Large-scale Randomness Study of Security Margins for 100+ Cryptographic Functions

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Abstract: The output of cryptographic functions, be it encryption routines or hash functions, should be statistically indistinguishable from a truly random data for an external observer. The property can be partially tested automatically using batteries of statistical tests. However, it is not easy in practice: multiple incompatible test suites exist, with possibly overlapping and correlated tests, making the statistically robust interpretation of results difficult. Additionally, a significant amount of data processing is required to test every separate cryptographic function. Due to these obstacles, no large-scale systematic analysis of the the round-reduced cryptographic functions w.r.t their input mixing capability, which would provide an insight into the behaviour of the whole classes of functions rather than few selected ones, was yet published. We created a framework to consistently run 414 statistical tests and their variants from the commonly used statistical testing batteries (NIST STS, Dieharder, TestU01, and BoolTest). Using the distributed computational cluster providing required significant processing power, we analyzed the output of 109 round-reduced cryptographic functions (hash, lightweight, and block-based encryption functions) in the multiple configurations, scrutinizing the mixing property of each one. As a result, we established the fraction of a function's rounds with still detectable bias (a.k.a. security margin) when analyzed by randomness statistical tests.

1 INTRODUCTION

Truly random data are essential in many cryptographic operations such as the generation of keys, unpredictable nonces or padding schemes. Flawed random generators producing partially predictable data can lead to factorization of TLS server keys (Heninger et al., 2012), compromise of RSA keys from electronic IDs (Bernstein et al., 2013) or theft of funds from cryptocurrency wallets (Ketamine, 2018). Similarly, cryptographic functions such as block ciphers or hash functions are expected to produce output indistinguishable from the truly random data as long as the attacker does not posses the key used, input data or both. Producing data with detectable biases suggests a susceptibility to linear or differential cryptanalysis (Matsui, 1993; Biham and Shamir, 2012).

Multiple randomness test batteries exist, the most common being NIST STS (Rukhin et al., 2010), Dieharder (Brown et al., 2013) and TestU01 (L'Ecuyer and Simard, 2007). However, when applied to pseudorandom generators, human cryptanalysts typically do not consider general-purpose statistical batteries to be a very useful method for identifying weaknesses of cryptographic functions. There are at least two reasons for this:

Firstly, the use of statistical testing batteries is far from easy in practice. Multiple test suites exist and with incompatible interfaces, using different approaches for the test interpretation and difficult to computationally scale enough to analyze more than a handful of analyzed functions. The results of statistical tests are also notoriously difficult to interpret: Research literature contains multiple examples of invalid results due to flawed test interpretations. It is unclear how sensitive the separate tests included in the different statistical batteries are and how well-suited they are for the domain of testing the output of cryptographic functions.

Secondly, existing tests are perceived as not sensitive enough and significantly falling behind the human-performed cryptanalysis. However, there is only limited empirical evidence for this – it holds only for the well-studied functions such as finalists of the AES or SHA-3 competitions. We aim to answer

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the question of whether automated analysis using randomness testing batteries is useful for initial cryptanalysis of round-reduced functions and how these automatically found security margins compare with the results found by human cryptanalysts.

A significant amount of earlier work has tackled the problem of automatic randomness analysis of cryptographic functions' output using statistical tests. However, they share the following limitations: 1) Only a limited set of statistical tests is used for evaluation (usually a single battery) and test results interpretation is often flawed (ignoring correlated tests, inappropriately adjusting the significance level, etc.). 2) Only a very narrow set of different outputs is tested (usually just a one or few functions).

Finally, but just as importantly, almost all these studies are difficult to replicate as the implementations are often not published and the computations performed (e.g., seed used for data sequence production) are not deterministic. The related work is revisited in more detail at the end of the paper in Section 5.

To address the outlaid issues while avoiding the common limitations, we designed and performed a wide analysis of 109 different cryptographic functions (or 130 different combinations of functions and their parametrizations) by assessing their outputs with statistical tests. The output data is generated by CryptoStreams[1] – our open-source framework for generating data from round-reduced cryptographic functions in multiple input configurations. Testing was done using 414 statistical tests from four randomness testing batteries (NIST STS (Rukhin et al., 2010), Dieharder (Brown et al., 2013), TestU01 (L'Ecuyer and Simard, 2007) and BoolTest (Sýs et al., 2017)), unified into a user-friendly tool called the Randomness Testing Toolkit (RTT)[2]. The analysis is fully automatic and replicable using these tools and the published starter seeds.

Our paper brings the following contributions in the areas of cryptographic function analysis and statistical testing of randomness:

1. *Methodology for round-reduced function bias assessment.* We define a new methodology for testing and deciding whether given round-reduced function produces biased data using randomness testing batteries with adjustable significance level α. The methodology aims to eliminate false positives and produce conservative results.
2. *Security margins analysis of cryptographic functions.* For 109 distinct round-reduced functions, we establish the security margin with respect to the perceived randomness of produced outputs. We then compare the results with manual cryptanalysis, as shown in Subsection 3.2. To the best of our knowledge, dataset being analyzed is superior in terms of number of functions, tested strategies and amount of data being tested to previous research in the field.

[1] https://github.com/ph4r05/CryptoStreams
[2] https://github.com/ph4r05/rtt-deployment

2 RANDOMNESS TESTS AS DISTINGUISHERS

Tests of randomness are proposed to detect specific types of patterns (bias from randomness) in the analysed data. For example, a very simple test called Monobit compares the counted frequency of binary ones and zeroes, which shall be roughly equal in a random stream. Each test can typically detect also other types of patterns, but some others remain undetected. For example, Monobit can detect bias from randomness for the sequence ('011 011 011...'), but the sequence ('01 01 01 ...') will not be detected as biased.

In practice, tests are grouped into testing suites called *batteries*. We analyze tests belonging to commonly used batteries NIST STS, Dieharder, TestU01, and recently introduced BoolTest. If tests are parameterized, the default settings from a battery are used. Some tests in these batteries consist of more variants (e.g., forward, backward, specific number of bits in a searched pattern), resulting in 414 test variants in total.

