

Wavelet Transform for the Analysis of Convolutional Neural Networks in Texture Recognition

Joao Batista Florindo ^a

*Institute of Mathematics, Statistics and Scientific Computing, University of Campinas,
Rua Sérgio Buarque de Holanda, 651, Cidade Universitária "Zeferino Vaz",
Distr. Barao Geraldo, CEP 13083-859, Campinas, SP, Brazil*

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Abstract: Convolutional neural networks have become omnipresent in applications of image recognition during the last years. However, when it comes to texture analysis, classical techniques developed before the popularity of deep learning has demonstrated potential to boost the performance of these networks, especially when they are employed as feature extractor. Given this context, here we propose a novel method to analyze feature maps of a convolutional network by wavelet transform. In the first step, we compute the detail coefficients from the activation response on the penultimate layer. In the second one, a one-dimensional version of local binary patterns are computed over the details to provide a local description of the frequency distribution. The frequency analysis accomplished by wavelets has been reported to be related to the learning process of the network. Wavelet details capture finer features of the image without increasing the number of training epochs, which is not possible, in feature extractor mode. This process also attenuates over-fitting effect at the same time that preserves the computational efficiency of feature extraction. Wavelet details are also directly related to fractal dimension, an important feature of textures and that has also recently been found to be related to generalization capabilities. The proposed methodology was evaluated on the classification of benchmark databases as well as in a real-world problem (identification of plant species), outperforming the accuracy of the original architecture and of several other state-of-the-art approaches.

1 INTRODUCTION

In the last years, convolutional neural networks (CNN) have been successfully employed in a variety of tasks in computer vision, including image classification (Krizhevsky et al., 2017), object recognition (Ren et al., 2017), image segmentation (Chen et al., 2018), video analysis (Ng et al., 2015), etc. Among such tasks, an important topic that also has benefited from CNNs is texture recognition (Cimpoi et al., 2016). Besides theoretical developments, CNN-based algorithms have also been applied to several real-world problems in which texture images play fundamental role, such as in medicine (Shin et al., 2016), remote sensing (Zhu et al., 2017), face recognition (Taigman et al., 2014), and others.

Despite the great performance of classical CNN architectures in texture recognition, it has been also shown that other strategies can leverage the power of

those neural networks, either increasing the classification accuracy or reducing computational requirements for training of the learning model. In particular, traditional texture representations that have been successful in the years before the popularization of deep learning have been recently rediscovered and combined with CNNs in hybrid models that have demonstrated interesting capabilities. A remarkable example is deep filter banks (Cimpoi et al., 2016), that use convolutional feature maps as local descriptors and classical encodings like Fisher vectors and “bag-of-visual-words”. In (Ji et al., 2020), the authors obtain high classification accuracy and performance by introducing constraints and compression techniques associated to local binary patterns (LBP) (Ojala et al., 2002) to a convolutional/recurrent neural network. In (Anwer et al., 2018), LBP maps are combined with the original image at different stages of the CNN improving the recognition capacity. In T-CNN (Andrearczyk and Whelan, 2016), CNN feature

^a  <https://orcid.org/0000-0002-0071-0227>

maps are also interpreted as filter banks and an energy measure is established to provide feature vectors for image representation. We should also mention here the “handcrafted” CNN descriptors, that use CNN-like architectures, but associated with pre-defined filter banks. Examples are NmzNet (Vu et al., 2017), ScatNet (Bruna and Mallat, 2013), PCANet (Chan et al., 2015), etc.

