Coupled PID-SDRE Controller of a Quadrotor: Positioning and Stabilization of UAV Flight

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Abstract: This work presents a coupled Proportional-Integral-Derivative and State-Dependent Riccati Equation (PID-SDRE) controller. PID angular position controller coupled to nonlinear infinite-time SDRE controller for speed stabilization is proposed. For the quadrotor modelling a full 6 degree of freedom (DoF) model is considered and described by nonlinear state-space approach. Also, a stable state-dependent parameterization (SDP) necessary for solution of the SDRE control problem is proposed. Solution of the SDRE control problem with adequate defined weighting matrices in the performance index shows the possibility of fast and precise quadrotor positioning with optimal stabilization of speeds. Two methods of optimal SDRE-based stabilization are proposed, tested, and compared.

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1 INTRODUCTION

Todays, Unmanned Aerial Vehicles (UAVs) have become an object of interest of industrial, businesses and governmental organizations. They are being adopted worldwide, especially by following sectors: military, commercial, personal and future technology. Briefly speaking, in places where man cannot reach or is unable to perform in a timely and efficient manner especially including danger zones and places.

Due to the development of UAV application, quadrotors has drawn full attention due to its advantages of flexibility, portability, versatility. The heart of each UAV is a control system, a brain which has to be optimal, robust, and intelligent (Chipofya, 2017; Sadeghzadeh, 2011; Sheng S, 2016; Stepien, 2019; Voos, 2006; Zhang, 2009).

Flight control of multi-role UAV is viewed as a difficult area of aerospace engineering (Hoffmann, 2007; Kim, 2020). Moreover, each flight control system of a quadcopter is nonlinear and coupled. The controller should be an independent system, which aims to create the best autopilot hardware. Most of now existing controllers are based on PID controllers (Chodnicki, 2018).

Modern optimal control theory proposes high performance and a rapidly emerging control technique called infinite-time state-dependent Riccati equation (SDRE) (Banks, 2007; Cloutier, 1996; Korayem, 2015). This is a suboptimal control methodology for nonlinear systems. The technique uses direct parameterization to bring the nonlinear system to a linear structure having state-dependent coefficients (SDC). The SDRE is then solved accordingly to the change of state trajectory to obtain a nonlinear feedback controller matrix, which coefficients, in other feedback gains, are the solution (Cimen, 2010; Heydari, 2015; Mracek, 1998).

Many practical implementations of quadrotor controllers are limited. When using a PID controllers to angular or linear positioning, for instance, there is no guarantee that angular or linear speeds became

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controlled to constant or zero. Then a combination of the PID with another controller (or sub-controller) should be provided to control the speed vector toward zero (Sadeghzadeh, 2011; Chodnicki, 2018).

The main contribution of this research is to develop the PID-SDRE closed-loop control system employing the 6 DoF UAV model. The PID as the main controller is used for angular position control. The internal speed sub-controller SDRE is used to stabilize of angular and linear speed control. The modelling and control design methodology presented is the concept proposed to design a high-performance and optimal flight controller for UAV. The nonlinear model of the drone and solution of the infinite-time suboptimal speed control problem is applied, analyzed and compared employing two SDRE-based methods (Banks, 2007; Cloutier, 1996; Stepien, 2019; Voos, 2006).

2 QUADROTOR DYNAMICS

The rigid body equations of motion are the differential equations that describe the evolution of basic states of a quadrotor. The quadrotor model presents Fig. 1.

Figure 1: Quadrotor model.

The quadrotor dynamics is generally defined using Newton's force and moment equations (Hoffmann, 2007; Kim, 2020; Chodnicki, 2018; Zhang, 2009). The force equation is following

$$\boldsymbol{F} = m(\dot{\boldsymbol{v}} + \boldsymbol{\omega} \times \boldsymbol{v}), \quad (1)$$

where $\boldsymbol{v}$ is the UAV linear speed vector, $\boldsymbol{\omega}$ is the angular speed vector, m is the UAV mass and $\boldsymbol{F}$ denotes the force vector. The moments equation describes all the moments acting on the UAV, equal to the rate of change of angular moment vector

$$\boldsymbol{M} = \boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega}, \quad (2)$$

where $\boldsymbol{I}$ is an aircraft symmetrical inertia matrix and $\boldsymbol{M}$ denotes moment vector. Considering vector $\boldsymbol{v}$ defined for all components in x, y and z direction and $\boldsymbol{\omega}$ for roll ϕ, pitch θ and yaw ψ angle

