Building a Fast CSP Solver Based on SAT

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Outline

- SAT, SAT solving, and SAT encodings
- Picat: a modeling language for SAT
- PicatSAT: Compiling CSP to SAT
  - Compiling arithmetic constraints
  - Decomposing global constraints
- Experimental results

The Satisfiability Problem (SAT)

Given a Boolean formula, the SAT problem is to determine if the formula is satisfiable. If yes, it finds an assignment for the variables that makes the formula satisfiable.

$$(x_1 \rightarrow x_2) \vee \neg((\neg x_1 \leftrightarrow x_3) \vee x_4) \wedge \neg x_2$$

$$\neg x_1 \vee x_2 \vee x_3$$

$$\neg x_1 \vee x_2 \vee \neg x_4$$

CNF

G.S. Tseytin: On the complexity of derivation in propositional calculus, 1969.

SAT Solving (CDCL)

1. Choice: Assign a value to a selected variable.

2. Unit propagation: Use this assignment to determine values for the other variables.

3. *Backjump*: If a conflict is found, add the negation of the conflict-causing clause as a new clause and backtrack to the choice that made the conflict occur.

4. Continue from step 1.

- Martin Davis, Hilary Putnam: A Computing Procedure for Quantification Theory, 1960.
- Martin Davis, George Logemann, Donald Loveland: A Machine Program for Theorem Proving, 1962.
- Joao Marques-Silva, Ines Lynce and Sharad Malik: Conflict-Driven Clause Learning SAT Solvers, 2009.

SAT Encodings

$$X :: \{a_1, a_2, \ldots, a_n\}$$

- Direct encoding

$$B_i \leftrightarrow X = a_i$$
$$B_1 \ \vee \ B_2 \ \vee \ \ldots \ \vee \ B_n$$
$$\texttt{at\_most\_one}([B_1, B_2, \ldots, B_n])$$

- Order encoding

$$B_i \leftrightarrow X \leq a_i$$

- Log encoding (sign-and-magnitude)

$$X.\mathrm{m} = <B_{k-1} \ldots B_1 B_0>$$
$$X.\mathrm{s} = 0 \text{ or } 1$$

- Johan de Kleer: A Comparison of ATMS and CSP Techniques,1989.
- James M. Crawford, Andrew B. Baker: Experimental Results on the Application of Satisfiability Algorithms to Scheduling Problems, 1994.
- Kazuo Iwama, Shuichi Miyazaki: SAT - Variable Complexity of Hard Combinatorial Problems, 1994.

CSP Solvers based on SAT

- BEE (order encoding)
- FznTini (log encoding)
- meSAT (order and direct encodings)
- PicatSAT (log and direct encodings)
- Savile Row (order and direct encodings)
- Sugar (order encoding) and its successors

- Naoyuki Tamura, Akiko Taga, Satoshi Kitagawa, Mutsunori Banbara: Compiling finite linear CSP into SAT, 2009.
- Jinbo Huang: Universal Booleanization of Constraint Models, 2008.
- Amit Metodi, Michael Codish: Compiling finite domain constraints to SAT with BEE, 2012.
- Mirko Stojadinovic, Filip Maric: meSAT - multiple encodings of CSP to SAT, 2014.
- Neng-Fa Zhou and Hakan Kjellerstrand: Optimizing SAT Encodings for Arithmetic Constraints, 2017.

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Picat

- picat-lang.org
  - Pattern-matching, Intuitive, Constraints, Actors, Tabling
- Core logic programming concepts
  - Logic variables (arrays and maps are terms)
  - Implicit pattern-matching and explicit unification
  - Explicit non-determinism
- Language constructs for scripting and modeling
  - Functions, loops, list comprehensions, and assignments
- Facilities for combinatorial search
  - Tabling for dynamic programming and planning
  - The `cp`, `sat`, `mip` , and `smt` modules for CSPs

- Neng-Fa Zhou, Hakan Kjellerstrand, Jonathan Fruhman: Constraint Solving and Planning with Picat, 2015.

Picat's Constraint Modules

import sat. import cp. import mip. import smt.