In this context, we propose the use of wavelet transforms as a technique for the analysis of CNN feature maps. Following ideas like those developed in (Cimpoi et al., 2016), we apply the wavelet transform over the penultimate fully-connected layer of the CNN. In the following, we calculated one-dimensional binary patterns (Tirunagari et al., 2017) of the detail signal to form the final texture representation. The frequency distribution of the neural response, here described by wavelet transform, is known to be related to the learning evolution (Rahaman et al., 2019). Injecting such complementary information to the image descriptors has potential to be very beneficial to a more robust representation. Furthermore, using the different radii for binary patterns ensures a second layer of multiscale analysis over localized frequencies. Together, these transformations of the activation response are capable of expressing the localized frequency at multiple scale levels, capturing a complete mapping of the CNN both in frequency and spatial domain.

The proposed methodology was assessed on the classification of four benchmark data sets: KTHTIPS-2b (Hayman et al., 2004), FMD (Sharan et al., 2009), UIUC (Lazebnik et al., 2005), and UMD (Xu et al., 2009). It was also applied to a practical problem, that of identifying Brazilian plant species based on the leaf surface texture (Casanova et al., 2009). The results are competitive with the state-of-the-art on texture recognition and suggest our proposal as a promising approach for practical purposes, especially for smaller datasets where the use of CNN as a feature extractor is more appropriate.

2 PROPOSED METHOD

2.1 Motivation

An interesting result obtained by theoretical studies on neural networks is that low frequencies (global viewpoint) are learned before high frequencies (fine details) along the training epochs (Rahaman et al., 2019). The reader can check, for example, Figure 2 in (Ronen et al., 2019) for an intuition. Evidences suggest that such “learning guided by frequency” phe-

nomena, confirmed both theoretically for toy models and empirically on realistic deep neural networks, does not occur by chance and that it could be explored further. In particular, if frequencies are important during the training, it makes sense to imagine that they are relevant at the output of the network as well. Given that the training usually stops after a fixed number of cycles, it is expected that the distribution of frequencies in the output response of the network is tightly related to the distribution on the input (image in our case). On the other hand, it is also well known in image processing and analysis that frequency analysis provides an alternative complementing viewpoint of the original spatial representation. This same information however has not been sufficiently explored on the network response.

Fourier transform is the most well known tool for frequency analysis and would be a natural candidate for the same analysis on neural networks. Nevertheless, a simple observation of the response distribution makes it clear how the information conveyed drastically varies depending on the observed region of the signal. This is a consequence both of the random initialization and the complexity of the neural network function. In this context, wavelet analysis is more suitable as it supports the spatial localization together with the frequency. We have in this way a more complete description of the frequency distribution on the network. Furthermore, the energy of wavelet details is known to be directly related to the fractal dimension of the signal (Mallat, 2008). Besides being an important feature in texture analysis (Xu et al., 2009), recent studies have also established connections between such dimension and the generalization performance of neural networks (Simsekli et al., 2020).

Figure 1 shows the wavelet scalograms and frequency distribution at the penultimate layer of a CNN, for three clearly different textures: an irregular profile T1 (“pebbles”), a homogeneous texture T2 (“fur”) and an artificial periodic material T3 (“corduroy”). At the bottom we see the average energy contributed by each frequency in each texture. T3 spectrum is more balanced, not surpassing the magnitude of the other samples with localized peaks. This corresponds to moderate contribution of a larger band of frequencies and is consequence of the multiscale regularity, corresponding to multiple frequencies arising. For T1 and T2, they alternate their influence in each frequency region. T2 has two wide peaks around 0.1 and 0.25, whilst T1 concentrates around 0.15 and 0.45. Such shift to lower frequencies in T2, when compared with T1, is a consequence of the regular patterns in T1, even though we do not have the same homogeneity presented by T3 now.