$$\begin{cases} \boldsymbol{v} = [u \quad v \quad w]^T \\ \boldsymbol{\omega} = [p \quad q \quad r]^T \end{cases}, \quad (3)$$

then equations of quadrotor aerodynamics can be defined for linear and angular speeds. In addition, because of a quadrotor symmetry, so in the inertia matrix the off-diagonal entries become zero, then

$$\boldsymbol{I} = diag(I_x, I_y, I_z). \quad (4)$$

The system of nonlinear equations that describes aircraft flight dynamics, considering gravity forces g and force due to the thrust F_T, is following

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rv - qw + gsin\theta + \frac{1}{m}(F_{Dx} + F_x) \\ pw - ru - gsin\emptyset cos\theta + \frac{1}{m}(F_{Dy} + F_y) \\ qu - pv - gcos\emptyset cos\theta + \frac{1}{m}(F_{Dz} + F_T + F_z) \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (-(I_z - I_y)rq + M_x)/I_x \\ (-(I_x - I_z)pr + M_y)/I_y \\ (-(I_y - I_x)pq + M_z)/I_z \end{bmatrix}, \quad (6)$$

where F_{Dx}, F_{Dy}, F_{Dz} denotes drag forces and M_x, M_y, M_z are applied angular moments. It is assumed that the torque and thrust caused by each rotor act particularly in the z axis of the quadrotor frame. Moment results from the thrust action of each rotor around the center of mass which induces a pitch and roll motion.

The relationship between the body-fixed angular speed vector $[p \quad q \quad r]^T$ and the rate of change of the Euler angles $[\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T$ can be determined by resolving the Euler rates into the body-fixed coordinate frame. Hence, to describe the orientation an Euler angle relationship is used from the transformation from the local horizontal to the body axes. The resulting kinetic equations are

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & sin\phi tan\theta & cos\phi tan\theta \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi sec\theta & cos\phi sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad (7)$$

where ϕ is a roll angle, θ is a pitch angle, and ψ is a yaw angle and $\sec\theta = 1/\cos\theta$.

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3 PID-SDRE CONTROL

Quadrotor is an unstable system. Therefore, a control and stabilization system in design should allow one to control the orientation in the system. Then, not only control of the space orientation, but also angular and linear speeds should be stabilized. Thus, two blocks of controllers are used: one for controlling space orientation by the angular position, and the next for stabilizing the angular and linear speeds.

This paper deals with coupled Proportional-Integral-Derivative and State-Dependent Riccati Equation (PID-SDRE) controller dedicated to orientation control and stabilization. The control system schema is presented in Fig. 2.

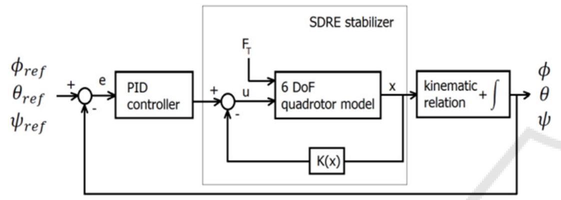

Figure 2: PID-SDRE control schema of quadcopter.

It consists of two control units. The orientation control system is realized in outer closed-loop systems using PID controller, but the speed stabilization problem is performed by the inner closed-loop subunit with feedback compensator employing infinite-time SDRE control technique. In this case, a thrust force F_T is set as constant and allows one to get desired altitude. The other variables contained in the Fig. 2 denote: $\boldsymbol{x} = [u \quad v \quad w \quad p \quad q \quad r]^T$ – state vector of the 6 DoF model, $\boldsymbol{u} = [F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z]^T$ – attitude control vector and error vector of the attitude angles $\boldsymbol{e} = [\phi_{ref} - \phi \quad \theta_{ref} - \theta \quad \psi_{ref} - \psi]^T$.

3.1 PID Attitude Controller

The closed-loop control system used to quadrotor space positioning consists of three independent controllers for roll, pitch and yaw angles.

