hat{\tau}_z$. In Eq. (1), the Green's function and the self-energy part of the $\hat{\Sigma}$ matrix have the form

$$\hat{G}=\begin{pmatrix}G^R & G\\ 0 & G^A\end{pmatrix},\quad \hat{\Sigma}=\begin{pmatrix}\Sigma^R & \Sigma\\ 0 & \Sigma^A\end{pmatrix}. \tag{2}$$

In turn, the quantities G and $G^{R,A}$ are (2×2) matrices consisting of the usual Green's function g and the Gor'kov function F:

$$G=\begin{pmatrix}g_1 & F_1\\ -F_2 & g_2\end{pmatrix},\quad \hat{\tau}_z=\begin{pmatrix}\tau_z & 0\\ 0 & \tau_z\end{pmatrix},\quad \check{\Delta}=\begin{pmatrix}\hat{\Delta} & 0\\ 0 & \hat{\Delta}\end{pmatrix}$$
$$\tau_z=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \hat{\Delta}=\begin{pmatrix}0 & \Delta_1\\ -\Delta_2 & 0\end{pmatrix}. \tag{3}$$

The interaction of the electrons with phonons and impurities is taken into account with the help of the self-energy part of $\hat{\Sigma}$. Assuming the interaction of the electrons with phonons and impurities to be weak, we obtain the following expression for the self-energy part of $\hat{\Sigma}$:

$$\hat{\Sigma}_{\mathbf{p}}(t,t')=\hat{\Sigma}_{\mathbf{p}}^{imp}(t,t')+\hat{\Sigma}_{\mathbf{p}}^{ph}(t,t'),$$
$$\hat{\Sigma}_{\mathbf{p}}^{imp}(t,t')=-\frac{inv}{2}\int d\Omega_{\mathbf{p}_1}\sigma_{\mathbf{pp}_1}\hat{G}_{\mathbf{p}_1}(t,t')-\frac{i}{2\tau_s}\int\frac{d\Omega_{\mathbf{p}_1}}{4\pi}\tau_z\hat{G}_{\mathbf{p}_1}(t,t')\tau_z,$$
$$\hat{\Sigma}_{\mathbf{p}}^{ph(R,A)}(t,t')=\frac{\nu g^2}{8}\int d\Omega_{\mathbf{p}_1}\{D_{\mathbf{p}-\mathbf{p}_1}(t'-t)G_{\mathbf{p}_1}^{R,A}(t,t')+D_{\mathbf{p}-\mathbf{p}_1}^{A,R}(t'-t)G_{\mathbf{p}_1}(t,t')\}: \tag{4}$$
$$\hat{\Sigma}_{\mathbf{p}}^{ph}(t,t')=\frac{\nu g^2}{8}\int d\Omega_{\mathbf{p}_1}\{D_{\mathbf{p}-\mathbf{p}_1}(t'-t)G_{\mathbf{p}_1}(t,t')$$
$$-(D_{\mathbf{p}-\mathbf{p}_1}^{R}(t'-t)-D_{\mathbf{p}-\mathbf{p}_1}^{A}(t'-t))(G_{\mathbf{p}_1}^{R}(t,t')-G_{\mathbf{p}_1}^{A}(t,t'))\},$$

where $\sigma_{\mathbf{pp}_1}$ is the scattering cross section of the electron by the impurity,

$$\hat{G}_{\mathbf{p}}(t,t')=\frac{1}{\pi}\int d\xi\hat{G}(t,t',\mathbf{R}+\mathbf{r}/2,\mathbf{R}-\mathbf{r}/2)e^{-i\mathbf{pr}}d^3\mathbf{r}, \tag{5}$$

$\xi=p^2/2m-\mu$, v is the velocity on the Fermi surface, $\nu=mp/2\pi^2$ is the density of states on the Fermi surface, τ_s is the time of flight of an electron with spin flip, n is the concentration of the impurities, g is the constant of electron-phonon interaction, and $D_{\mathbf{p}-\mathbf{p}'}(t-t',\mathbf{R})$ is the phonon Green's function.

In the derivation of the expression (4) for $\hat{\Sigma}$, we used the fact that the Green's functions $\hat{G}$ have a sharp maximum near the Fermi surface. The significant values of the parameter ξ always satisfy the condition $\xi\ll\mu$. In this region, the scattering cross section from impurities and the phonon Green's function are weakly dependent on ξ. Therefore, the scattering cross section from impurities and the phonon Green's function in the expression (4) depend only on the angles of the vector $\mathbf{p}$, which lies on the Fermi surface, and the electron Green's functions are integrated over the energy variable ξ. As in the static case,[3,4] it turns out to be possible to obtain equations (with quasiclassical accuracy) for the Green's functions integrated over the energy variable ξ. For this, we note that Eq. (1), which has the form $\hat{G}^{-1}\hat{G}=\hat{1}$, can be written in the form $\hat{G}\hat{G}^{-1}=\hat{1}$. In the latter equation, the operator $\hat{G}^{-1}$ acts on the variables $\mathbf{r}'$ and t'. We subtract one equation from the other and transform to the momentum representation in the coordinate difference $\mathbf{r}=\mathbf{r}'$. Using the independence of $\hat{\Sigma}$ of the variable ξ, we integrate the resultant equation over ξ. As a result, we obtain an equation for the Green's function (5) integrated over the energy variable ξ[2,5]:

$$\frac{1}{m}\mathbf{p}\frac{\partial\hat{G}}{\partial\mathbf{R}}+\hat{\tau}_z\frac{\partial\hat{G}}{\partial t}+\frac{\partial\hat{G}}{\partial t'}\tau_z+\hat{H}(t)\hat{G}-\hat{G}\hat{H}(t')$$
$$+i\int_{-\infty}^{\infty}dt_1\{\hat{\Sigma}(t,t_1)G(t_1,t')-\hat{G}(t,t_1)\hat{\Sigma}(t_1,t')\}=0, \tag{6}$$

where

$$\hat{G}=\hat{G}_{\mathbf{p}}(t,t',\mathbf{R}),\quad \hat{\Sigma}=\hat{\Sigma}_{\mathbf{p}}(t,t',\mathbf{R}),$$
$$\hat{H}(t)=-\frac{ie}{m}\mathbf{pA}(t)\hat{\tau}_z-i\check{\Delta}(t)+ie\varphi(t). \tag{7}$$

The Green's function G satisfies the normalization condition[2]

$$\int_{-\infty}^{\infty}dt_1\hat{G}_{\mathbf{p}}(t,t_1)\hat{G}_{\mathbf{p}}(t_1,t')=\hat{\delta}(t-t'). \tag{8}$$

