DOA-DETECTION GUIDED NLMS ADAPTIVE ARRAY

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ABSTRACT

In various adaptive array applications, the directions of arrival (DOAs) of the desired user signal are sparsely separated. As such, the desired beam-pattern has a sparse structure. We propose an NLMS based adaptive algorithm which exploits this sparse DOA structure and provides significantly improved convergence and tracking capabilities.

1. INTRODUCTION

The performance of an adaptive array is commonly measured by its steady state error (under time invariant conditions), convergence speed, tracking speed, computational cost, as well as its stability. The least mean square LMS, or its normalised equivalent NLMS algorithm, is the most commonly used algorithm for adaptation [1, 2]. This is due to its relatively low computational cost and very good stability properties. However, its main drawback is its relatively slow convergence and tracking speeds when the adaptive filter length is 'large' [2, 3]. In communications-based adaptive array applications this may occur, for example, with densely populated cellular communications cells, where long arrays are required to produce highly directional beam-patterns.

An approach to combat this 'parameter dimension' effect (within any adaptive application), is to incorporate dimension reduction techniques within the adaptive LMS/NLMS algorithm. This may be realised in a number of ways, depending on the characteristics of the system/channel being estimated or equalised. In the case of communications-based adaptive arrays, such as in cellular or WLAN applications, the spatial channel is often characterised by having a 'sparse' structure. That is, the desired user signal typically has only a small number of sparsely separated directions of arrival (desired-DOAs). Accordingly, a possible approach to dimension reduction involves transforming the adaptive system into the spatial or DOA domain and subsequently adaptively estimating only the dominant or 'active' desired-DOAs. In this paper we follow this approach. The key idea is the use of a criterion for accurately detecting the active desired-DOAs. Following the work of Homer *et. al.* [4, 5, 6], we propose an activity criterion which is based on the minimisation of a structurally consistent least squares (SC-LS) cost function. Ultimately, we propose an NLMS based adaptive array algorithm which, for DOA sparse communications channels, demonstrates significantly higher convergence and tracking speeds than the standard NLMS algorithm. Furthermore, this is achieved with only a moderate increase in computational cost and without compromising the (time-invariant channel) steady state error. Note, in this paper we assume the communication temporal channel is a (zero delay) non-time dispersive channel. Accordingly, we consider only adaptive arrays with a complex valued scalar weight applied to each array element, as illustrated in Figure 1.

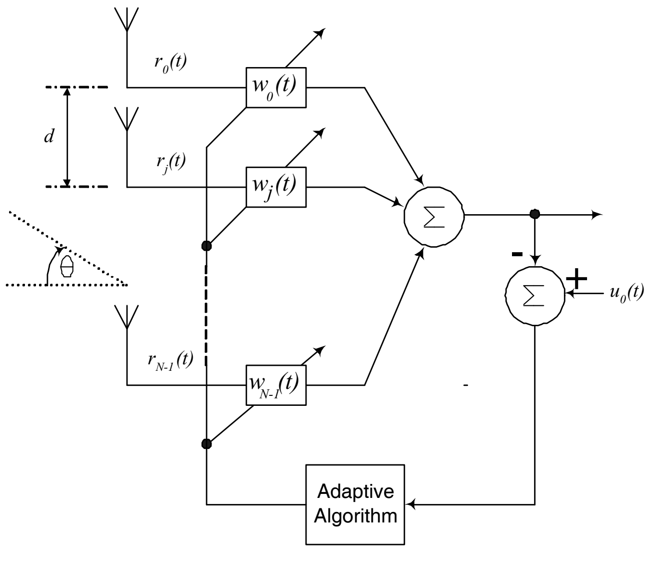

Figure 1: $N-$ element adaptive array.

This paper is organised as follows. The next section introduces notation and describes the standard NLMS algorithms operating in the array-weight and array-DOA domains. Section 3 uses the notion of a structurally consistent least squares cost function to derive an activity measure in the DOA domain. Section 4 describes the implementation of the algorithm. Section 5 presents some simulation results and we conclude in Section 6.