Randomness testing is based on statistical hypothesis testing procedure that computes the statistical distance between the analyzed dataset and a synthetic dataset produced by the idealized model (hypothesis). Tests compute a certain type of statistic of bits and evaluate how far is the computed statistic from the expected theoretical statistic for random data expressed typically as the p-value (L'Ecuyer and Simard, 2007; Marsaglia, 1995; Rukhin et al., 2010).

A small p-value below a chosen significance level α leads to the rejection of the hypothesis (bias detected). In such a case, the test forms a distinguisher as it can distinguish analysed data from random data. We can commit two types of error. Type I error when the test rejects the hypothesis and data being generated by an unbiased random generator. Type II error when the hypothesis is accepted although generated by a biased generator.

The probability of the Type I error is equal to α and is directly controllable. The probability of the Type II error β is unknown but related to α – larger/s-

maller α corresponds to smaller/larger β. Also, the size of the data affects both α and β.

Randomness testing is used for the two main areas: 1) To detect systematic bias or physical failures of the TRNGs (true random number generators) and 2) To demonstrate insufficient confusion and diffusion property (Menezes et al., 1996) of TRNGs. A physical device failure typically results in a constant or very low variability output, which is easy to detect even by simple tests like Monobit. A significantly more complex bias is present for TRNGs with an insufficient mixing property. The TRNG function designer typically improves the mixing property by increasing the number of internal rounds (Menezes et al., 1996). It should be noted that to conclude that a given generator is biased is a difficult problem due to the probabilistic nature of randomness test result. In practice, this problem is solved by repeated testing with additionally generated data until the observed bias is confirmed or bias disappears and the generator is declared unbiased. Besides the probabilistic nature of test results, there are other problems concerned with the settings of tests (trade-off between Type I and II errors, which is also affected by the volume of the analysed data), approximation used to compute p-values and incorrect assumptions about their uniform distribution, incorrect test implementation, etc. We discuss these problems in more detail in Section 4.1.

2.1 Building Function Testing Set

Every sequence we analyse in our experiments depends on a cryptographic function (and its parameterization) included in the benchmarking dataset and a structure of input data (counter – CTR, Low Hamming Weight counter – LHW, pair of inputs with one bit flipped and their parameterizations – SAC) processed by the function. More details on these are provided below in Section 2.1.2. The sequences are parameterized by the following variables:

1. cryptographic function: a) the number of internal rounds (on average, with range of five subsequent rounds per function), b) key for stream and block ciphers, seed for PRNG;
2. type (CTR, LHW or SAC) of processed input is parameterized by: a) *seed* – defines seed of fixed PRNG used to generate random blocks in SAC, b) *offset* – defines initial value of CTR or LHW counter,
3. the length of the output data generated (10 MB, 100 MB and 1000 MB).

The combination of used parameters – cryptographic function (defined by a number of rounds and a key in case of block ciphers), type of input (CTR, LHW, SAC) and output data size (10, 100, 1000 MB) – will be called *configuration* (shortly cfg) of the (potentially) detectable biased generator. Each configuration is used to generate three different sequences by using different values of seed, offset and byte.

The following two sections explain the selection of cryptographic functions, type of inputs and usage of their parameters in more details.

2.1.1 Round-reduced Cryptographic Functions

The basic set of cryptographic functions was collected mainly from various cryptographic function competitions like AES or SHA-3 with an addition of well-known functions with available source-code. Out of these, we included only functions with available test vectors and such that contain some internal round-based structure and can be therefore round-reduced to produce a function with a reduced complexity.

The rounds used for a particular function were iteratively selected so that there are at least two rounds before and after the last round for which at least one test detects a bias. In total, we analyzed 52077 round-reduced function configurations (complete list of analyzed functions is shown in Table 1).

2.1.2 Parameterized Input Data Generation

Three different data generation strategies are used to analyze confusion and diffusion properties of the target function output, namely *CTR*, *LHW* and *SAC*.

The *CTR* strategy generates blocks of a particular size, each containing the current block index. Intuitively, the high bits are set to zero while the low bits are iterating until the required amount of data is generated. Note that block ciphers with extremely small blocks, like 32-bit version SIMON and SPECK ciphers, can produce only $2^{32} \cdot 32$ bits ≈ 17.2 GB. However, even shorter streams are having issues with a too high uniqueness of blocks.

The *LHW* stands for Low Hamming Weight as it generates input blocks with a fixed low Hamming weight. The weight is derived from the block size as it is required to avoid cycling of the generator, i.e., depleting all options on the block size. If the tested function f has an input block size of 128 bits, and we need to generate 100 MB of data, we set the Hamming weight to 4, as $16\binom{128}{4} \approx 170$ MB. The idea behind the *LHW* strategy is to cover the whole input block with small changes only, keeping the total Hamming weight low, thus feeding the minimal possible entropy to a function. Both *CTR* and *LHW* serve as low-entropy input generators, allowing for inspection of confusion properties of the cryptographic functions.

The *SAC* strategy aims to test the Strict Avalanche Criterion (Webster and Tavares, 1985), where the tested function shall generate two seemingly uncorrelated output blocks despite only a single bit flip in the corresponding input blocks. It generates pairs of blocks, where the first block in the pair is randomly generated and the second one is almost the same except for a single bit flip at a randomly selected position. Both blocks are then used as an input to the tested function f. This strategy inspects mainly diffusion properties of the cryptographic functions.

The strongest generating strategy is *rnd*, which generates a random block using a given seed, with the PCG64 generator (O'Neill, 2014). Similarly, *ornd* strategy generates a random value only once, then repeats it each time a value is needed. The *ornd* usage: generate random plaintext block, then repeat the same value with different keys.

Let's define a *function configuration* as a tuple (function name, round). An *input configuration* defines input streams fed to the function being analyzed, e.g., plaintext, key, seed. Function generates output sequence that is analyzed with randomness testing batteries. We have analyzed basic function types: block ciphers, stream ciphers, hash functions, PRNGs. Each function type differs in input configurations being used.