In many problems, it suffices to express the Green's functions $G^{R,A}$ in terms of $\Delta(t)$ and $A(t)$ with the help of the formulas of the static approximation. The corrections to them due to the alternating field are usually small and can be found from perturbation theory. In the solution of Eqs. (6) for the Green's function G it is convenient to make use of the normalization condition (8). This condition allows us to express two matrix elements in the matrix $\hat{G}$ in terms of two others; therefore, it is convenient to introduce two distribution functions: f and f_1, with the help of which the Green's function can be represented in the form

$$G(t,t')=\int_{-\infty}^{\infty}dt_1\{G^R(t,t_1)\hat{f}(t_1,t')-\hat{f}(t,t_1)G^A(t_1,t')\},$$
$$\hat{f}=f+\tau_z f_1. \tag{9}$$

The Green's function G, written down in the form (9), satisfies the normalization condition (8) in the case of

arbitrary functions f and f_1. In slowly changing fields ($\omega \ll \Delta$) the distribution functions f and f_1 are fast functions of the time difference and slow functions of the sum of the times. We transform to the Fourier representation in the time difference:

$$G_\varepsilon(t)=2f(\varepsilon)\delta+2f_1(\varepsilon)\alpha-i\frac{\partial f}{\partial\varepsilon}\frac{\partial\gamma}{\partial t}+i\frac{\partial f}{\partial t}\frac{\partial\gamma}{\partial\varepsilon}; \tag{10}$$

$$2\alpha=G^R\tau_z-\tau_zG^A.\quad 2\delta=G^R-G^A.\quad 2\gamma=G^R+G^A. \tag{11}$$

We substitute Eq. (10) for the Green's function in Eq. (6). In addition, we take into account for the Green's function $G^{R,A}$ the equation that follows from the system (6):

$$\mathrm{Sp}\left\{\frac{\partial\alpha}{\partial t}-i\frac{\partial\hat H}{\partial t}\frac{\partial\delta}{\partial\varepsilon}\right\}=0. \tag{12}$$

As a result, we obtain a set of equations for the functions $f(p, \varepsilon, \mathbf{r}, t)$ and $f_1(p, \varepsilon, \mathbf{r}, t)$:

$$\begin{aligned}
&\frac{1}{2}\mathrm{Sp}\left\{\alpha\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\alpha f_1-i\frac{\partial f}{\partial\varepsilon}\frac{\partial\hat H}{\partial t}\delta\right.\\
&+\frac{1}{2}\frac{\partial\gamma}{\partial\varepsilon}\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial\varepsilon}\frac{\partial\hat H}{\partial t}\right)-\frac{1}{2}\frac{\partial\gamma}{\partial t}\frac{\partial}{\partial\varepsilon}\left(\frac{\partial f}{\partial\varepsilon}\frac{\partial\hat H}{\partial t}\right)\\
&\left.-i\frac{\partial\hat H}{\partial t}\frac{\partial}{\partial\varepsilon}\alpha f_1\right\}+I^{imp}+I_1^{ph}(f)+I_3^{ph}(f_1)=0.\\
&\frac{1}{2}\mathrm{Sp}\left\{\frac{\partial}{\partial t}\alpha f_1+\frac{\partial f_1}{\partial t}\hat\Delta\frac{\partial\delta}{\partial\varepsilon}+\alpha\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)f-2i\hat\Delta\gamma f_1\right.\\
&\left.-\frac{\partial f}{\partial\varepsilon}\frac{\partial\hat\Delta}{\partial t}\tau_z\gamma+e\frac{\partial\varphi}{\partial t}\frac{\partial f}{\partial\varepsilon}\alpha-\frac{i}{2}\frac{\partial f}{\partial\varepsilon}\frac{\partial^2\Delta}{\partial t^2}\tau_z\frac{\partial\delta}{\partial\varepsilon}\right\}\\
&+I_1+I_2^{ph}(f_1)+I_4^{ph}(f)=0,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
I^{imp}&=\frac{nv}{2}\int d\Omega_{p_1}\sigma_{pp_1}\mathrm{Sp}\{f_p-f_{p_1})\delta_p\delta_{p_1}\\
&+f_1(\mathbf{p})\delta_{p_1}\alpha_p-f_1(\mathbf{p}_1)\delta_p\alpha_{p_1}\}.\\
I_i^{imp}&=\frac{nv}{2}\int d\Omega_{p_1}\sigma_{pp_1}\mathrm{Sp}\{(f_p-f_{p_1})\tau_z\alpha_p\tau_z\delta_{p_1}\\
&+(f_1(\mathbf{p})-f_1(\mathbf{p}_1))\tau_z\alpha_{p_1}\tau_z\alpha_p\}.\\
I_l^{ph}&=\frac{ivg^2}{8}\int d\Omega_{p_1}\int\frac{d\varepsilon_1}{2\pi}J_l^{ph},\quad l=1.2.3.4:\\
J_1^{ph}(f)&=\mathrm{Sp}\{\delta_p(\varepsilon)\delta_{p_1}(\varepsilon_1)[D_{p-p_1}(\varepsilon_1-\varepsilon)(f_p(\varepsilon)-f_{p_1}(\varepsilon_1))\\
&+(D^R_{p-p_1}(\varepsilon_1-\varepsilon)-D^A_{p-p_1}(\varepsilon_1-\varepsilon))(1-f_p(\varepsilon)f_{p_1}(\varepsilon_1))]\}.\\
J_2^{ph}(f_1)&=\mathrm{Sp}\{\alpha_p(\varepsilon)\tau_z\alpha_{p_1}(\varepsilon_1)\tau_zD_{p-p_1}(\varepsilon_1-\varepsilon)(f_1(\mathbf{p},\varepsilon)-f_1(\mathbf{p}_1,\varepsilon_1))\\
&-(D^R_{p-p_1}(\varepsilon_1-\varepsilon)-D_{p-p_1}(\varepsilon_1-\varepsilon))\alpha_p(\varepsilon)[\alpha_{p_1}(\varepsilon_1)f_p(\varepsilon)f_1(\mathbf{p}_1,\varepsilon_1)\\
&+{}^1/_2(\alpha_{p_1}(\varepsilon_1)+\tau_z\alpha_{p_1}(\varepsilon_1)\tau_z)f_{p_1}(\varepsilon_1)f_1(\mathbf{p},\varepsilon)]\}.\\
J_3^{ph}(f_1)&=\mathrm{Sp}\{D_{p-p_1}(\varepsilon_1-\varepsilon)(f_1(\mathbf{p},\varepsilon)\delta_{p_1}(\varepsilon_1)\alpha_p(\varepsilon)\\
&-f_1(\mathbf{p}_1,\varepsilon_1)\delta_p(\varepsilon)\alpha_{p_1}(\varepsilon_1))-(D^R_{p-p_1}(\varepsilon_1-\varepsilon)-D_{p-p_1}(\varepsilon_1-\varepsilon))\\
&\times(f_1(\mathbf{p},\varepsilon)f_{p_1}(\varepsilon_1)\delta_{p_1}(\varepsilon_1)\alpha_p(\varepsilon)+f_1(\mathbf{p}_1,\varepsilon_1)f_p(\varepsilon)\delta_p(\varepsilon)\alpha_{p_1}(\varepsilon_1))\}.\\
J_4^{ph}(f)&=\mathrm{Sp}\{\tau_z\alpha_p(\varepsilon)\tau_z\delta_{p_1}(\varepsilon_1)\cdot[D_{p-p_1}(\varepsilon_1-\varepsilon)(f_p(\varepsilon)-f_{p_1}(\varepsilon_1))\\
&+(D^R_{p-p_1}(\varepsilon_1-\varepsilon)-D^A_{p-p_1}(\varepsilon_1-\varepsilon))(1-f_p(\varepsilon)f_{p_1}(\varepsilon_1))]\}.
\end{aligned} \tag{14}$$

