# Segmenting Maps by Analyzing Free and Occupied Regions with Voronoi Diagrams

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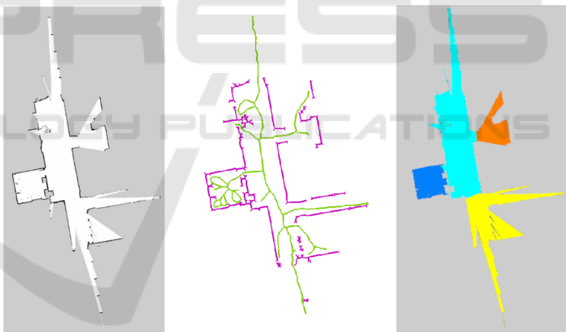
Keywords: Room Segmentation, Voronoi Diagram, Occupancy Grid Map, Indoor Environments, Mobile Robots.

Abstract: Traditional mapping techniques rely on metric properties, which represent indoor information with specific geometric characteristics. This fact highly differs from the way in which people interpret their surroundings. By geometrically segmenting occupancy grid maps into rooms, robots are brought closer to the way in which we understand indoor environments. In this work, Voronoi diagrams are proposed as the main tool to locate map partitions. As a novelty, they are extracted from free and occupied spaces to analyze their shape. This allows to locate narrow passages on free zones which coincide with protruding parts on occupied zones, indicating a nearby door. An additional advantage is the use of a varying threshold that depends on the map structure. This dynamic value can adjust to multiple scenarios, avoiding the use of a fixed threshold that cannot be generalized. Experiments have been conducted in multiple maps, showing the potential of the propose method.

## 1 INTRODUCTION

Indoor environments have been traditionally represented by occupancy grid maps because of their facility of being built with techniques such as Simultaneous Localization and Mapping (SLAM). A grid is used to partition the environment and each of its cells is then classified as either occupied, unoccupied or unknown. However, this way of representing indoor locations differs from the way in which we humans interpret our surroundings. Typical indoor scenarios are divided into rooms and corridors, to which people assign a different utility. That is why geometrically segmenting occupancy grid maps is nowadays a major subject of study. Knowing where these regions are gets robots a step closer to understanding indoor locations meaningfully. Additionally, partitions can be used for multiple applications such as extracting topological maps (Joo et al., 2010), applying coverage algorithms for cleaning robots (Kleiner et al., 2017) or performing multi-robot tasks (Wurm et al., 2008).

In this work, we propose an offline method based on the use of Voronoi diagrams as the main tool for segmenting occupancy grid maps into meaningful regions. The main purpose is segmenting static environments for further uses such as topological planning on robotic applications. Other works have already proposed these diagrams as a tool for geometrically segmenting indoor scenarios. However, they are only extracted from free space, similarly to the way in which they are used for path planning. A major drawback of these methods is that they typically require additional procedures such as combining Voronoi diagrams with image processing techniques or merging small regions after segmenting. The main novelty of the proposed method is using Voronoi diagrams to analyze the structure of both free and occupied zones. The diagram corresponding to free space allows us to locate narrow passages, where the occupied space



(a) (b) (c)

Figure 1: Representative example of the proposed method: (a) occupancy grid map, (b) Voronoi diagrams extracted from free regions (green) and occupied regions (magenta), (c) final map segmentation resulting from analyzing Voronoi diagrams properties.



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happens to have protruding areas. By locating these zones, we define a method that does not depend on other procedures and does not require any merging steps. Fig. 1 shows a representative example of the extracted Voronoi graphs from an occupancy grid map and the resulting map partitions. The use of the terms Voronoi diagram and Voronoi graph are used interchangeably in the rest of the paper.

The rest of the paper is divided as follows. Section 2 reviews related work on room segmentation. In Section 3, the proposed method based on Voronoi diagrams is described. Results obtained from applying this method to multiple occupancy grid maps is shown in Section 4. Finally, conclusions and future work are derived in Section 5.

