Supersymmetry method in localization theory[1)]

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The conductivity of electrons in a random potential is investigated. The Green's functions are written in the form of integrals over superfields whose components are classical boson and fermion fields. This makes it possible to carry out the averaging over the random potential from the very beginning, without using the replica technique. Through the use of the generalized Hubbard-Stratonovich transformation, the problem reduces to a nonlinear supertensor σ model. The supertensors are of rank 8×8 and contain both commuting and anticommuting elements. The magnetic and spin-orbit interactions lower the symmetry. Renormalizability is proved for the two-dimensional case, for which the Gell-Mann–Low function is calculated and makes it possible to determine the dependence of the conductivity on the frequency or on the size of the system.

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1. INTRODUCTION

When describing the behavior of a particle in a random potential field, it is important to take into account the quantum nature of the phenomenon. A sufficiently strong degree of disorder in a three-dimensional metal can lead to localization of all the states.[1] In one-dimensional and two-dimensional spaces, localization is possible at arbitrarily weak degree of disorder.[2-4] These phenomena are connected with repeated scattering of the particles by one and the same scattering center. Localization becomes easier with decreasing dimensionality because of the increase in the probability of return of the particle to the initial point.

The localization manifests itself most strongly in a one-dimensional disordered chain.[2,3] Analysis of perturbation theory for a chain shows that deviations from the two- and three-dimensional cases occur even in the first orders. For example, in the three-dimensional case, only non-intersecting diagrams are significant.[5] In the case of a one-dimensional chain all the diagrams are of the same order, which leads ultimately to localization.[3]

The analysis of the situation in the two-dimensional case and in thick wires is more complicated. Formally, in these systems, the contribution of individual intersecting diagrams is less than in non-intersecting ones. The summation of the specially chosen ladder diagrams, however, leads to the appearance of diffusion modes that are gapless at zero frequencies, and whose contribution in the one-dimensional and two-dimensional cases is divergent.[6] Large corrections to classical conductivity theory result from the existence of these modes and their interaction. To study the localization in low-dimensional and possibly also three-dimensional systems it is therefore very important to have a convenient description of the interaction of diffusion modes.

Direct calculation on the basis of standard perturbation theory is not convenient, since it is necessary to integrate in all the diagrams both over the diffusion and over the electron lines. A reasonable approach is one that permits integration over the electron lines prior to integration over the diffusion lines, so that the problem reduces to a study of the effective Lagrangian of the diffusion modes. Quite a number of methods of such a description have been proposed.[7-9] All these studies are based on the use of the replica technique, in which the initial system is replaced by n thermodynamic systems, after which the partition function is averaged over the random potential. To obtain the final result it is necessary to let n go to zero in the answer. A particularly fruitful idea was that of Wegner[10] concerning the violation of the symmetry between the replicas. This idea led subsequently to a rigorous derivation of the Lagrangians of the interacting modes.[11,12] Schäfer and Wegner[11] used the representation of the Green's functions with the aid of a continuum integral over Bose fields, while Efetov, Larkin, and Khmel'nitskii used the continual integral over Fermi fields.[12] As a result, the symmetry of the collective variable $Q_{\alpha\beta}$ that describes the diffusion turned out to be different. In particular, the symmetry group corresponding to the variable $Q_{\alpha\beta}$ obtained in Ref. 11 is noncompact, while that corresponding to $Q_{\alpha\beta}$ of Ref. 12 is compact. It appears that this difference should vanish in the limit as $n\to 0$.

Despite the progress made, the methods based on the replica technique are not fully satisfactory. The main reason is that no procedure has yet been developed for an analytic continuation from integer n to $n\to 0$. There was therefore no assurance that the results are correct (the only exception seems to be perturbation theory). Within the framework of the proposed methods[11,12] it is necessary to work with matrices $Q_{\alpha\beta}$ have dimensionalities $2n\times 2n$, which is a rather difficult problem. In addition, the approximations made, the applicability conditions of which are satisfied at finite n, are frequently not valid on going to $n\to 0$.

We propose below a method of describing diffusion modes, based on writing down the Green's functions with the aid of a continual integral both over Fermi and over Bose fields. This procedure, with the aid of mixed integration over super-fields that contain Fermi and Bose variables as components, make it possible to average over the random potential without using the method of replicas. The derivation of the Lagrangian of the diffusion modes is schematically similar to that

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represented in Refs. 11 and 12. Just as earlier, a collective variable appears and serves as the analog of the order parameter in the theory of phase transitions. Now, however, this variable Q has dimensions 8×8, and half the elements are Bose fields while the other half are Fermi fields. The obtained Lagrangian containing this variable belongs to the class of generalized σ models and has a high symmetry $SPU(8)/SPU(4)\times SPU(4)$, where $SPU(N)$ is the group of superunitary transformations. In the method developed, the existence of gapless diffusion modes is the consequence of violation of the supersymmetry. An effective Lagrangian is obtained and takes into account the external magnetic field, scattering by magnetic impurities, and spin-orbit interaction. The conductivity in two-dimensional space is calculated by way of example.

2. CHOICE OF MODEL AND AVERAGING OVER THE IMPURITIES

The behavior of the electron in a random potential can be described by a correlation density function $\rho(\mathbf{r},t)$:

$$\langle\rho(\mathbf{r},t)\rho(\mathbf{r}',t')\rangle-\langle\rho\rangle^2=\int d\omega e^{-i\omega(t-t')}\int\frac{n_\varepsilon-n_{\varepsilon+\omega}}{\omega}K(\omega)\,d\varepsilon, \tag{1}$$

where the angle brackets denote averaging over the positions of the impurities. The quantity $K(\omega)$ in (1) is the averaged two-particle Green's function, which is expressed in the usual manner in terms of the retarded G^R and advanced G^A single-particle Green's functions:

$$\begin{aligned}&K(\omega)=\langle G_\varepsilon^R(x,x')G_{\varepsilon+\omega}^A(x',x)\rangle,\\&G^{(R,A)}(x,y)=\sum_k\frac{\varphi_k^*(x)\varphi_k(x')}{\varepsilon-E_k\pm i\delta}.\end{aligned} \tag{2}$$

In this equation, $x=\mathbf{r},\alpha$, where $\mathbf{r}$ is the coordinate and α is the spin variable; E_k and φ_k are the eigenvalues and eigenfunctions of the electron in the impurity field:

$$H\varphi_k=E_k\varphi_k,\quad H=H_0+H_1,\quad \langle H_1\rangle=0. \tag{3}$$

The problem of obtaining the behavior of an electron in a random potential consists in solving Eq. (3), obtaining the single-particle Green's functions from the wave functions and the energies, and then averaging the product of two Green's functions (2). Since it is impossible to obtain the exact solution of Eq. (3), approximate methods are frequently used, in which the interaction with the impurities is accounted for by perturbation theory.[5,6] Averaging over the impurities is carried out in each term of the series. With this approach, the diffusion appears after the summation of ladder diagrams.