The function f_1 is usually small; therefore the second equation of (13) is written in the linear approximation (terms proportional to Ef_1 are omitted). In the derivation of the collision integrals I_i^{ph} from $\hat\Sigma^{ph}$, only terms proportional to D or $D^R - D^A$ are kept; these describe real processes with absorption or emission of phonons. The exchange of virtual phonons, which is described by components of the type $D^R + D^A$, leads to renormalization of the order parameter Δ.

If the order parameter Δ changes little over the distance which the electron traverses in a time of the order of Δ^{-1}, then we can use the following local approximation for the Green's function $G^{R,A}$:

$$\begin{aligned}
G^R&=\frac{1}{((\varepsilon+i\delta)^2-|\Delta|^2)^{1/2}}\begin{pmatrix}\varepsilon, & \Delta\\ -\Delta^*, & -\varepsilon\end{pmatrix},\\
G^A&=-\frac{1}{((\varepsilon-i\delta)^2-|\Delta|^2)^{1/2}}\begin{pmatrix}\varepsilon, & \Delta\\ -\Delta^*, & -\Delta\end{pmatrix}.
\end{aligned} \tag{15}$$

In this approximation, the quantity $2\gamma = G^R + G^A$ is equal to zero for $|\varepsilon| > |\Delta|$. This quantity enters into the equation for f_1; therefore, in the case of small gradients, when the phonon collision term is significant in the equation for f_1, the quantity $G^R + G^A$ must be described with account of the phonon collision term

$$\begin{aligned}
&\varepsilon(G^R\tau_z-\tau_zG^R)-\hat\Delta\hat G^R+G^R\hat\Delta+\Sigma^{ph(R)}G^R-G^R\Sigma^{ph(R)}=0,\\
&\Sigma^{ph(R)}=\frac{\nu g^2}{8}\int d\Omega_{p_1}\int\frac{d\varepsilon_1}{2\pi}\{D_{p-p_1}(\varepsilon_1-\varepsilon)G_{p_1}{}^R(\varepsilon_1)\\
&+D^A_{p-p_1}(\varepsilon_1-\varepsilon)G_{p_1}(\varepsilon_1)\}.
\end{aligned} \tag{16}$$

Near the transition temperature, account of the phonon collision term leads to the replacement of ε by $\varepsilon \pm i/2\tau_\varepsilon$ in formula (15) for $G^{R,A}$;

$$\tau_\varepsilon^{-1}=\frac{\pi\nu g^2}{2(sp)^2}\int_0^\infty\frac{d\varepsilon\,\varepsilon^2}{\mathrm{ch}(\varepsilon/2T)\,\mathrm{sh}(\varepsilon/2T)}=\frac{7\zeta(3)\pi\nu g^2T^3}{2(sp)^2}, \tag{17}$$

where s is the sound velocity in the normal metal, and $\zeta(3)$ is the Riemann zeta function.

In the case of small gradients and currents, the Green's functions and, consequently, the kinetic equation, have the same form at any concentration of the nonmagnetic impurities. The set of kinetic equations (13) transforms in this case into the equations obtained by Galaĭko and Shumeĭko.[6] At low concentrations of nonmagnetic impurities, when $\hat\Sigma^{imp}$ can be taken into account in the equations for $G^{R,A}$ by perturbation theory, the set of equations (13) transforms into the ordinary kinetic equation for the quasiparticles[7] at arbitrary current, but the gradient of the modulus of $|\Delta|$ in this case should be small.

The collision term I^{ph}, which describes the energy relaxation, is small (of the order T^3/Θ_D^2), however, account of I^{ph} is important, since without it there is degeneracy: in the static case, the arbitrary function $f(\varepsilon)$, which does not depend on the coordinates, satisfies the set of equations (13). The shape of the function $f(\varepsilon)$ is determined from the condition that I^{ph} vanish:

$$f_1=0,\quad f(\varepsilon)=\mathrm{th}\,(\varepsilon/2T).$$

Since the dependence of the function $f(\varepsilon)$ on the energy is determined by the term I^{ph}, nonlinear effects appear in weak alternating fields.

We represent the distribution function f in the form of a sum of two components: a large component $\langle\bar f\rangle$, which does not depend on the angles of the vector $\mathbf{p}$ and is weakly dependent on the coordinate, and a small component $\tilde f$, which does not have mean value over the angles or the coordinates. The functions f_1 and $\tilde f$ can be found from the set of equations (13) in the approximation

linear in the field. The equation for $\langle \bar{f} \rangle$, which is nonlinear in the field variable, is obtained by simple averaging of the first equation of (13) over the angles of the vector **p**. The order parameter Δ, the current density **j**, and the charge density ρ are expressed in terms of the Green's function G according to the formulas[2]

$$\mathbf{j}(t) = -\frac{ep}{4\pi}\int \frac{d\Omega_{\mathbf{p}}}{4\pi} \mathrm{Sp}\, \tau_3 \mathbf{p} G_{\mathbf{p}}(t,t),$$
$$\rho(t) = -ev\left\{2e\varphi(t) + \frac{\pi}{2}\int \frac{d\Omega_{\mathbf{p}}}{4\pi} \mathrm{Sp}\, G_{\mathbf{p}}(t,t)\right\}. \qquad (18)$$

The Green's function G can be expressed with the help of Eq. (10) in terms of the distribution functions f and f_1. The formulas (18), together with the kinetic equations (13) and the Maxwell equations, allow us to consider the problem of superconductivity in a low-frequency electromagnetic field of arbitrary amplitude.

