

On the analytical approach for modeling photovoltaic systems behavior

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A B S T R A C T

The 1-diode/2-resistors electric circuit equivalent to a photovoltaic system is analyzed. The equations at particular points of the $I-V$ curve are studied considering the maximum number of terms. The maximum power point as a boundary condition is given special attention. A new analytical method is developed based on a reduced amount of information, consisting in the normal manufacturer data. Results indicate that this new method is faster than numerical methods and has similar (or better) accuracy than other existing methods, numerical or analytical.

1. Introduction

The use of renewable energy is a big concern in modern societies, and among the different sources, photovoltaic energy is one of the most relevant in terms of increase of installed power. This fact, together with other more specific applications (e.g. satellites and spacecraft), has led scientists to study the behavior of photovoltaic cells and the methods to optimize their power generation. From the middle of the twentieth century, descriptions of the mechanisms that rule the conversion of solar radiation into electric power have been published [1–6]. In addition, great efforts have been exerted to develop equivalent electrical/mathematical models to analyze the behavior of solar cells under different conditions, mainly different radiation levels and cell temperatures.

An electrical model consists in a simple circuit whose behavior fits the real behavior of a solar cell (see Figs. 1 and 2). The use of these circuit models, together with the correct definition of the electric parameters involved, is extremely important to maximize the power extracted from the cell working under real conditions. Also, the use of equivalent circuit models makes the simulation of more complex power systems that include solar cell panels possible. It should be pointed out that sometimes these power systems can have a very complicated behavior (e.g. in space applications these systems include batteries and programmed power consumptions, with important temperature gradients and different radiation levels affecting the output voltage of the solar panels, and must be optimized to ensure the survival of the satellite/spacecraft).

The photoelectric effect is responsible for transforming the radiation on the solar cells into electric energy. In general, the easiest way to characterize a solar cell is by considering a current source connected in parallel to an ideal diode (see Fig. 2a) [1,2,5].

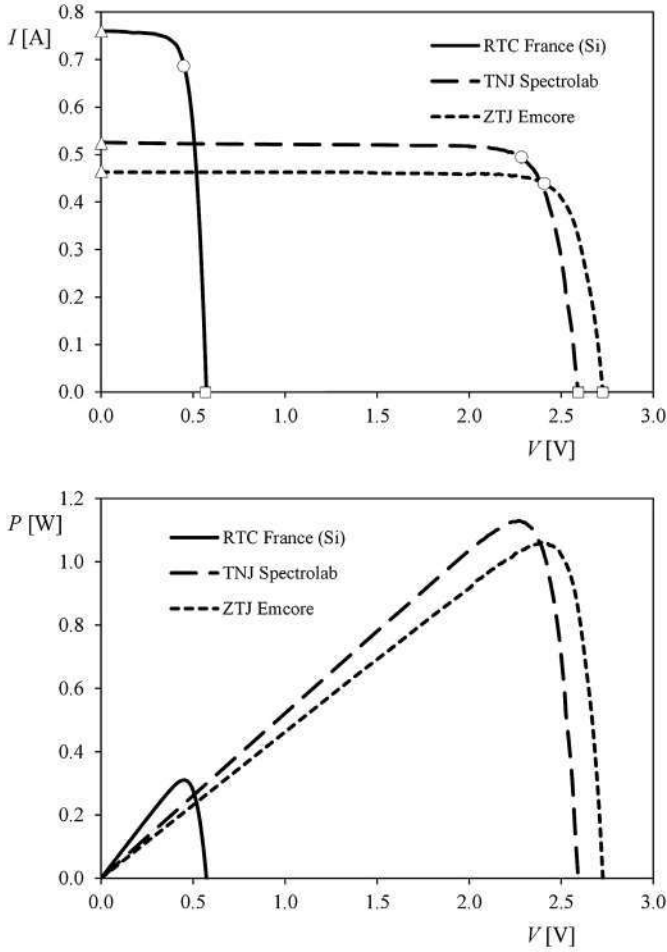


Fig. 1. Top: Current–voltage (I – V) curves of different solar cells: silicon (Si) cell from R.T.C. (La Radiotechnique Compelec, Paris, France), measured with 8096 microcomputer (Commodore, West Chester, Pennsylvania) at 33 °C [11]; TNJ triple-junction (GaInP2/GaAs/Ge) cell from Spectrolab (Sylmar, California, USA), measured at AM0 – 1353 W m²– and 28 °C; and ZTJ triple-junction (InGaP/InGaAs/Ge) cell from Emcore (Albuquerque, New Mexico, USA), measured at AM0 – 1353 W m²– and 28 °C. In every curve short circuit (triangles), maximum power (circles), and open circuit (squares) points are indicated. Bottom: Power curve of these solar cells. Data from TNJ and ZTJ solar cells extracted from the manufacturer datasheets.