The output of a PID controller is following $\boldsymbol{u}_{PID} = [M_{xPID} \quad M_{yPID} \quad M_{zPID}]^T$, and is equal to the PID control input to the plant, is calculated in the time domain from the feedback error as:

$$\boldsymbol{u}_{PID} = \boldsymbol{k}_P \boldsymbol{e} + \boldsymbol{k}_I \int \boldsymbol{e} dt + \boldsymbol{k}_D \frac{de}{dt}. \qquad (8)$$

The error signal e is a three-element vector fed to the PID controller, which computes proportional, derivative and integral of this error signal with respect to time. $\mathrm{k_P}$, $\mathrm{k_I}$, $\mathrm{k_D}$ are proportional, integral and derivative gain diagonal matrices:

$$\begin{aligned} \boldsymbol{k}_P &= diag(\boldsymbol{k}_{P\phi}, \boldsymbol{k}_{P\theta}, \boldsymbol{k}_{P\psi}), \\ \boldsymbol{k}_I &= diag(\boldsymbol{k}_{I\phi}, \boldsymbol{k}_{I\theta}, \boldsymbol{k}_{I\psi}), \\ \boldsymbol{k}_D &= diag(\boldsymbol{k}_{D\phi}, \boldsymbol{k}_{D\theta}, \boldsymbol{k}_{D\psi}). \end{aligned} \qquad (9)$$

The integral matrix gain $\boldsymbol{k}_I$ times the integral of the error vector plus the derivative matrix gain $\boldsymbol{k}_D$ times the derivative of the error vector are computed using its approximation and creating digital form of the PID. A standard formulation of digital PID that uses bilinear transformation of continuous integral and derivative action is employed (Kim, 2020; Sadeghzadeh, 2011).

3.2 SDRE Speed Compensator – Classic Approach

The state-dependent Riccati equation (SDRE) suboptimal control method is an efficient tool for control of the nonlinear 6 DoF quadrotor model. The technique with a further improved and modified approach is widely described in recent literature (Banks, 2007; Cimen, 2010; Mracek, 2006; Voos, 2006). The SDRE approach is used in the context of the nonlinear controller problem with a quadratic objective function defined as the sum of energy lost and delivered to the system, what is compatible with practical applications.

The infinite-time control problem consists of finding optimal control law that minimizes the following objective function defined for infinite control time:

$$J(\boldsymbol{u}) = \frac{1}{2}\int_0^\infty (\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} + \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u}) dt, \qquad (10)$$

subject to nonlinear dynamics for affine systems

$$\dot{\boldsymbol{x}} = \mathbf{F}(\boldsymbol{x}) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}. \qquad (11)$$

Nonlinear UAV dynamics (11) can be written using the state-dependent coefficient (SDC) form (Banks, 2007)

$$\dot{\boldsymbol{x}} = \mathbf{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}, \qquad (12)$$

where $\boldsymbol{Q}(\boldsymbol{x})$ is symmetric, positive semi-definite weighting matrix for states, $\boldsymbol{R}(\boldsymbol{x})$ is the symmetric, positive definite weighting matrix for control inputs. Equation (11) includes $\mathbf{F}(\boldsymbol{x})$ vector, which is piecewise continuous in time and smooth with respect to their arguments, which satisfy the Lipschitz condition. Considering (12), if the pair $\{\mathbf{A}(\boldsymbol{x}), \boldsymbol{B}(\boldsymbol{x})\}$ is a stabilizable parameterization of the system, then to check controllability of the affine system, this pair in the linear sense should be controllable for all $\boldsymbol{x}$.

Employing Hamiltonian theory (Cimen, 2010) the optimal control law is

$$\boldsymbol{u}_{SDRE} = -\boldsymbol{R}(\boldsymbol{x})^{-1}\boldsymbol{B}(\boldsymbol{x})^T\boldsymbol{K}(\boldsymbol{x})\boldsymbol{x}, \tag{13}$$

where $\boldsymbol{K}(\boldsymbol{x})$ is a state-dependent feedback compensator which can be obtained from solution of a state-dependent algebraic Riccati equation (SDARE)

$$\boldsymbol{K}(\boldsymbol{x})\boldsymbol{A}(\boldsymbol{x}) + \boldsymbol{A}(\boldsymbol{x})^T\boldsymbol{K}(\boldsymbol{x}) - \boldsymbol{K}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\boldsymbol{R}(\boldsymbol{x})^{-1}\boldsymbol{B}(\boldsymbol{x})^T\boldsymbol{K}(\boldsymbol{x}) + \boldsymbol{Q}(\boldsymbol{x}) = \boldsymbol{0}. \tag{14}$$

Equation (14) is in the form of algebraic SDRE (SDARE) for affine systems. Solution of the equation exactly results in suboptimal control because it neglects so-called "SDRE necessary condition for optimality" which tends to zero (Banks, 2007; Korayem 2015). The 6 DoF quadrotor model (5)-(6) in the form of (11) can be successfully rewritten in the SDC form (12) by finding stable parameterization for $\boldsymbol{A}(\boldsymbol{x})$. Then solution of the infinite-time SDRE problem seems to be formality in the context of UAV stabilization.