2. CONDUCTIVITY OF A PURE SUPERCONDUCTOR IN THE MIXED STATE

In an electric field **E**, the vortex structure moves with velocity **u**, which is connected with the electric field by the relation

$$\mathbf{u} = [\mathbf{E} \times \mathbf{B}] B^{-2}.$$

Carrying out a gauge transformation with phase χ defined by the equation

$$\partial\chi/\partial t = 2e\mathbf{u}\mathbf{A}\left(\mathbf{r} - \int \mathbf{u}\, dt\right), \qquad (19)$$

we obtain the result that the order parameter Δ and the vector potential **A** in such a situation change according to the law

$$\partial\Delta_1/\partial t = -\mathbf{u}\partial_-\Delta, \quad \partial\Delta_2/\partial t = -\mathbf{u}\partial_+\Delta_2, \quad \partial\mathbf{A}/\partial t = [\mathbf{u} \times \mathbf{H}]. \qquad (20)$$

A transport current flows over the superconductor in such motion. For the determination of the value of this current, we use the method described in Ref. 8.

The motion of the vortex lattice leads to the appearance of corrections to the order parameter and the vector potential, which can be found from Eqs. (18) and the Maxwell equations for the vector potential. In first order in the velocity, these equations have the form[8]

$$\hat{L}\begin{pmatrix}\Delta_1^{(1)}\\ \Delta_2^{(1)}\\ \mathbf{A}^{(1)}\end{pmatrix} = \frac{mp}{4\pi}\int \frac{d\Omega_{\mathbf{p}}}{4\pi}\int \frac{d\varepsilon}{2\pi}\begin{pmatrix} F_1^{(1)}\\ F_2^{(1)}\\ -ev\,(g_1^{(1)} - g_2^{(1)})\end{pmatrix} - \begin{pmatrix}0\\0\\ \langle \mathbf{j} \rangle\end{pmatrix}, \qquad (21)$$

where the corrections to the Green's functions $F_{1,2}^{(1)}$ and $g_{1,2}^{(1)}$ are determined by the formulas (10) and (13). Here the time derivatives are determined by the formulas (20). The mean current can be separated in the right side of Eq. (21), in order that the correction to the vector potential $\mathbf{A}^{(1)}$ not contain terms that increase with the coordinates. The mean current $\langle \mathbf{j} \rangle$ is assumed to be small in order that the change in the magnetic field produced by it be small over a distance of the order of the size of the cell. The operator L is found from the static equations and is equal to the second variational derivative of the free energy with respect to Δ and **A**.[8] In the case of lattice displacements and gauge transformations, the free energy does not change; therefore the result of the action of the operator $\hat{L}$ on the column $(\mathbf{e}\cdot\partial_-\Delta, \mathbf{e}\cdot\partial_+\Delta^*, [\mathbf{H}\times\mathbf{e}])$ is equal to zero. Here **e** is the displacement vector of the lattice. We multiply Eq. (21) from the left by the row $([\mathbf{B}\times\partial_+\Delta^*], [\mathbf{B}\times\partial_-\Delta], \mathbf{B}\cdot\mathbf{H})$ and average over the volume of the cell. As a result, the left side of Eq. (21) vanishes and we obtain the following expression for the transport current $\langle \mathbf{j} \rangle$:

$$\mathbf{B}^2\langle j \rangle = -\frac{mp}{4\pi}\int \frac{d\varepsilon}{2\pi}\int \frac{d\Omega_{\mathbf{p}}}{4\pi}$$
$$\times\left\langle \mathrm{Sp}\begin{pmatrix} ev(\mathbf{BH}), & [\mathbf{B}\times\partial_-\Delta]\\ -[\mathbf{B}\times\partial_+\Delta^*], & -ev(\mathbf{BH})\end{pmatrix} G^{(1)}\right\rangle. \qquad (22)$$

The angular brackets in (22) denote averaging over the cell.

We now consider superconductors with a large mean free path $\xi \ll l \ll \xi\varepsilon_F/\Delta$ for the electrons. We find the contribution to the conductivity, which is proportional to the free path length. Such a contribution comes only from corrections to the distribution function f. The corrections to the Green's functions $G^{R,A}$ and the distribution function f_1 remain finite in the limit as $l \to \infty$ and therefore will not be taken into account. As observed above, it is convenient to represent the distribution function f in the form of a sum of two components:

$$f = \langle f \rangle + \tilde{f}, \qquad (23)$$

where $\langle f \rangle$ does not depend on the angles of the vector **p** and on the coordinates. In the calculation of the conductivity in the approximation that is linear in the field, one must set $\langle f \rangle = \tanh(\varepsilon/2T)$. The set of equations (13) for the functions f and f_1 we shall solve by the classical trajectory method. Since $f_1 \ll \tilde{f}$, it follows from the second equation of the set (13) that

$$\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\tilde{f} = 0 \qquad (24)$$

and the function $\tilde{f}$ is constant on the trajectory. Integrating the first equation of the set (13) along the trajectory, we obtain

$$i\frac{\partial\langle f \rangle}{\partial\varepsilon}\int_{-\infty}^{\infty} \mathrm{Sp}\frac{\partial\hat{H}}{\partial t}\delta dl = nv\int d\Omega_{\mathbf{p}_1}\sigma_{\mathbf{pp}_1}(\tilde{f}_{\mathbf{p}} - \tilde{f}_{\mathbf{p}_1})\int_{-\infty}^{\infty}\mathrm{Sp}\delta_{\mathbf{p}}\delta_{\mathbf{p}_1}dl. \qquad (25)$$

Only a single component, proportional to $\tilde{f}$, remains in Eq. (22) for the mean current density in this approximation:

$$\mathbf{j} = -\frac{mp}{2\pi B^2}\int \frac{d\Omega_{\mathbf{p}}}{4\pi}\int \frac{d\varepsilon}{2\pi}$$
$$\times\langle \mathrm{Sp}\begin{pmatrix} ev(\mathbf{BH}), & [\mathbf{B}\times\partial_-\Delta]\\ -[\mathbf{B}\times\partial_+\Delta^*], & -ev(\mathbf{BH})\end{pmatrix}\tilde{f}\delta\rangle. \qquad (26)$$

The expressions (25) and (26) allow us to find the conductivity of pure superconductors at any temperature and in any magnetic field if the solutions of the static problem for the functions $G^{R,A}$ and Δ are known. In weak fields $H \ll H_{c2}$ (the distance between the vortices is much greater than ξ) at low temperatures, the conductivity in

the approximation that is linear in the field was found in Ref. 9:

$$\sigma=\frac{empr\tau\Delta^2}{2\pi^2B}\ln\frac{\Delta}{T},\quad T\ll T_c. \tag{27}$$