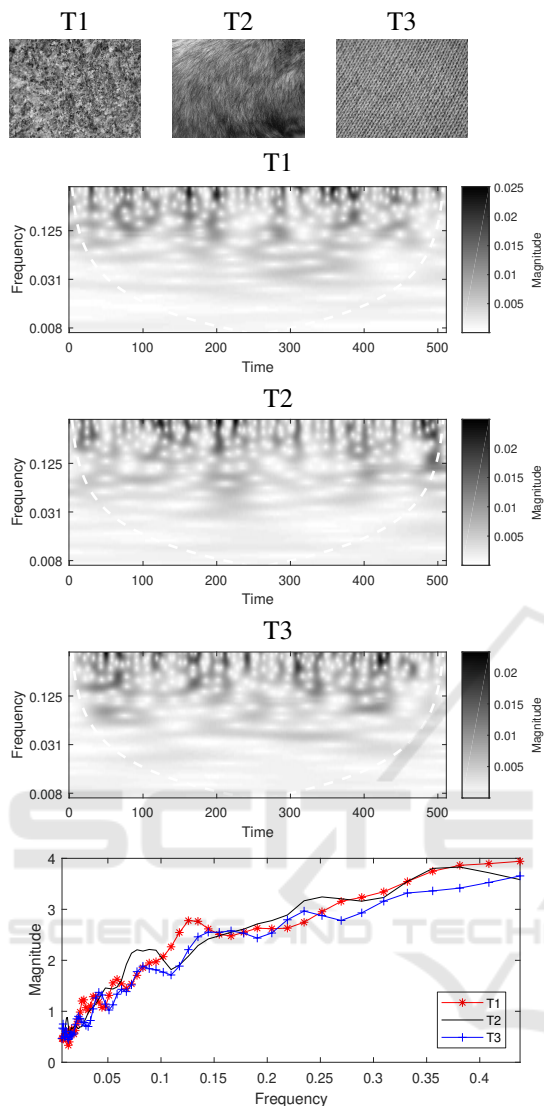


Figure 1: Three types of textures: irregular (T1), natural regular (T2), and artificially periodic (T3). Below we have the wavelet scalograms of each texture and the average magnitude on the frequency range.

Such remarkable behavior naturally points out to the possibility of exploring wavelet decomposition of neural network responses. The combination of wavelets with neural networks has in fact been explored in the literature (Zhang and Benveniste, 1992; Alexandridis and Zaprani, 2013; Fujieda et al., 2017). These works, however, usually focus on introducing new trainable elements into the network architecture. Here we opt for a significantly simpler strategy of applying multi-resolution wavelets to the network activation response, which besides having simpler implementation, allows for more in-depth studies in the future on how such responses are affected by

frequency response (a more elaborated version of the analysis in Figure 1).

2.2 Implementation

Based on these assumptions, the proposed method relies on the processing of the activation responses of neurons at the penultimate fully-connected layer. Here we use the VGGVD architecture as in (Cimpoi et al., 2016). Each image is processed by the already trained neural network outputting a vector of real values at that layer. Such vector is therefore processed by a composite operator formed by a combination of a discrete wavelet transform (Mallat, 2008) and local binary patterns in their 1D version, as described in (Tirunagari et al., 2017).

In the following we calculate the detail coefficients d_{jk} . In discrete domain they can be obtained by

$$d_{jk} = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} y_n \Psi_{j0k}(n), \quad j \geq j_0, \quad (1)$$

where j is the decomposition level, M is the length of y and ϕ and ψ are the wavelet functions. These coefficients can be easily obtained in any modern scientific programming language. Here we use *symlet4* wavelet functions (Mallat, 2008). Two levels of decomposition are employed here yielding the vectors d_1 and d_2 .

In the next step, those detail vectors are linearly combined with a predefined weight α :

$$D_\alpha = \begin{cases} (1 - \alpha)ds(y) + \alpha d_1. & \text{if } 0 \leq \alpha \leq 1, \\ (2 - \alpha)ds(d_1) + (\alpha - 1)d_2. & \text{if } 1 \leq \alpha \leq 2, \end{cases} \quad (2)$$

where ds is a downsampling function (here using a lowpass Chebyshev Type I infinite impulse response filter of order 8). The downsampling operation is necessary to make the combined vectors to have the same length.

