Technical Note

Improved analytical method to study the cup anemometer performance

Santiago Pindado, Alvaro Ramos-Cenzano and Javier Cubas

Abstract

The cup anemometer rotor aerodynamics is analytically studied based on the aerodynamics of a single cup. The effect of the rotation on the aerodynamic force is included in the analytical model, together with the displacement of the aerodynamic center during one turn of the cup. The model can be fitted to the testing results, indicating the presence of both the aforementioned effects.

Keywords: anemometer calibration, cup anemometer, cup aerodynamics, rotor aerodynamics, Fourier analysis, analytical model

(Some figures may appear in colour only in the online journal)

1. Introduction

Despite being invented in the 19th century, the cup anemometer remains the most commonly used wind speed sensor for the wind energy industry. This industrial sector is a massive consumer of cup anemometers as they are needed for wind turbine control and wind energy production forecast [1]. The cup anemometer was thoroughly studied during the 20th century, the first analysis being carried out in the last decades of the previous century (see in [2] a quite complete review of the research done in relation to this instrument). Leaving aside effects such as *overspeeding* or the vertical flow, the cup anemometer performance is generally associated with the transfer function that translates the rotation speed (or a related variable such as the anemometer's output signal) into a wind speed measurement [3, 4].

The 2-cup positions analytical model has been traditionally used to study the cup anemometer performance in both the steady and transitional states [5–7]. This model gives information based on the aerodynamic forces on a single cup, considering its angle in relation to the incoming wind flow but not any rotational effect. The 3-cup analytical model [8, 9] represents an improvement, not only in relation to the results [4], but also in relation to the possibilities of the model. The effects of rotor asymmetries (rotor damaged, or one cup missing) have been successfully studied with the 3-cup analytical model [4, 10, 11].

In the previous work 'Fourier analysis of the aerodynamic behavior of cup anemometers' [12], the 3-cup analytical model was used to analyze the cup anemometer performance (i.e. its rotational speed as a function of the wind speed), from the aerodynamics of a single cup, the normal-to-the-cup aerodynamic force coefficient being expressed in terms of Fourier harmonic series. Although the results were quite accurate when different cup shapes were analyzed, some differences arose when the ratio, $r_r = R_c/R_{rc}$, of the cup radius, R_c, to the cups' center rotation radius, R_{rc}, was considered. In figure 1 the results of that work are plotted. In the figure, the anemometer constant, K, defined as the wind speed, V, divided by the cups' center rotation speed, ωR_{rc}:

$$K = \frac{V}{\omega R_{rc}} = \frac{A_r f_r + B}{\omega R_{rc}} \sim \frac{A_r}{2\pi R_{rc}}, \qquad (1)$$

where, obviously, ω is the mean rotation speed of the anemometer (the rotation speed at a constant and uniform wind speed is composed by an average term plus several harmonic