- Constraints
  - Domain
    - X :: Domain
    - X notin Domain
  - Arithmetic
    - (X #= Y), (X #!= Y), (X #> Y), (X #>= Y), ...
  - Boolean
    - (X #/\ Y), (X #\/ Y), (X #<=> Y), (X #=> Y), (X #^ Y), (#~ X)
  - Table
    - table_in(VarTuple,Tuples), table_notin(VarTuple,Tuples)
  - Global
    - all_different(L), element(I,L,V), circuit(L), cumulative(...), ...
- Solver invocation: solve(Options,Vars)

Modeling Sudoku in Picat

```
A = {{_,6,_,1,_,4,_,5,_},
     {_,_,8,3,_,5,6,_,_},
     {2,_,_,_,_,_,_,_,1},
     {8,_,_,4,_,7,_,_,6},
     {_,_,6,_,_,_,3,_,_},
     {7,_,_,9,_,_,_,_,4},
     {5,_,_,_,_,_,_,_,2},
     {_,_,7,2,_,6,9,_,_},
     {_,4,_,5,_,8,_,7,_}}.
```

9	6	3	1	7	4	2	5	8
1	7	8	3	2	5	6	4	9
2	5	4	6	8	9	7	3	1
8	2	1	4	3	7	5	9	6
4	9	6	8	5	2	3	1	7
7	3	5	9	6	1	8	2	4
5	8	9	7	1	3	4	6	2
3	1	7	2	4	6	9	8	5
6	4	2	5	9	8	1	7	3

```
import sat.

sudoku(A) =>
    N = len(A),
    A :: 1..N,
    foreach(Row in 1..N)
        all_different(A[Row])
    end,
    foreach(Col in 1..N)
        all_different([A[Row,Col] : Row in 1..N])
    end,
    M = floor(sqrt(N)),
    foreach(Row in 1..M..(N-M+1), Col in 1..M..(N-M+1))
        Square = [A[Row+Dr,Col+Dc] : Dr in 0..M-1, Dc in 0..M-1],
        all_different(Square)
    end,
    solve(A).
```

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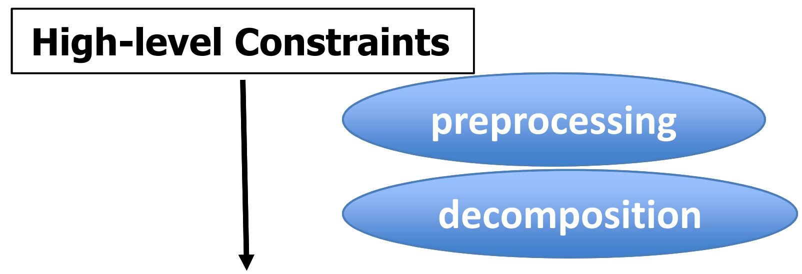
PicatSAT

- A compiler that translates CSP into CNF
- Adopts log and direct encodings
- Performs numerous optimizations
  - Preprocessing constraints to exclude no-good values (CP)
  - Common subexpression elimination (language compilers)
  - Logic optimization (hardware design)
  - Compile-time equivalence reasoning
- Employs efficient decomposers for global constraints
- Offered in Picat as a constraint module, named sat
- Written in Picat with more than 10,000 LOC
- One of the top solvers in recent solver competitions

The PicatSAT Compiler



Primitive Constraints

- Small Pseudo-Boolean (PB) constraint: $\Sigma_1^n (a_i \times B_i) \odot b$, $n =< 10$
- Boolean cardinality constraint: $\Sigma_i^n B_i \odot b$ (b is 1 or 2)
- $X :: D$
- $X \odot Y$
- $X + Y = Z$
- $X \times Y = Z$
- $Y =$ -X
- $\texttt{abs}(X) = Y$
- $X \ \texttt{div} \ Y = Z$
- $X \ \texttt{mod} \ Y = Z$
- Reified primitive constraint: $B \Leftrightarrow C$
- Implicative primitive constraint: $B \Rightarrow C$

- Constraints are made to be arc-consistent or interval consistent
- No primitive constraints are duplicated
- Avoid creating large-domain variables
- Avoid creating domains with negative values

Encoding Small PB Constraints

- Espresso

```
% X+Y = 2*U+V,
% where [X,Y,U,V] :: 0..1, UV ≠ 11

.i 4
.o 1
0000 0
0001 1
0010 1
0011 -
0100 1
0101 0
0110 1
0111 -
1000 1
1001 0
1010 1
1011 -
1100 1
1101 1
1110 0
1111 -
```