Finally, the descriptors are given by applying 1D LBP with radius r over D_α :

$$D_{\alpha,r} = LBP_r(D_\alpha). \quad (3)$$

The proposed method essentially explores two complementary fundamental viewpoints of the texture neural activation: the frequency and spatial distribution. At the same time it benefits from all the machinery of CNNs and their capacity of extracting features in highly complex hierarchical organizations. The efficient local description provided by LBP over the high frequency distribution captured by wavelet details has the effect of providing a detailed map of finer details in the output CNN function. According to studies like (Rahaman et al., 2019), this corresponds

precisely to characteristics learned in more advanced stages of the CNN. On the other hand, frequency domain is not restricted to a well-localized point on the loss surface, such that this type of analysis ends up being less susceptible to over-fitting than simply using a very large number of epochs in the CNN model.

3 EXPERIMENTS

We assess the performance of the proposed method in texture recognition over four benchmark datasets (KTH-TIPS-2b (Hayman et al., 2004), FMD (Sharan et al., 2009), UIUC (Lazebnik et al., 2005), and UMD (Xu et al., 2009)) and an application to the identification of Brazilian plant species (database 1200Tex (Casanova et al., 2009)).

KTH-TIPS2b database is composed by 4752 color images of 11 different materials (classes). Unlike classical texture databases, that are focused on visual appearance, here each target group may contain substantially different images. This is particularly challenging mainly for classical local patch approaches like LBP and bag-of-visual-words. Each material is further divided into 4 samples, each sample corresponding to particular settings of illumination, scale and perspective. Here we adopt the most challenging protocol of training on 1 sample of each class and testing on the remaining 3 samples.

FMD is also a material-based dataset. Comprises 10 classes with 100 color images per class and the training/testing protocol is a random split with 50% for training and 50% for testing, which is repeated 10 times to provide statistical measures.

UIUC and UMD, on the other hand, are typical texture-based databases. Both comprise 25 classes with 40 grayscale samples in each class. Both are also collected under non-controlled conditions and share variations in scale, illumination and viewpoint. The most remarkable difference is in resolution: whereas UIUC has smaller images with size 640×480 , UMD images has resolution 1280×960 .

1200Tex (Casanova et al., 2009) is a database of color texture images acquired by scanning the leaf surface of 60 species of Brazilian plants. The objective is to identify the respective species. A commercial scanner is used and conditions of illumination, pose and scale are strictly controlled. For each class (species) we have 60 image samples, each one corresponding to a non-overlapping window with size 128×128 . The training/testing split is the same one used in UIUC and UMD, i.e., 50% for training and 50% for testing.

For the classification we employed linear discriminant analysis (Bishop, 2006), mainly for its ease of interpretation and no need for tuning a large number of hyper-parameters. Given the high number of features, we also applied principal component analysis (Bishop, 2006). The number of components was defined by cross-validation over the training set.

4 RESULTS

Figure 2 shows the classification accuracies on the compared databases when parameters r and α are varied. In general, we observe that $r = 0$, i.e., the direct use of wavelet transform is a good choice in most scenarios. Nevertheless, some significant boost was obtained in KTH and UIUC by using $r = 2$ and $r = 3$, respectively. It can also be noticed that $r = 2$ and $r = 3$ are in general preferred over $r = 1$. Another interesting point is that the best performance is achieved usually for $\alpha \approx 1.0$, which corresponds to the first level of detail coefficients. Actually, we verified the accuracy for higher levels but no gain was obtained. Table 2 lists the best accuracies for each LBP radius, also confirming the effectiveness of using $r = 0$.

Tables 1 and 2 present the accuracies compared to state-of-the-art results published in the literatures on the same databases and using the same protocol. We separate KTH, FMD, UIUC, and UMD (benchmark data sets) from 1200Tex (application) due to the comparatively reduced number of results for the later in the literature. In both cases we notice the proposed descriptors outperforming several state-of-the-art methods, including advanced CNN-based solutions. This is more evident in the most challenging databases, i.e., KTH, FMD and 1200Tex. Even when the proposed descriptors were surpassed in a few situations in UMD and UIUC databases, they achieved higher accuracy in the other ones.

