

Mathematical Modeling of Management of Technosphere Safety in the Region

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Abstract. Mathematical modeling of management of technosphere safety of the region is a dynamic system described by a system of differential equations, which based on the synthesis and the law of object integrity preservation V.G. Burlov. Mathematical model take into account the conditions of normal functioning of the social, economic and technical-technological region's systems and emergency situations. The proposed model allows to solve the inverse task of management and to construct technosphere safety in the region with the given parameters, considering the possibilities of functioning social and economic region's systems.

Keywords: Technosphere Safety, Management Model, Law of Conservation of Integrity of the Object Model, Social, Economic and Technical-Technological Region's Systems.

1 Introduction

An effective mechanism for management of technosphere safety (TS) of the region as a dynamic system should be the nonlinear modeling conditions of functioning of the social, economic and technical-technological region's systems. The forecast of the behavior of the this region's systems in the normal and emergency modes of its operation is necessary to reduce the severity of the consequences of emergency situations. To make a forecast, models that take into account the interaction of all objects that make up the social, economic and technical-technological region's systems are required. The purpose of developing such models is allows to solve the inverse task of management and to construct TS in the region with the given parameters.

Social, economic and technical-technological region's systems are a managed dynamic systems that includes different subsystems. The composition of this region's systems are determined by the specifics of the development of the region in which it is located, and the totality of all objects located in the territory (population, jobs in the real economy, energy supply, etc.) with established links between them. The parameters that characterize the activities of each of these objects change over time and affect the entire region. Therefore, it is more correct to consider the management of TS

of the region as a complex dynamic system, changing in time under the influence of internal and external factors. The influence of the technosphere on the social and economic region's systems is an internal factor and can negatively affect the environment, public health, and the state of the economy.

The developing informatization processes have tasks: increasing efficiency of management in regions on the basis of system approach, forming the common information space solving operational and strategic tasks of region's management of TS.

Methodological basis for modeling of regional processes of management of TS is system analysis. Its main procedure is building generalized (integrated) regional model reflecting all factors and relations of a real system.

The region as a modeling object is characterized by:

- weak theoretical knowledge, quality nature of knowledge about the system, no theory of city's development;
- high uncertainty level of the source information. There is internal and external uncertainty. Internal uncertainty is a combination of factors which can not be controlled by a decision-maker fully, but he/she may influence them (e.g., domestic socio-economic environment, risk factors, etc.). External uncertainty is defined by interaction with environment - these are the factors which can be slightly controlled by a decision-maker (ecological, demographic, foreign policy situation, resources supply to the region from the outside, etc.);
- as a consequence, the results are of quality nature and make it possible to judge about development directions of the dynamic processes, analyze stability of dynamic processes.

Regional processes of management of TS should be analyzed and modeled considering the following factors:

- a region is seen as a complicated semi structured system, which system modeling assumes revealing a great number of complex interrelated cause and effect links between factors described in the system and which result of influence is not always obviously seen;
- regional systems are stochastic and should be studied in the conditions of uncertainty and ambiguity;
- a region include social system. It is vital to consider long-term interests of the society while decision-making. Regional development level should provide conditions for human life reproduction;
- a region is a dynamic system. Research of reproduction processes demands study of the system's development dynamics, growth processes analysis considering general life cycle of the region and its parts (population, enterprises, etc.).

Further integration of management processes and informatization in the social sphere, economic and TS makes it necessary to mathematical modeling of management of TS in the region.

2 Methods

2.1 Literature Review

Dynamical systems theory comprises a broad range of analytical, geometrical, topological, and numerical methods for analyzing differential equations and iterated mappings. Nonlinear systems of differential equations in mathematical modeling began to be considered in the first third of the XX century Lotka-Volterra. They modeled biological processes by introducing a large number of boundary conditions. One of the first descriptions of natural phenomena through a system of nonlinear differential equations was E.N. Lorenz and his followers [1-14]. E. N. Lorenz's discovery in 1963 said that the solutions to his equations never settled down to equilibrium or to a periodic state instead they continued to oscillate in an irregular, aperiodic fashion. Moreover, if he started his simulations from two slightly different initial conditions, the resulting behaviors would soon become totally different. The implication was that the system was inherently unpredictable, tiny errors in measuring the current state of the atmosphere would be amplified rapidly, eventually leading to embarrassing forecasts. In 1971 Ruelle and Takens proposed a new theory for the onset of turbulence in fluids, based on abstract considerations about strange attractors. A few years later, R.M. May [15] found examples of chaos in iterated mappings arising in population biology, and stressed the pedagogical importance of studying simple nonlinear systems.