The methods that turn out to be convenient are those which permit averaging over the impurities from the very beginning, and reduce the problem to an investigation of field-theoretical models. Such methods, however, are based on the use of the method of replicas.[13] If distances that exceed the mean free path play a major role, the problem then reduces to the Lagrangian of the interacting diffusion modes. We propose below a method that avoids the use of the method of replicas but yields nevertheless the effective Lagrangian of the interacting modes. The method is based on writing down the Green's functions with the aid of integration over superfields whose components are classical Bose and Fermi fields.

For the calculations that follow it is necessary to present the basic formulas that define the classical Fermi fields and the integrals over these fields. By integrals over Bose fields will be meant ordinary integrals. The classical fermion fields are described by two sets χ_i and χ_i^* of Grassmann anticommuting variables[14-16] satisfying the relations

$$\{\chi_i,\chi_j\}=\{\chi_i,\chi_j^*\}=\{\chi_i^*,\chi_j^*\}=0. \tag{4}$$

By virtue of the property (4), the square of any variable is zero

$$\chi_i^2=\chi_i^{*2}=0. \tag{5}$$

It is convenient to define the operation of complex conjugation for the Grassmann variables. This operation sets in correspondence each variable χ_i to a variable χ_i^*. By definition, we assume that the inverse operation is described by the formulas

$$(\chi_i^*)^*=-\chi_i. \tag{6}$$

The definition (6) differs in sign from the concept of complex conjugation for ordinary numbers. For anticommunting quantities, however, it turns out to be reasonable. For example, the quantity $\chi_i^*\chi_i$ is not changed by the action of such a conjugation operation:

$$(\chi_i^*\chi_i)^*=-\chi_i\chi_i^*=\chi_i^*\chi_i. \tag{7}$$

Integration with respect to Grassmann variables was first defined by Berezin and is described by the formulas

$$\int d\chi_i=\int d\chi_i^*=0,\quad \int\chi_i\,d\chi_i=\int\chi_i^*\,d\chi_i^*=1. \tag{8}$$

Integrals of several variables are understood as repeated integrals. It is assumed that the differentials $d\chi_i$ also satisfy the anticommutativity conditions.

From the definition (8) followed directly an important formula for the integration of the Gaussian exponential[15]

$$\int\exp(-\chi^*A\chi)\,d^n\chi^*d^n\chi=\det A \tag{9}$$

and a formula for the integration of an exponential with linear terms

$$\int\exp(-\chi^*A\chi-\chi^*\varkappa_1-\varkappa_2\chi)\,d^n\chi^*d^n\chi=\det A\exp(\varkappa_2A^{-1}\varkappa_1), \tag{10}$$

where

$$\chi=(\chi_1,\chi_2\ldots\chi_n),\quad \chi^*=(\chi_1^*,\chi_2^*\ldots\chi_n^*),$$
$$d^n\chi=\prod_i d\chi_i,\quad d^n\chi^*=\prod_i d\chi_i^*,$$

A is an $n\times n$ matrix.

We emphasize that the integral of a Gaussian exponential over fermion fields (9), (10) differs from the integral of the same exponential over boson fields. In the case of integration over boson fields the right-hand sides of (9) and (10) contain $(\det A)^{-1}$. This difference

allows us to write down the denominator in the expression for the Green's function (2) in the form

$$(\varepsilon-E_k\pm i\delta)^{-1}=i\int\chi_k\chi_k^*\exp\Big(-i\sum_k(\varepsilon-E_k\pm i\delta)$$
$$\times(\chi_k^*\chi_k+S_k^*S_k)\Big)\prod_k d\chi_k^*\prod_k d\chi_k\prod_k dS_k^*\,dS_k$$
$$=i\int S_kS_k^*\exp\Big(-i\sum_k(\varepsilon-E_k\pm i\delta)(\chi_k^*\chi_k+S_k^*S_k)\Big)$$
$$\times\prod_k d\chi_k^*\prod_k d\chi_k\prod_k dS_k^*\,dS_k. \qquad (11)$$

Using the representation (11), we express the Green's function $G^{R,A}$ (2) in the form of a continual integral over boson and fermion fields:

$$G^{R,A}(x,x')=i\int\chi(x)\chi^*(x')\exp\Big(-i\int[\chi^*(y)(\varepsilon-H\pm i\delta)\chi(y)$$
$$+S^*(y)(\varepsilon-H\pm i\delta)S(y)]dy\Big)D\chi^*D\chi DSDS^*;$$
$$\chi(x)=\sum_k\chi_k\varphi_k(x).\quad\chi^*(x)=\sum_k\chi_k^*\varphi_k^*(x). \qquad (12)$$

The advantage of expression (12) for the Green's function lies in the fact that it has no weighting denominator. This permits immediate averaging over the random potential. In principle, the exponential in (12) can be preceded by boson rather than fermion fields. The possibility of writing down a single-particle Green's function with the aid of an integral over fermion and boson fields was noted in Ref. 17. Another example of a disordered system that can be investigated with the aid of boson-fermion fields, is presented in Ref. 18.

Using expressions (2) and (12), we reduce the two-particle Green's function $K(\omega)$ to the form

$$K(\omega)=\Big\langle\int\chi^1(x)\chi^{1*}(x')\chi^2(x')\chi^{2*}(x)e^{-L}D\chi^*D\chi DS^*DS\Big\rangle, \qquad (13)$$

$$L=i\int dy\sum_{a=1}^{2}\Big[\chi^{a*}(y)\Big((\varepsilon-H)(-1)^{a+1}+\frac{\omega}{2}-i\delta\Big)\chi^a(y)$$
$$+S^{a*}(y)\Big((\varepsilon-H)(-1)^{a+1}+\frac{\omega}{2}-i\delta\Big)S^a(y)\Big]. \qquad (14)$$

For a more compact expression, it is convenient to introduce in place of the sets of fermion and boson fields the superfield ψ with the following components

$$\psi^a=\begin{pmatrix}u^a\\v^a\end{pmatrix};\quad u^a=\frac{1}{\sqrt2}\begin{pmatrix}\chi^{a*}\\\chi^a\end{pmatrix},$$
$$v^a=\frac{1}{\sqrt2}\begin{pmatrix}S^{a*}\\S^a\end{pmatrix},\quad\bar\psi=(C\psi)^T, \qquad (15)$$

where $a=1$ or 2 and T is the transposition operation.