## 2 RELATED WORK

Many of the works on room segmentation rely on extracting features from occupancy grid maps. Authors in (Joo et al., 2010) propose the extraction of corner features to partition an occupancy grid map. These features are selected by analyzing curvature and angle at each individual cell on the grid. Then, doorways are defined by segments that relate two features with a length smaller than 1.2 m. Additionally, genetic algorithms are used as a second check to remove unnecessary segments. Another method is presented in (Fermin-Leon et al., 2017), where the proposed tool for segmenting an occupancy grid map is its boundaries. Regions between the map's boundaries and the convex hull, named pockets, are iteratively partitioned to calculate their concavity. Those with a value bigger than a threshold are selected as division points. Room partitions are then created by joining them. In (Liu et al., 2018), free cells on the grid map are clustered into groups marked by randomly selected initial points according to the distance among them and proving that no occupied cells are in between. Resulting regions are merged by analyzing their connectivity using Breath-First Search algorithm. It can be noticed that most of these methods rely on heuristically determined thresholds, which could not be suitable for every scenario, or they require a second step in which exceeding divisions are removed or segmented regions are merged, adding extra computational costs.

Another typically used method is Watershed algorithm. It is based on the way in which rivers drain basins in nature, where deeper zones are first filled up until water from different basins touch each other. Authors in (Fabrizi and Saffiotti, 2000) propose the application of the Watershed algorithm on fuzzy grid maps, where each cell value ranges from 0 to 1 indicating the probability of being occupied. This method is adapted to work online in (Buschka and Saffiotti, 2002). In (Kleiner et al., 2017), the distance transform of a binary occupancy grid map is calculated before applying the Watershed segmentation procedure. Then, heuristics are applied for room and corridor merging. As it happened with previous methods, heuristics need to be included in order to merge regions after applying the segmentation procedure. The main disadvantage of the Watershed algorithm is that it tends to oversegment regions since it is highly sensitive to local minima. In order to mitigate this fact, authors in (Ryu, 2020) propose the use of morphological operations before segmenting. By combining erosion and dilation, an initial estimation of the number and location of the different rooms is computed. Due to these operations, the initial map size is lost, so regions need to be grown afterwards until they reach the original map size. This procedure results in a mislocalization of some doorways, since some regions grow faster than others.

Voronoi graphs are one of the most applied techniques for room segmentation. In (Thrun, 1998), the Voronoi graph of free zones is analyzed. Points of local minima on the graph are selected as critical points. Then, critical lines which represent doorways are formed by joining these points to their closest occupied cells. A similar procedure is presented in (Beeson et al., 2005), which computes gateway locations in real time. Voronoi graphs and morphological operations are combined in (Myung et al., 2009) to locate door positions. First, the door C-space is calculated by eroding free space by half of doors width. Then, it is overlapped with the Voronoi graph to locate doorways. Authors in (Hou et al., 2019) propose the combination of Voronoi diagrams and alpha-shapes. Every free cell on the map is assigned to one Voronoi edge, creating multiple small regions. Alpha shapes are finally computed on the map to calculate which areas should be merged. A review of some room segmentation approaches is presented in (Bormann et al., 2016), where the Voronoi graph-based method turned out to be the most stable one, being closer to the ground-truth segmentation. Voronoi graphs are proven to be a handy tool for region characterization. However, they still have the same problem of being conditioned to determining a fixed threshold, which does not always work properly on all possible scenarios, deriving into erroneous door location and oversegmentation.

Other segmentation methods rely on learning techniques. Authors in (Mozos et al., 2007) propose the use of the AdaBoost classifier to identify locations corresponding to corridors, rooms or doorways. Similarly, in (Goeddel and Olson, 2016) a CNN is applied. In the work presented in (Friedman et al., 2007), conditional random fields are applied to Voronoi graphs to label places. Authors in (Hiller et al., 2019) propose to concatenate two CNNs. Even though these methods are effective, their design is complex and computationally expensive since they require training procedures. This is why we mainly focus this work review on geometric procedures.