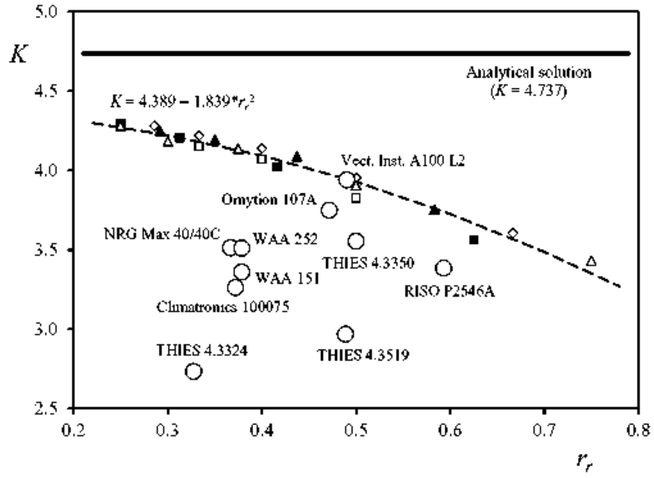

Figure 1. Anemometer factor, K, corresponding to different 90°-conical-cups rotors tested on an Ornytion 107 A anemometer [14], as a function of the ratio between the cups' radius and the cups' center rotation radius, $r_r = R_c/R_{rc}$. The symbols stand for the following cups' radius: $R_c = 20$ mm (open squares), $R_c = 25$ mm (closed squares), $R_c = 30$ mm (open triangles), $R_c = 35$ mm (closed triangles), $R_c = 40$ mm (open rhombi). The analytical solution (solid line), calculated with the 3-cup analytical model, has been added to the graph together with the results from calibrations performed on some commercial anemometers (open circles). A quadratic fit to this experimental data has also been added (dashed line). From [12].

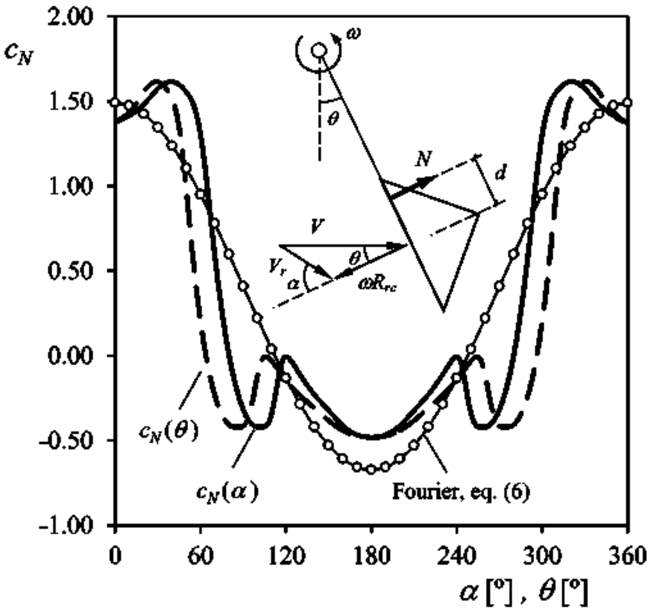

Figure 2. Normal aerodynamic force coefficient, c_N, of the Brevoort & Joyner Type-II (conical) cups [15], plotted as a function of the wind direction with respect to the cup, α, and with respect to the rotor position, θ (calculated for $K = 3.5$). The 1-harmonic Fourier series expansion of $c_N(\alpha)$, equation (6), has been added to the graph. Data from [10].

terms [10, 13]), A_r and B are the calibration constants of the anemometer referred to the rotation frequency, f_r, (and not to the output frequency, f, see [3]):

$$V = A_r f_r + B. \tag{2}$$

The previous equation is also known as the transfer function of the anemometer. Normally, the offset velocity, B, is not taken into account when calculating the anemometer factor, as it is smaller than the wind velocity. Therefore, it can be assumed that it only depends on the slope of the aforementioned transfer function, A_r, and the cups' center rotation radius, R_{rc}.

In figure 1, a quadratic equation has been fitted to the experimental data, this choice was done as for $r_r \to 0$ (either $R_c \to 0$, or $R_{rc} \to \infty$) and it seems reasonable to think that the solution should be asymptotic with zero slope. However, this quadratic expression is not clear [2]. Taking into account the testing results from [14], the slope of the cup anemometer calibration curve based on the rotation frequency, A_r, can be expressed as a function of R_{rc} and the cups' front area, S_c, ($S_c = \pi R_c^2$):

$$A_r = \frac{dA_r}{dR_{rc}} R_{rc} - S_c(\zeta + \eta S_c^{-\xi}), \tag{3}$$

therefore, the anemometer constant can be expressed as [2]:

$$K = \frac{1}{2\pi}\left(\frac{dA_r}{dR_{rc}} - \zeta' R_c r_r - \frac{\eta'}{R_c^{2\xi-1}} r_r\right) = \frac{1}{2\pi}\left(\frac{dA_r}{dR_{rc}} - \zeta' R_{rc} r_r^2 - \frac{\eta'}{R_c^{2\xi-1}} r_r\right). \tag{4}$$

Consequently, from the testing results it seems correct to suppose that the anemometer factor, K, depends, at least linearly, on the ratio r_r.