The equation that describes the behavior of the solar cell is then composed of two terms, one related to the source and the other to the p–n junction (which is, in fact, Shockley’s ideal diode equation) [5]:

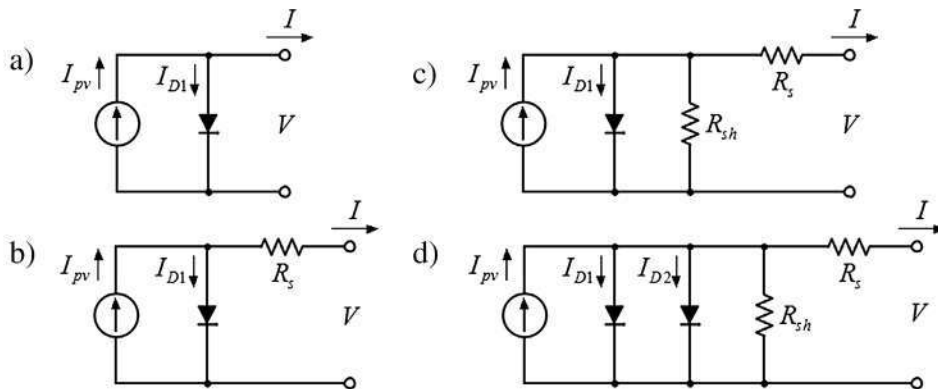


Fig. 2. Different circuit models to study the behavior of solar cells. (a) 1-diode; (b) 1-diode/1-resistor; (c) 1-diode/2-resistors; (d) 2-diodes/2-resistors.

$$I = I_{pv} - I_0 \left[\exp\left(\frac{qV}{kT}\right) - 1 \right]. \quad (1)$$

The first term of the expression above, I_{pv} , is the photocurrent delivered by the constant current source, the second term is the ideal recombination current from the diffusion and recombination of electrons and holes in p and n sides of the cell (Shockley diffusion theory), where I_0 is the reverse saturation current corresponding to it, T is the temperature and k is the Boltzmann constant. Finally, q is the charge of the electron. The last three constants are usually grouped into the so called thermal voltage, V_T :

$$V_T = \frac{kT}{q}. \quad (2)$$

To improve expression (1) and to better fit cell behavior, two resistors are usually added to the circuit (see Fig. 2c). One resistor (the shunt resistor, R_{sh}), represents the current leakage through the high conductivity shunts across the p–n junction and is added in parallel with the source and the diode. The other (the series resistance, R_s), is connected in series and represents the losses in cell solder bonds, interconnection, junction box, etc. [4,7]. Also, a non-dimensional constant, a , is added to the term of the recombination current in the p- and n-sides. This constant is called the ideality or quality factor (or sometimes emission coefficient), and it takes into account the deviation of the diodes from the Shockley diffusion theory (the value of this factor, a , is assumed to be constant and between 1 and 1.5 for one-junction cells [8,9], although some authors suggest that it depends on the ratio between the current, I , and voltage, V , of the cell [10]). The 1-diode/2-resistors circuit model is then defined by the expression:

$$I = I_{pv} - I_0 \left[\exp\left(\frac{V + IR_s}{aV_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}. \quad (3)$$

Another change to the solar cell model was proposed in 1961 by Wolf and Rauschenbach [4]. These authors suggested that the I – V characteristics of silicon solar cell are more accurately represented by a double exponential expression (see Fig. 2d), the second exponential standing for the current from the recombination of electrons and holes in the depletion region, which dominates at lower forward-bias voltages. The behavior of the solar cell can be then translated into the following equation:

$$I = I_{pv} - I_{01} \left[\exp\left(\frac{V + IR_s}{a_1 V_T}\right) - 1 \right] - I_{02} \left[\exp\left(\frac{V + IR_s}{a_2 V_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}, \quad (4)$$

where I_{01} and I_{02} are the saturation currents of each diode, and a_1 and a_2 the ideality factors that take into account the deviation of the diodes from the Shockley diffusion theory, a_1 is close to 1 while a_2 is frequently greater than 2.

The above 2-diode/2-resistors electrical model has been used by some authors to study solar cell performance [7,12–15]. However, it is quite complicated, since up to seven constants (only six if, as said, the hypothesis $a_1 \sim 1$ is assumed, or five if $a_2 \sim 2$ is also assumed) must be defined by experimental testing at defined levels of irradiance and temperature. On the other hand, it should be said that the simpler and more frequently used 1-diode/2-resistors model correctly represents the behavior of the solar cell around the maximum power point, that is, at high voltage levels [16,17]. This model simplifies the analysis of the solar cell behavior as a function of the different circuit variables [18–21], and has been used to analyze the effect of the irradiance and the temperature on the cell behavior [22,23].

An even more simplified version of the solar cell circuit, the 1-diode/1-resistor model (Fig. 2b), has been proposed by some authors to study a specific aspect of the solar cell, such as the series resistor or as its behavior under particular conditions [4,24]. Finally, it should be also mentioned that each model can be the best option depending on the solar cell characteristics [14,25]. The above theoretical approximations to solar cell behavior are widely accepted, nevertheless, some new models have arisen describing the solar cell as a multiple-zone element, with different electrical behavior within these zones [26,27].