In contrast to the methods described above, our proposal does not depend on a fixed heuristically determined threshold and does not need any merging steps. By analyzing properties of Voronoi diagrams from both free and occupied zones, doorway locations are estimated.

# 3 MAP SEGMENTATION

The proposed method is based on extracting features from occupancy grid maps by using Voronoi diagrams as the main tool. Fig. 2 shows a summary of the proposed steps for segmenting these maps into meaningful regions.

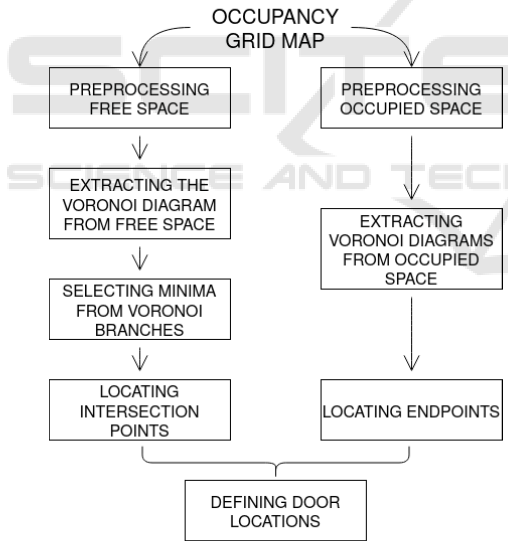



Figure 2: Methodology steps: the left branch corresponds to free space and the right branch corresponds to occupied space. By analyzing them with Voronoi diagrams, key points are extracted from both and are used to define door locations.

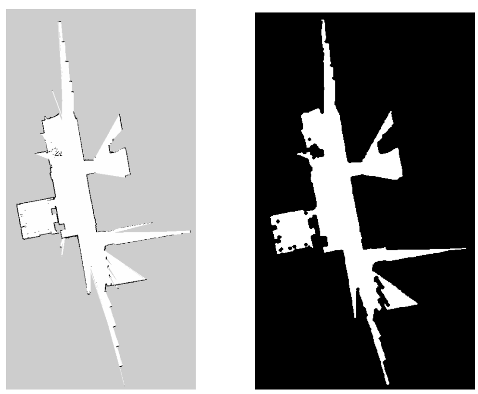
It can be seen that there are two main branches on the procedure, one for analyzing free space and the other one for occupied space. Finally, both are merged to define door locations. With respect to free space, steps are focused on locating narrow passages. For that reason, points of minimum value with respect to distance are selected on the Voronoi branches. Then, intersection points are defined by their two closest occupied cells on the map. With respect to occupied space, we look for protruding parts, which can be found by locating endpoints on Voronoi diagrams, given that they indicate a change in the structure of occupied zones. By analyzing the location of intersection points and endpoints, door locations are found. In the following subsections, a more detailed explanation of these steps is provided.

## 3.1 Analyzing Free Space

One of the main processes of the proposed method is analyzing free space. More specifically, it is intended to find narrow passages by using Voronoi diagrams. For that reason, an auxiliary binary image is created in which free space of the occupancy grid map is marked with value 1 and occupied and unknown spaces are marked with value 0. This image is preprocessed to remove noise, the Voronoi graph is extracted and points of minimum value with respect to distance are located on its branches to indicate possible door locations. Details on how these processes are carried out are provided below.

### 3.1.1 Preprocessing Free Space

Before extracting Voronoi diagrams, it is important to preprocess the corresponding binary image. This is because this type of diagrams are highly sensitive to noise, producing additional branches that do no provide any relevant information. For that reason, typical image processing techniques are applied to the map. First, an erosion process is conducted with a structuring element of rounded shape. Then, a median filter is applied to round corners (Zuo et al., 2020). A representative example of results can be seen in Fig. 3.