→

```
.i 4
.o 1
.p 5
1-00 1
-01- 1
01-0 1
110- 1
00-1 1
.e
```

$\neg X \vee U \vee V$

$Y \vee \neg U$

$X \vee \neg Y \vee V$

$\neg X \vee \neg Y \vee U$

$X \vee Y \vee \neg V$

Encoding the at-most-one Constraint

- Chen's two-product encoding

$$
\begin{array}{c|ccccc}
 & V_1 & V_2 & \dots\ V_j & \dots & V_q \\
\hline
U_1 & X_{11} & X_{12} & \dots & & X_{1q} \\
U_2 & X_{21} & X_{22} & \dots & & X_{2q} \\
. & & & & & \\
U_i & \dots & & X_{ij} & \dots & \\
. & & & & & \\
U_p & X_{p1} & X_{p2} & \dots & & X_{pq}
\end{array}
$$

$\boxed{\texttt{amo}([X_{11},\ X_{12},\ \dots,\ X_{pq}])}$

$\downarrow$

$\texttt{amo}([U_1,\ U_2,\ \dots,\ U_p])$

$\texttt{amo}([V_1,\ V_2,\ \dots,\ V_q])$

$X_{ij} \rightarrow U_i$ /\ V_j
for i in 1..p and j in 1..q

Jingchao Chen: A New SAT Encoding of the At-Most-One Constraint, 2010.

Sign-and-Magnitude Log Encoding

X :: D

- Each domain variable is encoded as a vector of Boolean variables
  - X.m = <B_{n-1},…,B_1,B_0>
  - X.s is the sign bit
- No negative zero is allowed
  - X.m = <0,…,0,0> $\Rightarrow$ X.s = 0

Sign-and-Magnitude Log Encoding (Example)

```
X :: [-2,-1,1,2]

X.m = <X1,X0>
X.s = S
```

- Naïve Encoding

$$\begin{array}{ll} \neg S \vee \neg X_1 \vee \neg X_0 & (X \neq -3) \\ \neg S \vee X_1 \vee X_0 & (X \neq -0) \\ S \vee X_1 \vee X_0 & (X \neq 0) \\ S \vee \neg X_1 \vee \neg X_0 & (X \neq 3) \end{array}$$

- Optimized Encoding (Using Espresso)

$$\begin{array}{l} \mathbf{X}_0 \vee \mathbf{X}_1 \\ \neg \mathbf{X}_0 \vee \neg \mathbf{X}_1 \end{array}$$

Breaking Arithmetic Constraints

- Combine power-of-2 terms

$$2^{k-1} \times X_{k-1} + \ldots + 2^0 \times X_0 \longrightarrow X \; :: \; 0..2^k - 1$$

- Move out non-linear terms
- Factor out terms with common coefficients
- Decompose the constraint (Huffman coding)

```
decompose(a_1 × X_1 + a_2 × X_2 + ... + a_n × X_n ⊙ b):
    add all the terms a_i × X_i into a priority queue Q
    while the constraint is not primitive:
        remove two terms a_i × X_i and a_j × X_j from Q
            where a_i = a_j, and
                X_i and X_j have the smallest domains
        post T = X_i + X_j
        add the term a_i × T into Q
    post the primitive constraint
```

The Comparison Constraint: X ≥ Y

- Signed comparison

$$
\begin{aligned}
&X.\mathrm{s} = 0 \land Y.\mathrm{s} = 1 \lor \\
&X.\mathrm{s} = 1 \land Y.\mathrm{s} = 1 \Rightarrow X.\mathrm{m} \leq Y.\mathrm{m} \lor \\
&X.\mathrm{s} = 0 \land Y.\mathrm{s} = 0 \Rightarrow X.\mathrm{m} \geq Y.\mathrm{m}
\end{aligned}
$$

- Unsigned comparison

$$X.\mathrm{m} = <X_{n-1}X_{n-2}\ldots X_1X_0>,\ Y.\mathrm{m} = <Y_{n-1}Y_{n-2}\ldots Y_1Y_0>$$

$$
\begin{aligned}
&T_0 \Leftrightarrow (X_0 \geq Y_0) \\
&T_1 \Leftrightarrow (X_1 > Y_1) \lor (X_1 = Y_1 \land T_0) \\
&\qquad\qquad \vdots \\
&T_{n-1} \Leftrightarrow (X_{n-1} > Y_{n-1}) \lor (X_{n-1} = Y_{n-1} \land T_{n-2})
\end{aligned}
$$