In (15), C is the "charge conjugation" matrix:

$$C^{ab}=\Lambda^{ab}\begin{pmatrix}c_1&0\\0&c_2\end{pmatrix}, \qquad (16)$$

where Λ is a diagonal matrix with components $\Lambda^{11}=-\Lambda^{22}=1$. The matrices c_1 and c_2 are of the form

$$c_1=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad c_2=\begin{pmatrix}0&1\\1&0\end{pmatrix}. \qquad (16a)$$

Here and elsewhere, the superior indices pertain to retarded or advanced parts. The conjugate field $\bar\psi$ is connected with the complex-conjugate ψ^* by the simple relation

$$\bar\psi=(\Lambda\psi^*)^T. \qquad (17)$$

In terms of the superfield, Eq. (14) can be rewritten as

$$L=i\int\bar\psi(x)(-\hat H_0+U(\mathbf r)+{}^1/_2\omega\Lambda)\psi(x)\,dx, \qquad (18)$$

where $U(r)$ is the impurity potential, and the operator $\hat H_0$ is given by

$$\hat H_0=-\varepsilon-\frac{1}{2m}\frac{\partial^2}{\partial\mathbf r^2}. \qquad (18a)$$

In the calculation of the averages over the impurity locations, the potential $U(r)$ is assumed to be a random quantity with a Gaussian δ-correlated distribution

$$\langle U(\mathbf r)\rangle=0,\quad\langle U(\mathbf r)U(\mathbf r')\rangle=\frac{1}{\pi\nu\tau}\delta(\mathbf r-\mathbf r'), \qquad (19)$$

where ν is the state density and τ is the free-path time. This distribution describes weakly interacting impurities of small size. In the general case one can verify that the results that follow remain valid upon a suitable redefinition of the constants.

In the case of a Gaussian distribution of a random potential, the averaging of (13) with the Lagrangian L (18) becomes very simple. After the averaging, expression (13) retains its form, while the Lagrangian L becomes

$$L=\int\Big[-i\bar\psi\hat H_0\psi+\frac{1}{2\pi\nu\tau}(\bar\psi\psi)^2+\frac{i(\omega-i\delta)}{2}\bar\psi\Lambda\psi\Big]dx. \qquad (20)$$

The Lagrangian (20) is similar to the Lagrangians used in field theory. In models described by such Lagrangians there can appear mean values $Q\sim\langle\psi\bar\psi\rangle$ and associated Goldstone modes. These mean values will be separated in the next section in a rather customary manner. Certain complications due to the need of taking into account both boson and fermion components are of no fundamental significance.

3. REDUCTION TO A NONLINEAR GENERALIZED σ MODEL

Further calculations with the Lagrangian (20) will be carried out under the assumption that the distances of importance are much larger than the mean free path. This is the situation in thick wires and two-dimensional systems, if the potential of the interaction with the impurities is small enough. In this case there exist mean values $Q\sim\langle\psi\bar\psi\rangle$ that vary slowly in space. To separate these slowly varying mean values we rewrite the interaction in (20) in the form

$$L_{int}=\frac{1}{2\pi\nu\tau}\sum_{\mathbf p_1+\mathbf p_2=\mathbf p_3+\mathbf p_4}(\bar\psi_{\mathbf p_1}\psi_{\mathbf p_2})(\bar\psi_{\mathbf p_3}\psi_{\mathbf p_4})\approx\sum_{\mathbf p_1,\mathbf p_2,\mathbf q}[(\bar\psi_{\mathbf p_1}\psi_{\mathbf p_2})(\bar\psi_{-\mathbf p_1+\mathbf q}\psi_{-\mathbf p_2-\mathbf q})$$
$$+(\bar\psi_{\mathbf p_1}\psi_{\mathbf p_2})(\bar\psi_{-\mathbf p_2-\mathbf q}\psi_{-\mathbf p_1+\mathbf q})+(\bar\psi_{\mathbf p_1}\psi_{-\mathbf p_1+\mathbf q})(\bar\psi_{\mathbf p_2}\psi_{-\mathbf p_2-\mathbf q})]. \qquad (21)$$

The regions with small q are specially separated in (21). Using the charge-conjugation operation (15), (16) and the commutation rules, it can be seen that the first and second terms in (21) are equal. Just as in Refs. 11 and 12, we use the Hubbard-Stratonovich transformation.[19] The simplest to separate is the last term of (21)

$$\exp\Big[-\frac{1}{2\pi\nu\tau}\sum_{p_1,p_2,q}(\bar\psi_{p_1}\psi_{-p_1+q})(\bar\psi_{p_2}\psi_{-p_2-q})\Big]$$
$$=\Big\{\int\exp\Big[-\frac{\pi\nu}{8\tau}\int E^2(\mathbf{r})\,d\mathbf{r}\Big]DE\Big\}^{-1}$$
$$\times\int\exp\Big[-\frac{i}{2\tau}\int E(\mathbf{r})\bar\psi(\mathbf{r})\psi(\mathbf{r})\,d\mathbf{r}-\frac{\pi\nu}{8\tau}\int E^2(\mathbf{r})\,d\mathbf{r}\Big]DE. \tag{22}$$

The integration in (22) is with respect to slowly varying real functions $E(\mathbf{r})$. Comparing (22) and (20), we can verify that $E(\mathbf{r})$ leads to only an insigificant change of energy, since it is a slow real function. Therefore the last term in (21) leads to no physical phenomena whatever and will be disregarded.

