

HEDGING VOLATILITY IN FOREIGN CURRENCIES

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Wide fluctuations in exchange rates are of continual concern to traders and investors in foreign currencies. One of the main issues not addressed by current risk management techniques is the phenomenon that volatility itself gyrates widely. These changes in volatility may have an adverse effect on risk management strategies, which typically assume that volatility is constant. In this article we propose the creation of new derivative instruments, namely, options (and futures) on volatility, designed to help in the management of unexpected volatility fluctuations.

We describe the construction of a volatility index to reflect the volatility of a specific exchange rate. The index, based on implied volatilities from currency options, will serve as an underlying instrument on which derivative products can be written. We discuss a variety of issues in valuing such options, and present a simple binomial example.

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Since the late 1960s, currencies of most Western economies have been allowed to float in value against other currencies. Fixed exchange rates were replaced by variable exchange rates determined by market forces (with only occasional intervention by the central banks implementing national monetary policy).

Variable exchange rates created the need for currency risk management in international transactions. One outcome was the establishment of markets to trade futures and options on foreign currencies, which allow firms to trade away risk directly associated with unpredictable changes in the value of foreign currencies. Another outcome is a stream of studies

trying to explain the nature of the risk of exchange rates. The objective of these studies has been to find ways to manage currency risk more effectively by identifying the probability distribution of changes in the value of currencies and by pricing the foreign exchange-based instruments.

A common assumption underlying most pricing models for foreign exchange (FX) options is that volatility is constant over the life of the option (see, for example, Garman and Kohlhagen [1983] and Grabbe [1983]). More recent studies on foreign exchange rates have shown that the standard deviation, or volatility, is not stationary (see, for example, Tucker and Scott [1987] and Wasserfallen and Zimmerman [1987]).¹

Changes in volatility could be the result of changes in the policies of central banks or changes in the prices of important raw materials like oil. Ito and Rolly [1987] show that the volatility of the dollar/yen exchange rate is affected by U.S. money supply changes. Johnson and Schneeweis [1992] find that macroeconomic information releases (such as announcements about the CPI, industrial production, or the money supply) explain the change in volatility between trading hours and non-trading hours for the yen, British pound, and deutschemark exchange rates.

Established derivative instruments are useful in hedging against, or in speculating on, changes in the level of the market. They do not, however, provide a means to hedge, or speculate on changes in market volatility. In this article we provide a rationale for creating new derivative instruments that handle the risk of changes in volatility and explore the use and valuation of such instruments.

I. WHY DO CURRENCY TRADERS NEED OPTIONS AND FUTURES ON VOLATILITY?

Many currency traders and investors who use options strategies find themselves exposed to volatility risk as a result of frequent changes in the volatilities of currency markets. There are a number of circumstances where options and futures on volatility may be attractive.

Consider, for example, an investor engaged in a covered-call strategy in the deutschemark (DM). Suppose the exchange rate (S) is 0.625 \$/DM, the strike price (K) is 0.625, the three-month T-bill rate

(r) is 3%, the three-month foreign rate (f) is 9%, time to maturity (t) is three months, and volatility (σ) is 10%. The dollar-denominated call option on the DM, using the FX version of the Black-Scholes model, has a value of 0.82, and the hedge ratio (delta) is 0.39. An increase in volatility to 15%, assuming no change in the exchange rate, would significantly increase the value of the option; the call price would be 1.42, and delta would increase to 0.43.

The primary effect of the change in volatility is the loss produced by the large increase in the trader's liability (the written call). To the extent that the change in volatility is correlated with the change in the exchange rate, there may be a secondary effect, an additional change in delta. The unexpected loss resulting from one or both effects may be hedged using a call option on volatility.

When volatility reaches historically low levels, many investors and traders speculate on an increase in volatility. They "buy" volatility by purchasing a straddle, allowing them to benefit from high volatility (which may include a jump in the FX rate). The proposed volatility options and futures provide an alternative that is both more efficient and less costly to implement because it is focused only on volatility. One way to describe a volatility option is to think of it as an option on an option that is always at-the-money-forward (ATMF), since the sole determinant of such an option is volatility (see Brenner and Subrahmanyam [1993]).