The Addition Constraint: X+Y = Z

- Unsigned addition (ripple-carry adders)

$$\begin{array}{rcccc} & X_{n-1} & \ldots & X_1 & X_0 \\ + & Y_{n-1} & \ldots & Y_1 & Y_0 \\ \hline Z_n & Z_{n-1} & \ldots & Z_1 & Z_0 \end{array}$$

Carriers are used

- Signed addition

$$\begin{array}{l} X.\mathtt{s} = 0 \wedge Y.\mathtt{s} = 0 \Rightarrow Z.\mathtt{s} = 0 \wedge X.\mathtt{m}+Y.\mathtt{m} = Z.\mathtt{m} \\ X.\mathtt{s} = 1 \wedge Y.\mathtt{s} = 1 \Rightarrow Z.\mathtt{s} = 1 \wedge X.\mathtt{m}+Y.\mathtt{m} = Z.\mathtt{m} \\ X.\mathtt{s} = 0 \wedge Y.\mathtt{s} = 1 \wedge Z.\mathtt{s} = 1 \Rightarrow X.\mathtt{m}+Z.\mathtt{m} = Y.\mathtt{m} \\ X.\mathtt{s} = 0 \wedge Y.\mathtt{s} = 1 \wedge Z.\mathtt{s} = 0 \Rightarrow Y.\mathtt{m}+Z.\mathtt{m} = X.\mathtt{m} \\ X.\mathtt{s} = 1 \wedge Y.\mathtt{s} = 0 \wedge Z.\mathtt{s} = 0 \Rightarrow X.\mathtt{m}+Z.\mathtt{m} = Y.\mathtt{m} \\ X.\mathtt{s} = 1 \wedge Y.\mathtt{s} = 0 \wedge Z.\mathtt{s} = 1 \Rightarrow Y.\mathtt{m}+Z.\mathtt{m} = X.\mathtt{m} \end{array}$$

The Full Adder

$$X_i + Y_i + C_{in} = C_{out} Z_i$$

$$X_i \vee \neg Y_i \vee C_{in} \vee Z_i$$
$$X_i \vee Y_i \vee \neg C_{in} \vee Z_i$$
$$\neg X_i \vee \neg Y_i \vee C_{in} \vee \neg Z_i$$
$$\neg X_i \vee Y_i \vee \neg C_{in} \vee \neg Z_i$$
$$\neg X_i \vee C_{out} \vee Z_i$$
$$X_i \vee \neg C_{out} \vee \neg Z_i$$
$$\neg Y_i \vee \neg C_{in} \vee C_{out}$$
$$Y_i \vee C_{in} \vee \neg C_{out}$$
$$\neg X_i \vee \neg Y_i \vee \neg C_{in} \vee Z_i$$
$$X_i \vee Y_i \vee C_{in} \vee \neg Z_i$$

A Specialized Adder for X+1 = Y

$$\begin{array}{rccccc} & X_{i+4} & X_{i+3} & X_{i+2} & X_{i+1} & X_i \\ + & & & & & C_{in} \\ \hline C_{out} & Y_{i+4} & Y_{i+3} & Y_{i+2} & Y_{i+1} & Y_i \end{array}$$

- Use full adders
  - Use one carry variable for each bit position
  - Need 50 clauses
- Use specialized adders
  - Use one carry variable for every 5 bit positions
  - Need 25 clauses

The Multiplication Constraint: X*Y = Z

- The Shift-and-Add Algorithm

X.m*Y.m = Z.m   X.m = <Xn-1,…,X1,X0>

$$
\begin{array}{l}
X_0 = 0 \Rightarrow S_0 = 0 \\
X_0 = 1 \Rightarrow S_0 = Y \\
X_1 = 0 \Rightarrow S_1 = S_0 \\
X_1 = 1 \Rightarrow S_1 = (Y << 1) + S_0 \\
\vdots \\
X_i = 0 \Rightarrow S_i = S_{i-1} \\
X_i = 1 \Rightarrow S_i = (Y << i) + S_{i-1} \\
\vdots \\
X_{n-1} = 0 \Rightarrow S_{n-1} = S_{n-2} \\
X_{n-1} = 1 \Rightarrow S_{n-1} = (Y << (n-1)) + S_{n-2} \\
Z = S_{n-1}
\end{array}
$$