The formula (27) is valid in order of magnitude even at $T\sim T_c$. Near T_c, we can find the temperature dependence of the conductivity if we take it into account that in Eqs. (25) and (26) the values $\varepsilon\sim\Delta$ are significant and the dimension $\xi\sim v/\Delta$ is important. With account of the observations made, we obtain for the conductivity the expression

$$\sigma=\sigma_0+C_0\frac{empr\tau\Delta^3}{\pi^3TB}, \tag{28}$$

where σ_0 is the conductivity of the normal metal and C_0 is a number of order unity. Equation (28) is applicable at temperatures that are not too close to T_c, while the free path length of the electrons is greater than the size of the vortex ($\tau\Delta\gg1$). Near the critical field H_{c2}, the conductivity is found in Ref. 10:

$$\frac{\sigma}{\sigma_0}=1+\frac{2\langle|\Delta|^2\rangle}{eHv^2}\left[\ln\left(\frac{(2\gamma eHv^2)^{1/2}}{\pi T}\right)+\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+\frac{1}{4\pi T\tau}\right)\right],\quad T\ll T_c,$$

$$\frac{\sigma}{\sigma_0}=1+\frac{\pi^{5/2}(1+\sqrt{2})\langle|\Delta|^2\rangle}{32T(eHv^2)^{1/2}},\quad T_c-T\ll T_c, \tag{29}$$

$\ln\gamma=0.577$ is the Euler constant.

3. DEPENDENCE OF THE CONDUCTIVITY ON THE ELECTRIC FIELD

At low temperatures, the dependence of $\langle f\rangle$ on ε is determined not by the temperature but by the electric field in comparatively weak electric fields. The equation for $\langle f\rangle$ is obtained by averaging the first of Eqs. (13) over the coordinates and angles:

$$\left\langle -i\frac{\partial}{\partial\varepsilon}\mathrm{Sp}\left(\frac{\partial\hat H}{\partial t}\hat f\hat\delta\right)\right\rangle=-2I_1^{ph}. \tag{30}$$

In the derivation of Eq. (30), the equation (12) was employed. With logarithmic accuracy, Eq. (25) for the distribution function f can be solved at $\varepsilon\ll\Delta$. This solution has the form

$$f=\frac{4\tau|\Delta|}{\pi v}\frac{\partial\langle f\rangle}{\partial\varepsilon}\frac{(\mathbf{u}[\mathbf{B}\times\mathbf{p}])}{|[\mathbf{B}\times\mathbf{p}]|}\int_0^{v/\Delta}\frac{d\rho}{\rho}\Delta(\rho). \tag{31}$$

The Green's function $G^{R,A}$ entering into Eq. (30) should be replaced by its value in a static magnetic field, found in Ref. 11. As a result, Eq. (30) reduces to the form

$$\frac{\partial}{\partial\varepsilon}\left\{\left(\ln\frac{\Delta_0}{|\varepsilon|}\right)^{-1}\frac{\partial\langle f\rangle}{\partial\varepsilon}\right\}=\gamma\int_{-\infty}^{\infty}d\varepsilon_1(\varepsilon_1-\varepsilon)^2\,\mathrm{sign}(\varepsilon_1-\varepsilon)$$
$$\times\left\{\left(\ln\frac{\Delta_0}{|\varepsilon|}\right)^{-1}+\left(\ln\frac{\Delta_0}{|\varepsilon_1|}\right)^{-1}\right\}\left\{(\langle f(\varepsilon)\rangle-\langle f(\varepsilon_1)\rangle)\,\mathrm{cth}\frac{\varepsilon_1-\varepsilon}{2T}\right.$$
$$\left.+1-\langle f(\varepsilon)\rangle\langle f(\varepsilon_1)\rangle\right\}, \tag{32}$$

where

$$\gamma=\frac{\pi mpg^2v^2\ln(\tau\Delta_0)}{512\Delta^3\tau^2u^2(sp)^2}\left(\int_0^{v/\Delta}\frac{d\rho}{\rho}\Delta(\rho)\right)^{-2}. \tag{33}$$

At sufficiently large vortex velocities u, the values $\varepsilon\gg T$ are important. Here, Eq. (32) is nondimensionalized and its solution is a function of the dimensioness parameter $\varepsilon\gamma^{1/5}$, in particular, when this parameter is much greater than unity, we have

$$1-\langle f\rangle=A\exp\left[-(\varepsilon/T^*)^{5/4}\right],$$

where

$$T^*=(16\gamma/75)^{-1/5}\sim\Delta[T_c\tau_\varepsilon(T_c)(j/j_c)^2]^{1/5},$$

j_c is the critical pair-breaking current, and A is a constant of order unity.

At low temperatures, the size of the vortex is proportional to T.[11] This is connected with the fact that the distribution function at small ε depends on the ratio ε/T. In a strong electric field, when $T^*\gg T$, the size of the vortex increases and becomes proportional to T^*. In similar fashion, T should be replaced by T^* under the logarithm in the conductivity (Eq. (27)) in the presence of a strong electric field. Thus, over a broad range of electric fields $\Delta>T^*>T$, τ^{-1}, the conductivity depends logarithmically on the value of the electric field.

4. EMISSION OF MONOCHROMATIC PHONONS IN VORTEX MOTION

A nonequilibrium energy distribution of the electrons is produced by vortex motion and causes emission of nonequilibrium phonons. Although the effective temperature of the electrons $T^*\ll\Delta$, a broad spectrum of phonons with width $\omega\sim T^*$ is radiated. An interesting phenomenon appears when the effective temperature T^* becomes of the order of the gap width Δ. In this case, the electrons, becoming heated inside the vortex, reach the energy $\varepsilon=\Delta$. After this, they leave the region of the vortex and if the vortex density is low, the subsequent heating is greatly weakened. Accumulation of excitations with energy ε near Δ takes place, and these excitations recombine with emission of a phonon of frequency close to 2Δ. The width and intensity of the radiation can be found from Eq. (30) for the distribution function. The left side of this equation is the diffusion equation. The diffusion coefficient for the energy is determined by the behavior of a order parameter Δ and the function $G^{R,A}$ near the vortex axis at distances of the order of v/Δ. Therefore, it depends weakly on the energy and in order of magnitude can be replaced by its value at $\varepsilon\ll\Delta$:

$$D_\varepsilon\approx2\tau u^2\Delta^2eB.$$

At sufficiently low vortex density, the basic relaxation process in Eq. (30) at $\varepsilon>\Delta$ is the recombination of the excitations with emission of a phonon with energy close to 2Δ. Equation (30) for $\varepsilon>\Delta$ in this case reduces to the form

$$\frac{\partial}{\partial\varepsilon}\left(D_\varepsilon\frac{\partial f}{\partial\varepsilon}\right)=-\frac{2\Delta^3mpg^2}{\pi(sp)^2(\varepsilon-\Delta)^{1/2}}(1-f_\varepsilon)\int_\Delta^\infty\frac{d\varepsilon_1}{(\varepsilon_1-\Delta)^{1/2}}(1-f_{\varepsilon_1}). \tag{34}$$