Once the circuit model has been chosen to study a particular solar cell (or solar cell array), it must be adjusted (that is, the value of the circuit parameters must be estimated as accurately as possible). These calculations can be based on the calibration results of the cell, that is, once the I - V curve has been measured under certain irradiance and temperature conditions in a laboratory, the parameters of the model can be adjusted to give the best possible fit to this curve [11,28–33]. However, sometimes the only information available to adjust the selected circuit model comes from the manufacturer, and it is limited to only certain points of the I - V curve (short circuit, open circuit, and maximum power points, see Fig. 1) [8,18,34–38]. Finally, with regard to the methods developed to adjust the parameters of the selected circuit model, some of them are numerical [14,15,19,20,39,40] whereas some others are analytical [2,41–43].

Analytical methods have the advantage of being simple and fast. However, such methods are normally based on experimental characteristics of the I - V curve, that is, they require extensive testing results [44]. On the other hand, some authors have developed numerical methods to adjust the electric circuit parameters to the mentioned characteristic point of the curve [8,9,38]. This approach is quite interesting, as it uses only a few data to allow final users to analyze the performance of photovoltaic systems.

In the present work an analytical method for photovoltaic equivalent electric circuit parameters extraction is presented. This methodology is based only on manufacturers' data. As far as the

authors know, and according to a recent review [44], this approach to the parameter extraction problem does not seem to have been studied as yet. The analytical procedure is explained in Section 2 of the present work, whereas some application examples are included in Section 3, together with a comparison with the results from other numerical and analytical methods.

2. Solar cell modeling, the 1-diode/2-resistors model

A brief discussion on this model is included here as the basis to explain the work performed in the present paper. As stated above, the 1-diode/2-resistors circuit model is one of the most commonly used to study the behavior of solar cells and photovoltaic systems. The equation that describes the solar performance (that is, the relationship between electric current, I , and voltage V) of this model is Equation (3) [20], reproduced here for convenience:

$$I = I_{pv} - I_0 \left[\exp\left(\frac{V + IR_s}{aV_T}\right) - 1 \right] - \frac{V + IR_s}{R_{sh}}, \quad (5)$$

where I_{pv} is the photovoltaic current, I_0 is the saturation current of the diode, R_s is the series resistance, R_{sh} is the shunt resistance, and a the ideality factor of the diode. Finally, the thermal voltage of the cell, V_T , is a known quantity defined by Equation (2).

Obviously, prior to the use of this model to simulate the cell behavior it is necessary to identify the five parameters, I_{pv} , I_0 , R_s , R_{sh} and a . To calculate them, five boundary conditions extracted from the solar cell I - V curve are needed. These boundary conditions can be obtained either from the manufacturer's data, or by testing the solar cell. As an example of the data normally available, the values of the most representative points (short circuit: $V = 0$, $I = I_{sc}$; open circuit: $V = V_{oc}$, $I = 0$; and maximum power: $V = V_{mp}$, $I = I_{mp}$ points) of the measured I - V curve from different solar cells are included in Table 1, together with the temperature during the test. These data must be translated into boundary conditions to obtain the five parameters of the model. The values of the 1-diode/2-resistors circuit model parameters related to the cells from Table 1 are included in Table 2. These parameters were obtained by the authors both numerically or analytically depending on each case.

As mentioned, most solar cells manufacturers include in the specifications datasheet, at the least, information with regard to the most representative points of the I - V curve, that is, short circuit, open circuit, and the maximum power point. With these three points (indicated in Fig. 1 for three different solar cells), it is possible to derive four boundary conditions for Equation (5). Then, if no other information with regard to the I - V curve is available either from the manufacturer or from a testing regime, it is possible to obtain a solution that represents an approximation to the solar cell performance, as a function of the ideality factor, a . For single junction solar cells, it is suggested to give an initial value of this factor in the bracket [1, 1.5], in order to reduce the number of parameters to four [8,9,38]. As the curvature of the I - V curve is

Table 1

I - V curve data (short circuit $-V = 0$, $I = I_{sc}$, open circuit $-V = V_{oc}$, $I = 0$, and the maximum power $-V = V_{mp}$, $I = I_{mp}$ points; the slopes of the I - V curve at the open circuit and short circuit points, R_{s0} and R_{sh0}) of several solar cells. (CdS Cadmium sulfide solar cell (*)); BSC silicon solar cell –SAT, Paris, France–; GSC silicon solar cell –Radio M. J., Paris, France–; Q6LM™ silicon cell –Q-Solar, Calgary, Canada–).