Equivalence Reasoning

- Equivalence reasoning is an optimization that reasons about a possible value for a Boolean variable or the relationship between two Boolean variables.

$$
\begin{array}{ll}
X = \mathrm{abs}(Y) & \Rightarrow X.\mathrm{m} = Y.\mathrm{m},\ X.\mathrm{s} = 0 \\
X = \text{-}Y & \Rightarrow X.\mathrm{m} = Y.\mathrm{m},\ X.\mathrm{s} = Y.\mathrm{s} = 0 \rightarrow X.\mathrm{m} = 0 \\
X = Y \bmod 2^K & \Rightarrow X_0 = Y_0,\ X_1 = Y_1,\ \ldots,\ X_{k-1} = Y_{k-1} \\
X = Y \ \mathrm{div}\ 2^K & \Rightarrow X_0 = Y_K,\ X_1 = Y_{K+1},\ \ldots
\end{array}
$$

- No clauses are needed to encode X = abs(Y).

Constant Propagation on X+Y = Z

$$\boxed{X_i + Y_i = C_{out} Z_i}$$

$$\begin{aligned}
&\textbf{Rule-1} : X_i = 0 \Rightarrow C_{out} = 0 \wedge Z_i = Y_i. \\
&\textbf{Rule-2} : X_i = 1 \Rightarrow C_{out} = Y_i \wedge Z_i = \neg Y_i. \\
&\textbf{Rule-3} : Z_i = 0 \Rightarrow C_{out} = X_i \wedge X_i = Y_i \\
&\textbf{Rule-4} : Z_i = 1 \Rightarrow C_{out} = 0 \wedge X_i = \neg Y_i.
\end{aligned}$$

- Example-1

$$\begin{array}{cccc}
 & X_2 & X_1 & X_0 \\
+ & 1 & 0 & 0 \\
\hline
Z_3 & Z_2 & Z_1 & Z_0
\end{array}
\quad \Rightarrow \quad
\begin{aligned}
X_0 &= Z_0 \\
X_1 &= Z_1 \\
\neg X_2 &= Z_2 \\
X_2 &= Z_3
\end{aligned}$$

- Example-2

$$\begin{array}{cccc}
 & X_2 & X_1 & X_0 \\
+ & Y_2 & Y_1 & Y_0 \\
\hline
1 & 0 & 1 & 1
\end{array}
\quad \Rightarrow \quad
\begin{aligned}
\neg X_0 &= Y_0 \\
\neg X_1 &= Y_1 \\
X_2 &= Y_2 \\
X_2 &= 1 \\
Y_2 &= 1
\end{aligned}$$

Constant Propagation on X*Y = Z

$$
\begin{aligned}
&X_0 = 0 \Rightarrow S_0 = 0\\
&X_0 = 1 \Rightarrow S_0 = Y\\
&X_1 = 0 \Rightarrow S_1 = S_0\\
&X_1 = 1 \Rightarrow S_1 = (Y << 1) + S_0\\
&\vdots\\
&X_i = 0 \Rightarrow S_i = S_{i-1}\\
&X_i = 1 \Rightarrow S_i = (Y << i) + S_{i-1}\\
&\vdots\\
&X_{n-1} = 0 \Rightarrow S_{n-1} = S_{n-2}\\
&X_{n-1} = 1 \Rightarrow S_{n-1} = (Y << (n-1)) + S_{n-2}\\
&Z = S_{n-1}
\end{aligned}
$$

Rule 5 : $X_i = 0 \Rightarrow$ copy all of the bits of S_{i-1} into S_i.

Rule 6 : $X_i = 1 \Rightarrow$ copy the lowest i bits of S_{i-1} into S_i.

Rule 7 : $X.m = <X_{n-1} \ldots X_i 0 \ldots 0> \wedge X_i = 1 \quad \Rightarrow$
$Z_i = Y_0 \wedge Z_k = 0$ for $k \in 0..(i-1)$.

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all_different(L)

$$\texttt{all\_different}([V_1, V_2, \ldots, V_n])$$

- Standard

  $V_i \neq V_j$ for $i, j = 1, \ldots, n, i < j$.