Management of TS of the region should be carried out on the basis of the results of modeling the processes of socio-economic development in the framework of the selected concept of management.

O. Bezborodova and other suggest the territorial technosphere is a dynamic system described by a system of differential equations Lotka-Volterra, therefor should be the elimination of inoperable States using: the formation of a set of informative parameters; control and registration of values of informative parameters; creation of a database of normative and actual values of informative parameters, formation of control actions [16].

N. N. Lychkina and other offers methods of combining composite system-dynamic and agent-based models, allowing us to investigate the dynamics of socio-economic processes by a cyclical interaction of processes of individual and group behavior of economic and social agents at the micro level with the basic processes of socio-economic system at the macro level [17, 18].

T.G. Penkova and other suggest some criteria of emergency risk assessment using expert knowledge about danger levels [19, 20]. M.D. Molev and other purpose modeling of environmental safety of industrially developed regions of Russia was achieved via joint use of such mathematical methods as the integrated system analysis, synthesis of alternatives, algorithmization of processes and generalization of experimental data [21].

In article [3] represents the analysis of using the possibilities of the scenario analysis methods and modeling in the process of solving the planning and management problems of measures to ensure the man-made safety of a wide range of potentially dangerous production and infrastructure facilities [22].

The mathematical model of the management of TS in the region must take into account the conditions of normal functioning and critical situations, when the effectiveness of the decisions taken depends on the state of the emergency object and the values parameters that characterize this state.

An important place in the mathematical modeling of processes in the social, economic and technical-technological region's systems is occupied by nonlinear mathematical models that most fully and accurately describe existing processes. When studying the state stability of the region, the models are most adequate, for the description of which nonlinear systems of differential equations are needed.

Modeling of management on the basis of synthesis and the law of preservation of integrity of object is presented in works of V. G. Burlov, O. M. Lepeshkin and other [23-25].

2.2 Mathematical Modeling of Management of Technosphere Safety in the Region

The dynamic model based on the synthesis is formalized as a system of nonlinear differential equations. Three main system-forming indicators of activity of the region, corresponding according to the law of preservation of integrity of object of V. G. Burlov [23-25] to three basic interconnected properties are defined ("objectivity", "integrity", "variability" or "object", "purpose", "action").

The dynamic mathematical model of the energy sector management in the region based on the synthesis is formalized as a system of nonlinear differential equations. Three main system-forming indicators of activity of the region to three basic interconnected properties:

- indicator of social system of the region "x" (number of population= birth rate-death rate+ migration balance);
- indicator of economic system of the region "y"(number of jobs in the real economy= high-technology jobs + other jobs);
- indicator of technical and technological system of the region "z" (energy supply in the region=produced fuel and energy resources-consumed fuel and energy resources).

Formula (1) describes a system of differential equations of three systems of the region:

$$\begin{cases} \frac{dx}{dt} = ax - bxy + qxz; a, b, q > 0; \\ \frac{dy}{dt} = -py + cxy + \gamma yz; c, p, \gamma > 0; \\ \frac{dz}{dt} = \mu z - \tau xz - \delta yz; \delta, \mu, \tau > 0. \end{cases} \quad (1)$$

x, indicator of the number of population;

y, indicator of the number of jobs in the real economy;

z , indicator of the energy supply in the region;
 a , the coefficient of demographic activity;
 b , the coefficient of negative attitude of people to childbearing;
 q , coefficient of provision of energy in the region;
 c , coefficient of people's interest in economic development;
 p , coefficient of development of the real sector of the economy;
 γ , coefficient of energy supply of workplaces;
 μ , coefficient of development of energy supply in the region;
 τ , coefficient of compliance of the population with energy supply;
 δ , coefficient of compliance of the economy's development with energy supply.