A **hash function** takes an input string and transforms it to the output block. An input configuration is thus a configuration of an input stream of blocks x_i fed to the hash function. E.g., CTR configuration for function f generates a stream of values: $(f(x_0), f(x_1), \cdots) = (f(x_i))_i$. Hash functions use configurations: {*ctr, lhw, sac*}.

PRNG takes an input seed and generates long stream of output data. One testing approach is to generate a seed randomly and analyze a long output sequence of the PRNG. However, it does not yield useful results as PRNG internal state changes when generating output sequences. It is difficult to spot randomness biases for randomness testing batteries in this setting. Moreover, PRNGs usually do not have internal round structure so it is not possible to analyze weakened function versions, we have to test PRNG in a full strength. We thus have two approaches when testing PRNGs. 1) Test N bytes with a fixed seed, 2) an alternative testing approach by repeating the following: reseed PRNG with an input stream block, capture B output bytes, reseed and capture again: $(f(\text{seed}{=}s_i)[:B])_i$. An input configuration defines seed stream. PRNGs testing strategies: {*zero, ctr.seed, lhw.seed, sac.seed, rnd.seed*}.

Stream ciphers are similar to PRNGs in the testing context. Feeding a specific plaintext to the stream cipher does not bring benefit from the testing perspective as it is only XORed with the *keystream* generated by the stream function, adding unnecessary entropy. We thus only test the keystream itself, feeding zero vector plaintext to the function.

Stream ciphers usually have internal round structure that makes analysis of a long keystream from the weakened cipher possible. Input strategy *zero*: generate a random key k_0, generate a long keystream with the k_0. The output sequence is: $(f(\text{key}{=}r_0)[:N])$, where N is the desired keystream length. A *.key* testing strategy generates a sequence of first B bytes of the keystream for key k_i: $(f(\text{key}{=}k_i)[:\text{B}])_i$. Input strategy defines how (k_i) is generated. Stream function testing strategies: {*zero, ctr.key, lhw.key, sac.key, rnd.key*}.

Block Ciphers: Take two inputs of a fixed length: key, plaintext and output a fixed length block. We use two different input configuration types. Input configuration: fix a random key k_0, derived from a seed, generate input blocks x_i. $(f(\text{key}{=}k_0, x_i))_i$. Testing strategies: {*zero, ctr, lhw, sac*}.

Alternative approach is to change a key and observe the function behavior. This testing strategy can reveal e.g. sensitivity to weak keys. Key configuration defines how (k_i, x_i) sequence is generated. The tested output is then $(f(\text{key}{=}k_i, x_i))_i$. The simplest key strategy is to generate k_i via CTR mode and keep $x_i = 0$. We call such strategy *ctr.key..zero.inp*. Block cipher testing strategies: {*zero, ctr, lhw, sac, rnd.key*}, $\cup$ {*ctr.key, lhw.key, sac.key*} $\times$ {*..zero.inp*, *..ornd.inp*}. A stronger strategies with *..ornd.inp* suffix generate a random plaintext block once and reuses the same value with different keys, i.e., $x_i = x_j =$ randNext(seed).

Seeds: Using given strategies we obtain an output sequence for a function and input configuration. Then randomness testing batteries are used to assess whether the sequence contains statistical biases. If enough biases are found (defined later) we conclude the hypothesis about an uniform output sequence distribution for the function configuration being rejected, in short, function at a round r was rejected.

In order to reduce false-positives we use 3 different input configurations parameters per the function configuration F_c. I.e., when using CTR generator, we use an offset to generate 3 different counter sequences. Sequence 1 starts at offset 0, sequences 1, 2 have offsets o_i, o_2 such that sequences do not overlap on the lengths used for testing. Technically, if the output length per one block is B bytes, setting offset O is done by setting the most significant byte of the counter to O, e.g., $(S_2)_0 = 2 * 2^{8*8}, (S_2)_1 = 2 * 2^{8*8} + 1, \cdots$

LHW generator offset is realized by setting initial LHW state so that the combination space is partitioned to 3 disjoint parts. Technically, we compute a number of combinations for a weight w on B bits as $L = \binom{w}{B}$. Offsets are then $o_i = L * \frac{i}{3}$. Using ranking algorithm we then compute o_ith combination for $\binom{w}{B}$ and use it as a starting position for LHW. As SAC uses randomly generated values, we randomly generate seeds s_i so that SAC generates sequences S_i differ.

F_c is rejected if at least 2 out of 3 output sequences are rejected. The reason for the usage of three different seeds is to make the likelihood of the (unwanted) usage of a key with the value degrading the function confusion and diffusion properties (called *weak key*) very small. The weak keys are very infrequent or shown to be non-existent for common cryptographic functions with a full number of internal rounds. However, they are significantly more likely to occur with the reduced number of rounds we are using.

CryptoStreams is highly configurable; we preconfigured more than ten additional general testing strategies. We provide the list of all our strategies and their description at the GitHub repository[3]. The total number of the data configurations analyzed is: 6264x 10 MB, 6526x 100 MB and 4569x 1 GB. In total, we analyzed 5160.4 GB of data.

3 RESULTS

The following section summarizes basic observations from collected testing data.

3.1 Study Limitations

We used default settings of tests as such parameters are commonly used in practice. Usage of other non-standard parameters may provide a different result. Due to high computational costs only {10, 100, 1000} MB data sizes were tested. Using larger data sizes could reveal more subtle biases and detect more rounds. Also, some tests were not run as they require more data for analysis, e.g., Test U01 BigCrush.

From all possible data generation strategies, we used only a small subset that we deemed the most promising, as defined in Section 2.1.2. New generation strategies may reveal another unexpected biases. Also, no detailed analysis of patterns found for particular cryptographic functions was performed.