Optimal allocation of portfolio assets among different markets and currencies depends to some extent on the volatility of each currency. A change in volatility, *ceteris paribus*, may lead to a change in the desired proportion allocated to that specific currency. Volatility options may provide the means of protecting the asset allocation mechanism against adverse volatility changes.

Japanese warrants have become popular investment instruments in the United States and Europe. From the perspective of a non-Japanese investor, these warrants are subject to two major risks. The first can be identified as stock market risk, while the second is foreign currency risk. As contingent claims with built-in leverage, warrants are very sensitive to the change in the volatility of the underlying processes. Right now the possibilities of reducing volatility risk are both lim-

ited and expensive, because they require following a closely monitored dynamic strategy.

Most dynamic strategies for portfolio-protected investments are based on the assumption of stationary volatility. Portfolio insurance typically uses futures contracts to protect portfolios of securities against losses, relying on a hedge ratio based on models such as Black and Scholes, which assume that volatility is either constant or varies in a predictable manner. Dynamic strategies adopting these assumptions are vulnerable to major losses (and gains) with fluctuations in the volatility parameter. The crash of October 19, 1987, bears witness to this phenomenon. A volatility option could substantially reduce the risk of employing dynamic strategies in markets where alternative static strategies are deficient.

Assume, for example, that a portfolio insurance strategy requires an initial allocation of 50% in the foreign currency and 50% in cash. This allocation is based on an estimate of 20% volatility in the rate of change of the foreign currency. An increase in volatility of 5% may raise the effective insurance premium (i.e., the cost of the dynamic strategy) by 25% (assuming a ninety-day policy). This risk could be avoided by buying a call on volatility, which will cost only a fraction of the potential change in the insurance premium.²

We can demonstrate application of options on volatility in terms of a familiar options strategy, the *protective put strategy*. Assume a currency trader anticipating an increase in the volatility of FX is long the three-month 1.60 (DM/\$) at-the-money straddle on the \$. The value of the straddle with 15% volatility is 6.90 pfennigs. If volatility drops to 10%, the value of the straddle drops to 6.56 pfennigs, a drop of about 30% in the option value. To protect against such a drop the trader would buy a put option on volatility struck at 15%. Assuming that the volatility index itself has a 20% volatility, the value of such an option may be about 0.4 cents. In other words, the package that includes the at-the-money straddle and the volatility put would cost about 10 pfennigs.

For 10 pfennigs, the trader will benefit in any of the following circumstances: an increase in volatility, an upward jump in the price of FX, or a downward jump in the price of FX. The trader is protected against a decline in the volatility of FX.

Alternatively, the same trader who is long the straddle could sell a call on volatility. Selling this call entails giving up the potential gain from the increase in the straddle value due to an increase in volatility, but preserves the potential gain from a change in the price of FX, in either direction. The only loss is attributable to the time decay. It is interesting to note that the buyer of a volatility call will be a straddle seller who would like to reduce risk from a volatility increase. The volatility options, however, will not protect against a jump in the exchange rate.³

The creation of volatility-based instruments is a two-stage process. First, a volatility index must be created to reflect the volatility of the exchange rate of a specific foreign currency in terms of the home currency. Then, the index serves as an underlying asset on which options and futures can be written. These new volatility-based contracts would open up the possibility for investors to hedge against changes in volatility, without having to employ complicated dynamic strategies.

II. A CURRENCY VOLATILITY INDEX

A major problem in creating a market for volatility options is construction of a volatility index. There is no universally accepted method to measure the volatility of the currency market. Moreover, no single index can reflect the volatility changes for all currencies simultaneously, and be used by all traders to hedge their specific positions.

It is clear that a volatility index based only on one pair of currencies, such as \$/DM, will provide a measure that is specific to these currencies but may not provide a good indication of the \$/£ volatility. We expect that at least one index will be constructed for each pair of leading currencies. Natural candidates are combinations of the U.S. dollar, the Japanese yen, the British pound, and the German mark.