- Use `at_most_one`
  - Let $D = D_1 \cup D_2 \cup \ldots \cup D_n$
  - If $|D| > n$: $\forall_a \in D$: $\texttt{at\_most\_one}([V_1 = a, V_2 = a, \ldots, V_n = a])$
  - If $|D| = n$: $\forall_a \in D$: $\texttt{exactly\_one}([V_1 = a, V_2 = a, \ldots, V_n = a])$
- A hybrid of log and direct encodings

Table Constraints

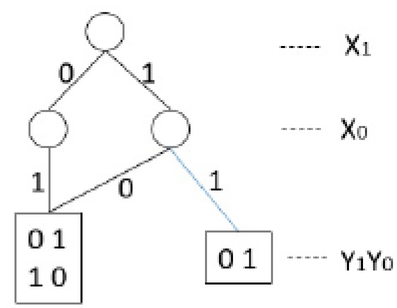
X	Y
1	1
1	2
2	1
2	2
3	1

X_1	X_0	Y_1	Y_0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1

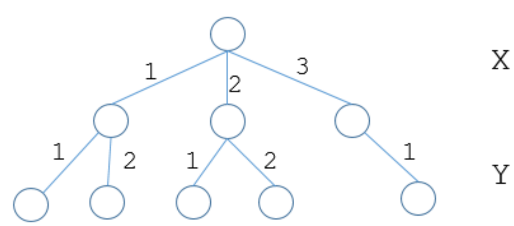
- Truth table encoding

$$\neg X_1 \lor \neg X_0 \lor Y_0$$
$$X_1 \lor X_0$$
$$Y_1 \lor Y_0$$
$$\neg Y_1 \lor \neg Y_0$$

- BDD and truth table



- Trie encoding



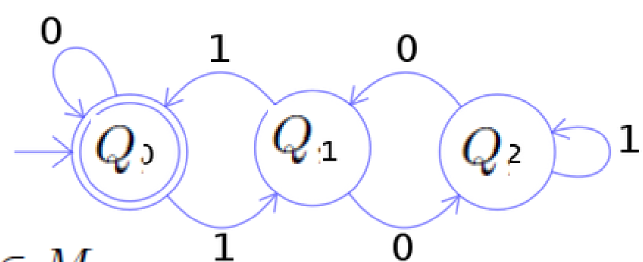
The element and regular Constraints

- $\texttt{element}(I,[E_1,E_2,\ldots,E_n],V)$
  - If E_i's are ground

    $(I,V) \in [(1,E_1),(2,E_2),\ldots,(n,E_n)]$
  - If at least one E_i is variable

    for each $i \in 1..n$: $I = i \Rightarrow V = E_i$

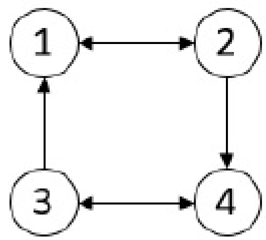
- $\texttt{regular}(L,Q,S,M,Q_0,F)$
  - Introduce state variables $Q_1 \ldots Q_n$
  - Transition constraints: $(Q_i, L_i, Q_{i+1}) \in M$
  - Preprocessing



Gilles Pesant: A Regular Language Membership Constraint for Finite Sequences of Variables, 2004.
Claude-Guy Quimper, Toby Walsh: Global Grammar Constraints, 2006.

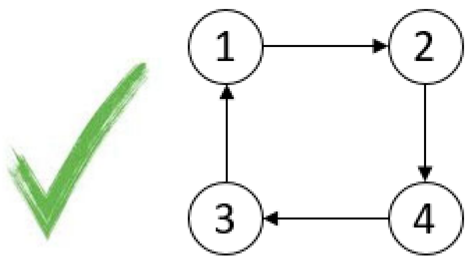
The Global Constraint circuit(G)

$$\texttt{circuit}(G)$$
$$G = [V_1, V_2, \ldots, V_n]$$

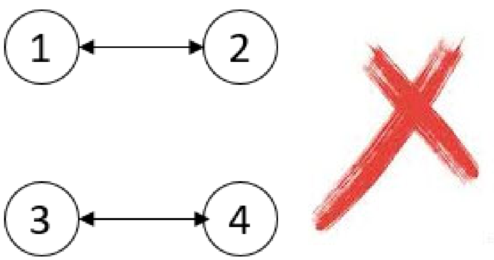


V_1 :: [2]
V_2 :: [1,4]
V_3 :: [1,4]
V_4 :: [3]

G = [2,4,1,3]



G = [2,1,4,3]



- Alexander Hertel, Philipp Hertel, and Alasdair Urquhart: Formalizing dangerous SAT encodings, 2007.
- Andrew Johnson: Stedman and Erin Triples encoded as a SAT Problem, 2018.
- Fangzhen Lin and Jicheng Zhao: On tight logic programs and yet another translation from normal logic programs to propositional logic, 2003.
- Neng-Fa Zhou: In Pursuit of an Efficient SAT Encoding for the Hamiltonian Cycle Problem, 2019.