In the aforementioned work 'Fourier analysis of the aerodynamic behavior of cup anemometers', a mathematical development was carried out based on the changing local wind flow angle along the cup chord when the cup is rotating, but with no concluding results. In the present study the analytical method developed in that previous work is revised, and new terms are included in the main equation. Therefore, the aim of the present work is to improve analytically the method already described in [12] in order to obtain a better correlation with testing results, especially in relation to the effect of the r_r on the anemometer factor K.

The first term introduced is a shift in the phase angle of the normal-to-the-cup aerodynamic force coefficient simplified expression, the second one being the displacement, d, of the aerodynamic center of the cup (i.e. where the normal-to-the-cup aerodynamic force is applied) from the cup center, in relation to the rotor's position angle (see the sketch included in figure 2).

If friction is not considered, the rotor dynamics is defined by the following equation:

$$\begin{aligned} I\frac{d\omega}{dt} &= \frac{1}{2}\rho S_c R_{rc} V_r^2(\theta)\left(1 - \frac{d(\alpha(\theta))}{R_c} r_r\right) c_N(\alpha(\theta)) \\ &+ \frac{1}{2}\rho S_c R_{rc} V_r^2(\theta + 120°)\left(1 - \frac{d(\alpha(\theta + 120°))}{R_c} r_r\right) c_N(\alpha(\theta + 120°)) \\ &+ \frac{1}{2}\rho S_c R_{rc} V_r^2(\theta + 240°)\left(1 - \frac{d(\alpha(\theta + 240°))}{R_c} r_r\right) c_N(\alpha(\theta + 240°)) \end{aligned} \tag{5}$$

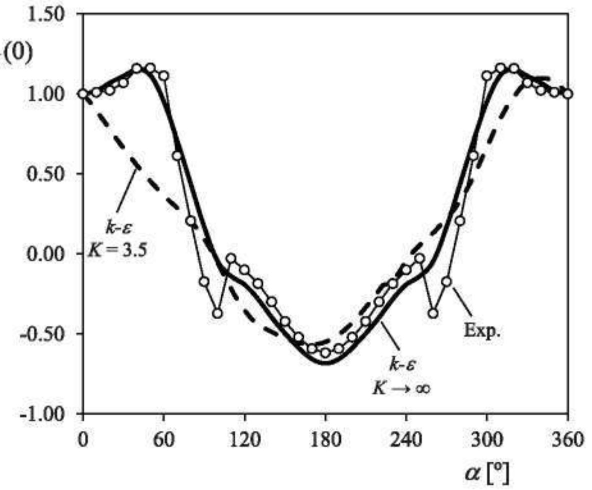

Figure 3. Non-dimensional normal aerodynamic force coefficient, $c_N/c_N(0)$ of a conical cup calculated using CFD (k-ε, turbulence model), as a function of the wind speed angle with respect to the cup, α [16]. Static ($K \to \infty$) and rotational ($K = 3.5$) cases. The experimental result corresponding to the static case [12] have been added to the graph.