Reference	Cell	V_{oc} {V}	I_{sc} {A}	V_{mp} {V}	I_{mp} {A}	R_{sh0} { Ω }	R_{s0} { Ω }	T {K}
Kennerud, 1969 [20]	CdS	0.420	0.804	0.316	0.698	20	0.08	
Charles, 1981 [21]	BSC	0.536	0.1023	0.437	0.0925	1000 ± 30	0.45 ± 0.01	300
Charles, 1981 [21]	GSC	0.524	0.561	0.390	0.481	25.9 ± 0.8	0.162 ± 0.005	307
Lo Brano, 2010 [43]	Q6LM	0.608	7.665	0.513	7.174	9.967	0.00443	298

(*) No information concerning the manufacturer of this cell was included in Ref. [20].

Table 2
1-Diode/2-resistors circuit model parameter values from different solar cells.

Reference	Cell	R_s (Ω)	R_{sh} (Ω)	I_{pv} (A)	I_0 (A)	a
Kennerud, 1969 [20]	CdS	0.03	20.3	0.805	$1.84 \cdot 10^{-5}$	1.37
Charles, 1981 [21]	BSC	0.070 ± 0.009	1000 ± 50	0.1023 ± 0.0005	$(1.10 \pm 0.05) \cdot 10^{-7}$	1.51 ± 0.07
Charles, 1981 [21]	GSC	0.08 ± 0.01	26 ± 1	0.5625 ± 0.0005	$(6 \pm 3) \cdot 10^{-6}$	1.72 ± 0.08
Lo Brano, 2010 [43]	Q6LM	$7.7315 \cdot 10^{-5}$	9.9672	7.65549	$7.87236 \cdot 10^{-8}$	1.28670

affected by this parameter, its value may be adjusted once the other parameter values have been calculated [40].

The short-circuit conditions once introduced in Equation (5), lead to the following expression [20]:

$$I_{sc} = I_{pv} - I_0 \left[\exp\left(\frac{I_{sc}R_s}{aV_T}\right) - 1 \right] - \frac{I_{sc}R_s}{R_{sh}}. \quad (6)$$

In Table 3 the values of each term of this expression are included. These values, obtained from different references (see Tables 1 and 2), can be used to evaluate the relative importance of each term. As a result, the second term of the right side of (6) can be neglected [20,45], and the expression above can be rewritten as:

$$I_{pv} = \frac{R_{sh} + R_s}{R_{sh}} I_{sc}. \quad (7)$$

From the open circuit conditions, Equation (5) can be rewritten as [20].

$$0 = I_{pv} - I_0 \left[\exp\left(\frac{V_{oc}}{aV_T}\right) - 1 \right] - \frac{V_{oc}}{R_{sh}} \quad (8)$$

Again, it is possible to estimate the magnitude of each term of the above equation (see Table 4). Some authors leave aside the last term of expression (8) [8,37], whereas others keep it [20,42,45]. Taking into account that the value of this term is higher than the lowest one remaining in expression (6), it has been conserved in the present calculations. Finally, from Equations (7) and (8), the saturation current, I_0 , can be reduced to:

$$I_0 = \frac{(R_{sh} + R_s)I_{sc} - V_{oc}}{R_{sh} \exp\left(\frac{V_{oc}}{aV_T}\right)} \quad (9)$$

If Equation (5) is evaluated at the maximum power point, the following expression is obtained:

$$I_{mp} = I_{pv} - I_0 \left[\exp\left(\frac{V_{mp} + I_{mp}R_s}{aV_T}\right) - 1 \right] - \frac{V_{mp} + I_{mp}R_s}{R_{sh}}. \quad (10)$$

Once more, analyzing relative magnitude of the different terms of the expression above, it is possible to neglect the second term inside the brackets. Finally, the following expression without dependence of I_{pv} and I_0 can be derived from Eqs. (7), (9) and (10) [45]:

$$I_{sc} - \left(I_{sc} - \frac{V_{oc} - R_s I_{sc}}{R_{sh}} \right) \left[\exp\left(\frac{V_{mp} + I_{mp}R_s - V_{oc}}{aV_T}\right) \right] - \frac{V_{mp} + I_{mp}R_s - I_{sc}R_s}{R_{sh}} = I_{mp}. \quad (11)$$

The fourth condition to be analyzed to obtain the four parameters is the power peak at the maximum power point. This condition has been obviated by some authors in analytical formulations [41,42]. However, for a better simulation around the maximum power point it is quite important to force the solution to satisfy it. As it is well known, the electric power can be expressed as:

$$P = IV, \quad (12)$$

which, differentiated once with respect to V , results in Ref. [20]:

$$\frac{\partial P}{\partial V} = V \frac{\partial I}{\partial V} + I = 0. \quad (13)$$

Imposing the peak power condition ($P = 0$) to the equation above, the following expression is obtained [20,46]:

$$\left(\frac{\partial I}{\partial V} \right) \Big|_{[I_{mp}, V_{mp}]} = -\frac{I_{mp}}{V_{mp}}. \quad (14)$$