The remaining terms in (21) can be split into slowly varying parts only by integrating over the matrix. We rewrite (21), separating the fermion (u) and boson (v) components of the supervector ψ (15). Discarding the last term and using the equality of the first two terms, we obtain

$$L_{int}=\frac{1}{\pi\nu\tau}\sum_{p_1,p_2,q}[(\bar u_{p_1}u_{p_2})(\bar u_{-p_2-q}u_{-p_1+q})$$
$$+(\bar v_{p_1}v_{p_2})(\bar v_{-p_2-q}v_{-p_1+q})+2(\bar u_{p_1}u_{p_2})(\bar v_{-p_2-q}v_{-p_1+q})]. \tag{23}$$

The splitting of the first two terms into slow parts in (23) can be carried out by integrating over the Bose fields. However, to split the third term in (23), integration with respect to the Fermi fields is necessary. Integrating by using Eq. (10) for the Gaussian integral over Fermi fields, and the corresponding equation for the integral over the Bose fields, we arrive at the equations

$$\exp(-L_{int})=P_1P_2P_3,$$
$$P_1=\int\exp\Big(-\frac{1}{\tau}\sum_{a,b=1}^{2}\sum_{p,q}\Big(\bar u_p{}^aA_q{}^{ab}u_{-p-q}{}^bi^{b-a}+\frac{\pi\nu}{4}\mathrm{Sp}\,A_q^{ab+}A_{-q}^{ab}\Big)\Big)\,DA,$$
$$P_2=\int\exp\Big(-\frac{1}{\tau}\sum_{a,b=1}^{2}\sum_{p,q}\Big(\bar v_p{}^aB_q{}^{ab}v_{-p-q}{}^bi^{b-a+1}+\frac{\pi\nu}{4}\mathrm{Sp}\,B_q^{ab+}B_{-q}^{ab}\Big)\Big)\,DB, \tag{24}$$
$$P_3=\int\exp\Big(-\frac{1}{\tau}\sum_{a,b=1}^{2}\sum_{p,q}\Big(\bar u_p{}^a\Sigma_q{}^{ab}v_{-p-q}{}^bi^{2b-a-1}$$
$$+\bar v_p{}^b\Sigma_q^{ab+}u_{-p-q}{}^ai^{a-1}+\frac{\pi\nu}{2}(-1)^a\mathrm{Sp}\,\Sigma_q^{ab+}\Sigma_{-q}^{ab}\Big)\Big)\,D\Sigma.$$

In (24) the integration is over the 2×2 matrices A^{ab}, B^{ab}, and Σ^{ab}. The elements of the matrices A^{ab} and B^{ab} are ordinary numbers, while those of Σ^{ab} are Grassmann variables. All these matrices satisfy the conditions that arise when the charge-conjugate and complex-conjugate quantities are separated in the integrals:

$$A^{ab+}=A^{ba}=c_1A^{ab\,T}c_1{}^T, \tag{25}$$
$$B^{ab+}=B^{ba}=c_2B^{ab\,T}c_2{}^T, \tag{26}$$
$$\Sigma^{ab+}=c_2\Sigma^{ab\,T}c_1{}^T, \tag{27}$$

For the sake of clarity, we write down the matrices A^{ab}, B^{ab}, and Σ^{ab} that satisfy the conditions (25)–(27) in explicit form:

$$A^{ab}=\begin{pmatrix}A_1{}^{ab} & A_2{}^{ab}\\ -A_2{}^{ab*} & A_1{}^{ab*}\end{pmatrix},\qquad B^{ab}=\begin{pmatrix}B_1{}^{ab} & B_2{}^{ab}\\ B_2{}^{ab*} & B_1{}^{ab*}\end{pmatrix}.$$
$$\Sigma^{ab}=\begin{pmatrix}\Sigma_1{}^{ab} & \Sigma_2{}^{ab}\\ -\Sigma_2{}^{ab*} & -\Sigma_1{}^{ab*}\end{pmatrix},\quad A_1{}^{ab}=A_1{}^{ba*},\quad A_2{}^{ab}=-A_2{}^{ba},\quad B_1{}^{ab}=B_1{}^{ba*},\quad B_2{}^{ab}=B_2{}^{ba}. \tag{28}$$

Altogether the matrices A, B, and Σ contain 16 independent complex variables, half of them boson and the other half fermion. The equality of the number of boson and fermion variables makes it unnecessary to write in (24) the weighting denominators, since their product is equal to unity. We note that the integrals in (24) always converge. The matrix A has a symmetry similar to that used to analyze the problem by the method of replicas with the aid of fermions,[12] and the matrix B analogously with the aid of bosons.[11]

The rather cumbersome expression (24) can be written in a more compact form by introducing the concept of the supermatrix. Such a supermatrix Q consists of blocks (superelements)

$$Q^{ab}=\begin{pmatrix}q_1{}^{ab} & q_3{}^{ab}\\ q_4{}^{ab} & q_2{}^{ab}\end{pmatrix}. \tag{29}$$

The quantities q_i^{ab} are also matrices, with q_1^{ab} and q^{ab} consisting of bosons, and q_3^{ab} and q_4^{ab} fermions. It is easy to verify that the product of any number of supermatrices is also a supermatrix of the form (29). Since half of the elements are anticommutative, the rules for operating with supermatrices differ somewhat from those for ordinary matrices. We define the supertransposition operation

$$(Q^{sT})^{ab}=\begin{pmatrix}(q_1{}^{ba})^T & -(q_4{}^{ba})^T\\ (q_3{}^{ba})^T & (q_2{}^{ba})^T\end{pmatrix}. \tag{30}$$

Using the definition (30) and the rules for commutation of the elements of the supermatrix, we can verify that the following equality holds

$$(Q_1Q_2)^{sT}=Q_2{}^{sT}Q_1{}^{sT}, \tag{31}$$

where Q_1 and Q_2 are arbitrary supermatrices of the form (29). The definition (30) allows us to introduce the operations of the charge and Hermitian superconjugations

$$\bar Q=CQ^{sT}C^T, \tag{32}$$

where C is defined by Eq. (16)

$$Q^{s+}=(Q^{sT})^*. \tag{33}$$

From (31)–(33) it follows immediately that

$$\overline{Q_1Q_2}=\bar Q_2\bar Q_1, \tag{34}$$
$$(Q_1Q_2)^{s+}=Q_2{}^{s+}Q_1{}^{s+}. \tag{35}$$