The backbone of the model is a system of differential equations and three dimensionless relative indicators: social, economic and technical-technological. Nine coefficients of the system of differential equations implement mechanisms of management for the processes of ensuring TS in the region.

Methods of nonlinear dynamics allow you to simulate fast, non-equilibrium processes (so-called phase transitions) in economic systems, related to the transition from one stable States in others. It is necessary to proceed to models aimed at describing non-equilibrium processes using the nonlinear dynamics apparatus.

It should be noted that there is no analytical solution this kind systems of equations, so the solution is possible only through the use of numerical methods that replace the continuous problem with a discrete one. The Cauchy problem for this kind of equations is described in detail in the theory of oscillations, when it is necessary to find continuous $0 \leq t \leq T$ variables trajectory $x = x(t)$ $y = y(t)$ $z = z(t)$, when $t > 0$ and initial condition .

$$\begin{cases} \frac{dx}{dt} = f_1(t, x(t)), 0 < t \leq T, x(0) = x_0; \\ \frac{dy}{dt} = f_2(t, y(t)), 0 < t \leq T, y(0) = y_0; \\ \frac{dz}{dt} = f_3(t, z(t)), 0 < t \leq T, z(0) = z_0. \end{cases} \quad (2)$$

when

$$f_1(t, x) = ax - bxy + qxz; \quad (3)$$

$$f_1(t, y) = -py + cxy + \gamma yz; \quad (4)$$

$$f_1(t, z) = \mu z - \tau xz - \delta yz; \quad (5)$$

where the specified functions are not explicitly time-dependent $f(x(t), y(t), z(t))$.

For this type of problem, it is advisable to use the Runge-Kutta method.

$$\begin{cases} x_{i-1} = x_i + \int_{t_i}^{t_{i-1}} f_1(t, x) dt; \\ y_{i-1} = y_i + \int_{t_i}^{t_{i-1}} f_2(t, y) dt; \\ z_{i-1} = z_i + \int_{t_i}^{t_{i-1}} f_3(t, z) dt. \end{cases} \quad (6)$$

The essence of this method is to replace the functions $f_1(t, x)$, $f_2(t, y)$, $f_3(t, z)$, with some approximation, the more accurate the approximate value of the integrand is, the more accurately the integral will be calculated, i.e., the more accurately it will be defined x_{i-1} , y_{i-1} , z_{i-1} .

Thus, it is obvious that the solution of the considered system of equations is most appropriate by applying the Runge-Kutta method, which is quite simple and gives acceptable accuracy results.

In system of nonlinear differential equations need determine the coefficients of the backbone parameters (x, y, z) obtained functional dependence of $l_i = f(K_{i1}, K_{i2})$, which are described by smooth functions, so after their decomposition in a number Taylor obtained justification of ratios social, economic and technical-technological systems of the region through the definition of functional dependencies using common dependencies. This made it possible to identify the parameters of the model and are represented by formulas:

$$\alpha = K_{r0} - K_r y_r + K_{r0} - K_s y_s + K_{ms0} - K_{ms(y)} y_{ms} - K_{ms(z)} z_{ms}; \quad (7)$$

$$b = -K_r \sigma_r + K_s \sigma_s + K_{ms(y)} \sigma_{ms(y)}; \quad (8)$$

$$q = K_{ms(z)} \omega_{ms(z)}; \quad (9)$$

$$p = K_{htrm0} - K_{htrm} x_{htrm0} + K_{drm0} - K_{drm} z_{drm}; \quad (10)$$

$$c = -K_{htrm} \sigma_{htrm} - K_{drm} \sigma_{drm}; \quad (11)$$

$$\gamma = -K_{drm} \phi_{drm}; \quad (12)$$

$$\mu = K_{pre} x_{pre} - K_{pre0} + K_{poe0} + K_{poe} y_{poe}; \quad (13)$$

$$\tau = K_{pre} \phi_{pre}; \quad (14)$$

$$\delta = K_{poe} \omega_{poe}. \quad (15)$$

To determine the above coefficients, an indexing system was developed and introduced, providing good visibility (table 1).