[3]https://github.com/ph4r05/SecurityMarginsPaper

3.2 Security Margins of Cryptographic Functions

The number of rounds for which a distinguisher was automatically constructed by at least one test can be used to establish the security margin of a given cryptographic function included in the CryptoStreams battery. We also performed an extensive literature survey to identify the highest number of rounds for which any distinguisher was published.

Classical techniques in cryptanalysis, such as linear or differential cryptanalyses, require determining some bias in the ciphertext, which then leads to a recovery of (some) bits of key, plaintext or both. The designers of cryptographic functions try to make the function complex enough to hide any such bias, usually by increasing the number of function's internal rounds. For cryptographic functions now considered secure (like AES), neither the general-purpose tests nor the custom tests by human cryptanalyst were shown to detect bias in a function with a full number of rounds as specified by its designers.

Still, we can weaken a function complexity/strength (e.g., in the number of rounds), while testing the output for a bias presence. If the number of rounds where the bias is detectable is too close to the full number of rounds (unweakened function), the function shall be considered less secure and in demand for strength improvement (e.g., by increasing the number of rounds). We define the *security margin* as the difference between the number of rounds, for which a distinguisher can be still constructed and the total number of rounds of that function. More precisely:

$$\text{sec. margin}(f) = 1 - \frac{\max(i) : f_i \text{ output is non-random}}{n},$$

where f_i stands for function f limited to i rounds and n is the total number of rounds of the function f as specified by its authors. For example, there exists a distinguisher for a 3-round AES, but not for the 4-round one. AES-128 has 10 rounds in total, so the resulting security margin against analyzed randomness tests is 70%.

Security Margins per Function Type. Results in Table 2 indicate that security margins of all function types are similar when considering only input type strategies. However, key type strategies reduce security margins significantly. Also, hash functions have typically greater security margins than block and stream ciphers using key strategies.

Measured security margins of MPC hash functions are high, indicating that testing batteries might

Table 1: The security margin for each tested cryptographic function included in the CryptoStreams testbed. The table depicts the maximal number of rounds for which some bias was reliably detected for at least one data configuration used (red bar). Additional rounds with a practical distinguisher published in research literature are shown as a pink bar (typically larger than randomness tests). If no published and practical (complexity $< 2^{80}$) attack is found, sign "$-$" is used. The numerical value of rounds is encoded as *rounds_by_RTT/rounds_by_literature/rounds_total*. The average bar shows the median of observed percentage security margin with first and third quantile in the parentheses. MPC functions (Aly et al., 2020) are cryptographic functions optimizing their arithmetic complexity (used for example in zero-knowlege proof systems or multi-party computation protocols).

Function	Security margin	Function	Security margin	Function	Security margin
Hash functions		CAMELLIA	4/8/18	RECT.K80	8/18/25
Abacus	0/-/280	CAST	3/9/12	RECT.K128	8/14/25
ARIRANG	3/4/4	FANTOMAS	2/5/12	R.RUNNER.K80	3/8/10
AURORA	2/-/17	GOST	29/20/32	R.RUNNER.K128	5/8/12
BLAKE	1/4/14	IDEA	6/4/8	SPARX-B64	2/8/24
Blender	0/-/32	KASUMI	3/8/8	SPARX-B128	3/8/32
BMW	0/-/16	KUZNYECHIK	2/4/10	SPECK	10/15/32
Boole	3/16/16	LBLOCK	11/24/32	TEA	32/5/32
Cheetah	4/12/16	LEA	8/8/24	TWINE	9/23/35
CHI	0/-/20	LED	7/-/48	XTEA	8/8/32
CubeHash	0/-/8	MARS	0/8/16	**MPC functions**	
DCH	1/4/4	MISTY1	1/6/8	GMiMC.S45a	1/-/121
DynamicSHA	10/-/16	NOEKEON	2/4/16	GMiMC.S128e	1/-/342
DynamicSHA2	17/17/17	PICCOLO	6/5/25	LowMC.S80a	4/-/12
ECHO	2/4/8	PRIDE	12/19/20	LowMC.S80b	4/-/12
ESSENCE	9/14/32	PRINCE	4/6/12	LowMC.S128a	4/-/14
Gost	1/5/32	RC5-20	5/17/20	LowMC.S128b	120/-/252
Grostl	2/-/10	RC6	5/5/20	LowMC.S128c	20/-/128
Hamsi	0/-/3	ROBIN	16/16/16	LowMC.S128d	16/-/88
JH	6/10/42	ROBIN*	3/-/16	MiMC.S45	1/-/116
Keccak	4/5/24	SEED	2/-/16	MiMC.S80	1/-/204
Lesamnta	3/32/32	SERPENT	3/5/32	MiMC.S128	1/-/320
Luffa	7/8/8	SHACAL2	21/44/80	Poseidon.S80b	0/-/8
MCSSHA-3	0/-/1	SIMON	19/26/68	Poseidon.BLS12	0/-/8
MD5	25/-/64	DES	16/16/16	Rescue.S45a	0/-/10
MD6	10/16/104	TRIPLE-DES	16/-/16	Rescue.S128e	0/-/10
RIPEMD160	14/48/80	TWOFISH	3/16/16	RescueP.128a	0/-/27
Sarmal	0/-/16	**Stream ciphers**		RescueP.128b	0/-/27
SHA-1	18/80/80	Chacha	3/6/20	RescueP.128c	0/-/14
SHA-2	14/31/64	DECIM	7/-/8	RescueP.128d	0/-/14
SHA-3	4/5/24	F-FCSR	5/5/5	RescueP.S80a	0/-/18
Shabal	0/-/1	Fubuki	0/-/4	RescueP.S80b	0/-/18
SHAvite3	2/-/12	Grain	11/13/13	RescueP.S80c	0/-/9
SIMD	0/-/4	HC-128	0/-/1	RescueP.S80d	0/-/9
Skein	4/17/72	Hermes	2/-/10	Starkad.S80b	1/-/8
Tangle	80/80/80	LEX	3/-/10	Starkad.S128e	1/-/8
Tangle2	80/-/80	MICKEY	0/-/1	Vision.S45a	0/-/10
TIB3	0/-/16	Rabbit	0/-/4	Vision.S128d	0/-/10
Tiger	2/19/23	RC4	1/-/1	**PRNGs**	
Twister	6/9/9	Salsa20	2/6/20	Std.LCG	1/1/1
Whirlpool	1/10/10	SOSEMANUK	8/-/25	Std.MTwister	1/1/1
Block ciphers		Trivium	3/5.8/8	Std.SubCarry	1/1/1
AES	3/6/10	TSC-4	14/-/32	U01.ULCG	1/1/1
ARIA	2/4/12	CHASKEY	3/7/16	U01.UMRG	1/1/1
BLOWFISH	5/4/16	HIGHT	11/18/32	U01.XorShift	1/1/1