In practical implementations, when the dynamics of the system become complicated it seems to be difficult to obtain a solution quickly, due to controller sampling time. It becomes necessary to approximate the solution. However, by employing advanced signal processors and dedicated solution algorithms based on Taylor series methods or interpolation methods (Banks, 2007), the control technique can be successively realized in practical implementation. The computational effort can be also reduced by implementing modified technique, proposed bellow.

3.3 SDRE Speed Compensator – Modified Approach

In the proposed modified approach, the controller is formulated as in the classic SDRE form (11), but the SDC parameterized form uses a separated form of matrix $\boldsymbol{A}(\boldsymbol{x})$:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A}_1 + \boldsymbol{A}_2(\boldsymbol{x}))\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}, \tag{15}$$

where $\boldsymbol{A}_1$ is a state-independent and $\boldsymbol{A}_2(\boldsymbol{x})$ is a state-dependent part of $\boldsymbol{A}(\boldsymbol{x})$, respectively. Then feedback compensator can also be defined as sum of state-independent (constant) and state-dependent parts $\boldsymbol{K}(\boldsymbol{x}) = \boldsymbol{K}_1 + \boldsymbol{K}_2(\boldsymbol{x})$, what results in a control law as

$$\boldsymbol{u}_{SDRE} = -\boldsymbol{R}^{-1}\boldsymbol{B}^T(\boldsymbol{K}_1 + \boldsymbol{K}_2(\boldsymbol{x}))\boldsymbol{x}. \tag{16}$$

As described in the paper (Stepien, 2019), the procedure for solving SDARE (14) can be simplified. The modified approach makes possibility solving algebraic Riccati equation SDARE for $\boldsymbol{K}_1$ and $\boldsymbol{K}_2(\boldsymbol{x})$ employing Moore-Penrose pseudoinverse (Barata, 2013).

$$\boldsymbol{A}_1{}^T\boldsymbol{K}_1 + \boldsymbol{K}_1\boldsymbol{A}_1 - \boldsymbol{K}_1\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{K}_1 + \boldsymbol{Q} = \boldsymbol{0}, \tag{17}$$

$$\boldsymbol{K}_2(\boldsymbol{x}) = [\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T]^{+}\boldsymbol{A}_2(\boldsymbol{x}), \tag{18}$$

Equation (17) is state-independent, hence it needs to be solved only once whole the control process. Thanks to this simplification, in comparison to the classic SDRE approach, the computational effort is strongly reduced. Then control law implementation may become much easier in a real control system.

4 SIMULATIONS

The nonlinear 6 DoF quadrotor model is applied to check the described infinite-time SDRE control for positioning and stabilization when the UAV try to find desired position during flight or take-off. Governing equations that describe the UAV aerodynamics are given by (5)-(6), but for the control purpose, state-dependent parameterization SDC is necessary. When considering the UAV flight dynamics, parametrized model (12) based on system (5) and (6) with gravity and drag compensation, can be described in SDC form

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}0 & r & -q & 0 & 0 & 0\\ -r & 0 & p & 0 & 0 & 0\\ q & -p & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & (I_z - I_y)q/I_x\\ 0 & 0 & 0 & 0 & 0 & (I_x - I_z)p/I_y\\ 0 & 0 & 0 & 0 & (I_y - I_x)p/I_z & 0\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ p\\ q\\ r\end{bmatrix} +$$

$$\begin{bmatrix}\frac{1}{m} & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{1}{m} & 0 & 0 & 0 & 0\\ 0 & 0 & \frac{1}{m} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{I_x} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{I_y} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{I_z}\end{bmatrix}\left(\begin{bmatrix}F_x\\ F_y\\ F_z\\ M_x\\ M_y\\ M_z\end{bmatrix} - \begin{bmatrix}-gsin\theta + \frac{1}{m}F_{Dx}\\ gsin\emptyset cos\theta + \frac{1}{m}F_{Dy}\\ gcos\emptyset cos\theta + \frac{1}{m}(F_{Dz} + F_T)\\ 0\\ 0\\ 0\end{bmatrix}\right), \tag{19}$$

where control vector generally consists of rolling, pitching, yawing moments and force vector with trust generated from UAV rotors.