2. PRELIMINARIES

The configuration we consider throughout this paper is shown in Figure 1. We consider an N element uniformly spaced linear array. For notational simplicity, we assume only a 2-dimensional spatial system. That is, all the received user signals (desired and interfering) lie in the same

2-dimensional plane, and that the linear adaptive array lies within this plane. We assume: the uniform antenna element spacing is $d \leq \ /2$ where is the wavelength of the narrow-band transmitted user signals; and each antenna element is isotropic.

We consider an 'equivalent sampled complex baseband' system. That is, we assume all signals are sampled and complex basebanded. At sampling instant t: $u_0(t)$ is the (complex basebanded) transmitted desired user signal; $u_i(t),\ i = 1,2,...,n$ is the transmitted i^{th} interfering user signal; $r_j(t),\ j = 0,1,...,N-1$ is the signal received at the j^{th} antenna array element; $s_j(t)$ is the noise signal at the j^{th} antenna array element.

We assume: each user transmitted signal $u_i(t),\ i = 0,1,2,...,n$ is described by a zero mean, bounded, wide sense stationary process of variance $\ _u^2$; the different user transmitted signals are uncorrelated with each other; the noise signal of each antenna element is a zero mean, bounded, wide sense stationary white process of variance $\ _s^2$; the noise signals are uncorrelated with each other and uncorrelated with the user signals.

We assume that the i^{th} $(i = 0,1,2,...,n)$ user transmitted signal arrives from a 'small' number m_i of sparsely separated directions; and each of these directions is characterised by an angle of arrival $\ _{i,k}, k = 1,2,...,m_i$ and a complex valued gain $g_{i,k}$. We choose the direction perpendicular to the linear array line as the zero angle ($= 0$ radians) direction.

Note: for the sake of simplicity (notation and algorithm development), we have assumed the DOA characteristics of the communication channel are time-invariant. It needs to be emphasised that the proposed DOA-detection guided NLMS adaptive array algorithm is suitable for time-varying DOA channels, as well as for time-invariant DOA channels.

Accordingly, the complex baseband signal received at the j^{th} array element is:

$$r_j(t) = \ _{i=0}^{n}\ _{k=1}^{m_i} u_i(t) g_{i,k} \exp(-\mathrm{j}(2\ \ d j \sin[\ _{i,k}]/\ \ +\ \ _{i,k})) + s_j(t) \quad (1)$$

where $\mathrm{j} = \sqrt{-1}$, and $\ _{i,k}$ is the phase at the $j = 0$ element of the i^{th} user signal arriving from direction $\ _{i,k}$.

The signal output by the adaptive array is:

$$\begin{aligned} v(t) &= \ _{j=0}^{N-1} r_j(t) w_j^*(t) = R^T(t) W^*(t), \\ W(t) &= [w_0(t), w_1(t), ..., w_{N-1}(t)]^T, \\ R(t) &= [r_0(t), r_1(t), ..., r_{N-1}(t)]^T \end{aligned}$$

and where $w_j(t)$ is the scalar weight applied at the j^{th} array element at sample time t. The standard NLMS adaptation equation for the array weight vector is:

$$W(t+1) = W(t) + \frac{\ }{R^T(t) R^*(t) + \ } R(t) e^*(t).$$

where: * denotes complex conjugate, $e(t) = u_0(t) - v(t)$ and , are small positive constants.