not be directly usable for this function family. We hypothesize this is due to usage of algebraic building blocks which are difficult to detect with randomness testing batteries. To verify the claim, we tested a simple function $f(x) = x^3 \pmod p$, where p is a 255-bit prime. The function f was fed with a strongly biased input distribution - normal distribution to produce 1 GB of output data. The output was tested with testing batteries (after applying rejection sampling transformation, described in Section 4.2). There was no bias detected in any of 10 tested streams. We thus conclude that even a simple algebraic function such as f can make the output indistinguishable from PRNG stream for testing batteries. In fact, the described function f is a basic building block of MiMC hash function.

Table 2: Aggregate security margins (SM, average and median values) for various function types and testing methods.

Function type	**Input SM [%]**		**Key SM [%]**	
	avg	med	avg	med
Hash	76.37	84.52	-	-
Block cipher	78.84	80.62	68.86	74.64
Stream cipher	80.77	90	64.59	70
MPC	93.94	100	95.85	100

Used Methods. Figure 1 displays both absolute and relative numbers per input generation methods for breaking the highest broken round (top-round) of the functions. From the results we can conclude that the *lhw* strategy is very effective in breaking top-rounds, both in input and key variants.

Also note that *key* variants have better relative success rate than input variants, indicating that cryptographic functions are more prone to biases when low-entropy keys are used compared to low-entropy inputs. Also, strategy *rnd.key* is the most difficult to detect as the entropy fed to the key of the function is high, it still managed to detect 37% of configurations it was used on.

Note that strategy *zero* is used for all stream functions with zero input as they generate long keystream with fixed random keys.

Considering input methods coupled with data sizes we observe that large data set 1000 MB dominate detection capabilities in all main methods {*lhw, ctr, sac*}. Interestingly, {*lhw, lhw.key*} on 10 MB outperform *ctr* on 100 MB.

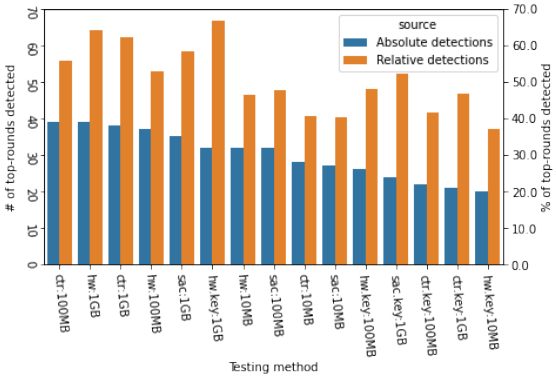
Key Method Dominance. We observed that in 24 (44.44%) cases the key variant was better than input variant for given inputs, same performance was seen in 19 cases (35.19%), input variant was better in 11 cases (20.37%). In some scenarios, the key to the input advantage is quite significant.

For example, Triple-DES is detected only to round 3 with input strategies, but the key strategy manages to detect all 16 rounds, even *rnd.key*, which is the most difficult strategy to detect. That means there are serious biases in Triple-DES output for some keys. A hypothesis is that weak keys known for Triple-DES were in the input stream, causing the ciphertext to contain biases. On the other hand, the SIMON function was detected with key methods up to 14 rounds but input methods reached 19 rounds. Figure 1c shows key to input method advantage with respect to the maximum round detected per given function.

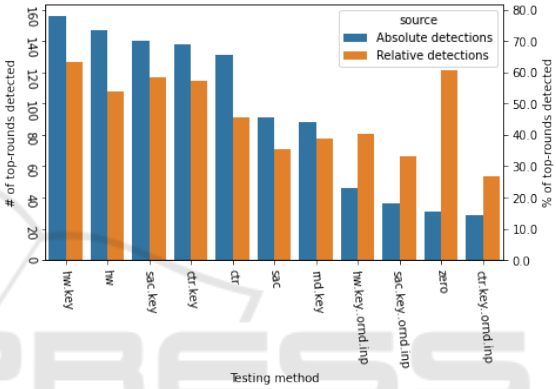
RC4 is known to contain biases on the beginning of the keystream. Experiments were not able to detect biases using *zero* strategy, i.e., using keystream with a random key. However, all tested key methods detected biases with 100 MB of data and more, even the most difficult *rnd.key* strategy.

Data Size. This paragraph focuses on detection capabilities when all variables are fixed, besides length of the input data stream. We take results over all detected rounds, not just a top-round detections. The most observed event was that changing input data stream did not increase detection capability, with 1694 detections (80.94%).

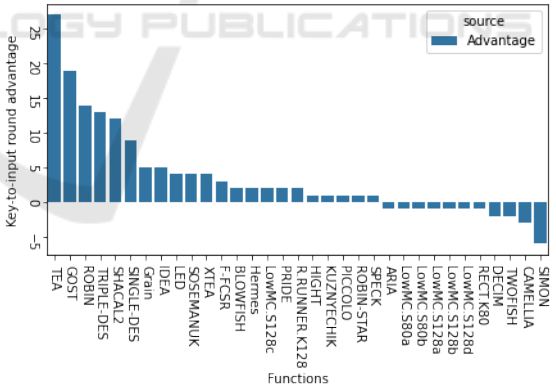
In general, one would assume that bigger the in-



(a) Testing methods breaking top-rounds, with sizes.