The charge-conjugation operation plays an important role in the description of the properties of quadratic forms. If ψ_1 and ψ_2 are arbitrary supervectors of the form (15), then the following equality holds

$$\bar\psi_1Q\psi_2=\bar\psi_2\bar Q\psi_1. \tag{36}$$

It is useful to introduce the concept of the superspur[16]

$$\mathrm{SSp}\,Q=\sum_a(q_1{}^{aa}-q_2{}^{aa}). \tag{37}$$

The superspur defined by (37) is invariant to cyclic permutations:

$$\mathrm{SSp}\,Q_1Q_2Q_3=\mathrm{SSp}\,Q_3Q_1Q_2 \tag{38}$$

and is not changed by supertransposition

$$\mathrm{SSp}\,Q=\mathrm{SSp}Q^{sT}. \tag{39}$$

In supermatrix notation, expressions (24) take the form

$$\exp(-L_{int})=\int\left[\exp\left(-\frac{1}{\tau}\int\left(\bar{\psi}Q\psi+\frac{\pi\nu}{4}\mathrm{SSp}\,Q^2\right)\right)d\mathbf{r}\right]DQ. \tag{40}$$

The supermatrix Q in (40) satisfies the self-adjoint conditions

$$Q=\bar{Q} \tag{41}$$

and consists of four superelements made up of the matrices A, B, and Σ (24)

$$Q^{11}=\begin{pmatrix} A^{11} & \Sigma^{11} \\ \Sigma^{11+} & iB^{11}\end{pmatrix},\quad Q^{12}=i\begin{pmatrix} A^{12} & i\Sigma^{12} \\ \Sigma^{21+} & iB^{12}\end{pmatrix},$$
$$Q^{22}=\begin{pmatrix} A^{22} & i\Sigma^{22} \\ i\Sigma^{22+} & iB^{22}\end{pmatrix},\quad Q^{21}=-i\begin{pmatrix} A^{12+} & \Sigma^{21} \\ i\Sigma^{12+} & iB^{12+}\end{pmatrix}. \tag{42}$$

Substitution of (21) and (40) in (20) yields an expression for the Lagrangian L in (13). Using the slow variation of the supermatrix Q in space, we can integrate (13) over the boson and fermion fields. We note that all integrals over the superfield ψ converge. Gaussian integration with respect to ψ is carried out with the aid of the equation

$$\int\exp(-\bar{\psi}P\psi)\,d\bar{\psi}\,d\psi=\exp\left(\frac{1}{2}\mathrm{SSp}\ln P\right), \tag{43}$$

where $P=\bar{P}$ is a self-adjoint supermatrix.

Equation (43) can be proved by successive integration over the boson and fermion fields. Using (13), (15), (20), (40), and (43), we obtain

$$K(\omega)=\frac{(\pi\nu)^2}{64}\int\mathrm{SSp}(k(1-\Lambda)(1-\tau_3)Q(\mathbf{r})(1+\Lambda)$$
$$\times(1-\tau_3)kQ(\mathbf{r}'))\exp(-F[Q])DQ, \tag{44}$$

where k is a superelement of the form (29) with components

$$k=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{45}$$

The matrix τ_3 is also of the form

$$\tau_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{46}$$

but is now in the space of the matrices A, B, and Σ.

The functional of the free energy in (44) is

$$F[Q]=\int d\mathbf{r}\left[-\frac{1}{2}\mathrm{SSp}\ln\left(iH_0-\frac{i\omega\Lambda}{2}-\frac{Q}{\tau}\right)+\frac{\pi\nu}{4\tau}\mathrm{SSp}\,Q^2\right]. \tag{47}$$

We note that the integral over Q in (44) with the functional $F[Q]$ (47) converges as before.

In two- and three-dimensional sufficiently pure samples, such that the inequality $\varepsilon\tau\gg1$ is satisfied, and in sufficiently thick wires with cross section much larger than atomic, the minima of the energy (47) are significant. Varying (47) with respect to Q and using the property (37), we obtain

$$Q=\frac{1}{\pi\nu}\int\hat{G}(\mathbf{p})\,d\mathbf{p}. \tag{48}$$

$$\hat{G}(\mathbf{p})=\left(i\varepsilon-\frac{ip^2}{2m}+\frac{i\omega\Lambda}{2}+\frac{Q}{\tau}\right)^{-1}. \tag{49}$$

At $\omega\neq0$, the solution of (49) is of the form $Q=\Lambda$. This extremum point does not belong to the set of supermatrices of the type (42) with conditions (25), (26), and (27), since the matrices B^{aa} in (42) are Hermitian with real eigenvalues. A similar disparity arises when the method of replicas is used in the boson representation.[11] To reach the saddle point it is necessary to shift the contour of the integration over the elements of the matrices B^{aa} into the complex plane. This procedure is described in detail in Ref. 11. At $\omega=0$ the solution of (48), (49) turns out to be strongly degenerate. Just as in Ref. 11, we obtain Q^{11} and Q^{22} at the saddle points from the specified Q^{12}. The solution, which goes over continuously at $\omega\neq0$ into $Q=\Lambda$, takes the form

$$Q=W+\Lambda(1-W^2)^{1/2};\quad W=\begin{pmatrix}0 & Q^{12}\\ Q^{21} & 0\end{pmatrix}, \tag{50}$$

where Q^{12} and Q^{21} are defined in (42). The supermatrix W has eight independent complex variables.

At the saddle point, besides the condition (41), there is satisfied the equality

$$\bar{Q}=KQ^{st}K;\quad K=\begin{pmatrix}1 & 0\\ 0 & k\end{pmatrix}, \tag{51}$$

where k is the superelement (45). Equation (51) can be verified by comparison with Eqs. (25)–(27), (42), and (50). For the supermatrix Q (50) there exists another convenient representation:

$$Q=\bar{U}\Lambda U, \tag{52}$$

where U is a superunitary matrix satisfying the conditions

$$\bar{U}U=1,\quad \bar{U}=KU^{st}K. \tag{53}$$

Thus, at the saddle point the extremal solution (52) is connected with the transformation group $U(8)/U(4)\times U(4)$. This degeneracy is due to the supersymmetry of the initial Hamiltonian (20). The supermatrix Q (52) plays the role of the order parameter and contains Grassmann variables. Calculating the free energy (47) at the minimum, we verify that it is equal to zero. Anticommutative variables do not enter in the physical quantities. A similar situation obtains in superconductivity, where the order parameter is a complex quantity, but all the thermodynamic quantities are real.