Taking also into account Equation (5) differentiated once with respect to V , it is possible to obtain the following expression from Equation. (14):

$$\frac{dI}{dV} = -\frac{I_0}{aV_T} \left(1 + \frac{dI}{dV} R_s \right) \left[\exp\left(\frac{V + IR_s}{aV_T}\right) \right] - \frac{1}{R_{sh}} \left(1 + \frac{dI}{dV} R_s \right). \quad (15)$$

On the other hand, taking into account expressions (7), (9) and (11), it is possible to derive from Equation. (15) an implicit expression of the series resistor, R_s , as a function of the initial parameters:

$$\frac{aV_T V_{mp} (2I_{mp} - I_{sc})}{(V_{mp} I_{sc} + V_{oc} (I_{mp} - I_{sc})) (V_{mp} - I_{mp} R_s) - aV_T (V_{mp} I_{sc} - V_{oc} I_{mp})} = \exp\left(\frac{V_{mp} + I_{mp} R_s - V_{oc}}{aV_T}\right). \quad (16)$$

And finally, the above expression combined with Equation (11) leads to the final expression of the shunt resistor, R_{sh} , as a function of R_s and the initial parameters:

$$R_{sh} = \frac{(V_{mp} - I_{mp} R_s) (V_{mp} - R_s (I_{sc} - I_{mp}) - aV_T)}{(V_{mp} - I_{mp} R_s) (I_{sc} - I_{mp}) - aV_T I_{mp}}. \quad (17)$$

As stated above, the use of the above equations is based on a previous estimation of the ideality factor, a . An alternative to this procedure consists in using other specific characteristics of the $I-V$ curve, such its slopes at the short circuit and open circuit points, R_{sh0} and R_{s0} respectively:

Table 3
Values of every term from Equation (6), obtained for different solar cells (calculated with coefficients from Tables 1 and 2).

Reference	Cell	I_{sc}	I_{pv}	$I_0 \left[\exp\left(\frac{I_{sc}R_s}{aV_T}\right) - 1 \right]$	$\frac{I_{sc}R_s}{R_{sh}}$
Kennerud, 1969 [20]	CdS	0.8040	0.8050	$1.56 \cdot 10^{-5}$	$1.19 \cdot 10^{-3}$
Charles, 1981 [21]	BSC	0.1023	0.1023	$2.21 \cdot 10^{-8}$	$7.16 \cdot 10^{-6}$
Charles, 1981 [21]	GSC	0.5610	0.5625	$1.01 \cdot 10^{-5}$	$1.73 \cdot 10^{-3}$
Lo Brano, 2010 [43]	Q6LM	7.6650	7.6555	$1.42 \cdot 10^{-9}$	$5.95 \cdot 10^{-5}$

Table 4

Values of every term from Equation (8), obtained for different solar cells (calculated with coefficients from Tables 1 and 2).

Reference	Cell	I_{pv}	$I_0 \left[\exp\left(\frac{V_{oc}}{aV_T}\right) \right]$	I_0	$\frac{V_{oc}}{R_{sh}}$
Kennerud, 1969	CdS	0.8050	0.7996	$1.84 \cdot 10^{-5}$	$2.07 \cdot 10^{-2}$
Charles, 1981	BSC	0.1023	0.1006	$1.10 \cdot 10^{-7}$	$5.36 \cdot 10^{-4}$
Charles, 1981	GSC	0.5625	0.5992	$6.00 \cdot 10^{-6}$	$2.02 \cdot 10^{-2}$
Lo Brano, 2010	Q6LM	7.6555	7.6660	$7.87 \cdot 10^{-8}$	$6.10 \cdot 10^{-2}$

$$R_{sh0} = - \left(\frac{\partial V}{\partial I} \right) \Big|_{I=I_{sc}}, \quad (18)$$

$$R_{s0} = - \left(\frac{\partial V}{\partial I} \right) \Big|_{V=V_{oc}}, \quad (19)$$

These slopes of the I - V curve have frequently been used in calculations to fit the electric model behavior to the experimental results [20,21]. With regard to the practical use of these parameters, empirical methods to estimate them have been recently developed for photovoltaic arrays [47].

To calculate the ideality factor, a , it seems better to use the slope at the short circuit point, R_{sh0} , instead of the slope at the open circuit point, R_{s0} , as the additional boundary condition. In this way, the final expressions of the method have a more simple form. It should also be mentioned that some authors prefer to use this boundary condition in their calculations to fit their models to the I - V curve [18,45], instead of the slope of the curve at the open circuit point (expression (19)).