(a) (b)

Figure 3: Preprocessing of free space: (a) original occupancy grid map, from which free space is selected, (b) resulting binary image after preprocessing free space.

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#### 3.1.2 Extracting the Voronoi Diagram from Free Space

Once the binary image has been correctly preprocessed, the Voronoi diagram is calculated. As an approximation, the image skeleton is proposed. It can be represented by the set of points corresponding to the centers of the maximal disks enclosed in free areas, touching its boundaries at two or more points (Saha et al., 2016). Since each disk is tangent to map boundaries at multiple points, and being the position of the center equidistant to them, the skeleton represents paths equidistant to objects. This definition is similar to the Voronoi diagram, which ensures a graph as far as possible from obstacles. Morphological operations are used to obtain the skeleton of the image. More specifically, MATLAB's function bwskel (MathWorks, 2022) is applied. Fig. 4 shows results of applying this method to the preprocessed map shown in Fig. 3. Finally, nodes are removed from the graph, leaving only its branches separately. Those with less that 10 pixels are additionally removed. Fig. 4 shows the resulting labeled branches.

(a) (b)

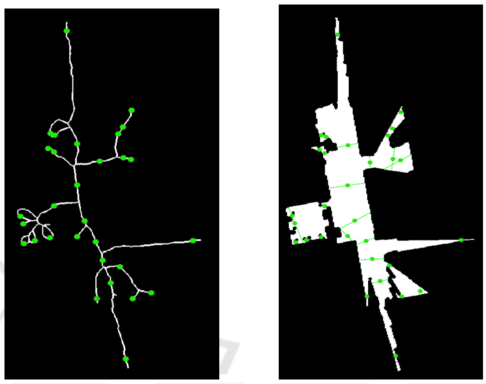
Figure 4: Extracted Voronoi diagram by using the image skeleton: (a) resulting image skeleton, (b) labeled branches.

#### 3.1.3 Selecting Minima from Voronoi Branches

Branches extracted on the previous step are used to locate points corresponding to possible doorways. For that reason, the distance transform of free space is used to assign the distance to the closest occupied cell for each branch point. Then, every branch is separately analyzed. For each of them, the smallest value of its points is found and pixels with such value $\pm 2$ pixels of margin are selected. This is done to help the creation of pixel sets, avoiding loose pixels. The result is a set of line segments corresponding to narrow passages on the scenario, which can be seen as promising door locations. For every segment, centroid and orientation are saved.

#### 3.1.4 Locating Intersection Points

The next step for locating doors is finding intersection points. They are defined as the closest occupied points on the binary map to the centroids of the previously computed line segments in perpendicular direction with respect to their orientation, one on each sense. By joining the two points corresponding to the same centroid, a line segment is defined, which corresponds to promising door locations. Fig. 5 shows a representative example, where minimum value points are marked with green dots and line segments are drawn by joining each pair of intersection points.



(a) (b)

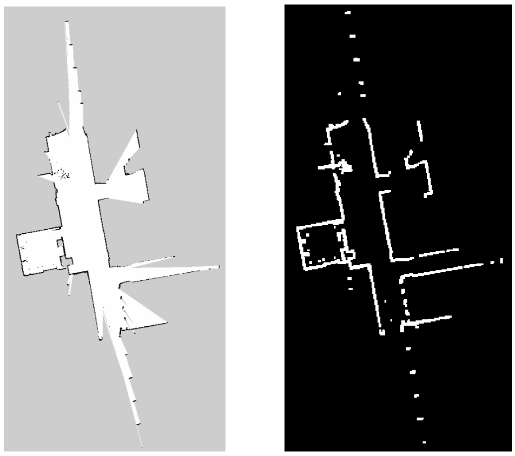
Figure 5: Promising door locations: (a) Points of minimum value on the Voronoi graph, (b) derived line segments corresponding to promising door locations.