Distance Encoding for circuit

$$\texttt{circuit}([V_1, V_2, \ldots, V_n])$$

- Variables
  - $H_{ij} = 1$ if the arc (i,j) is in the cycle.
  - Use a domain variable P_i for vertex i's position.
- Constraints
  - Channeling constraints

    For each $i \in 1..n,\ j \in 1..n$: $H_{ij} \Leftrightarrow V_i = j$ (1)
  - Degree constraints

    For each $j \in 1..n$: $\sum_{i=1}^{n} H_{ij} = 1$ (2)
  - Visit vertex 1 first: $P_i = 1$

    For each $i \in 2..n$:

    $H_{1i} \Rightarrow P_i = 2$ (3")

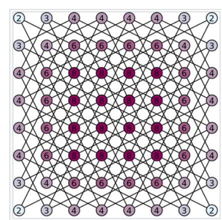
    $H_{i1} \Rightarrow P_i = n$ (4")
  - Transition constraints

    For each $i \in 2..n,\ j \in 2..n,\ i \neq j$:

    $H_{ij} \Rightarrow P_j = P_i + 1$ (5")

Preprocessing circuit

- HCP can be treated as a single-agent path finding problem.
  - Path heuristics: 
    - The agent cannot reach a vertex at time t if there are no paths of length t-1 from vertex 1 to the vertex.
    - The agent cannot occupy a vertex at time t if there are no paths of length n-t+1 from the vertex to vertex 1.



  - The Hall's theorem
    - If he start vertex 1 has exactly two neighbors, then the agent must visit one of the neighbors at time 2, and visit the other neighbor at time n.

The cumulative Constraint

$$\texttt{cumulative}([S_1, S_2, \ldots, S_n], [D_1, D_2, \ldots, D_n], [R_1, R_2, \ldots, R_n], Limit)$$

- Occupation constraints

  for each time t_i and each task j:

  $$O_{ij} \leftrightarrow S_j \leq t_i < S_j + D_j$$

- Resource constraints

  for each time t_i:

  $$\sum_{j=1}^{n} O_{ij} * R_j \leq Limit$$

- Time points
  - Time decomposition: all the time points in the make span
  - Task decomposition: only the start or end points

- Andreas Schutt, Thibaut Feydy, Peter J. Stuckey, and Mark G.Wallace. Explaining the cumulative propagator, 2011.

Outline

- SAT, SAT solving, and SAT encodings
- Picat: a modeling language for SAT
- PicatSAT: Compiling CSP to SAT
  - Compiling arithmetic constraints
  - Decomposing global constraints
- Experimental results

Experimental Results (MiniZinc Challenge, Free Search)

Benchmark	Picat19	Picat18	or-tools	HaifCSP	Choco	Chuffed
cargo (5)	3	3	3	3	0	3
concert-hall-cap(5)	4	3	4	0	4	4
elitserien (5)	2	0	5	5	4	5
gfd-schedule (5)	5	4	5	4	0	4
largescheduling(5)	0	0	0	0	0	0
mapping (5)	3	3	4	**5**	2	3
neighbours (5)	4	4	2	0	1	2
on-call-rostering(5)	3	3	**4**	0	0	0
oocsp_racks (5)	5	5	4	5	1	5
opt-cryptanalysis(5)	5	5	0	0	0	0
proteindesign12(5)	3	0	1	1	1	3
racp (5)	2	1	3	1	1	3
rotating-workforce(5)	**5**	5	4	1	3	5
seat-moving (5)	**5**	4	3	3	2	4
soccer-computational(5)	**5**	5	4	5	3	5
steiner-tree (5)	3	3	4	4	2	4
team-assignment(5)	3	3	3	3	1	3
test-scheduling(5)	2	3	3	0	3	2
train (5)	2	2	2	2	2	2
vrplc (5)	3	2	4	1	0	2
Total (100)	**67**	58	62	43	30	59
Benchmark	Picat19	Picat18	or-tools	HaifCSP	Choco	Chuffed