In summary, the presented results corroborate our expectations of an improved performance of CNNs when associated to the wavelet representation. The details coefficient were capable of extracting a fine-grained description of the image, but avoiding over-fitting due to the non-localized behavior of frequency distribution. Altogether, this was the rationale behind a more complete CNN representation for the texture image, that was responsible for more robust descriptors and, as a consequence, higher classification accuracies in general.

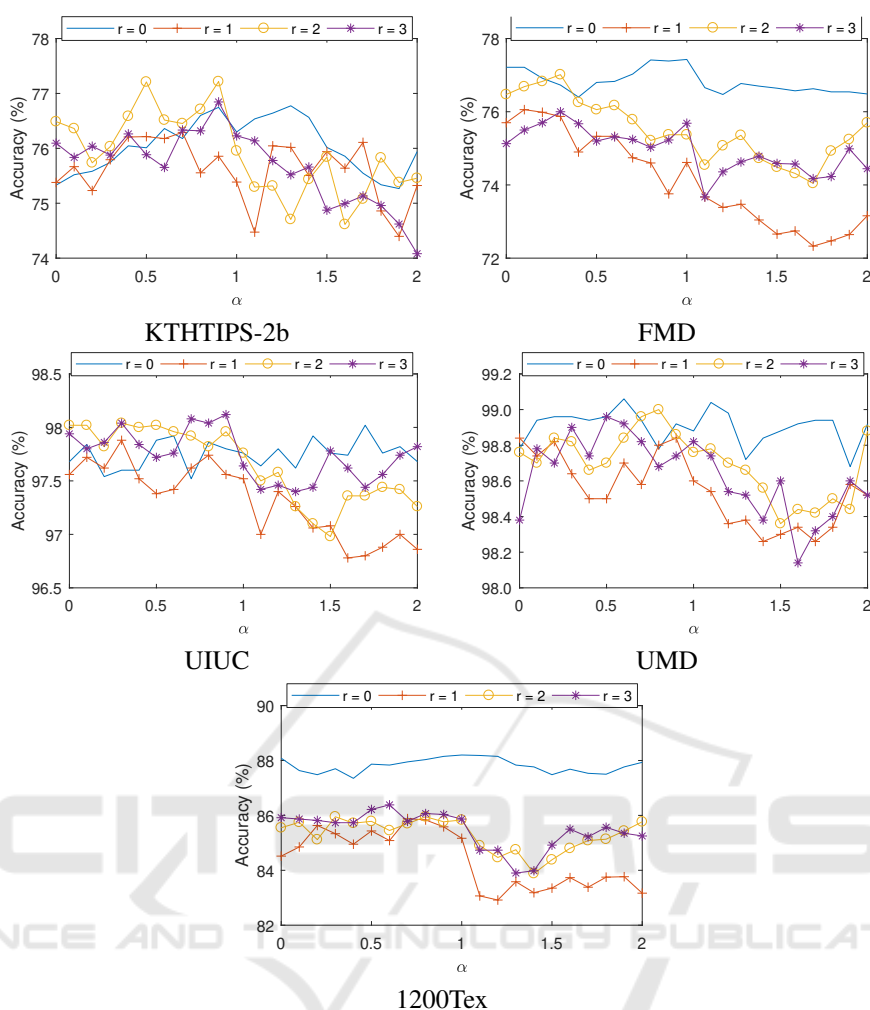


Figure 2: Accuracies of the proposed method for different hyperparameters on the texture databases. Even though $r = 0$ works fine in most cases, LBP improved the accuracy in more complicated scenarios, e.g., in KTHTIPS-2b textures.