A volatility index must be updated frequently, so historical volatility is not a good candidate for our index. The changes in a rolling historical volatility produced by adding and deleting one observation per day, or even hourly, are quite small.

A natural candidate is an index based on the series of implied volatilities from currency options traded on major exchanges. For the index to be more

reliable, the underlying options should be liquid and frequently traded. Even then, statistical procedures must be applied in order to smooth the noise element and achieve a reliable estimate of volatility changes. One way to reduce noise in the estimation is to infer the volatility from options traded in different markets such as spot options traded on the Philadelphia Exchange (PHLX) and futures options traded on the International Monetary Market (IMM).

It is also feasible to create an overall volatility index for, say, the U.S. dollar against all the other leading currencies. Such an index is useful especially when changes in dollar volatility spill over to all major economies. For example, the FINEX in the U.S. trades derivative contracts based on a dollar index representing the weighted-average dollar price of a basket of currencies used by U.S. corporations. The volatility of this dollar index may be of interest to many American companies that trade internationally. A similar currency basket index could be constructed for the DM or the British pound. The volatility of such a currency index could be a useful vehicle for hedging the positions of those who have portfolio commitments in multiple foreign currencies.

Our research on equity volatility supports a currency volatility index based on implying the volatility from a synthetic thirty-day, at-the-money, currency call option. The synthetic option should be recalculated every few minutes, based on actual prices of traded calls, say, on the German mark. Only short-term and next-term options, and only those series that are close-to-the-money, should be involved in the calculation.

The thirty-day at-the-money synthetic call will be interpolated from prices of calls from four series, with weights based on a pricing model for currency calls. First, the two slightly-in-the-money, short-term, and next-term series will be interpolated to yield a thirty-day slightly-in-the-money option. Second, a thirty-day slightly-out-of-the-money synthetic option is calculated. Third, the two thirty-day synthetic calls are combined to yield the thirty-day at-the-money call.⁴

Let us denote the currency spot exchange rate by S , and the two closest-to-the-money striking prices by K_1 and K_2 , so that $K_1 < S < K_2$. C_1^A , C_1^M and C_2^A , C_2^M denote the actual (A) and model (M) call prices for the current month and the next month, respectively. The

first step is therefore to calculate

$$C(\tau = 30, K_1) = C_1^A(K_1)W + C_2^A(K_1)(1 - W)$$

where

$$W = \frac{C^M(\tau = 30, K_1) - C_2^M(K_1)}{C_1^M(K_1) - C_2^M(K_1)}$$

The same procedure is applied to calculate $C(\tau = 30, K_2)$. The third step is to calculate the synthetic at-the-money thirty-day call, to be denoted by C^* , and

$$C^* = C(\tau = 30, K_1)v + C(\tau = 30, K_2)(1 - v)$$

where

$$v = \frac{C^M(\tau = 30, K = S) - C^M(\tau = 30, K_2)}{C^M(\tau = 30, K_1) - C^M(\tau = 30, K_2)}$$

C^* now reflects actual prices of traded options, and provides information on the volatility of the exchange rate. The pricing model can be used once more to calculate the implied standard deviation for C^* .

In order to reduce the noise in this estimation procedure, a combination of implied volatilities from synthetic call and put options should be considered. A moving average procedure over the latest estimates can also be used to reduce the noise further without affecting the basic changes in volatility.

III. VALUATION OF CURRENCY VOLATILITY OPTIONS: AN EXAMPLE

While valuation of volatility options is a complicated issue, a price will be determined in the marketplace that will balance the demand and supply for volatility insurance. The main issue in valuation of these options is the stochastic nature of volatility itself. In principle, we cannot use the no-arbitrage approach, because we cannot construct a riskless hedge that will take care of both stochastic exchange rates and stochastic volatility. A similar problem arises when one tries to model options on FX when volatil-

ity is assumed to be stochastic (see, for example, Heston [1991]).

One approach to tackle the problem is to assume a stochastic process for volatility and use risk-neutral valuation to solve for the option price. Since, in general, an analytical solution is not available, numerical procedures will be used. Unlike the Black-Scholes model, the validity of such a model depends on the risk attitudes of market participants. To the extent that people are risk-neutral, such a valuation may be appropriate.