(b) Figure 1a with aggregated sizes



(c) Key to input method detection advantage for breaking the highest round of the given function. Methods axis shows number of test methods rejecting the same round. Entries with no advantages are skipped for better readability.

Figure 1: Methods breaking top-level rounds and key-to-input methods advantage in breaking the highest rounds.

put stream the easiest is to spot biases for the randomness tests. In particular, it means that given configuration was not detected on lower input size but it was detected by all higher data sizes. This turned out to be the case for 397 detections (18.97%). For example, AES round 3 using *lhw.key* strategy was not detected on 10 MB input but since 100 MB. Similarly for Blowfish round 3 with *sac* strategy.

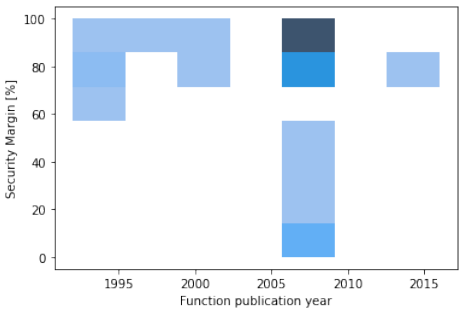
We also observed an inverse effect, i.e., detection was successful on a lower data size, but higher data sizes did not detect the stream. This happened in a single case (0.05%). Our hypothesis is that that bias was lost in a noise as data size increased, i.e., it was probably a fluke. The case is Kasumi round 3 with the *lhw.key* strategy, bias was observed only in the 10 MB input stream. Considering no other strategy could detect the same function configuration, we can assume the detection was a fluke. Rest of the cases are fluctuations where bias was either observed or undetected on a 100 MB data size, observed in a single case (0.05%).

Table 3 shows a result subset of a maximal detected round depending on a chosen method and increasing input size for the method. Apparently, the highest detected round is strongly dependent on the mentioned parameters.

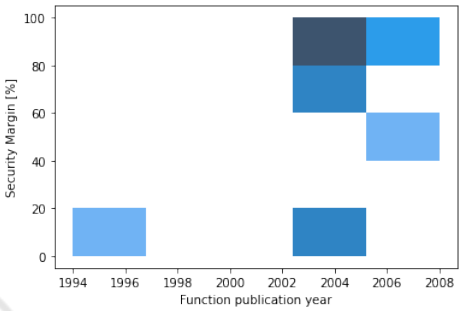
Table 3: Maximal breaking rounds for selected (function : method) pairs depending on different input sizes.

Function : method	**10 MB**	**100 MB**	**1 GB**
MD5 : lhw	17	20	25
: sac	13	16	20
DES : lhw	4	6	7
: rnd.key	16	16	16
SIMON: ctr	17	18	19
: lhw	15	17	17
: lhw.key	11	11	12
GOST : ctr.key	10	11	12
: lhw.key	21	25	29
: lhw.key..ornd.inp	20	22	27
: rnd.key	1	1	1

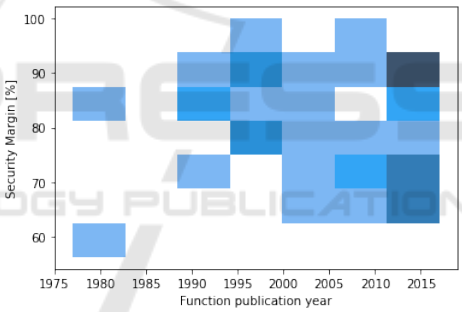
Function Publication Year. Figure 2 shows security margin heat maps (the darker the more functions belong to the bin) for particular function types based on the year of a publication of the function. Note that hash and stream function year variance is low as majority of our function dataset comes from eStream and SHA-3 competitions, thus many of the functions were published in the same year. From the figures we can conclude there is a high variance in security margins. Yet, we cannot conclude that older functions have lower security margins.



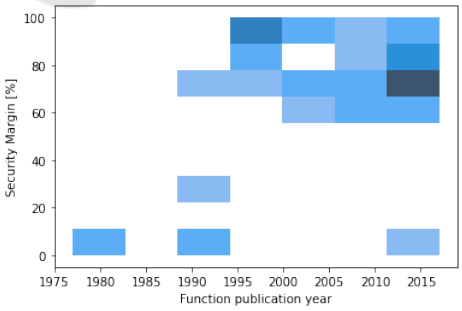
(a) Security margin for hash functions



(b) Security margin for stream ciphers



(c) Security margin for block ciphers, input methods



(d) Security margin for block ciphers, key methods

Figure 2: Security margin heat maps based on the publication year of the functions analyzed. The results generally show increased number of published functions around the related competitions (e.g., AES or SHA-3), better security margin for the functions introduced later, but with exceptions of the broken ones.

Bias Strength Measure. There are several ways to define a measured bias strength. E.g., ratio of randomness tests detecting the sequence as biased on a level α, minimal p-value observed, number of rejected seeds, etc. Intuitively, lower rounds of a cryptographic function should have stronger biases compared to higher rounds. Experiments confirm that this is true for majority of tested functions. To demonstrate this, lets observe ratio of rejecting tests for the SIMON function with the *lhw* input strategy on 100 MB of data, starting from round 11: 0.88, 0.83, 0.75, 0.68, 0.44, 0.37, 0.22, 0.07 (round 18), while round 19 is first with no bias detected with only 3/735 rejecting tests. On the other hand, there are functions which do not have such smooth bias progression, for example LED using *lhw* with 100 MB, rounds $\{1,2,3\}$ are all detected with 0.93 tests, round 4 has 0 test detections.

4 RANDOMNESS TESTING FOR MASSES

We created a practical, generally usable tool for randomness testing, the Randomness Testing Toolkit (RTT), which integrates four randomness testing batteries (NIST STS, Dieharder, TestU01, and BoolTest), under simple and unified command-line and web-based interface. The main benefits are: a) the simplicity of the randomness testing for the user, b) preconfigured batteries and test configurations[4] c) unified result interpretation, and d) significant parallelization to test large amounts of data.