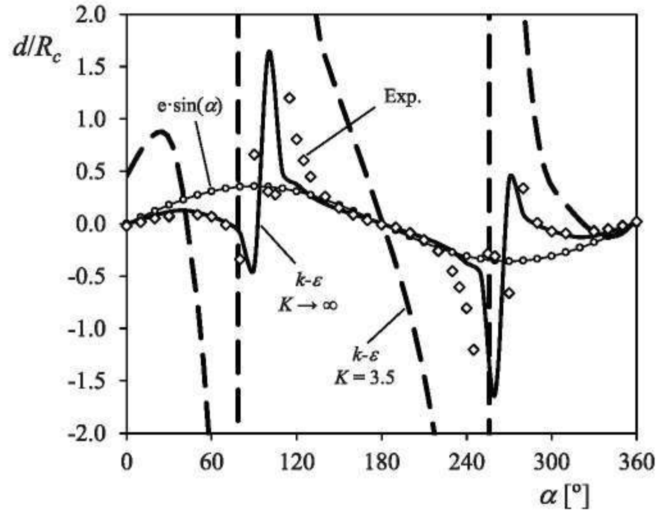

Figure 4. Non-dimensional shift of the normal-to-the-cup aerodynamic force, measured from the cup center (see also figure 2), d/R_c, as a function of the wind speed angle with respect to the cup, α. In the graph the results from CFD calculations (solid –$K \to \infty$–, and dashed lines –$K = 3.5$, with $r_r = 0.5$–) are included [16], together with testing results from a previous work [14], and the 1-harmonic Fourier approximation (equation (8)) to these experimental results.

which takes into account the contribution of each cup to the aerodynamic moment on the rotor. In the above equation, I is the rotor's moment of inertia, V_r is the wind speed relative to the cups, c_N is the aerodynamic normal force coefficient, α is the local wind direction with respect to the cups, θ is the angle of the rotor with respect to a reference line, and d is the shift of the aerodynamic normal force application point from the center of the cup (see the sketch in figure 2). The aerodynamic normal force coefficient, c_N, can be defined (in a first approximation) taking the averaged value and the first harmonic term of its decomposition in the Fourier series [12] (see figure 2):

$$c_N(\alpha) = c_0 + c_1 \cos(\alpha). \tag{6}$$

In a recent CFD (computer fluid dynamics) academic work [16], it was found that the above expression should include a phase angle, δ, that is:

$$\begin{aligned} c_N(\alpha) &= c_0 + c_1 \cos(\alpha + \delta) \\ &= c_0 + c_1 \cos(\delta)\cos(\alpha) - c_1 \sin(\delta)\sin(\alpha) \\ &= c_0 + c_{11}\cos(\alpha) - c_{12}\sin(\alpha). \end{aligned} \tag{7}$$

In figure 3 the effect of the rotation speed on the c_N coefficient related to a conical cup (at $K = 3.5$ rate, with $r_r = 0.5$), as a function of the wind direction with respect to the aforementioned cup is shown. In the figure, the differences of this case in relation to the case of the same case but without considering the rotation effects (that is, $K \to \infty$) can be clearly noted. In the mentioned academic work, the phase angle regarding a conical cup was found to be $\delta \sim 11°$–$13°$ for different values of the anemometer factor, K, the values of c_0 and c_1 coefficients being also not specially altered when compared to the static configuration of the cup (i.e. $K \to \infty$). Similar changes in the aerodynamic normal force coefficient when a cup is rotating have been also reported by Dahlberg *et al* [17] and Potsdam *et al* [18].

As already mentioned, the second term introduced in the equations is the shift, d, of the normal-to-the-cup aerodynamic force, N, from the center of the cup (see figure 2). This shift is caused by the aerodynamic (pitching) moment on the cup. In our former works this was not considered [12, 14]. However, this shift is taken into account in an interesting work published in 1895 by Chree [19]. As far as the authors know, this work represents the first analytical attempt to explain the *overspeeding* phenomenon, which had been previously observed in cup anemometer measurements in the field. In figure 4 this shift is shown as a function of the wind angle with respect to the cup. Three cases are included in the graph, the experimental results corresponding to the testing series measured in 2012 on a static conical cup [14], and the CFD results corresponding to a static ($K \to \infty$) and a rotating cup ($K = 3.5$, with $r_r = 0.5$) [16]. The results corresponding to the static cups are anti-symmetrical with respect to $\alpha = 180°$, whereas a phase angle can be observed regarding the shift d on the rotating cup. Also, a greater shift was calculated in this last case, probably produced by the 'forced' rotation of the cup (the solution for a rotor equipped with this cup had a lower value of rotation speed, $K = 5.1$). Therefore, taking into account what was previously mentioned, a reasonable first approximation to the non-dimensional shift of the normal-to-the-cup aerodynamic force, N, from the center of the cup, d/R_c, as a function of the wind speed angle with respect to the cup, α, could have the following expression:

$$\begin{aligned} \frac{d(\alpha)}{R_c} &= e\sin(\alpha + \gamma) = e\cos(\gamma)\sin(\alpha) + e\sin(\gamma)\cos(\alpha) \\ &= e_{11}\sin(\alpha) + e_{12}\cos(\alpha). \end{aligned} \tag{8}$$