Table 1. Indexing and description of parameters

i-th in- dex	Parameter denotes the share			Primary value the coefficient	Derivative of a function	Given initial values			Value of the parameter with <i>i</i> -th index
	ϕ_i	σ_i	ω_i	K_{i0}	K_i	x_i	y_i	z_i	
<i>r</i>	-	+	-	+	+	-	+	-	birth rate
<i>s</i>	-	+	-	+	+	-	+	-	death rate
<i>ms</i>	-	-	-	+	-	-	+	+	migration balance
<i>ms(y)</i>	-	+	-	-	+	-	-	-	migration balance due to economic conditions
<i>ms(z)</i>	-	-	+	-	+	-	-	-	migration balance due to energy supply
<i>htrm</i>	-	+	-	-	+	-	+	-	high-technology jobs
<i>drm</i>	-	+	-	+	+	-	+	-	other jobs
<i>pre</i>	+	-	-	+	+	+	-	-	produced fuel and en- ergy resources
<i>poe</i>	-	-	+	+	+	-	-	+	consumed fuel and energy resources

Reducing the system of equations under consideration to a matrix form showed that it is quadratic:

$$\dot{X} = L \cdot X_q \cdot 1 + X_q \cdot Q \cdot X_q^T \cdot \quad (16)$$

where the first part is linear in nature:

$$L \cdot X_q \cdot 1 = \begin{pmatrix} a & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & \delta \end{pmatrix} \cdot \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} ax \\ -py \\ \mu z \end{cases} \quad (17)$$

the second has a quadratic nature:

$$\begin{aligned} X_q \cdot Q \cdot X_q^T &= \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \cdot \begin{pmatrix} 0 & b & q \\ c & 0 & \gamma \\ \mu & \tau & 0 \end{pmatrix} \cdot \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \\ \Rightarrow \begin{cases} 0xx - bxy + qxz \\ cyx - oyy + \gamma yz \\ -\tau zx - \delta zy + 0zz \end{cases} &= \begin{cases} -bxy + qxz \\ cxy + \gamma yz \\ -\tau xz - \delta yz \end{cases} \end{aligned} \quad (18)$$

The presence of a quadratic component makes it advisable to analyze the solution based on the classical theory of oscillations. Accordingly, for the analysis of the system of equations, it is advisable to use phase portraits that sufficiently fully and succinctly reflect the properties of the function under consideration. In this case, a phase portrait is understood as the totality of all its trajectories depicted in the space of phase variables.

Accordingly, for the possibility of analyzing the results of applying the numerical solution method, it is proposed to use phase portraits that sufficiently fully and succinctly reflect the properties of the system under consideration.

For a more reasonable assessment and analysis of local bifurcations of phase portraits near singular points and limit cycles, it is necessary to consider only those values of input parameters when the system of differential equations degenerates from a state of stable equilibrium or goes into chaos.

It is worth noting that the system of differential equations under consideration has stability points where the derivatives are zero $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0, \frac{dz}{dt} = 0$.

Such points are called special points of this differential equation. The system of equations under consideration may have many special points. Accordingly, it is necessary to consider all of them.

The quadratic structure allows you to show that there are only 5 stability points

Taking the opportunity when $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0, \frac{dz}{dt} = 0$ it is appropriate to consider the following algebraic system of equations:

$$\begin{cases} ax - bxy + qxz = 0; \\ -py + cxy + \gamma yz = 0; \\ \mu z - \tau xz - \delta yz = 0. \end{cases} \quad (19)$$

By converting the equation system to the following form:

$$\begin{cases} x(a - by + qz) = 0; \\ y(-p + cx + \gamma z) = 0; \\ z(\mu - \tau x - \delta y) = 0. \end{cases} \quad (20)$$

It is not difficult to make sure that the following points are special:

Special point №1:

$$x = 0, y = 0, z = 0. \quad (21)$$

It is obvious that the existence of this point has no physical meaning for the system in question. Since all system – forming indicators are equal to zero, they are absent. The system is degenerate.