As discussed earlier, in order to exploit the sparse spatial characteristics of the desired user's signal DOAs, we carry out the NLMS adaptation in the DOA domain. The corresponding NLMS algorithm is:

$$W_f(t+1) = W_f(t) + \frac{N\ }{R_f^T(t) R_f^*(t) + \ } R_f(t) e^*(t),$$

where $R_f(t) = [r_{f,0}(t), r_{f,1}(t), ..., r_{f,N-1}(t)]^T = FT[R(t)]$, $W_f(t) = [w_{f,0}(t), w_{f,1}(t), ..., w_{f,N-1}(t)]^T = FT[W(t)]$, $FT[X(t)]$ denotes the Fourier transform of the vector X at time t, and $e(t) = u_0(t) - v(t) = u_0(t) - R^T(t) W^*(t) = u_0(t) - R_f^T(t) W_f^*(t)$.

We define $W^{(0)} = [w_0^{(0)}, w_1^{(0)}, ..., w_{N-1}^{(0)}]$ as the desired weight-domain vector, that is the weight vector which minimises the cost function of (2) below. Similarly, we define $W_f^{(0)} = FT[W^{(0)}] = [w_{f,0}^{(0)}, w_{f,1}^{(0)}, ..., w_{f,N-1}^{(0)}]$ as the desired DOA-domain vector. In many cases, $W_f^{(0)}$ will show a sparse structure. The same is not generally true for $W^{(0)}$. The desired vector $W^{(0)}$ or $W_f^{(0)}$ depends both on the DOAs of the desired user signal and the interfering user signals.

3. ACTIVITY DETECTION

As discussed earlier, our proposed NLMS based adaptive array algorithm incorporates an activity criterion for detecting the active (or existing) desired-DOAs.The activity criterion we employ is derived from the following structurally consistent least squares based cost function [4]:

$$V_{SCLS}(T) = V_{LS}(T) + m\ \ _u^2 \log T \quad (2)$$

where: T is the data length or current (time) sample number; $V_{LS}(T) = \ _{t=1}^{T} |u_0(t) - R_f^T(t) W_f^*|^2$; $\ _u^2 =$ variance of $u_0(t)$; $W_f =$ estimated array DOA-domain vector, which contains only m active/nonzero elements.

In general, minimisation of $V_{SCLS}(T)$ requires examination and comparison of a large number $\binom{N}{m}$ of index sets with $\binom{N}{m} = \ _{m=1}^{N} \frac{N!}{(N-m)!m!}$. To circumvent this large comparison problem, we begin by introducing an assumption, which is not necessarily valid but which greatly simplifies the cost function analysis. We then include a number of modifications to offset the effects of the simplifying assumption.

Assume the DOA-domain received signal vector $R_f(t)$ has uncorrelated elements. Then, for sufficiently large T, we may approximate $V_{SCLS}(T)$ of (2) by [4]:

$$\begin{aligned} \hat{V}_{SCLS}(T) &= \ _{t=1}^{T} v^2(t) - \ _{k=1}^{m} [X_{a_k}(T) - \ _u^2 \log T] \quad (3) \\ X_{a_k}(T) &= \frac{|\ _{t=1}^{T} u_0(t) r_{f,a_k}^*(t)|^2}{\ _{t=1}^{T} |r_{f,a_k}(t)|^2} \quad (4) \end{aligned}$$

where $|.|$ denotes modulus, and $a_k (k = 1,2,...m)$ are the unknown indices of the active elements of the desired weight vector $W_f^{(0)}$.

It is apparent that $\hat{V}_{SCLS}(T)$ is minimised by (and hence the indices of the desired active elements correspond to) those indices j which satisfy:

$$X_j(T) > L(T) \quad (5)$$

where

$$L(T) = \ _u^2 \log T \approx \frac{\log T}{T} \ _{t=1}^{T} |u_0(t)|^2.$$

Equation 5 provides us with a suitable activity criterion for the case in which the elements of $R_f(t)$ are uncorrelated. This activity criterion, however, is not suitable for the more general case in which the $R_f(t)$ vector elements are correlated. This is because the correlation causes neighbouring indices to contribute significantly to the numerator term of $X_j(T)$.

To reduce this coupling effect from neighbouring indices, we propose the following three modifications. These modifications are based on the work of Homer *et. al.* [5, 6] which focusses on NLMS adaptive temporal channel estimators, as opposed to NLMS adaptive arrays.