Once the value of R_{sh0} has been estimated, the additional boundary condition at the short circuit point can be expressed as [42]:

$$\frac{1}{R_{sh}} - \frac{1}{R_{sh0} - R_s} + \frac{I_0}{aV_T} \exp\left(\frac{R_s I_{sc}}{aV_T}\right) = 0. \quad (20)$$

Taking into account a magnitude analysis of the terms included in the above equation (see Table 5), it can be simplified as:

$$\frac{1}{R_{sh}} - \frac{1}{R_{sh0} - R_s} = 0, \quad (21)$$

which leads to the following expression:

$$R_{sh0} = R_{sh} + R_s. \quad (22)$$

Some authors [18,41,42,45], have simplified the above equation as:

$$R_{sh0} = R_{sh}. \quad (23)$$

Nevertheless, in the present work the whole expression (22) has been kept, because in the case of solar panels the ratio R_s/R_{sh} seems to grow directly as a function of the number of cells connected in series, so there may be cases where R_s should not be neglected.

Table 5

Values of every term from Equation (20), obtained for different solar cells (calculated with coefficients from Tables 1 and 2).

Reference	Cell	$\frac{1}{R_{sh}}$	$\frac{1}{R_{sh0} - R_s}$	$\frac{I_0}{aV_T} \exp\left(\frac{I_{sc} R_s}{aV_T}\right)$
Kennerud, 1969	CdS	0.049	0.050	$8.64 \cdot 10^{-4}$
Charles, 1981	BSC	0.001	0.001	$3.38 \cdot 10^{-6}$
Charles, 1981	GSC	0.038	0.039	$3.53 \cdot 10^{-4}$
Lo Brano, 2010	Q6LM	0.100	0.100	$2.42 \cdot 10^{-6}$

Table 6

Manufacturer's data from a KC200GT solar array [8,9]. The corresponding values from the equivalent electric circuit calculated with the proposed method have also been included.

	Manufacturer's data	Present method
I_{mp} {A}	7.61	7.6100
V_{mp} {V}	26.3	26.3000
P_{mp} {W}	200.143	200.1430
I_{sc} {A}	8.21	8.2100
V_{oc} {V}	32.9	32.9000
N	54	54

With Equations (22) and (17) it is possible to derive an expression that gives the ideality factor as a function of constants already known, R_s and the slope R_{sh0} :

$$aV_T = \frac{(V_{mp} - I_{mp}R_s)(V_{mp} + (I_{mp} - I_{sc})R_{sh0})}{(V_{mp} - I_{mp}R_{sh0})}. \quad (24)$$

Finally, introducing Equation (24) in expression (16) and reorganizing the remaining terms, the value of the series resistor, R_s , is obtained:

$$R_s = \frac{(A - B)V_{mp}}{(A + B)I_{mp}} + \frac{B}{(A + B)} \frac{V_{oc}}{I_{mp}}, \quad (25)$$

where:

$$A = (V_{mp} + (I_{mp} - I_{sc})R_{sh0}) \log\left(\frac{V_{mp} + (I_{mp} - I_{sc})R_{sh0}}{V_{oc} - I_{sc}R_{sh0}}\right), \quad (26)$$

and,

$$B = V_{mp} - R_{sh0}I_{mp}. \quad (27)$$

3. Results and discussion

In Table 6 the manufacturer's data of a KC200GT solar array (Kyocera, Scottsdale, Arizona), is included [8,9]. These parameters were measured at 25 °C and A.M1.5 (1000 W m⁻²). In Table 7 the results obtained with the present method are included, together with the ones from the numerical method proposed by Villalva et al. (2009). The procedure followed for the present calculations can be summarized as:

- Estimate the parameter a . In this case, the value of the ideality factor was chosen to be the same selected by Villalva et al. (2009). See Table 7.
- Obtain R_s from expression (16).
- Obtain R_{sh} from expression (17).
- Obtain I_0 from expression (9).
- Obtain I_{pv} from expression (7).

It can be observed in Table 7 that the results are very similar to the ones obtained by the other authors. Also, in Table 6 the values of

Table 7

Parameters of 1-diode/2-resistors circuit model adjusted with the presented methodology to the KC200GT solar array. The results from Refs. [8,9], have also been included in the table.

Model	a	R_s {Ω}	R_{sh} {Ω}	I_0 {A}	I_{pv} {A}
Present method – analytical–	1.3	0.2308	597.3855	$9.7631 \cdot 10^{-8}$	8.2132
Villalva (2009) – numerical–	1.3	0.221	415.405	$9.825 \cdot 10^{-8}$	8.214

Table 8

I - V curve from a Photowatt-PWP 201 solar module composed of 36 solar cells connected in series [11].

Photowatt-PWP 201 solar module	
V [V]	I [A]
-1.9426	1.0345
0.1248	1.0315
1.8093	1.0300
3.3511	1.0260
4.7622	1.0220
6.0538	1.0180
7.2364	1.0155
8.3189	1.0140
9.3097	1.0100
10.2163	1.0035
11.0449	0.9880
11.8018	0.9630
12.4929	0.9255
12.649	0.9120
13.1231	0.8725
14.2221	0.7265
14.6995	0.6345
15.1346	0.5345
15.5311	0.4275
15.8929	0.3185
16.2229	0.2085
16.5241	0.1010
16.7987	-0.0080
17.0499	-0.1110
17.2793	-0.2090
17.4885	-0.3030

Table 10

Parameters of 1-diode/2-resistors circuit model adjusted with the presented methodology to the Photowatt-PWP 201 solar module behavior (Table 8). The results from other authors have also been included in the table.