### 3.2 Analyzing Occupied Space

Another main process of the proposed method is analyzing occupied regions. It is intended to find protruding zones by using Voronoi diagrams. The first step is creating a binary image where occupied zones are marked with value 1 and free and unknown regions are marked with value 0. Then, it is preprocessed, Voronoi diagrams are extracted and endpoints are located. A more detailed explanation of these steps is given below.

#### 3.2.1 Preprocessing Occupied Space

As it happened for free space, it is essential to preprocess the binary image corresponding to occupied space before extracting Voronoi diagrams. Image processing techniques are again applied. In this case, we are interested in highlighting protruding areas. For that reason, rounding corners needs to be avoided. In contrast to the previous case, the selected structuring element for applying image processing techniques is a square with which a dilation process is performed. A representative example of results can be seen in Fig. 6.

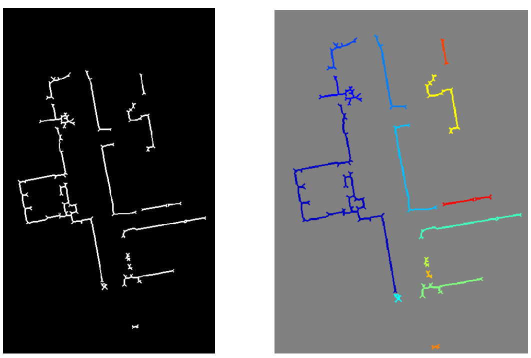


(a) (b)

Figure 6: Preprocessing of occupied space: (a) original occupancy grid map, where occupied space is marked in black, (b) resulting binary image after preprocessing occupied space.

#### 3.2.2 Extracting Voronoi Diagrams from Occupied Space

Voronoi graphs are extracted from the binary image corresponding to occupied space in the same way as explained in Sec. 3.1.2, where skeletons are used as an approximation. In this case, multiple Voronoi graphs are obtained, each one corresponding to a different occupied zone, in contrast to free space, where a single Voronoi diagram was obtained since all free space is connected. Additionally, those branches with less than 20 pixels are removed. In this way, small mapped zones normally corresponding to mapping errors due to sensor noise are removed. Fig. 7 shows the resulting diagrams obtained from the binary map shown in Fig. 6 and a labeled image with a different color assigned for each of the diagrams.



(a) (b)

Figure 7: Extracting Voronoi graphs from occupied space: (a) resulting Voronoi graphs, each one corresponding to a different occupied zone, (b) labeled graphs.

#### 3.2.3 Locating Endpoints

Voronoi graphs extracted from occupied zones are a helpful tool for locating protruding parts, which are placed near door locations. Every change in the shape of occupied zones results into a new branch on the Voronoi graphs. By locating these branches, protruding parts can be easily defined. In order to do so, endpoints are selected as representative points on Voronoi graphs. They are defined as pixels with exactly one pixel neighbor using connectivity 8.

### 3.3 Defining Door Locations

Door locations are defined as narrow passages in free space, where protruding parts occur in occupied zones. Intersection points defined in Sec. 3.1.4 indicate narrow passages found on free space, whereas endpoints defined in Sec. 3.2.3 define protruding parts in occupied zones. By relating these two sources of information, a final estimation of where doors are can be made. For that reason, the first step is locating the closest endpoint to each intersection point on free space. By joining the two resulting endpoints, another line segment is defined. Fig. 8 shown an example of the resulting line segments, where the green line is defined by joining intersection points $IP_1$ and $IP_2$ (free space information) and the pink line is defined by joining the two closest endpoints $EP_1$ and $EP_2$, respectively (occupied space information).