The solution of Eq. (34) is the Bessel function

$$1-f=Ay^{1/3}K_{1/3}(y), \tag{35}$$

where

$$y^{1/3}=\frac{\varepsilon-\Delta}{\mu}, \quad \mu^{-2}=\frac{128\Gamma(^1/_3)\Delta^3 mpg^2}{27\pi 2^{2/3}(sp)^2 D_\varepsilon}A, \tag{36}$$
$$A=(1-f(\Delta))\Gamma(^7/_6)2^{1/3}/\pi^{1/6}\Gamma(^1/_3).$$

The boundary value of the distribution function $f(\Delta)$ is determined by the diffusion of the excitations inside the vortex. If $T^* \gg \Delta$, then $f(\Delta) \ll 1$. If $T^* \ll \Delta$, then $1-f(\Delta)$ is exponentially small. At $T^* \sim \Delta$, the order parameter inside the vortex changes greatly but the value of Δ at large distances changes very little.

The frequency distribution of phonons takes near $\omega = 2\Delta$ the form

$$\frac{\partial N_\omega}{\partial t}=\frac{2\Delta^3 mp}{\pi(sp)^2}\frac{mpg^2}{2\pi^2}\iint\limits_{\Delta}^{\infty}\frac{d\varepsilon d\varepsilon_1}{(\varepsilon-\Delta)^{1/2}(\varepsilon_1-\Delta)^{1/2}}\delta(\varepsilon+\varepsilon_1-\omega)(1-f_\varepsilon)(1-f_{\varepsilon_1}). \tag{37}$$

The characteristic width of this distribution is of the order of μ (Eq. (36)). At $T^* \sim \Delta$, this width is

$$\mu^2 \sim eBv^2/\tau\Delta \sim \Delta^2(\tau\Delta)^{-1}B/H_{c2}. \tag{38}$$

Thus, at low temperatures and infrequent vortices, all the energy of the electric field is transformed into the energy of almost monochromatic phonons with frequency $\omega = 2\Delta$.

5. SUPERCONDUCTORS WITH SHORT ELECTRON FREE PATH LENGTH

In superconductors with a short free path length, the kinetic equation for the distribution function transforms in the diffusion equation into a function that is independent of the angles of the vector **p**. This equation is easiest to obtain by starting from the general equation (6) the Green's function $\hat{G}$. In this case, the function $\hat{G}$ can be represented in the form

$$\hat{G}_{\mathbf{p}}=\hat{G}+\mathbf{p}\hat{\mathbf{G}}_1, \quad \mathbf{p}\hat{\mathbf{G}}_1 \ll \hat{G}, \tag{39}$$

where the functions $\hat{G}$ and $\hat{\mathbf{G}}_1$ do not depend on the angles of the vector **p**. Using the normalization condition (8), we get the following expression for the function $\hat{\mathbf{G}}_1$

$$\hat{\mathbf{G}}_1=-\frac{\tau_{tr}}{m}\{ie\mathbf{A}\hat{\tau}_z\delta(t-t')+\hat{G}\partial\hat{G}\}, \tag{40}$$

where

$$\partial=\partial/\partial\mathbf{r}-ie\mathbf{A}\hat{\tau}_z, \quad \hat{G}\hat{G}\equiv\int dt_1\hat{G}(t,t_1)\hat{G}(t_1,t').$$

We substitute Eq. (39) for the function $\hat{G}_{\mathbf{p}}$ in Eq. (6) and average over the angles of the vector **p**:

$$-D\frac{\partial}{\partial\mathbf{r}}\{ie\mathbf{A}\hat{\tau}_z\delta(t-t')+\hat{G}\partial\hat{G}\}+\hat{\tau}_z\frac{\partial\hat{G}}{\partial t}+\frac{\partial\hat{G}}{\partial t'}\hat{\tau}_z$$
$$+(-i\check{\Delta}(t)+ie\varphi(t))\hat{G}-\hat{G}(-i\check{\Delta}(t')+ie\varphi(t')) \tag{41}$$
$$+ieD\{\mathbf{A}(t)\hat{\tau}_z\hat{G}\partial\hat{G}-(\hat{G}\partial\hat{G})\mathbf{A}(t')\hat{\tau}_z\}=-i(\hat{\Sigma}^{ph}\hat{G}-\hat{G}\hat{\Sigma}^{ph}),$$

$D=vl_{tr}/3$ is the diffusion coefficient. The equations determining $G^{R,A}$ and G are contained in Eq. (41).

We express the Green's function G as before, with the aid of two distribution functions, in the form

$$G=\int dt_1\{G^R\hat{f}-\hat{f}G^A\}, \tag{42}$$
$$\hat{f}=f+\tau_z f_1. \tag{43}$$

In a slowly varying electromagnetic field, the equations for the distribution functions $f(\varepsilon, t)$ and $f_1(\varepsilon, t)$ have the form

$$-D\frac{\partial}{\partial\mathbf{r}}\mathrm{Sp}\left\{\frac{\partial f}{\partial\mathbf{r}}(1-G^RG^A)\right\}-D\frac{\partial}{\partial\mathbf{r}}(f_1\mathbf{j}_\varepsilon)+2\frac{\partial f}{\partial t}\mathrm{Sp}\,\alpha$$
$$+\frac{\partial f}{\partial\varepsilon}\left\{eD\frac{\partial\mathbf{A}}{\partial t}\mathbf{j}_\varepsilon-2\,\mathrm{Sp}\frac{\partial\hat{\Delta}}{\partial t}\delta\right\}+4I_1^{ph}(f)=0,$$
$$-D\frac{\partial}{\partial\mathbf{r}}\mathrm{Sp}\left\{\frac{\partial f_1}{\partial\mathbf{r}}(1-\tau_zG^R\tau_zG^A)\right\}-D\frac{\partial f}{\partial\mathbf{r}}\mathbf{j}_\varepsilon+2\frac{\partial}{\partial t}(f_1\,\mathrm{Sp}\,\alpha) \tag{44}$$
$$-4if_1\,\mathrm{Sp}(\gamma\hat{\Delta})+2\frac{\partial f}{\partial\varepsilon}\mathrm{Sp}\left\{e\frac{\partial\varphi}{\partial t}\alpha-\frac{\partial\hat{\Delta}}{\partial t}\tau_z\gamma+\frac{i}{2}\tau_z\frac{\partial^2\hat{\Delta}}{\partial t^2}\frac{\partial\delta}{\partial\varepsilon}\right\}+4I_2^{ph}(f_1)=0,$$

where $\mathbf{j}_\varepsilon=\mathrm{Sp}\,\tau_\varepsilon(G^R\partial G^R-G^A\partial G^A)$.