5 CONCLUSIONS

This work presented a new method for texture recognition extracting wavelet details from the penultimate layer of a CNN. Such coefficients were also subjected to LBP analysis to improve even further its ability of capturing the complexity of a visual texture.

The performance of the proposal was evaluated in terms of accuracy on the classification of benchmark datasets and on a practical problem of identifying plant species using the leaf surface texture. In this tasks, the proposed method outperformed several state-of-the-art approaches for texture recognition, attesting its potential as a powerful representation for texture images.

The success of the proposed methodology can be expected and explained in theoretical terms by a novel

perspective over the CNN offered by the localized frequency analysis of wavelets. Detail coefficients describe fine-grained characteristics of the image at the same time that frequency domain prevents over-fitting as the feature is not exactly localized in space.

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Table 1: Accuracies for different databases: KTH2b, FMD, UIUC, and UMD. Several state-of-the-art approaches are outperformed by the proposed method.

Method	KTH2b	FMD	UIUC	UMD
VZ-MR8 (Varma and Zisserman, 2005)	46.3	22.1	92.9	-
LBP (Ojala et al., 2002)	50.5	-	88.4	96.1
VZ-Joint (Varma and Zisserman, 2009)	53.3	23.8	78.4	-
BSIF (Kannala and Rahtu, 2012)	54.3	-	73.4	96.1
CLBP (Guo et al., 2010a)	57.3	43.6	95.7	98.6
LBP _{riu2} /VAR (Ojala et al., 2002)	58.5	-	84.4	95.9
PCANet (NNC) (Chan et al., 2015)	59.4	-	57.7	90.5
RandNet (NNC) (Chan et al., 2015)	60.7	-	56.6	90.9
ScatNet (NNC) (Bruna and Mallat, 2013)	63.7	-	88.6	93.4
DeCAF (Cimpoi et al., 2014)	70.7	60.7	94.2	96.4
SIFT + BoVW (Cimpoi et al., 2014)	58.4	49.5	96.1	98.1
FC-CNN VGGM (Cimpoi et al., 2016)	71.0	70.3	94.5	97.2
FC-CNN AlexNet (Cimpoi et al., 2016)	71.5	64.8	91.1	95.9
FC-CNN VGGVD (Cimpoi et al., 2016)	75.4	77.4	97.0	97.7
RAMBP (Alkhatib and Hafiane, 2019)	68.9	46.8	94.8	98.6
H2OEP (Song et al., 2021)	64.2	-	-	-
SWOBP (Song et al., 2020)	66.4	-	-	-
SLGP (Song et al., 2018b)	53.6	-	-	-
LBPC (Singh et al., 2018)	50.7	-	-	-
LETRIST (Song et al., 2018a)	65.3	-	97.7	98.8
BRINT _{CPS} (Pan et al., 2019)	-	-	92.2	93.5
MRELBP _{CPS} (Pan et al., 2019)	-	-	95.2	94.2
DSTNet (Florindo, 2020)	61.0	-	93.6	98.5
2D-LTP (Xiao et al., 2019)	-	49.0	-	-
Proposed	77.2	77.7	98.1	99.0

Table 2: State-of-the-art accuracies for 1200Tex. Outperforming computationally intensive methods like FV-CNN is a remarkable achievement.

Method	Accuracy (%)
LBPV (Guo et al., 2010b)	70.8
Network diffusion (Gonçalves et al., 2016)	75.8
FC-CNN VGGM (Cimpoi et al., 2016)	78.0
FV-CNN VGGM (Cimpoi et al., 2016)	83.1
Gabor (Casanova et al., 2009)	84.0
FC-CNN VGGVD (Cimpoi et al., 2016)	84.2
SIFT + BoVW (Cimpoi et al., 2014)	86.0
FV-CNN VGGVD (Cimpoi et al., 2016)	87.1
DSTNet (Florindo, 2020)	79.3
Proposed	88.2

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