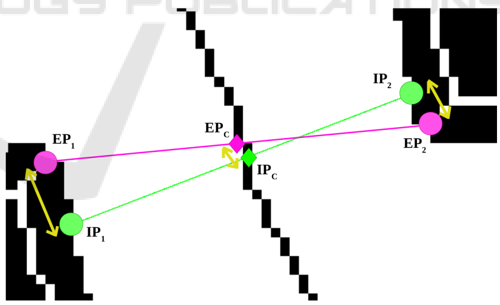



Figure 8: Information linkage for defining door locations. The green segment joins intersection points $IP_1$ and $IP_2$, obtained from free space, and the pink segment joints endpoints $EP_1$ and $EP_2$, obtained from occupied space. $IP_C$ and $EP_C$ are the central points of the segments, respectively. Yellow bidirectional arrows indicate distance between paired points.

Once information from free and occupied spaces is related, distances between each intersection point - endpoint couple as well as distances between the two central points of each line segment are analyzed (yellow arrows from Fig. 8). In order to do so, a threshold is proposed to compare distances. The main advantage of the proposed threshold is that it varies according to the map in which the segmentation procedure is being applied, contrary to the majority of segmentation works in which thresholds are generalized for all kinds of scenarios. By varying its value according to the structure of the occupancy grid map, it is better adjusted to data that is being processed, so door locations are better estimated. The proposed threshold value is an estimation of walls thickness. In this case, the distance transform is applied on occupied spaces to assign the distance to the closest free cell for each point on Voronoi graphs obtained from occupied space. Then, the mean value of these points multiplied by two (since skeletons are found in the middle of occupied zones) is selected, which indicates how wide occupied zones are. For each grouped data, the following conditions need to be accomplished in order to be accounted as a door location:

- $d_e(IP_1, EP_1) \leq thres$
- $d_e(IP_2, EP_2) \leq thres$
- $d_e(IP_1, EP_1) \leq \frac{thres}{2}$

where $d_e$ is the euclidean distance. Additionally, none of the line segments should intersect with any Voronoi graph from occupied space. If these conditions are met, a door location is finally defined by the line segment joining the two intersection points.

# 4 EXPERIMENTAL RESULTS

In this section, the proposed method is compared against other map segmentation procedures. More specifically, three methods are compared against our proposal. The first two are morphological operations and the traditional use of Voronoi diagrams as presented in (Bormann et al., 2016). The third method is based on Watershed (Fabrizi and Saffiotti, 2000) applied on the distance transform of an occupancy grid. The selected dataset for the experiments is obtained from (Bormann et al., 2016), where both furnished and unfurnished scenarios with varying sizes can be found, as well as ground-truth segmentations. A total of 18 furnished and 18 unfurnished scenarios are selected from the complete number of available maps. Evaluation metrics are also computed according to the methods proposed in (Bormann et al., 2016): execution times, precision and recall. As explained in the original work, precision is high if estimated rooms fit inside ground-truth rooms, which means that it is an indicator of undersegmentation. The opposite applies for recall, which is high when ground-truth rooms fit inside estimated rooms, being an indicator of oversegmentation. Both values need to be high for a good performance of the methods. Moreover, they need to be balanced in order to ensure a robust operation in different situations. Computations have been performed on an AMD Ryzen 7 4700u CPU.

Results for execution times, precision and recall on unfurnished scenarios are provided in Table 1 and some representative segmented maps are shown in Fig. 9. Morphological operations turn out to be the worst method, providing the lowest precision and recall values. With respect to execution times, traditional Voronoi graphs need a significantly higher amount of time to be executed, which produces the method to be unsuitable for online applications. The proposed method however significantly reduces computation times by analyzing Voronoi graphs from both free and occupied spaces. Only Watershed is faster than the proposed method. Additionally, it has a higher precision value and only a slightly lower recall. For a better comparison of both methods, they are tested on furnished scenarios.