At small deviations δQ from the equilibrium value, the free energy is quadratic in these deviations:

$$F=\frac{1}{4\tau^2}\mathrm{SSp}\int\frac{d^dk}{(2\pi)^d}\left\{\int\frac{d^dp}{(2\pi)^d}[\hat{G}_p\delta Q_k\hat{G}_{p+k}\delta Q_{-k}+\pi\nu\tau\delta Q_k\delta Q_{-k}]\right\}$$
$$+\frac{i\omega\pi\nu}{4}\int\mathrm{SSp}\,\Lambda Q\,d\mathbf{r}. \tag{54}$$

A distinction must be made between longitudinal fluctuations of Q, which change the eigenvalues, and transverse ones, in which only the matrix U in (52) fluctuates. For transverse fluctuations we have

$$Q\delta Q+\delta QQ=1.$$

The longitudinal fluctuations alter greatly the free energy, and can therefore be neglected.

Homogeneous transverse fluctuations do not change the free energy at all in the low-frequency limit. The functional F of the free energy, which describes these fluctuations, contains at low frequencies only gradients

of Q and terms linear in the frequency

$$F=\frac{\pi\nu}{8}\int \mathrm{SSp}[D(\nabla Q)^2-2i\omega\Lambda Q]d\mathbf{r}, \tag{55}$$

where $D=\overline{v_x^2\tau}/2$ is the diffusion coefficient.

The functional (55) describes Goldstone modes whose existence is the consequence of the spontaneous breaking of the supersymmetry and of the existence of the order parameter Q. A system with free energy (55) belongs to the class of nonlinear σ models. We note the formal analogy between expression (55) and the corresponding expressions obtained in investigations by the method of replicas.[11,12] Of course, the matrices Q have everywhere different structures.

To calculate the propagator of the free diffusion mode it suffices to use the first terms of the expansion of Q in terms of W in (50). Substituting this expansion in (55) we obtain

$$F_0\approx\frac{\pi\nu}{8}\int[D\,\mathrm{SSp}(\nabla W)^2+i\omega\,\mathrm{SSp}\,W^2]d\mathbf{r}. \tag{56}$$

From (56) and (54) we get

$$K_0(\omega)=2\pi\nu/(D\mathbf{k}^2+i\omega). \tag{57}$$

The propagator $K_0(\omega)$ corresponds to the usual diffusion equation.

4. MAGNETIC AND SPIN-ORBIT INTERACTIONS

We have considered so far the scattering of electrons by ordinary impurities. The system was in this case invariant to time reversal. When account is taken of the spin degrees of freedom, invariance to rotations in spin space is added. The high degree of symmetry of the free energy (55) with respect to the transformations of U (53) is due to the invariance of the system to time reversal and to spin rotations. An external magnetic field and magnetic impurities destroy the symmetry with respect to time reversal. The symmetry with respect to rotations in spin space is violated by the interaction with the magnetic impurities and by the spin-orbit interactions. When all these perturbations are accounted for, the Hamiltonian takes the form

$$H=\varepsilon\left(\hat{\mathbf{p}}-\frac{ie}{c}\mathbf{A}\tau_3\right)-\varepsilon+\sigma[\nabla V_{so},\mathbf{p}]. \tag{58}$$

$$H=\hat{U}+\mathbf{U}_s\sigma+\sigma[\nabla U_{so}\mathbf{p}]. \tag{59}$$

In (58) and (59) $\mathbf{A}$ is the vector potential, V_{so} is the spin-orbit interaction in the absence of impurities, U_s is the interaction with the magnetic impurities, and U_{so} is the impurity spin-orbit interaction. The matrix τ_3 defined by (46) arises in the transformation of the χ, χ^*, S, S^* Lagrangian into the Lagrangian expressed in terms of ψ and $\bar{\psi}$.

If the entire interaction is weaker than scattering by ordinary impurities, the influence of the latter can be calculated independently. All the calculations are similar to those in Ref. 12. Repeating the derivation of the functional (55) in the presence of a magnetic field, we obtain

$$F=\frac{\pi\nu}{8}\int \mathrm{SSp}\left[D\left(\nabla Q+\frac{e\mathbf{A}}{c}[Q,\tau_3]\right)^2-2i\omega\Lambda Q\right]d\mathbf{r}. \tag{60}$$

The free energy (60) contains, besides the gradient term, the commutator $[Q,\tau_3]$. Because of this term, some of the diffusion modes are no longer gapless. By substituting (42) in (60) we can verify that the only remaining Goldstone excitations are A_1, B_1, and Σ_1 in (28). The A_2, B_2, and Σ_2 fluctuations are suppressed within the limit of long waves by the magnetic fields. If the Hamiltonian does not depend on the spins, all the results are separately applicable for particles with spin up and with spin down. To study the spin interactions it is necessary to double the number of variables. The anticommuting (u^a) and the commuting (v^a) components of the supervector ψ are of the form

$$u^a=\frac{1}{\sqrt{2}}\begin{pmatrix}\chi^{a*}\\ i\sigma_y\chi^a\end{pmatrix},\quad v^a=\frac{1}{\sqrt{2}}\begin{pmatrix}S^{a*}\\ i\sigma_yS^a\end{pmatrix}; \tag{61}$$

$$\bar{\psi}=(C\psi)^T,\quad c_1=\begin{pmatrix}0 & i\sigma_y\\ i\sigma_y & 0\end{pmatrix},\quad c_2=\begin{pmatrix}0 & -i\sigma_y\\ i\sigma_y & 0\end{pmatrix}.$$

It is assumed that the supermatrix C is expressed in terms of c_1 and c_2 in accord with Eq. (16). All the equations of the preceding section remain unchanged if the supermatrix C is taken to mean the expression (61). In particular, the symmetry of Q is determined as before by Eqs. (41) and (51). After averaging over the random fields of the magnetic impurities U_s the Lagrangian (20) acquires an additional term

$$\mathcal{L}_s=\frac{1}{6\pi\nu\tau_s}(\bar{\psi}\tilde{\sigma}\psi)^2,\quad \tilde{\sigma}=\sigma\otimes\tau_3,$$

$$\langle U_s^i(\mathbf{r})U_s^j(\mathbf{r}')\rangle=\frac{\delta_{ij}}{3\pi\nu\tau_s}\delta(\mathbf{r}-\mathbf{r}'). \tag{62}$$

Integration over the superfields ψ leads to an additional term in the free energy

$$F_s=\frac{\pi\nu}{3\tau_s}\int \mathrm{SSp}(Q\tilde{\sigma})^2\,d\mathbf{r}. \tag{63}$$

In the long-wave limit, an anisotropy of the form (63) leads to suppression of the excitations A_2, B_2, and Σ_2 in (28). [All the elements in the matrices (28) are now spin quaternions.] In addition, only unity elements remain in the spin quaternions A_1, B_1, and Σ_1. Therefore the action of the magnetic impurities is equivalent to the action of the magnetic field.