2. Model development and results

From equation (5) and averaging its value along one turn, which is equal to zero [4, 8, 9, 12, 20], it is possible to derive the following expression:

$$0=\frac{1}{2\pi}\int_0^{2\pi}V_r^2(\theta)\left(1-\frac{d(\alpha(\theta))}{R_c}r_r\right)c_N(\alpha(\theta))\mathrm{d}\theta \quad (9)$$

which, taking into account [12]:

$$V_r(\theta)=\sqrt{V^2+(\omega R_{rc})^2-2V\omega R_{rc}\cos(\theta)}, \quad (10)$$

and equations (7) and (8) lead to:

$$\begin{aligned}0=&\int_0^{2\pi}\left(1+\frac{1}{K^2}-\frac{2}{K}\cos(\theta)\right)\\&(1-r_r(e_{11}\sin(\alpha)+e_{12}\cos(\alpha)))\cdot\\&(c_0+c_{11}\cos(\alpha)+c_{12}\sin(\alpha))\mathrm{d}\theta\end{aligned} \quad (11)$$

As indicated in equation (5), there is a relationship between the position angle of the rotor, θ, and the wind flow angle with respect to the cup, α, which depends on the ratio of the wind speed to the rotation speed, that is, the anemometer constant:

$$\tan(\alpha)=\frac{K\sin(\theta)}{K\cos(\theta)-1}. \quad (12)$$

Regarding the trigonometric expressions included in equation (11), the following relationship has been well established to work with a good enough level of accuracy [12]:

$$\cos(\alpha)=\eta_0+\eta_1\cos(\theta)+\eta_2\cos(\theta)^2+\eta_3\cos(\theta)^3, \quad (13)$$

where coefficients η_0, η_1, η_2, and η_3 are expressed as a function of anemometer factor K:

$$\begin{aligned}\eta_0&=\frac{-1}{\sqrt{1+K^2}};\ \eta_1=\frac{K}{\sqrt{1+K^2}}-\frac{1}{K^2-1};\\ \eta_2&=\frac{1}{\sqrt{1+K^2}};\ \eta_3=\frac{K^2}{K^2-1}-\frac{K}{\sqrt{1+K^2}}.\end{aligned} \quad (14)$$

Besides, for the present work it was necessary to develop a similar relationship for the sine function:

$$\sin(\alpha)=\xi_1\sin(\theta)+\xi_2\sin(2\theta), \quad (15)$$

where coefficients ξ_1, and ξ_2 are also expressed as a function of the anemometer factor K:

$$\xi_1=\frac{K}{\sqrt{1+K^2}};\ \xi_2=\frac{K^2}{2}\left(\frac{1}{\sqrt{K^2-1}}-\frac{1}{\sqrt{1+K^2}}\right). \quad (16)$$

Differences between equation (13) and the exact values of $\cos(\alpha)$ are lower than 0.044 ($K=2.5$) and 0.015 ($K=3.5$), whereas the differences between equation (15) and the exact values of $\sin(\alpha)$ are lower than 0.093 ($K=2.5$) and 0.048 ($K=3.5$).