Metrics for furnished scenarios using Watershed and the proposed method are collected in Table 2. Visual results are also provided in Fig. 10, where it can be clearly seen how our method outperforms Watershed. Due to to presence of furniture, Watershed highly oversegments scenarios, creating multiple rooms where there is a single one. This does not happen in our method, which is significantly less affected by the presence of clutter. Numerical results support these observations. Precision and recall on our method maintain similar results for both furnished and unfurnished scenarios. However, in the case of Watershed, recall descends to a 44.1%, which means that scenarios are highly oversegmented. Precision and recall turn out to be highly unbalanced, producing an erratic segmentation performance.

Table 2: Metrics for furnished scenarios.

| | Watershed | Ours |
|---|---|---|
| Runtime [s] | **0.21** | 4.72 |
| Precision | **91.6 ± 11.4** | 83.9 ± 16.8 |
| Recall | 44.1 ± 9.5 | **70.2 ± 6.8** |

Table 1: Metrics for unfurnished scenarios.

| | Morphological | Voronoi | Watershed | Ours |
|---|---|---|---|---|
| Runtime [s] | 5.55 | 109.43 | **0.33** | 2.06 |
| Precision | 75.1% ± 17.7% | 83.9% ± 14.1% | **94.0% ± 9.4%** | 89.3% ± 11.9% |
| Recall | 80.3% ± 9.4% | **90.5% ± 8.9%** | 81.1% ± 9.0% | 82.1% ± 8.8% |