Modification 1: Replace $X_j(T)$ by:

$$\tilde{X}_j(T) = \frac{| \ _{t=1}^{T}\{e(t) + w_{f,j}^*(t) r_{f,j}(t)\} r_{f,j}^*(t)|^2}{\ _{t=1}^{T} |r_{f,j}(t)|^2}. \quad (6)$$

Modification 2: Replace $L(T)$ by:

$$\tilde{L}(T) = \frac{\log T}{T} \ _{t=1}^{T} |e(t)|^2. \quad (7)$$

Modification 3: Include an exponentially forgetting factor: $(1- \ )$, $0 < \ \ll 1$ within the summation terms of $\tilde{X}_j(T)$ and $\tilde{L}(T)$.

Importantly, the inclusion of Modification 3, in addition to reducing the correlation coupling effect, also improves the applicability of the DOA-detection guided NLMS adaptive array to DOA time-varying systems. This capability is demonstrated in the simulation section.

4. DOA-DETECTION GUIDED NLMS ADAPTIVE ALGORITHM

The proposed algorithm is as follows.

Initialisation:

(a) For each array element index j, initialise $b_j(0) = d_j(0) = w_{f,j}(0) = 0$.

(b) Initialise: $q(0) = C(0) = 0$.

At each sample interval T:

(a) Standard signal operations:

$$\begin{aligned} R_f(T) &= FT[R(T)], \ FT = \text{Fourier transform} \\ v(T) &= R_f^T(T) W_f^*(T) \\ e(T) &= u_0(T) - v(T). \end{aligned}$$

(b) Activity threshold calculation:

$$\begin{aligned} q(T) &= (1- \ )q(T-1) + |e(T)|^2 \\ C(T) &= (1- \ )C(T-1) + 1 \\ \tilde{L}(T) &= q(T)\log\{C(T)\}/C(T) \end{aligned}$$

(c) Activity measure calculation, for element index j:

$$\begin{aligned} b_j(T) &= (1- \ )b_j(T-1) \\ &\quad + [e(T) + w_{f,j}^*(T) r_{f,j}(T)] r_{f,j}^*(T) \\ d_j(T) &= (1- \ )d_j(T-1) + |r_{f,j}(T)|^2 \\ \tilde{X}_j(T) &= \frac{|b_j(T)|^2}{d_j(T)}. \end{aligned}$$

(d) Application of activity criterion, for element j: If $\tilde{X}_j(T) > \tilde{L}(T)$ then label j as an active element index a_k; otherwise label j as an inactive element index.

(e) NLMS adaptation, for element j: If $j = a_k$ (that is, corresponds to a detected active index) then:

$$\begin{aligned} w_{f,j}(T+1) &= w_{f,j}(T) \\ &+ \frac{N}{\ _{a_k} |r_{f,a_k}(T)|^2 + \ } r_{f,j}(T) e^*(T) \end{aligned}$$

where $\ _{a_k}$ =summation over all detected active indices. If $j \neq a_k$ then

$$w_{f,j}(T) = 0.$$

4.1 Computational Complexity

We measure computational complexity by the number of multiplications per sample interval (MPSI). The standard array-weight domain NLMS algorithm requires $3N+2$ MPSI; while the corresponding standard array-DOA domain NLMS algorithm requires $3N+2+(N/2)\log N$ MPSI. The proposed DOA-Detection guided NLMS algorithm requires $6N + \hat{m} + 5 + (N/2)\log N$ MPSI. [Note: This assumes the values of $\tilde{L}(k)$ and $\log\{\tilde{L}(k)\}/\tilde{L}(k)$ are available from a look-up table.] Hence, for sufficiently long and practical arrays ($1 \ll N < 100$) and sufficiently DOA sparse channels ($N \gg \hat{m}$), the computational cost of the proposed DOA detection guided NLMS algorithm is approximately two to three times that of the standard NLMS algorithms.