Model	a	R_s (Ω)	R_{sh} (Ω)	I_0 (A)	I_{pv} (A)
Present method – analytical–	45.1958	1.3535	559.6804	$1.3214 \cdot 10^{-6}$	1.0342
Phang et al. (1984) – analytical– [42]	63.3689	0.0832	561.0340	$6.4049 \cdot 10^{-5}$	1.0319
Easwarakhanthan (1986) [11]	48.4500	1.2057	549.4500	$3.2875 \cdot 10^{-6}$	1.0318
AlHajri et al. (2012) [51]	48.2889	1.2053	714.2857	$3.1756 \cdot 10^{-6}$	1.0313
Bouzidi et al. (2007) [48]	48.1862	1.2030	555.5556	$3.0760 \cdot 10^{-6}$	1.0339
AlRashidi et al. (2011) [31]	48.5862	1.1968	555.5556	$3.4360 \cdot 10^{-6}$	1.0441
Wei et al. (2011) [49]	52.2430	1.0755	1850.1000	$8.3010 \cdot 10^{-6}$	1.0286
El-Naggar et al. (2012) [50]	48.8211	1.1989	833.3333	$3.6642 \cdot 10^{-6}$	1.0331
Peng et al. (2013) [30]	48.3221	1.2132	625.0000	$3.2212 \cdot 10^{-6}$	1.0313
Gong (2013)	48.6428	1.2013	981.9822	$3.4823 \cdot 10^{-6}$	1.0305

$$R_{s0} = C_s \frac{V_{oc}}{I_{sc}}, \quad (28)$$

$$R_{sh0} = C_{sh} \frac{V_{oc}}{I_{sc}}, \quad (29)$$

where $C_s = 0.11175$ and $C_{sh} = 34.49692$. Based on the above expressions $R_{s0} = 1.817$ and $R_{sh0} = 561.034$ for the analyzed solar array. The procedure followed for the present calculations can be summarized as follows:

- Estimate R_{sh0} with expression (31).
- Obtain R_s from expression (25).
- Obtain a from expression (24).
- Obtain R_{sh} from expression (17).
- Obtain I_0 from expression (9).
- Obtain I_{pv} from expression (7).

In Table 10 the calculated parameters of the 1-diode/2-resistors circuit model equivalent to the mentioned solar module are included. In this table the results obtained numerically by other authors have been also included, this information has been summarized by AlRashidi et al. [31]. In Fig. 3, the difference between the calculated and the benchmark experimental I - V curve from Table 8, $I-I_{curve}$, is shown for the mentioned solar module. Results from the other authors have also been included in the figure. It can be observed that with the proposed analytical method based on manufacturers' data, it is possible to derive the 1-diode/2-resistors circuit model parameters, obtaining similar results to other more complicated numerical methods.

To globally compare the results, some authors have used the non-dimensional standard deviation, SD , as defined by Easwarakhanthan [11]:

$$SD = \sqrt{\frac{1}{m} \sum_{i=1}^m \left(\frac{I_{calc,i}}{I_i} - 1 \right)^2}, \quad (30)$$

where $I_{calc,i}$ and I_i are, respectively, the current calculated with the electric model and the measured current at a certain point, i , of the I - V curve, and m is the number of points on the curve. This

the characteristic points of the I - V curve, obtained from the calculated equivalent circuit have been included. As expected, they are quite exact, validating all the simplifications made within the development of the present method.

As said, an additional validation has been made to check this method, based on a solar panel benchmarking I - V curve from Refs. [11], see Table 8. This experimentally measured I - V curve corresponds to a Photowatt-PWP 201 solar module (Photowatt, Bourgoin-Jallieu, France), composed of 36 solar cells connected in series and measured at 45 °C. It has been widely used by different authors as benchmark to check electric circuit models and methods developed for parameter extraction [30–33,48–51]. The three characteristic points of the I - V curve are included in Table 9, these points were extrapolated in Ref. [11] from the experimental data. In order to check the present method, the results from a 1-diode/2-resistors equivalent circuit calculated from the data included in Table 9 are compared to the results from Table 8. Also, the results are compared to the ones resulting from Phang et al. (1984) analytical method, based on the mentioned three characteristic points of the I - V curve (estimating the slopes of the I - V curve at the open circuit and short circuit points with these data). In this case, the starting point is the estimation of the slope R_{sh0} , instead of the ideality factor, a . Following the empirical procedure described by Orioli and Di Gangi (2013), the slopes of the I - V curve at the open circuit and short circuit points are [47]:

Table 9

Values of the current and voltage at short circuit, open circuit, and maximum power points (I_{sc} , V_{oc} , V_{mp} , I_{mp}), calculated for a Photowatt-PWP 201 solar module (see Ref. [11]) from experimental data.