As defined in (19) and shown in Fig. 2, the thrust acts positively along the positive body *z*-axis. A quadcopter can either hover or adjust its altitude by applying equal thrust to all four rotors, where two of these motor spin clockwise, while the other two spin counter clockwise. To adjust its yaw, or make it turn left or right, the quadcopter applies more thrust to one set of motors generating yawing moment. To pitch it and roll it, on the other hand are adjusted by applying more thrust on one rotor and less to the other opposing rotor generating pitching and rolling moments.

Accordingly to the control schema proposed in Fig. 2, the control applied to the quadrotor $\boldsymbol{u} = [F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z]^T$ is a sum of PID control and SDRE stabilization, where controller outputs are rolling, pitching and yawing moments, as

$$\boldsymbol{u} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{u}_{PID} \end{bmatrix} - \boldsymbol{u}_{SDRE}, \tag{20}$$

where F_x, F_y, F_z obtained from SDRE controller are assumed to be zero. The UAV properties used with certain assumptions and indicated values to be able to perform further calculations in the chapter due to the model (5)-(6) and (19) are following: m=5,35 kg, I_x= 0,04 kg·m^2, I_y= 0,14 kg·m^2, I_z= 0,17 kg·m^2.

Employing described quadrotor model, the PID-SDRE control technique is applied to control the UAV attitude, considering infinite-time horizon SDRE control for stabilization. The control speed and final positioning error depend on PID gains, but stabilization is optimal and works accordingly to the SDRE technique.

The PID-SDRE method is chosen, because the UAV should rapidly answer for user commands, moreover the path of flight must be sometimes rapidly stabilized when unexpected external forces try to change its position and orientation during flying action. Considering above, the control problem consists of finding UAV state dynamics and PID-SDRE controls for prescribed orientation θ_{ref}=45°, ϕ_{ref}=30°, ψ_{ref}=15° during take-off with reference speed $\boldsymbol{x_{ref}} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$ and initial speed $\boldsymbol{x_0} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$.

In association with the dynamics (19), the PID controller gains are:

$$\begin{aligned} \boldsymbol{k}_P &= diag(0.3; 0.3; 0.3), \\ \boldsymbol{k}_I &= diag(0.1; 0.1; 0.1), \\ \boldsymbol{k}_D &= diag(0.001; 0.001; 0). \end{aligned} \tag{21}$$

and quadratic cost functional weighting matrices in (10) are chosen as

$$\boldsymbol{Q} = 2 \cdot \begin{bmatrix} 100 & 0 & 10 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 10 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0.2 & 2 \end{bmatrix} \text{ and } \boldsymbol{R} = 0.5 \cdot \boldsymbol{I}_{6\times 6} . \tag{22}$$

Simulations are done to show the performance of the control designed in section 3. The quadrotor state dynamics, in other words, UAV response including its orientation to the desired angle position is shown below. Firstly simulations are performed for the UAV controlled by PID only, neglecting SDRE stabilizer.

Next, simulation is performed for the full PID-SDRE controller (Fig. 2) to show how the UAV can be stabilized in the context of angular and linear speeds. To check and compare described in previous section SDRE-based methods: classic and modified, simulations are performed for both proposed SDRE stabilizers.

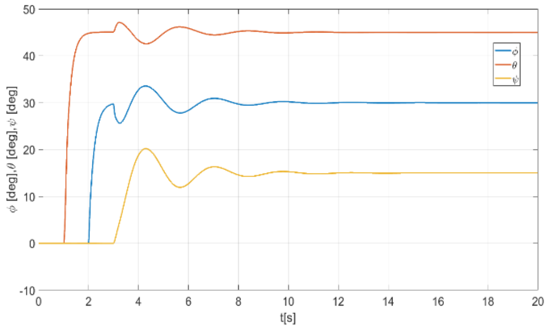

Figure 3: Angular position response, PID control.

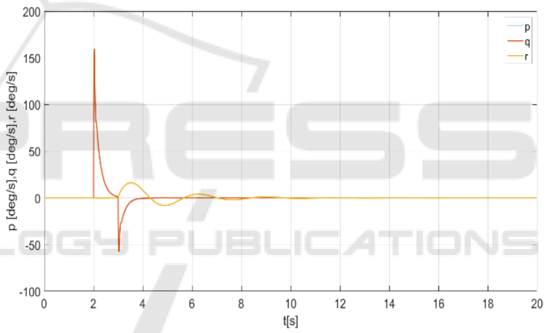

Figure 4: Angular speed response, PID control.