The coefficients of Eq. (44) are written in the approximation linear in the electric field. In some cases, this approximation is sufficient for the solution of nonlinear problems. In problems where the linear response is sought, Eqs. (44) transform into the equations of Schmid and Schön.[12] At temperatures close to T_c, the size of the vortex is large $\xi^2 \gg D/\Delta$ and we can use the local approximation (15) for the Green's functions $G^{R,A}$. In the region $|\varepsilon| > |\Delta|$ the function $f_1 = 0$ and the equation for the distribution function f takes the form

$$-D\frac{\partial^2 f}{\partial\mathbf{r}^2}+\frac{\partial f}{\partial t}\frac{\partial(\varepsilon^2-|\Delta|^2)^{1/2}}{\partial\varepsilon}-\frac{\partial f}{\partial\varepsilon}\frac{\partial(\varepsilon^2-|\Delta|^2)^{1/2}}{\partial t}=-I_1^{ph}(f). \tag{45}$$

For the distribution function f in Eq. (45), it is necessary to derive the boundary condition on the boundary of the sample, where $|\varepsilon| = |\Delta|$. Integrating Eq. (44), we find that, in a narrow region near the boundary, the quantity

$$\frac{\partial f}{\partial\mathbf{r}}\mathrm{Sp}(1-G^RG^A)=4\frac{\partial f}{\partial\mathbf{r}}\theta(|\varepsilon|-|\Delta|)=\mathrm{const} \tag{46}$$

is conserved. Under the barrier, Eq. (46) vanishes. Consequently,

$$\left.\frac{\partial f}{\partial\mathbf{n}}\right|_{\mathrm{rp}}=0. \tag{47}$$

In the collision integration I^{ph} the component corresponding to the departure of the particles from the state with energy ε depends on $f(\varepsilon)$ and in the components corresponding to the arrival of particles, values $\varepsilon_1 \sim T$ are important. The distribution function $f(\varepsilon)$ differs from the equilibrium function in the energy range $\varepsilon \sim \Delta$. Near the transition temperature, when $\Delta \ll T$, the distribution function $f(\varepsilon_1)$ in the collision integral I^{ph} can be replaced by its equilibrium value. In this case,

$$I_1^{ph}(f)=\tau_\varepsilon^{-1}\varepsilon(\varepsilon^2-|\Delta|^2)^{-1/2}(f(\varepsilon)-\mathrm{th}\,(\varepsilon/2T)), \quad |\varepsilon|>|\Delta|,$$

where τ_ε is determined by the formula (17).

In superconductors with a small free path length, the expression for the transport current (22) takes the form

$$B^2\langle j\rangle = -\frac{mp}{4\pi}\int\frac{d\varepsilon}{2\pi}$$
$$\times\left\langle \mathrm{Sp}\begin{pmatrix} 0, & [B\times\partial_-\Delta] \\ -[B\times\partial_+\Delta^*], & 0\end{pmatrix} G - eD\,\mathrm{Sp}\,\tau_z(G^R\partial G + G\partial G^A)\right\rangle. \quad (48)$$

At temperatures close to the transition temperature, the expression (48) for the transport current is considerably simplified. In this case, we can use the expressions (15) for the Green's functions $G^{R,A}$. These are valid in magnetic fields not too close to the critical field H_{c2}. As a result, we obtain

$$\langle j\rangle = \frac{mp}{4\pi B^2}\left\langle \int\limits_{|\varepsilon|>|\Delta|}\frac{d\varepsilon}{2\pi}(\varepsilon^2-|\Delta|^2)^{-1/2} f\left[B\times\frac{\partial|\Delta|^2}{\partial \mathbf{r}}\right]\right\rangle. \quad (49)$$

In the approximation linear in the electric field, we can neglect the collision term I^{ph} in Eq. (45) at temperatures not too close to T_c, when $D\xi^{-2}\gg\tau_\varepsilon^{-1}$. For a single vortex, Eq. (45) is conveniently written in cylindrical coordinates (ρ, φ):

$$D\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\tilde f)\right) = \frac{\partial\,\mathrm{th}(\varepsilon/2T)}{\partial\varepsilon}u\frac{\partial(\varepsilon^2-\Delta^2)^{1/2}}{\partial\rho},$$
$$f = \mathrm{th}\frac{\varepsilon}{2T} + \tilde f(\rho)\cos\varphi. \quad (50)$$

The solution of this equation is of the form

$$f - \mathrm{th}\frac{\varepsilon}{2T} = \cos\varphi\,\frac{u}{D}\frac{\partial\,\mathrm{th}(\varepsilon/2T)}{\partial\varepsilon}\frac{1}{2}\int\limits_\rho^\infty d\rho_1\rho_1[(\varepsilon^2-|\Delta|^2)^{1/2}+C]. \quad (51)$$

where the coefficient C is found from the condition of the vanishing of the derivative of the distribution function at the point at which $|\Delta| = |\varepsilon|$ in the case $|\varepsilon| < \Delta_\infty$, or on the boundary of the cell at $|\varepsilon| > \Delta_\infty$:

$$C = \begin{cases} \dfrac{2}{\rho_\varepsilon^2}\displaystyle\int\limits_0^{\rho_\varepsilon} d\rho\,\rho(\varepsilon^2-\Delta^2)^{1/2}, & |\varepsilon|<\Delta_\infty \\ -(\varepsilon^2-\Delta_\infty^2)^{1/2}, & |\varepsilon|>\Delta_\infty \end{cases}, \quad (52)$$

where ρ_ε is found from the equation $|\varepsilon| = |\Delta(\rho_\varepsilon)|$.