Photowatt-PWP 201 solar module	
V_{oc} [V]	16.7785
I_{sc} [A]	1.0317
V_{mp} [V]	12.6490
I_{mp} [A]	0.9120

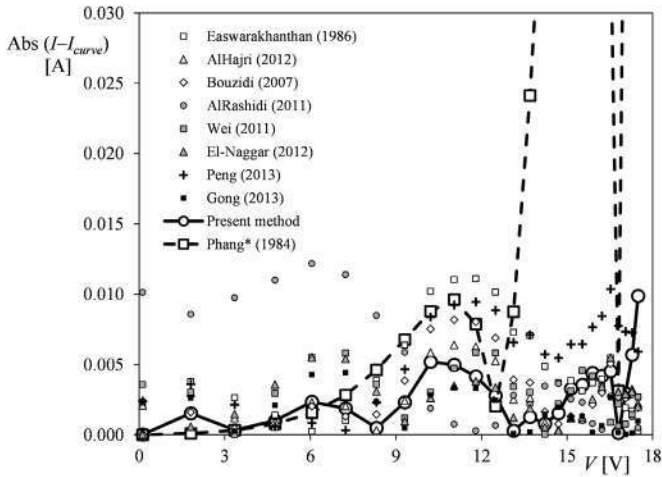


Fig. 3. Difference between the calculated and the experimental I - V curves with regard to the Photowatt-PWP 201 solar module. (*) Some points with regard to the Phang method are out of scale.

Table 11

Non-dimensional standard deviation, ξ , calculated, using the proposed method, with respect to the experimental values of the I - V curves of the R.T.C. France silicon cell, and the Photowatt-PWP 201 solar module. Results from other numerical methods from the available literature have also been included. See also Fig. 3.

Model	Photowatt-PWP 201 solar module	
	ξ	ξ (*)
Present method	$2.85 \cdot 10^{-3}$	$2.86 \cdot 10^{-3}$
Phang (1984) [42]	$3.44 \cdot 10^{-2}$	$2.05 \cdot 10^{-2}$
Easwarakhanthan (1986) [11]	$5.90 \cdot 10^{-3}$	$8.42 \cdot 10^{-3}$
AlHajri (2012) [51]	$3.23 \cdot 10^{-3}$	$4.15 \cdot 10^{-3}$
Bouzidi (2007) [48]	$3.89 \cdot 10^{-3}$	$5.53 \cdot 10^{-3}$
AlRashidi (2011) [31]	$6.02 \cdot 10^{-3}$	$2.22 \cdot 10^{-3}$
Wei (2011) [49]	$3.61 \cdot 10^{-3}$	$4.04 \cdot 10^{-3}$
El-Naggar (2012) [50]	$2.84 \cdot 10^{-3}$	$2.44 \cdot 10^{-3}$
Peng (2013) [30]	$6.16 \cdot 10^{-3}$	$7.49 \cdot 10^{-3}$
Gong (2013)	$2.20 \cdot 10^{-3}$	$2.02 \cdot 10^{-3}$

parameter can compare results from different I - V curves. However, it excessively takes into account the errors around the open circuit point. As a solution, it seems better to define a non-dimensional standard deviation, ξ , based on the RMSE parameter proposed by Askarzadeh [32,33], divided by the short circuit current, I_{sc} , in order to make it non-dimensional:

$$\xi = \frac{\text{RMSE}}{I_{sc}} = \frac{1}{I_{sc}} \sqrt{\frac{1}{m} \sum_{i=1}^m (I_{cal,i} - I_i)^2}. \quad (31)$$

In Table 11 the values of the comparison parameter, ξ , are included for the solar module. Two different calculations have been done, in the first the voltage range $[0, V_{oc}]$ is taken into account, whereas in the second only a $\pm 10\%$ bracket of the open circuit voltage, V_{oc} , around the maximum power point is taken into account. The results suggest that, for the studied solar panel, with a good knowledge of the characteristic points of the I - V curve and applying the present analytical method, it is possible to define an equivalent circuit that reproduces the experimental I - V curve with similar accuracy to the ones calculated with numerical methods.

4. Conclusions

In the present study, a new analytical method to extract the parameters of a 1-diode/2-resistors electric circuit equivalent to a

photovoltaic array is developed and analyzed. The major conclusions resulting from this work are:

- The method is based on the manufacturer's datasheet of the photovoltaic system (short circuit, maximum power and open circuit points).
- Terms in the equations neglected in the analytical models of previous authors are considered relevant enough to be introduced in the final expressions without increasing their complexity.
- The first derivative of the power with respect to the output voltage equal to zero is considered as a boundary condition in the method developed in the present study. As far as the authors know, this boundary condition is not considered in other analytical methods.
- The above two conditions ensure better results around the maximum power point.
- Results indicate that the calculated equivalent electric circuit reproduces the I - V curve with similar, and even better, accuracy to numerical methods with higher computational requirements.

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