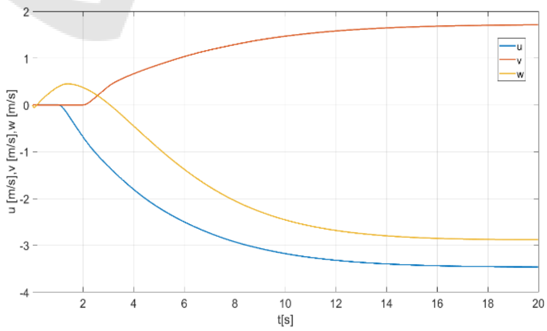

Figure 5: Linear speed response, PID control.

Figs. 3-5 show the closed-loop response of the PID controller of the quadrotor. Simulations are performed for angular positioning with θ_{ref}=45°, ϕ_{ref}=30°, ψ_{ref}=15°, programmed sequentially at 1, 2 and 3 sec. When look at Fig. 3, the quadrotor is successively controlled with a small overshot by PID reducing angular speed toward zero (Fig. 4). However, control system does not consider linear speeds, and the UAV moves in airspace.

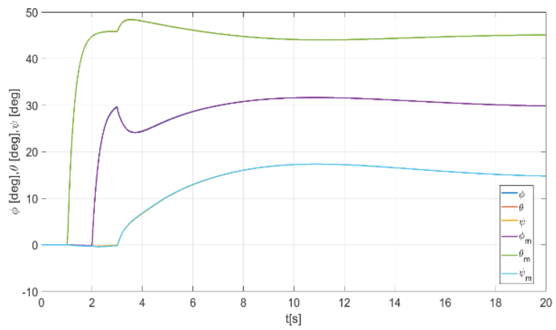

Figure 6: Angular position response, PID-SDRE control.

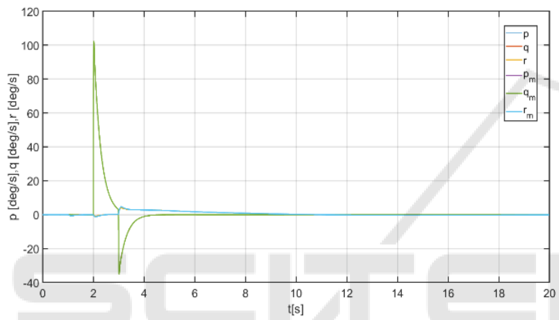

Figure 7: Angular speed response, PID-SDRE control.

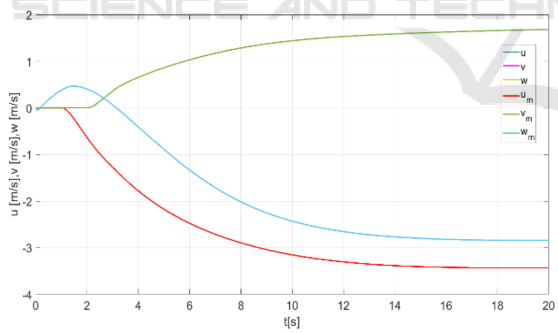

Figure 8: Linear speed response, PID-SDRE control.

When considering proposed PID-SDRE control, Fig. 6-8 shows that quadrotor can be successfully controlled to referenced angles zeroing angular speed and reducing overshoots. It allows to operate with different UAV orientation at non-zero linear speed stabilizing angular positioning task. The PID-SDRE technique is examined for classic and modified SDRE approach. Simulation results are the same. It proofs the usefulness and correctness of the methods presented and used.

5 CONCLUSIONS

The hybrid PID-SDRE control technique for the UAV-quadrotor infinite-time control problem is formulated and solved. The UAV nonlinear 6 DoF, state-dependent parametrized model is proposed. The PID fine-tuned control methodology with an optimal nonlinear feedback speed stabilizer, performing attitude control and stabilization task is analyzed. The effectiveness of the presented technique is demonstrated on numerical example where the UAV response is found using two different SDRE-based techniques.

The results presented demonstrate that in the future, the proposed control technique will be successively applied to real-time UAV flight control systems. Moreover, an approach based only on SDRE technique, neglecting PID, will be strongly developed.

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