Experimental Results (XCSP Competition, COP)

Benchmark	Picat19	Picat18	Choco	Concrete	Scop-both	Scop-order
Auction(16)	2	2	2	2	n/a	n/a
Bacp(24)	8	**8**	4	4	n/a	n/a
CrosswordDesign(13)	1	2	3	3	n/a	n/a
Fapp(18)	4	**7**	6	6	n/a	n/a
GolombRuler(13)	6	6	6	**7**	n/a	n/a
GraphColoring(11)	7	6	5	**7**	n/a	n/a
Knapsack(14)	10	6	6	6	n/a	n/a
LowAutocorrelation(14)	10	**10**	3	3	n/a	n/a
Mario(10)	7	7	10	10	n/a	n/a
NurseRostering(21)	2	2	2	2	n/a	n/a
PeacableArmies(14)	6	4	4	4	n/a	n/a
PizzaVoucher(10)	8	**7**	3	4	n/a	n/a
PseudoBoolean-opt(13)	5	**5**	3	3	n/a	n/a
QuadraticAssignment(19)	3	2	2	2	n/a	n/a
Rcpsp(16)	**16**	14	12	12	n/a	n/a
Rlfap(25)	22	5	5	5	n/a	n/a
SteelMillSlab(17)	4	2	2	2	n/a	n/a
StillLife(13)	13	**13**	3	3	n/a	n/a
SumColoring(14)	5	**5**	3	3	n/a	n/a
Tal(10)	7	3	4	4	n/a	n/a
TemplateDesign(15)	12	11	11	10	n/a	n/a
TravelingTournament(14)	2	2	2	2	n/a	n/a
TravellingSalesman(12)	3	**3**	1	2	n/a	n/a
COP Total (346)	163	132	102	106	n/a	n/a
Benchmark	Picat19	Picat18	Choco	Concrete	Scop-both	Scop-order

Experimental Results (XCSP Competition, CSP)

Benchmark	Picat19	Picat18	Choco	Concrete	Scop-both	Scop-order
Bibd(12)	8	8	4	4	8	8
CarSequencing(17)	17	17	11	2	13	17
ColouredQueens(12)	4	**4**	3	3	3	3
Crossword(13)	4	2	**5**	3	3	3
Dubois(12)	12	12	7	4	12	12
Eternity(15)	7	6	**7**	6	6	6
Frb(16)	6	3	3	4	5	5
GracefulGraph(11)	6	6	6	6	6	6
Haystacks(10)	8	10	4	3	10	10
Langford(11)	9	9	7	3	9	9
MagicHexagon(11)	4	3	4	**5**	3	3
MisteryShopper(10)	10	10	10	**11**	10	10
PseudoBoolean-dec(13)	7	4	4	4	7	8
Quasigroups(16)	16	7	6	7	**8**	8
Rlfap(12)	12	12	11	8	12	12
SocialGolfers(12)	8	8	6	6	8	8
SportsScheduling(10)	5	4	3	4	4	4
StripPacking(12)	12	11	7	6	11	11
Subisomorphism(11)	7	2	**7**	4	3	3
CSP Total (236)	162	138	115	93	141	146
Total (583)	325	270	217	199	n/a	n/a
Benchmark	Picat19	Picat18	Choco	Concrete	Scop-both	Scop-order

Summary of PicatSAT

- Adopts log and direct encodings
- Performs numerous optimizations
  - Preprocessing constraints to exclude no-good values (CP)
  - Common subexpression elimination (language compilers)
  - Logic optimization (hardware design)
  - Compile-time equivalence reasoning
- Employs efficient decomposers for global constraints
- Available on picat-lang.org

Thanks

- **Other contributors**
  - Roman Barták
  - Agostino Dovier
  - Jonathan Fruhman
  - Sanders Hernandez
  - Håkan Kjellerstrand
  - Jie Mei
  - Yi-Dong Shen
  - Taisuke Sato
- **Funding**
  - **NSF** CCF1018006 and CCF1618046
  - **PSC CUNY**
  - **NYC Media Lab**