Taking into account all the above expressions, equation (11) can be rewritten as:

$$\begin{aligned}0&=f(K,c_0,c_{11})+r_r(c_0e_{12}g_1(K)+c_{11}e_{12}g_2(K)+c_{12}e_{11}g_3(K))\\&=f(K,c_0,c_1\cos(\delta))\\&\quad+r_re((c_0g_1(K)+c_1\cos(\delta)g_2(K))\sin(\gamma)\\&\quad+c_1g_3(K)\sin(\delta)\cos(\gamma)),\end{aligned} \quad (17)$$

where:

$$\begin{aligned}f(K)&=\left(1+\frac{1}{K^2}\right)\left(c_0+c_{11}\left(\eta_0+\frac{1}{2}\eta_2\right)\right)-c_{11}\frac{1}{K}\left(\eta_1+\frac{3}{4}\eta_3\right)\\&=\left(1+\frac{1}{K^2}\right)\left(c_0+c_1\cos(\delta)\left(\eta_0+\frac{1}{2}\eta_2\right)\right)\\&\quad-c_1\cos(\delta)\frac{1}{K}\left(\eta_1+\frac{3}{4}\eta_3\right),\end{aligned} \quad (18)$$

and:

$$g_1(K)=\left(1+\frac{1}{K^2}\right)\left(\eta_0+\frac{1}{2}\eta_2\right)-\frac{1}{K}\left(\eta_1+\frac{3}{4}\eta_3\right), \quad (19)$$

$$\begin{aligned}g_2(K)=&\left(1+\frac{1}{K^2}\right)\left(\eta_0^2+\frac{1}{2}(2\eta_0\eta_2+\eta_1^2)+\frac{3}{8}(2\eta_1\eta_3+\eta_2^2)+\frac{5}{16}\eta_3^2\right)\\&-\frac{2}{K}\left(\eta_0\eta_1+\frac{3}{4}(\eta_0\eta_3+\eta_1\eta_2)+\frac{5}{8}\eta_2\eta_3\right),\end{aligned} \quad (20)$$

$$g_3(K)=-\left(1+\frac{1}{K^2}\right)\left(\frac{1}{2}\xi_1^2+\frac{1}{2}\xi_2^2\right)+\frac{1}{K}\xi_1\xi_2. \quad (21)$$

The above equation (17) can be solved, for a given value of r_r, if:

- variables c_0 and c_1 (related to the aerodynamic performance of a non-rotation cup) are known, and
- the respective behavior of variables δ, e and γ, as a function of the anemometer factor, K, is also known.

From previous works [10, 12], $c_0=0.348$ and $c_1=1.152$ for 90° conical cups. Besides, it seems reasonable to consider the value $\delta=12°$ for the phase angle related to the aerodynamic forces on a rotating 90° conical cup, based on the aforementioned results from [16]. If equation (17) is now fitted to testing results from [11], some insights can occur. In figure 5 the mentioned testing results related to a Climatronics 100 075 anemometer equipped with different 90°-conical-cups rotors have been included, together with the fitting of equation (17) to them. The testing results are basically the same as the ones included in figure 1, as they correspond to a similar testing campaign carried out with the same cup sets but a different anemometer. The fitting process was performed using the Fitteia program [21]. The fitting included in the figure was obtained for values of e and γ, with a specific relationship between them, see figure 6. As can be observed by comparing the analytical results obtained before (see figure 1), the method developed in the present work fits better to the testing results and reproduce the effect of the ratio, r_r, on the anemometer factor, K, (i.e. the cup anemometer performance). See also in figure 5 (referred to the right Y-axis of the graph), the percentage error defined as:

$$\varepsilon=\frac{K-K_{\text{testing}}}{K_{\text{testing}}}. \quad (22)$$

of the present analytical approximation and the one regarding the initial one ($K=4.737$ [12]). It can be observed in the