In the case of sufficiently weak spin-orbit interaction there is added to the Lagrangian (20) a term of the form

$$\mathcal{L}_{so}=-\frac{1}{2}\left\langle\left(\int\bar{\psi}(\mathbf{r})\sigma[\nabla(V_{so}+U_{so}),\hat{\mathbf{p}}]\psi(\mathbf{r})d\mathbf{r}\right)^2\right\rangle.$$

After integration with respect to ψ we obtain an additional contribution to the free-energy functional

$$F_{so}=\pi\nu\sum_{i=1}^{3}\frac{1}{\tau_{so}^{(i)}}\mathrm{SSp}(Q\sigma^i)^2,$$

$$\frac{1}{\tau_{so}^{(i)}}=\pi\nu\langle([\nabla U_{so},\mathbf{p}]^i)^2+[\nabla V,\mathbf{p}]^2\rangle\tau. \tag{64}$$

The last equation takes into account the possible anisotropy of the spin-orbit interaction.

As a result of the term (64), the only Goldstone excitation remaining are those corresponding to unity elements in the spin quaternions. We then obtain in place of (28)

$$A^{ab}=\begin{pmatrix} A_1^{ab} & A_2^{ab} \\ A_2^{ab*} & A_1^{ab*} \end{pmatrix}, \quad B^{ab}=\begin{pmatrix} B_1^{ab} & -B_2^{ab} \\ B_2^{ab*} & B_1^{ab*} \end{pmatrix}, \qquad (65)$$
$$\Sigma^{ab}=\begin{pmatrix} \Sigma_1^{ab} & \Sigma_2^{ab} \\ -\Sigma_2^{ab*} & \Sigma_1^{ab*} \end{pmatrix}.$$

A_i, B_i, and Σ_i in (65) are unity elements in the spin quaternions.

The results show that the magnetic field, the magnetic impurities, and the spin-orbit interactions take the form of anisotropies with different symmetries in the free-energy functional, and lower the initial symmetry of the energy (55). A similar effect is exerted by these interactions on the free energy that arises in the method of replicas.[12]

5. RENORMALIZATION GROUP

By way of example of concrete calculations with the aid of the method described in the preceding section, we consider the problem of the conductivity of a two-dimensional disordered metal. We shall use for the calculations the free-energy functional (55). The asymmetry of the order parameter Q in (55) depends on the presence of magnetic and spin-orbit interactions and is determined by the equations of the preceding sections. We use the renormalization-group method in a form similar to that proposed by Polyakov for the investigation of the vector σ model.[20] Such a method was already used in Ref. 12 for the tensor model.

We break up the superunitary supermatrix $U(\mathbf{r})$ in (52) into a product of a fast and slow part $U_0(\mathbf{r})$ and $\tilde{U}(\mathbf{r})$, respectively:

$$U(\mathbf{r})=U_0(\mathbf{r})\tilde{U}(\mathbf{r}). \qquad (66)$$

We assume that $U_0(\mathbf{r})$ and $\tilde{U}(\mathbf{r})$ satisfy the conditions (53). Substituting (52) and (66) in (55) we obtain

$$F=\frac{1}{t}\int \mathrm{SSp}[(\nabla Q_0)^2+2[\nabla Q_0, Q_0]\Phi+[Q_0,\Phi]^2-2i\tilde{\omega}\tilde{U}\Lambda\tilde{U}Q_0]d\mathbf{r},$$
$$Q_0=\tilde{U}_0\Lambda U_0, \quad \Phi=\nabla\tilde{U}\cdot\tilde{U}, \qquad (67)$$
$$1/t=D\pi\nu/8, \quad \tilde{\omega}=\omega/D.$$

The supermatrix Φ (67) satisfies the equation $\Phi=-\bar{\Phi}$. We integrate (44), with the free energy defined by (67), with respect to the fast variable $Q_0(r)$. As a result of the integration, the energy F in (67) is replaced by the energy $\tilde{F}$ that describes the slow fluctuations

$$\tilde{F}=-\ln\int e^{-F}DQ_0. \qquad (68)$$

To simplify the calculations it is convenient to choose from the outset the gauge of the supermatrices $\tilde{U}$ and Φ. We assume that $\tilde{U}$ in (66) and (67) is close to unity. Then the only nonzero superelements in the supermatrix Φ are Φ^{12} and Φ^{21}, which are interconnected by the anti-self-adjointness condition. Extremely important in the calculations is how to separate the fast and slow variables and cut off the diverging integrals in an invariant manner. To eliminate the slow changes in Q_0 with momenta $k<\lambda$, we add to (67) a term of the form

$$F_{\mathrm{reg}}=-\frac{2}{t}\int \lambda^2\,\mathrm{SSp}\,\Lambda Q_0\,dr. \qquad (69)$$

The cutoff of the integrals that diverge at large momenta will be carried out with the aid of dimensional regularization. Using the representation (50), expanding Q_0 in terms of W, and retaining in the functional F (67) and (69) the number of terms necessary to obtain the first two orders, we get

$$F\approx F_0+F_0'+F_1+F_2+F_J,$$
$$F_0=\frac{1}{t}\int \mathrm{SSp}[(\nabla W)^2+\lambda^2W^2]dr,$$
$$F_0'=\frac{1}{2t}\int \mathrm{SSp}\left[(\nabla W)^2W^2+(\nabla W\cdot W)^2+\frac{\lambda^2}{2}W^4\right]dr,$$
$$F_1=-\frac{2}{t}\int \mathrm{SSp}(W\nabla W\cdot W\Phi\Lambda)dr, \qquad (70)$$
$$F_2=\frac{2}{t}\int \mathrm{SSp}\left[(W\Phi)^2+W^2\Phi^2+\frac{1}{4}W^4\Phi^2\right.$$
$$\left.-\frac{1}{4}(W^2\Phi)^2+\frac{i\tilde{\omega}}{2}W^2\tilde{Q}+\frac{i\tilde{\omega}}{8}W^4\tilde{Q}\right]dr,$$
$$F_J=\frac{1}{t}\int \mathrm{SSp}[(\nabla\tilde{Q})^2-2i\tilde{\omega}\Lambda\tilde{Q}]dr.$$