4.1 Test Detection Evaluation

The goal of our evaluation is to correctly (with minimal Type I and II errors) identify biased generators defined by the parameters (cryptographic function, round, key, strategy and size). P-values are the only results of tests we use in our evaluation to identify biased/unbiased generators. Interpretation of a p-value (rejection/non-rejection of a hypothesis – shortly fail/pass) is based on the significance level α chosen by a tester. P-value below α indicates biased statistic that is interpreted as "generator of the analysed data fails the given test". Choice of the significance level is crucial since it affects the probabilities of both Type I and II errors.

Batteries use a conservative value α_{FAIL} is used to minimize Type I error, i.e. false-positive. Conservative α determines a bigger Type II error as both errors are related. The only way how to decrease Type II error, for fixed Type I error, is to increase the volume of analysed data.

Each test of the original batteries computes two types of p-values: a) standard "first-level" p-value computed by a test for one sequence, b) "second-level" p-value computed by uniformity test (*Kolmogorov-Smirnov*, *Anderson-Darling*, etc.) for a sequence of first-level p-values, checking whether first level p-values are uniformly distributed on the [0,1) interval.

In the RTT, the result of each test is a second-level p-value. It is computed typically for several tens or several hundreds of (first-level) p-values computed by the test. Some tests compute several second-level p-values (over the same set of first-level p-values) using different uniformity tests. The p-values (first-level or second-level) computed by the original batteries (and also by the RTT) are not exact due to approximation methods used in the tests. The sequence fails a given test if at least one of the second-level p-values is smaller than the significance level. Approximation of the *null* distribution is used in the computation of first-level p-values. This introduces some small errors that are accumulated in the second-level p-values. To find suitable significance level α, we analyzed reference random data produced by full-round AES-128 to determine number of false positives, taking approximation errors into account.

The initial analysis of data showed that three tests behave differently than others. The *smultin_MultinomialBitsOver* from the Rabbit sub-battery of TestU01 has a significantly biased proportion of small p-values. Random Excursion and Random Excursion Variant tests from the NIST STS compute different numbers of first-level p-values for a fixed size of the sequence hence significance of their results varies. We excluded the results of these three tests from all our experiments (including one with the reference data).

BoolTest (Sys et al., 2017) battery comes in two versions. BoolTest2 returns directly a p-value and it works without precomputations. BoolTest1 is evaluated using confidence intervals of Z-scores from empirically computed reference distibution (for each BoolTest setting) using 10^5 of random sequences. This gives the significance level $\alpha_{BT1} = 10^{-5}$ for the BoolTest, for RTT we use the same $\alpha_{RTT} = 10^{-5}$.

4.2 Function Bias Classification

As mentioned in Section 2.1.2, we claim the input configuration is rejected if at least 2 of 3 tested seed

[4]E.g., TestU01 is a randomness testing library that requires user to develop a testing program, choose which tests to run, etc.

variants are rejected. This section describes the decision methodology for a seed variant to be claimed rejected. We set global $\alpha = 10^{-7}$ to minimize false-positives across our dataset.

Reference Runs, Cut-off Threshold. We ran RTT tests on 75 000 reference input configurations, i.e., full 10 round AES-128 using *ctr* input mode with a random key. The reference test data results show behaviour of statistical tests on a good-quality pseudo-random input data, namely tests p-value distribution on a reference data and distribution of number of simultaneously failing tests per analyzed sequence with $\alpha_T = 10^{-5}$. From the reference test data we can establish a cut-off criteria on a number of simultaneously failing test per sequence run to be 5% of performed test with $\alpha \leq 10^{-7}$. Reference runs showed that the maximal number of simultaneously failing tests with α_T for a single sequence is 3 out of 216 tests (1.38%, such event occurred 9 times, 0.016%). As the number of analyzed sequences in our cryptographic function dataset is smaller (54 500), the cut-off limit is set conservatively enough to avoid false-positives. Ideally, a further study of reference p-value distributions and number of test rejections per tested sequence would be needed to model the distribution and to avoid computation of reference data in sizes of the tested set. It would also help setting cut-off limit precisely for given α. The reference data study is left for a future work.

Hommel Correction. In the ideal case, the proportion of failed sequences for a given test is given by the significance level α for an unbiased generator. In such a case, the number of failed sequences is given by $n * \alpha$ for number of sequences n. It is clear that the probability that a sequence (generated by an unbiased or unbiased generator) fails one of the used tests increases with the number of tests m. It is possible to compute the actual significance level α_m (sequence fails m tests if it fails at least of the tests) for m tests although we used α as the significance level for each test using p-value correction methods.

If the number of tests rejecting the seed is below the cut-off threshold, we apply the Hommel correction (Hommel, 1988) on all p-values collected from the seed run, using given α to tell whether tested sequence is rejected. Hommel correction is used to decide null-hypothesis rejection using set of positively-correlated p-values representing results from randomness tests and their associated alternate hypotheses. The correction keeps the probability of a false-positive of the p-value set below α. Note that p-values are positively correlated as the tests test the same input sequence and also due to overlap of features being assessed by the tests, i.e., a particular bias pattern can cause multiple tests to reject the input stream. To the best of our knowledge, this is a new method for assessing biases in output of round-reduced cryptographic functions using different positively-correlated randomness tests.

BoolTest1 Rejection. BoolTest1 works with a precomputed reference distribution, thus the α_{BT1} is fixed to an inverse to a number of reference results. As $\alpha_{BT1} < \alpha$ BoolTest1 would never help reject the tested sequence. We thus use observation from cut-off threshold analysis. There are always 36 BoolTest1 tests executed per tested sequence. From the reference data we observed that BoolTest1 results have negligible correlation and rejection happens in $36n * \alpha_{BT1}$ on average. BoolTest1 is thus removed from Hommel correction and we use BoolTest1 cut-off threshold 3 to consider output sequence rejected with $\alpha \leq 10^{-7}$. Due to small correlation among BoolTest1 test fails, distribution of a number of simultaneously failing tests per input can be easily simulated without need to run BoolTest1 and approximate it with Beta-Binomial distribution.