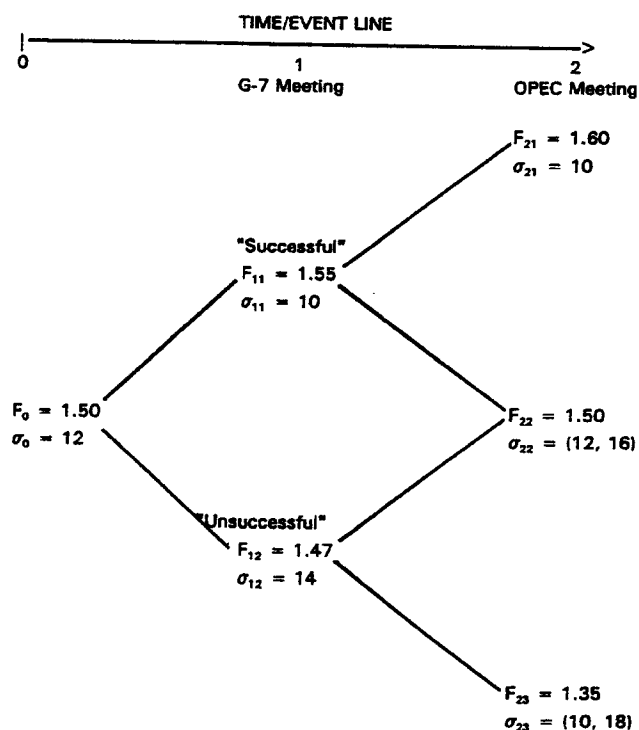
Another important factor in the model is the relationship between the level of the exchange rate and the stochastic process that governs volatility. In general, any valuation model will require estimating the correlation between the two variables. To illustrate how volatility options could be valued, we provide a simple binomial example where the processes for the volatility and the exchange rate are somewhat correlated.

Let us assume that a G-7 meeting is expected to be held next month (time 1), and that one month later (time 2) an OPEC meeting will take place. If the G-7 meeting is "successful," the dollar is expected to appreciate against the deutschemark from 1.50 to 1.55, and its volatility is expected to be 10% (on an annual basis). If the meeting is "unsuccessful," we expect the exchange rate to be 1.47, and the volatility to be 14%. In this example we expect one of two outcomes to take place, with each outcome characterized by two parameters: the exchange rate, and the volatility of the subsequent exchange rates.

The OPEC meeting, the following month, can also result in two outcomes: the price of oil increases or decreases. If the G-7 meeting at $t = 1$ is successful, and the price of oil increases, we may expect a further strengthening of the dollar, to 1.60, and no change regarding future volatility, $\sigma_{11} = \sigma_{21} = 10\%$. If, however, the OPEC meeting results in a decrease in the price of oil, we would observe a decline in the exchange rate to 1.50, but volatility may either be 12% or 16%, with equal probability.

Similarly, if the G-7 meeting is unsuccessful, and the price of oil increases, the exchange rate will increase to 1.50 (from 1.47), and volatility may assume one of two values, 12% or 16%. If, however, the price of oil decreases, the dollar will slip further to 1.35,

EXHIBIT 1
EXCHANGE RATES F_{ij} AND THE STANDARD DEVIATION σ_{ij}
AT TIME i ($i = 1, 2$) AND STATE j ($j = 1, 2, 3$)