5. SIMULATIONS

Simulations were conducted to compare the performance of the standard weight vector domain and DOA domain NLMS adaptive array algorithms with that of the proposed DOA detection guided NLMS adaptive array algorithm.

The simulation conditions were as follows.

Signal wavelength $= 20$mm; Array element spacing $d = \ /2$; Number of array elements $N = 64$; Adaptation constants: $= 0.001$, $= 0.1$, $= 0.99$.

Desired signal u_0: random binary real valued signal (generated using Matlab: $u_0 = sign(randn(1,2000))$).

First interfering signal u_1: random binary real valued signal with amplitude twice that of desired signal (generated using Matlab: $u_1 = 2 * sign(randn(1,2000))$).

Second interfering signal u_2: random binary real valued signal with amplitude equal to that of desired signal (generated using Matlab: $u_2 = sign(randn(1,2000))$).

Antenna element noise signal s_j: complex valued random Gaussian signal with variance $\ _s^2 = 2$ (generated using Matlab: $s_j = randn(1,2000) + \mathrm{j} * randn(1,2000)$, where $\mathrm{j} = \sqrt{-1}$).

Table 1 shows the simulation DOA parameters of the desired and interfering signals. A time sequence of 2000 sample intervals was considered, with some of the DOA parameters undergoing a sudden change at sample interval 1001.

Figures 2,3 show the results of the simulations. Shown are the average of ten similar simulations. Figure 2 is a plot of the squared error $|e(t)|^2$ over time (sample number), for each of the three NLMS algorithms. Figure 3 is a plot of the array beampattern for each of the algorithms at either $t = 1000$ (*i.e.* just before the sudden time variation) or $t = 2000$.

Table 1: Desired signal and interfering signal parameters

Desired Signal DOAs, sample times: t=1:1000				
Angles, $\{\ _{0,k}\}_{k=1}^{4}$	75^o	35^o	-20^o	-55^o
Gains, $\{g_{0,k}\}_{k=1}^{4}$	$0.5e^{j\ /3}$	$1.0e^{j}$	$0.8e^{j\ /9}$	0.5
Desired Signal DOAs, sample times: t=1001:2000				
Angles, $\{\ _{0,k}\}_{k=1}^{4}$	75^o	10^o	-20^o	-75^o
Gains, $\{g_{0,k}\}_{k=1}^{4}$	$0.5e^{j\ /3}$	$1.0e^{j}$	$1e^{j\ /9}$	0.5
First Interfering signal DOAs, sample times: t=1:1000				
Angles, $\{\ _{1,k}\}_{k=1}^{3}$	55^o	5^o	-70^o	
Gains, $\{g_{1,k}\}_{k=1}^{3}$	$1.0e^{j\ /3}$	$1.0e^{j\ /7}$	0.5	
First Interfering signal DOAs, sample times: t=1001:2000				
Angles, $\{\ _{1,k}\}_{k=1}^{3}$	55^o	25^o	-55^o	
Gains, $\{g_{1,k}\}_{k=1}^{3}$	$1.0e^{j\ /3}$	$1.0e^{j\ /7}$	0.5	
Second Interfering signal DOAs, sample times: t=1:1000				
Angles, $\{\ _{2,k}\}_{k=1}^{5}$	50^o	$+10^o$	-35^o	
Gains, $\{g_{2,k}\}_{k=1}^{5}$	$1.2e^{j\ /3}$	$0.8e^{j1.6}$	$0.8e^{j}$	
Second Interfering signal DOAs, sample times: t=1001:2000				
Angles, $\{\ _{2,k}\}_{k=1}^{5}$	50^o	0^o	-35^o	
Gains, $\{g_{2,k}\}_{k=1}^{5}$	$1.2e^{j\ /3}$	$0.8e^{j1.6}$	$0.8e^{j}$	