We substitute Eq. (51) for the distribution function in (48) and obtain

$$\sigma/\sigma_0 = \beta(T)H_{c2}(T)/B; \quad (53)$$
$$\beta(T) = \frac{\pi^3(1-T/T_c)^{-1/2}}{(14\zeta(3))^{1/2}}\left\{\int\limits_1^\infty d\varepsilon\int\limits_0^\infty dx\,x[(\varepsilon^2-\bar\Delta^2)^{1/2}-(\varepsilon^2-1)^{1/2}]^2\right.$$
$$\left.+\int\limits_0^1 d\varepsilon\left[\int\limits_0^{x_\varepsilon} dx\,x(\varepsilon^2-\bar\Delta^2)+\frac{2}{x_\varepsilon^2}\left(\int\limits_0^{x_\varepsilon} dx\,x(\varepsilon^2-\bar\Delta^2)^{1/2}\right)^2\right]\right\} = 4.1(1-T/T_c)^{-1/2};$$
$$\bar\Delta(x) = \Delta(x\xi)/\Delta_\infty, \quad \xi^2 = \pi D/8(T_c-T).$$

In a number of researches[13,2,14] the incorrect boundary condition $f_{boun}=0$ is used. Such a boundary condition gives the value 2.1 for the numerical coefficient in Eq. (53). In Refs. 13 and 14, the values 1.1 and 2.85 were respectively obtained for this coefficient.

The increase of the coefficient $\beta(T)$ as the transition temperature is approached is due to the strong change in the distribution function of the excitations. This growth continues as long as the size of the vortex is not too great and the time of diffusion the excitations over the vortex core is less than the energy relaxation time. At $T_c - T \sim \tau_\varepsilon^{-1}$, these times become equal. Upon further approach to T_c the departure of the distribution function from the equilibrium value decreases. In this case, the first term in Eq. (45) is small in comparison with I^{ph} and the distribution function is of the form

$$f = \mathrm{th}\frac{\varepsilon}{2T} + \frac{\tau_\varepsilon}{2\varepsilon}\frac{\partial\,\mathrm{th}(\varepsilon/2T)}{\partial\varepsilon}\left(\mathbf{u}\frac{\partial|\Delta|^2}{\partial\mathbf{r}}\right). \quad (54)$$

Substituting this value for the distribution function in Eq. (49), we obtain the following expression for the conductivity σ:

$$\sigma = \frac{mp\tau_\varepsilon}{8\pi TB^2}\left\langle|\Delta|\left(\frac{\partial|\Delta|}{\partial\mathbf{r}}\right)^2\right\rangle. \quad (55)$$

For a single vortex we have

$$\frac{\sigma}{\sigma_0} = \frac{H_{c2}(T)}{B}\frac{\pi^2/2}{(7\zeta(3))^{1/2}}\left(1-\frac{T}{T_c}\right)^{1/2}\int\limits_0^\infty dx\,x\left(\frac{\partial\bar\Delta}{\partial x}\right)^2$$
$$= 2.7\frac{H_{c2}}{B}\tau_\varepsilon T_c\left(1-\frac{T}{T_c}\right)^{1/2}. \quad (56)$$

As the transition temperature is approached, the "anomlous" contribution to the conductivity found above, a contribution that is connected with the change in the excitation distribution function, decreases. In the region $\tau_\varepsilon \sim \Delta^{-1}$ the contribution to the conductivity connected with the change in the Green's function $G^{R,A}$ becomes important.

CONCLUSION

Because of the large energy relaxation time, nonlinear effects appear in comparatively weak electric fields, when the transport current is still much smaller than the critical pair-breaking current. A strong change in the excitation distribution function, which accompanies vortex motion, leads not only to a nonlinear volt-ampere characteristic, but also to emission of nonequilibrium phonons. In particular, in magnetic fields $B \ll H_{c2}$, at low temperatures, almost all the energy of the electric field can be transformed into the energy of nearly monochromatic phonons with frequency $\omega = 2\Delta$. For a quantitative description of nonlinear effects at arbitrary temperatures and magnetic fields, it is necessary to solve the self-consistent problem: to compute the Green's functions $G^{R,A}$ of the static problem for a given order parameter $\Delta(\mathbf{r})$. These functions are substituted in the kinetic equations for f and f_1. The values found for the distribution function f and the Green's function $G^{R,A}$ are substituted in the equation that determines the order parameter Δ for self-consistency.

In different limiting cases, for example in those considered above, the dependence of the transport current on the temperature and the values of the electric and magnetic fields can be found analytically, apart for the numerical coefficients.

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Quasilocal vibrations produced in dilute $V_{1-x}Pt_x$ alloys under conditions of a strong restructuring of phonon spectrum of V

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Results are presented of the measurements of the spectra of inelastic incoherent scattering of neutrons by $V_{1-x}Pt_x$ alloys ($x = 5$ and 7 at.%). In the low-frequency regions of the spectra, quasilocal vibrations (QLV) are observed and are connected with introduction of the heavy impurity Pt atoms into the V. The energy position of the QLV is shifted in the region of higher values in comparison with those expected in isotopic-substitution model. Introduction of the Pt atoms leads also to a shift of the entire spectrum towards higher frequencies. The shift of the QLV and of the entire spectrum depends on the concentration of the impurity atoms. The energy position of the QLV can be satisfactorily explained within the framework of isotopic substitution using a renormalized spectrum of the original lattice.

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INTRODUCTION

Of great significance in the understanding of the physical properties of real crystals, is the investigation of alloys with sufficiently small contents of impurity atoms, when the impurity atoms can be regarded as isolated. In this case the theoretical interpretation of the experimental data becomes greatly simplified.

Even the first investigations of a number of dilute alloys, carried out both with the aid of the neutron-scattering method and by measuring the low-temperature heat capacity, [1–3] have shown that in all cases new vibrational states are produced, due to the introduction of the impurity atoms. Depending on the value of the mass-difference parameter $\varepsilon = 1 - M_1/M_0$ (M_0 and M_1 are respectively the masses of the matrix and impurity atoms,) the introduction of impurity atoms leads to formation of either quasilocal vibrations (QLV) or local vibrations (LV). It turns out here that the energy position of the observed additional singularities in the spectra cannot be explained in most cases within the framework of the simplest model of isotopic substitution.

The introduction of a new parameter[4] τ determined by means of a best fit of the experimental results to the calculation data yields information concerning a certain effective change of the local force constants.

However, in the study of alloys of non-isoelectronic elements one observes not only the formation of impurity states, but also a noticeable deformation of the entire spectrum of the initial lattice, which manifests itself in a shift of the spectrum towards higher energies. The deformation is larger the greater the difference between the electronic properties of the alloyed elements and the higher the concentration of the impurity atoms. Thus, introduction of the parameter τ, which takes into account the change of the local force constants, does not explain the influence of the deformation of the entire spectrum on the formation of the QLV.

From among the vanadium-base alloys investigated by us, [5,6] the strongest restructuring of the initial spectrum should be expected for alloys with platinum impurity. In addition, in disordered solid solutions of V with Pt there is no noticeable change of the lattice

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