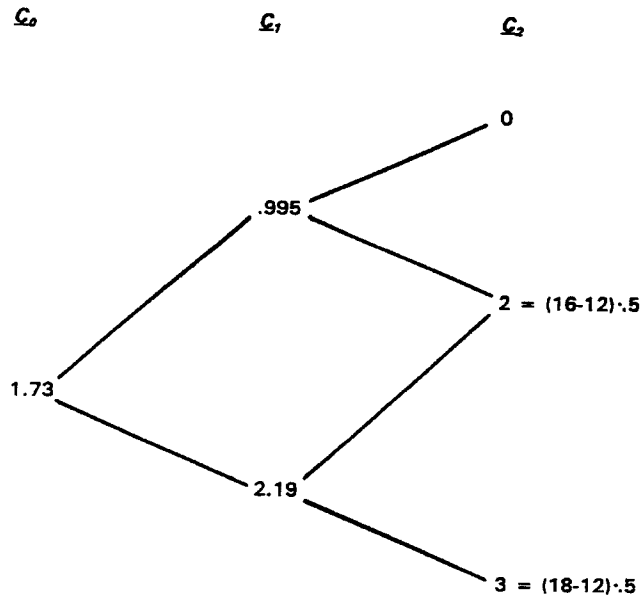
Notes: σ_{ij} is in annualized percentage points. σ_{22} and σ_{23} assume two possible values each (numbers in parentheses).

and volatility will have a wider distribution; it will be either higher at 18% or lower at 10% with equal probability. All possible outcomes are described in Exhibit 1.

In the example, no option on the exchange rate can provide a hedge against uncertain volatility. Only a two-period call option on volatility could provide such a hedge. Volatility is not a traded asset, so the volatility option cannot be priced simply by ruling out arbitrage. Rather, we assume that investors are risk-neutral and apply risk-neutral valuation to the option at hand.

To value such an option, using a 12% strike price, we make some additional simplifying assumptions. The options are European, and the domestic and foreign interest rates are the same at 0.5% per month. Using the binomial tree in Exhibit 1, we

EXHIBIT 2
VALUE-TREE FOR A TWO-PERIOD CALL OPTION ON
VOLATILITY WITH A STRIKE LEVEL OF 12%



show, in Exhibit 2, the payoffs of the volatility call option at time 2.

At time 1, however, we have two uncertainties: first, the uncertainty of the exchange rate (e.g., moving from 1.47 either to 1.50 or to 1.35), and then the uncertainty about the level of volatility at either of the exchange rates. For example, if we get to $F_{22} = 1.50$, we may have a volatility of either 12% or 16%. Thus, to value the option at time 1 we use a 0.5 probability that σ_{22} , for example, will be 16%. We then use the probability of getting to node (2, 2) from node (1, 2), which is the same as the risk-neutral probability of getting from F_{12} to F_{22} .

Hence, the value of the volatility call at time 1, state 2, C_{12} is given by:⁵

$$C_{12} = [C_{22}P + C_{23}(1 - P)] / (1 + 0.005)$$

where

$$P = \frac{(1 + 0.005 - 0.005) - (1.35/1.47)}{(1.5/1.47) - (1.35/1.47)} = 0.8$$

Thus,

$$C_{12} = [(2 \times 0.8) + (3 \times 0.2)] / 1.005 = 2.19$$

Similarly:

$$C_{11} = [(0 \times 0.5) + (2 \times 0.5)] / 1.005 = 0.995$$

Using C_{11} and C_{12} , the value of the option at time 0, C_0 is given by:

$$C_0 = [(0.995 \times 0.375) + (2.19 \times 0.625)] / 1.005 = 1.73$$

The value of the volatility option is about 1.73 pfennigs. It provides insurance against changes in volatility above 12%. The alternative of buying or selling straddles is not equivalent to buying or selling volatility options, because a trader who sells straddles to benefit from a potential decline in volatility undertakes additional risk, gamma risk, due to a potential large shift in the value of the currency. Volatility options deal solely with volatility risk.

IV. CONCLUSIONS

The large swings in FX volatility in the past few years demonstrate the need for financial instruments for hedging changes in volatility. An instrument meeting this need is exchange-traded futures and options on an FX volatility index. FX traders could establish long or short positions on volatility by trading volatility futures, and limit or expand their volatility positions by using volatility options.

ENDNOTES

The authors would like to thank Bill Silber and June Dilevsky for their helpful suggestions and comments. This article is based on the ideas presented in Brenner and Galai [1989].

¹This phenomenon has been observed in other markets such as the stock market.

²The exact magnitude of the savings requires using a model that values volatility options, which we discuss in Section III.

³To reduce the risk due to a price jump, the trader should hold a "butterfly" spread instead of a straddle.

⁴A similar procedure for a synthetic call is suggest-