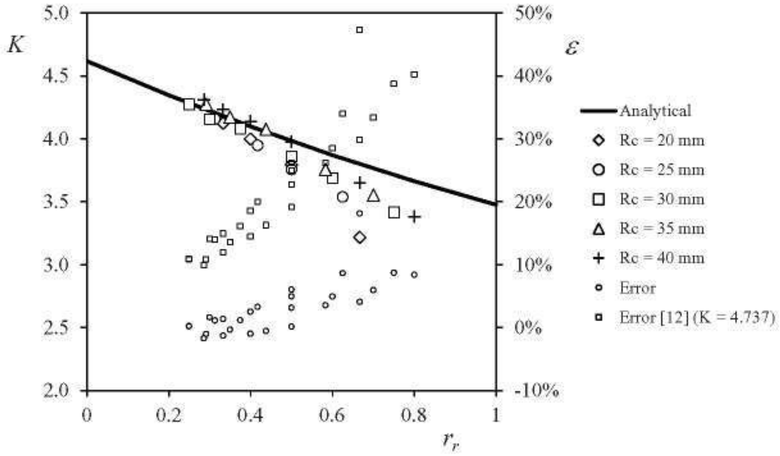

Figure 5. Anemometer factor, K, corresponding to calibrations of a Climatronics 100075 cup anemometer equipped with different rotors [10], as a function of the ratio between the cups' radius and the cups' center rotation radius, $r_r = R_c/R_{rc}$. The fitting of the analytical method developed in the present work to these results is also included (solid line). The errors (equation (22)) of the present analytical approximation and the previous one [12], are included and referred to the right Y-axis.

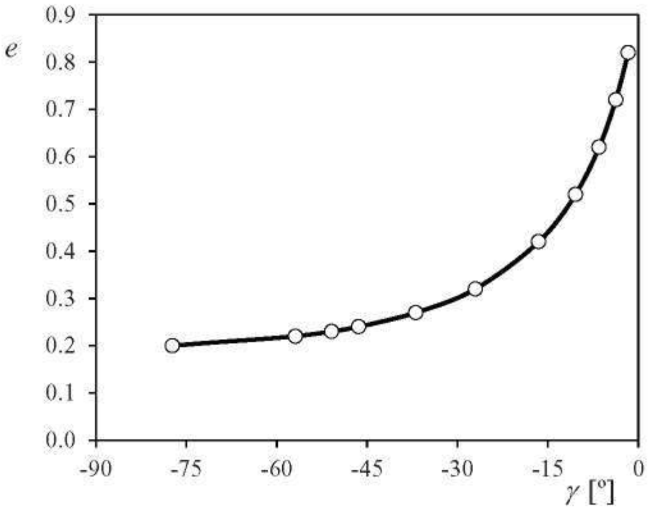

Figure 6. Relationship between the variable $e = d/R_c$, and the phase angle γ that produce the best fitting of the analytical method developed in the present work and the testing results (see figure 5).

figure that the present approximation represents an improvement. However, the fitting is less accurate for the higher values of the ratio, r_r. A possible explanation for this behavior (higher rotation speed, and therefore lower anemometer factors), lies in two factors: a better aerodynamic efficiency, and/or a reduction of the rotor's moment of inertia.

3. Conclusions

In this work an improvement to the 3-cup analytical model for studying cup anemometer performance is included. The following effects are taken into account in the present method:

- the phase angle of the aerodynamic force with respect to the rotor's position angle, due to the rotation, and
- the displacement from the cup's center of the normal-to-the-cup aerodynamic force as a function of the wind direction with respect to the mentioned cup.

The proposed methodology shows good agreement with the experimental results, reflecting the effect of the ratio of the cup radius, R_c, to the cups' center rotation radius, R_{rc}, that is, $r_r = R_c/R_{rc}$, on the anemometer factor, K. This effect was observed in a previous work [12], but was not properly included in the analytical calculations.