Prime Order Functions. Randomness testing batteries are suited to test byte-aligned data. However, cryptographic functions used in context of MPC / zero-knowledge proofs using algebraic construction can work in prime fields $\mathbb{F}_p$, where p is a prime. Such function F output vectors $\mathbb{F}_p^m$. Testing batteries thus cannot be naïvely applied to output values $x_i < p$, as interval $[p, 2^{8\lceil lg(p)/8 \rceil})$ is not covered by the function output. Thus batteries trivially reject the output even though the output is uniformly distributed on the interval $[0, p)$. We thus need to apply an transformation T to the function output that transforms uniform prime order elements x_i to a uniform byte-aligned outputs x'_j before testing it with the batteries. The simplest approach is to use a rejection sampling, i.e., find byte-aligned boundary $M = 2^{8\lfloor lg(p)/8 \rfloor} - 1 \leq p$. Output x is then ignored if $x > M$. Rejection sampling strategy requires more data to be generated as portion of the generated data is discarded. We also used more advanced transformations based on inverse function sampling, i.e., directly stretching uniform distribution from $[0, p)$ to $[0, M]$ using auxiliary randomness so all F output values are used.

5 RELATED WORK

In this section, we firstly discuss published works in cryptanalytical statistical testing, secondly the analysis of tests of randomness in RTT. There exist many other statistical batteries except for those used in RTT (NIST STS, Dieharder and TestU01): Donald Knuth (Knuth, 1969), Diehard (Marsaglia, 1995), Crypt-X suite (Caelli et al., 1998), PractRand (Doty-Humphrey, 2014), RaBiGeTe (Piras, 2004), CryptoStat (Kaminsky and Sorrell, 2013), YAARX (Biryukov and Velichkov, 2014), ENT (Walker, 2008), SPRNG (Mascagni and Srinivasan, 2000), gjrand (Jones, 2007) and the BSI test suite (Schindler and Killmann, 2002).

There are two cryptanalytic approaches that are based on the randomness testing. In the first approach, a cryptographic function (hash, block or stream cipher) is turned into a PRNG, and this PRNG is used to generate a sequence of bits/bytes and a test of randomness is applied to the sequence. In the second approach, the cryptographic function (and its randomness) is analysed directly. Tests of randomness are applied here to check whether the given function forms a random function or a random Boolean function. Nice overview of papers covering both approaches can be found in (Kaminsky, 2019), section "Related Work". Next, we will list only sources not included in (Kaminsky, 2019) or papers that are the most relevant to our approach.

One of the evaluation criteria for AES candidates was "their demonstrated suitability as random number generators." Therefore, AES candidates were evaluated by NIST STS batteries under several testing scenarios. Murphy in (Murphy, 2000) described the methodology of testing and its weaknesses. Soto in (Soto, 1999) reported randomness evaluation of fifteen AES candidates under nine categories of data: Key Avalanche, Plaintext Avalanche (*SAC* with zero key), Plaintext/Ciphertext Correlation (*RPC*), Cipher Block Chaining Mode, Random Plaintext/Random Keys (stream consisting of key, plaintext and ciphertext tuple), Low Density Plaintext (*LHW*), Low Density Keys, High Density Plaintext (inversed *LHW*), and High Density Keys. In the succeeding work (Soto and Bassham, 2000), Soto and Bassham expanded recent results with a study of round reduced candidates with longer keys and the same data types (LHW, SAC, etc). Kubicek et al. analysed the round reduced TEA using evolutionary algorithms (Kubícek et al., 2016).

Hernandez-Castro and Barrero in (Hernandez-Castro and Barrero, 2017) evaluate tests in the Ent battery using genetic algorithms. Authors of NIST STS analysed the correlation of tests results in order to eliminate redundant tests. NIST performed a study to determine the dependence between the NIST STS tests (Rukhin et al., 2010). They applied principal components analysis of m (no value specified) p-values and extracted 161 factors, equal to the number of tests (default settings of tests were not used). They claimed that "there is no large redundancy among our tests". Yet some other works also analysed the correlation of NIST STS tests (e.g., (Sýs et al., 2015)) and showed a correlation between NIST STS's tests leading to an estimate of the higher number of failing test needed for rejection of randomness hypothesis. Eskandari et al. in (Eskandari et al., 2018) automatically construct distinguishers for 30 crypto primitives using bit division property method and a new Salvatore framework.

6 CONCLUSIONS

Our paper provides the most extensive analysis of the power of the commonly used randomness statistical tests and their variants on the data produced by 109 cryptographic hash functions and block ciphers published to date. To perform the analysis of 414 tests, we modified the code of every function to make the number of internal rounds configurable and executed it on three types of highly redundant input blocks to produce a stream of testing data.

All the tests were integrated under the same interface allowing for an efficient evaluation using a distributed computation cluster. We also designed a unified framework for evaluating biases' presence in the output of a cryptographic function using different randomness testing batteries. Our approach is reproducible, and new functions can be later tested and compared with existing results.

The security margin – as the ratio between the total number of function rounds and rounds with bias still detectable (distinguisher) – was established for all the tested functions and compared with the distinguishers published in the research literature. While being around 77% on average (up to 23% of internal rounds still distinguishable), there is high variability among the functions analyzed. SHA-3 exhibits a security margin of 83%, and ten functions, including Blake, Gost, Skein, Mars, Serpent, Shacal2, and XTEA have even more than 90%. Contrary to functions with a large security margin, the functions detected with a very small margin (Arirang, DCH, DynamicSha2, Luffa, and Twister) were also shown to be weak by existing research literature. The randomness tests can be therefore seen as the simple automated first step in cryptanalysis.

All our tools and results, including tests results from both reference runs and cryptographic function dataset, are publicly available[5].

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