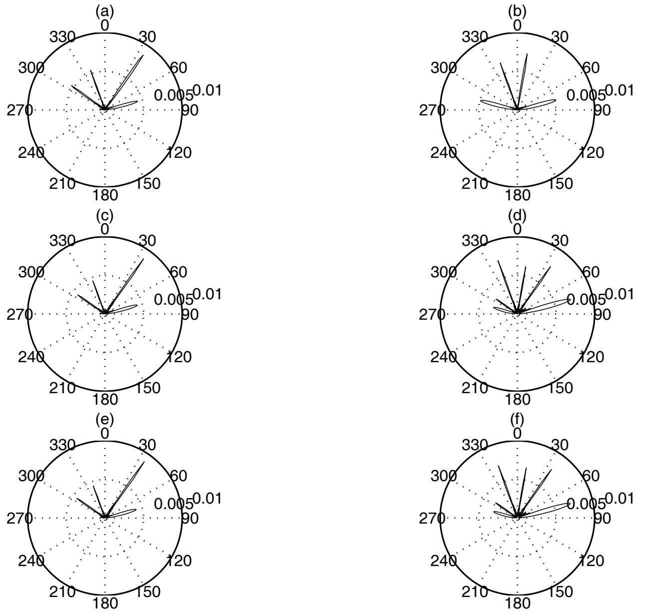

Figure 3: Plot of array beampattern for: proposed NLMS algorithm at (a) $t = 1000$ and (b) $t = 2000$; standard NLMS algorithm in DOA domain at (c) $t = 1000$ and (d) $t = 2000$; standard NLMS algorithm in weight domain at (e) $t = 1000$ and (f) $t = 2000$.

The results indicate that the proposed algorithm shows a significantly enhanced convergence rate and tracking ability over the standard NLMS algorithms. The ability of the proposed algorithm to accurately track the desired DOAs is clearly demonstrated by Figure 3(b). The attained beampattern at $t = 2000$ displays maxima "only" in the current desired DOAs. In comparison, at $t = 2000$ the standard NLMS algorithms display maxima also in the previous ($t = 1 : 1000$) desired DOAs.

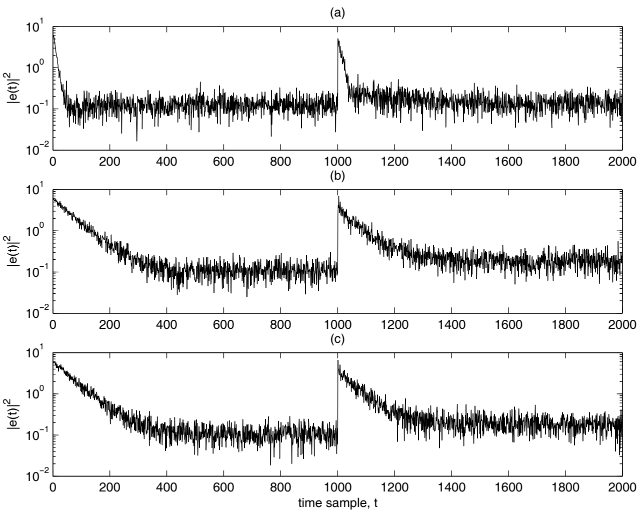

Figure 2: Plot of squared error $|e(t)|^2$ over time for: (a) proposed NLMS algorithm, (b) standard NLMS algorithm in DOA domain, (c) standard NLMS algorithm in weight domain.

6. CONCLUSIONS

We have proposed a detection guided NLMS adaptive array algorithm that operates in the DOA domain. This algorithm incorporates an activity criterion, which is based on a structurally consistent version of the least squares cost function. The activity criterion is employed to detect the active DOAs of the desired user signal. NLMS estimation is then only applied to the (complex) gain coefficients of these detected DOAs. Simulation results show that this algorithm exhibits significantly better convergence and tracking capabilities than the standard NLMS adaptive array algorithms.

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