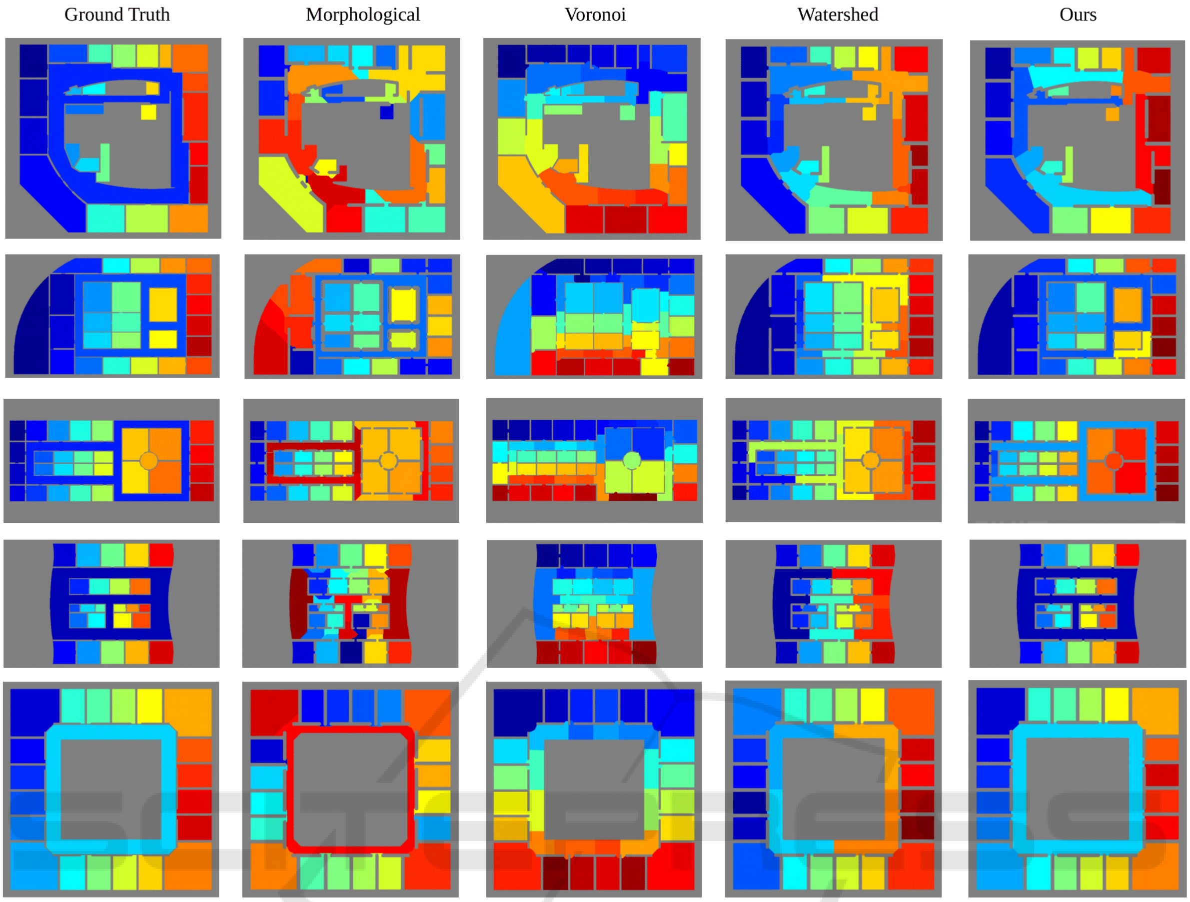



Figure 9: Examples of segmentation results for five unfurnished scenarios from (Bormann et al., 2016).

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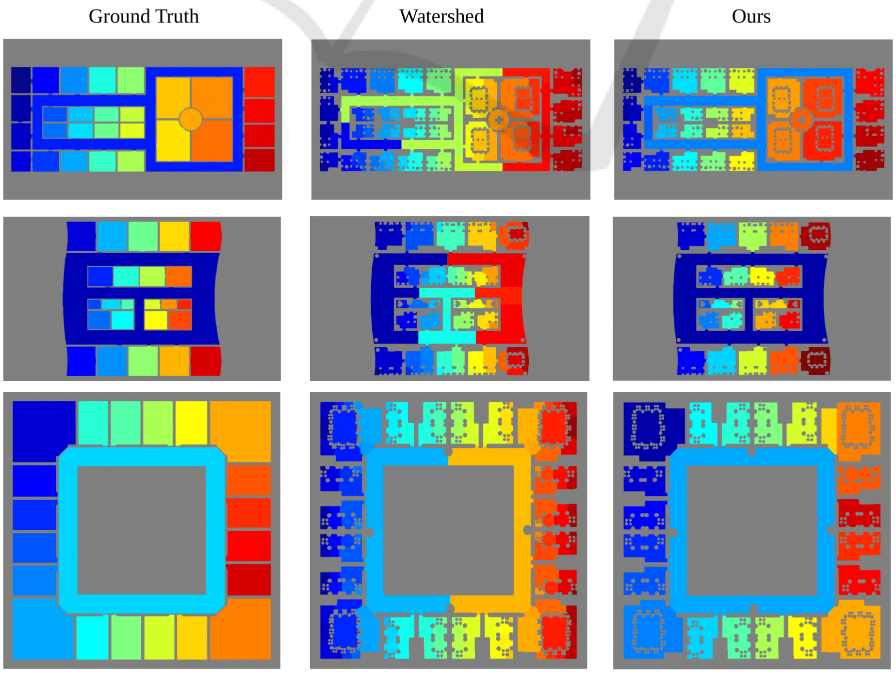


Figure 10: Examples of segmentation results for three furnished scenarios from (Bormann et al., 2016).

# 5 CONCLUSIONS AND FUTURE WORK

In this work, Voronoi diagrams have been used as the main tool for segmenting indoor scenarios into rooms. By extracting them from both free and occupied spaces, segmentation results have outperformed state of the art techniques, with high invariability in furnished environments. With respect to other methods, no additional steps have been required to unify areas after segmenting. Additionally, a method has been proposed to extract a threshold dependent on the scenario in which it is being applied, adjusting to the specific needs of each map.

In future work, it is intended to use the proposed method for locating doors. By knowing where doors are, the main challenge is modifying robots behaviour to facilitate door trespassing.

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