$\tilde{Q}$ in (70) is determined by the expression

$$\tilde{Q}=\tilde{U}\Lambda\tilde{U}.$$

There are no terms linear in W in (70), since W varies rapidly. Assuming that $\lambda^2\gg\tilde{\omega}$, retaining the first two orders in t, we reduce (68) to the form

$$\tilde{F}=F_0+\langle F_2\rangle_0-\tfrac{1}{2}\langle F_1^2\rangle_0-\langle F_0'F_2'\rangle_0+F_J. \qquad (71)$$

F_2' in (71) stands for the part of the functional F_2 quadratic in W, and the angle brackets $\langle\cdots\rangle_0$ denote averaging with the functional F_0 from (70). The quantity F_J describes the contribution from the Jacobian. The Gaussian integrals are calculated in accord with Wick's theorem. The following equations, which are verified by direct calculation, are useful:

$$\langle WPW\rangle_0=\frac{t}{8k^2}\left\{\alpha\Lambda[\Lambda,\bar{P}]+\frac{1}{2}(1+\alpha^2)(\mathrm{SSp}\,P-\Lambda\,\mathrm{SSp}\,\Lambda P)\right.$$
$$\left.+\frac{1}{2}(1-\alpha^2)\tau_3(\mathrm{SSp}\,\tau_3P-\Lambda\,\mathrm{SSp}\,\Lambda\tau_3P)\right\}, \qquad (72)$$
$$\langle\mathrm{SSp}(WP_1)\cdot\mathrm{SSp}(WP_2)\rangle_0=\frac{t}{8k^2}[P_1,\Lambda]\cdot[P_2,\Lambda].$$

In (72), P is an arbitrary supermatrix, while P_1 and P_2 are self-adjoint ($P_1=\bar{P}_1$, $P_2=\bar{P}_2$) supermatrices. The superelements in all these matrices have the same structure as the superelements W. The values of the coefficient α in (72) are -1, 0, and $+1$ for potential scattering, magnetic interactions, and spin-orbit interactions, respectively.

Using (72) and calculating the mean values in (71), we obtain

$$\tilde{F}=\frac{1}{t}\int\left\{\mathrm{SSp}(\nabla Q)^2\left[1+\frac{\alpha t}{8}\int\frac{d^dk}{(2\pi)^dk^2}+\frac{t^2}{64}(1-\alpha^2)\left(\frac{1}{d}-\frac{1}{2}\right)\right.\right.$$
$$\left.\left.\times\int\frac{1}{(k_1^2+\lambda^2)(k_2^2+\lambda^2)}\frac{d^dk_1d^dk_2}{(2\pi)^{2d}}\right]-2i\tilde{\omega}\,\mathrm{SSp}\,\Lambda Q\right\}dr. \qquad (73)$$

The averaging over the angles in the integral that results from the third term in (71) has already been carried out in Eq. (73), whose nonlogarithmic terms have been discarded. The quadratic divergences that arise in the second-order calculation are cancelled by the contribution of the Jacobian. We note an interesting feature of the model in question. Only the "tempera-

ture" t is renormalized in it, and the effective frequency $\tilde{\omega}$ remains constant. This property is preserved in all orders and is due to the particle-number conservation law. Equation (74) enables us to write down the Gell-Mann–Low function $\beta(t)$:

$$\beta(\tilde{t}) \equiv d\tilde{t}/d\ln(1/\lambda) = \varepsilon\tilde{t} + \alpha\tilde{t}^2 - {}^1/_2\tilde{t}^3(1-\alpha^2); \quad \tilde{t} = t/16\pi. \tag{74}$$

Equation (74) shows that there are no terms of order t^3 in $\beta(t)$ in the cases of potential and spin-orbit scattering. The solution of (74) leads to a logarithmic pole for potential scattering, and to a zero-charge situation for the spin-orbit interaction. In the magnetic case there is no term of order t^2, but the term of order t^3 is not equal to zero. The results for the magnetic and potential scatterings agree with the conclusions arrived at by Wegner with the aid of the method of replicas and by using the results of Ref. 21. The statement made in Refs. 12 and 22 that there is no t^3 term in the magnetic case is incorrect. The error is due to the noninvariant cutoff used in these papers at large momenta. The first order for the spin-orbit scattering coincides with the result obtained in Ref. 12.

The diffusion coefficient D and consequently also the conductivity are inversely proportional to t (67). Therefore solution of the Gell-Mann–Low equation (74) enables us to determine the dependences of these quantities on the frequency and on the size of the system. We note that calculations for pure potential scattering were carried out for the spinless particles, while those for spin-orbit scattering were made for particles with spin. The transition from spinless particles to particles with spin is effected by a simple redefinition of D and t.

6. CONCLUSION

It was shown in the preceding sections that the problem of the electron conductivity in a random potential is equivalent to the problem of the thermodynamics of a supertensor field. Despite certain complications due to the fact that half of the supermatrix elements are fermions, the principal rules of operation with supermatrices are similar to the rules for ordinary matrices. In contrast to the methods based on the replicas, the difficulty connected with the assumption concerning the number of replicas does not arise here. This has made possible a rigorous corroboration of the renormalization-group equation in the two-dimensional case. Of course, this advantage is not very significant in the calculation of the perturbation-theory series terms, but turn out to be very important in the investigation of more complicated problems. The use of the supersymmetrical representation of the Green's functions has made it possible to express the problem of the conduction in few-dimensional systems in terms of fluctuations of the order parameter Q. The use of the concept of such an order parameter can be substantial also in the investigation of three-dimensional disordered metals. In contrast to phase-transition theory, the symmetry group corresponding to Q is not compact. The known nonanalyticity of the conductivity in the low-frequency region is possibly due to this noncompactness. The magnetic and spin-orbit interaction manifest themselves as special anisotropies of different symmetry. A change in symmetry leads to a dependence of the conductivity on these interactions. Although the components of the order parameter contain Grassmann variables, the physical quantities should be ordinary numbers without even an even number of Grassmann variables. This is the usual situation in the theory of phase transitions. In particular, in superconductivity theory the order parameter is a complex number, while all the thermodynamic quantities are real.

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[1)]By supersymmetry is meant below a general symmetry with respect to transformations that intermix fermions and bosons. In field theory this term is used in a narrower sense.

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