References

[1] Coquilla R V and Obermeier J 2008 Calibration speed range for rotating anemometers used in wind energy applications *Proc. of the 46th AIAA Aerospace Sciences Meeting and Exhibit* (*January*) (Reno, NV: American Institute of Aeronautics and Astronautics) pp 2–7

[2] Sanz-Andrés A, Pindado S and Sorribes F 2014 Mathematical analysis of the effect of the rotor geometry on cup anemometer response *Sci. World J.* **2014** 1–23

[3] Pindado S, Vega E, Martínez A, Meseguer E, Franchini S and Pérez I 2011 Analysis of calibration results from cup and propeller anemometers. Influence on wind turbine annual energy production (AEP) calculations *Wind Energy* **14** 119–32

[4] Pindado S, Cubas J and Sorribes-Palmer F 2014 The cup anemometer, a fundamental meteorological instrument for the wind energy industry. Research at the IDR/UPM institute *Sensors* **14** 21418–52

[5] Schrenk O 1929 Über die Trägheitsfehler des schalenkreuz-anemometers bei schwankender windstärke *Zeitschrift fur Tech. Phys.* **10** 57–66

[6] Wyngaard J C 1981 Cup, propeller, vane, and sonic anemometers in turbulence research *Annu. Rev. Fluid Mech.* **13** 399–423

[7] Westermann D 1996 Overspeeding—über das eigentümliche Tiefpaßverhalten von Schalensternanemometern (overspeeding measurements of cup anemometers) *DEWI Mag.* **9** 56–63

[8] Ramachandran S 1969 A theoretical study of cup and vane anemometers *Q. J. R. Meteorol. Soc.* **95** 163–80

[9] Kondo J, Naito G I and Fujinawa Y 1971 Response of cup anemometer in turbulence *J. Meteorol. Soc. Japan* **49** 63–74

[10] Pindado S, Cubas J and Sorribes-Palmer F 2015 On the harmonic analysis of cup anemometer rotation speed: a principle to monitor performance and maintenance status of rotating meteorological sensors *Measurement* **73** 401–18

[11] Pindado S, Cubas J and Sorribes-palmer F 2015 On the analytical approach to present engineering problems : photovoltaic systems behavior, wind speed sensors performance, and high-speed train pressure wave effects in tunnels *Math. Problems Eng.* **2015** 1–17
[12] Pindado S, Pérez I and Aguado M 2013 Fourier analysis of the aerodynamic behavior of cup anemometers *Meas. Sci. Technol.* **24** 065802
[13] Pindado S, Cubas J and Sanz-Andrés A 2013 Aerodynamic analysis of cup anemometers performance. The stationary harmonic response *Sci. World J.* **2013** 1–11
[14] Pindado S, Pérez J and Avila-Sanchez S 2012 On cup anemometer rotor aerodynamics *Sensors* **12** 6198–217
[15] Brevoort M J and Joyner U T 1935 *Experimental Investigation of the Robinson-Type Cup Anemometer. NACA TN-513* (Langley Field, VA: National Advisory Committee for Aeronautics, Langley Aeronautical Lab.)
[16] Ramos Cenzano A 2014 *Análisis Mediante Cálculo Numérico (CFD) Del Comportamiento De Anemómetros De Cazoletas* (Madrid: Universidad Politécnica de Madrid) http://oa.upm.es/32407/ (accessed 24/08/2015)
[17] Dahlberg J-Å, Gustavsson J, Ronsten G, Pedersen T F, Paulsen U S and Westermann D 2001 Development of a standardised cup anemometer suited to wind energy applications—(Classcup) Contract JOR3-C 798-0263. The Aeronautical Research Institute of Sweden, Sweden; Risø National Laboratory, Denmark; German Wind Energy Institute, Germany. Publishable Final Report 1–37
[18] Potsdam M, Cicolani L, Gassaway B and Mattle D 2013 Aerodynamic analysis and flight simulation of an anemometer for rotational stabilization of a helicopter slung load *Proc. of the 31st AIAA Applied Aerodynamics Conf.* (*San Diego, CA*) pp 1–22
[19] Chree C 1895 Contribution to the theory of the Robinson cup-anemometer *Lond. Edinburgh Dublin Philos. Mag. J. Sci.* **40** 63–90
[20] Busch N E and Kristensen L 1976 Cup anemometer overspeeding *J. Appl. Meteorol.* **15** 1328–32
[21] Sebastião P J 2014 The art of model fitting to experimental results *Eur. J. Phys.* **35** 015017