Special point №2:

$x = 0, (y, z - \text{solutions of equations});$

$$\begin{cases} y(-p + \gamma z) = 0 \\ z(\mu - \delta y) = 0 \end{cases} \Rightarrow \begin{cases} -p + \gamma z = 0 \\ \mu - \delta y = 0 \end{cases} \quad (22)$$

It is obvious that the existence of this point has no physical meaning for the system in question. Since there is no social system. The system cannot exist when $x = 0$.

Special point №3:

$y = 0$, (x, z - solutions of equations);

$$\begin{cases} x(a + qz) = 0; \\ z(\mu - \tau x) = 0 \end{cases} \Rightarrow \begin{cases} a + qz = 0; \\ \mu - \tau x = 0 \end{cases} \quad (23)$$

It is obvious that the existence of this point has no physical meaning for the system in question. Since there is no economy. The system cannot exist when $y = 0$.

Special point №4:

$z = 0$, (x, y - solutions of equations);

$$\begin{cases} x(a - by) = 0; \\ y(-p + cx) = 0 \end{cases} \Rightarrow \begin{cases} a - by = 0; \\ -p + cx = 0 \end{cases} \quad (24)$$

The existence of this point is possible. Theoretically, a state can exist without a military system. The system can exist when $z = 0$. This case is not considered.

Special point №5:

$x \neq 0, y \neq 0, z \neq 0$. (x, y, z - solutions of equations);

$$\begin{cases} x(a - by + qz) = 0; \\ y(-p + cx + \gamma z) = 0; \\ z(\mu - \tau x - \delta y) = 0. \end{cases} \quad (25)$$

It is possible to exist only this special point (Special point №5), the physical meaning of which characterizes the state as being in a state of equilibrium. This particular point is taken as the focus of the system of equations. Accordingly the system of equations for finding foci will take the following form:

$$\begin{cases} x(a - by + qz) = 0; \\ y(-p + cx + \gamma z) = 0; \\ z(\mu - \tau x - \delta y) = 0. \end{cases} \quad (26)$$

The inequality of at least one of the system-forming indicators to zero does not make physical sense, so the solutions (21-23) will be considered degenerate. Equation (24) makes sense when there are no military expenditures, or their impact on the economy

is insignificant (however, $y = \frac{a}{b}, x = \frac{p}{c}$), this case is not considered. Therefore, only the non-degenerate system of equations (25) is of practical significance.

As noted earlier, the existence of a system is possible only if the system-forming indicators are not equal to zero $x \neq 0, y \neq 0, z \neq 0$. . At the same time, it should be taken into account that the system will be stable when the increment for each indicator is equal to zero $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0, \frac{dz}{dt} = 0$, which also reduces the amplitude of fluctuations of system-forming indicators at acceptable input parameters.

Accordingly, the state of equilibrium in the framework of the system under consideration is fulfilled under the following conditions:

$$\begin{cases} \frac{dx}{dt} = 0, x \neq 0; \\ \frac{dy}{dt} = 0, y \neq 0; \\ \frac{dz}{dt} = 0, z \neq 0. \end{cases} \quad (27)$$

Accordingly, to find the bifurcation lines, it is necessary to consider the following mathematical interpretation of the system of equations under consideration:

$$\begin{cases} \frac{dx}{dt} = f(x, y, z, \bar{a}, \bar{b}, \bar{q}); \\ \frac{dy}{dt} = f(x, y, z, \bar{p}, \bar{c}, \bar{\gamma}); \\ \frac{dz}{dt} = f(x, y, z, \bar{\mu}, \bar{\tau}, \bar{\delta}). \end{cases} \quad (28)$$

where $\bar{a}, \bar{b}, \bar{q}, \bar{p}, \bar{c}, \bar{\gamma}, \bar{\mu}, \bar{\tau}, \bar{\delta}$ are the input parameters of the system of equations under consideration.

Accordingly, it is necessary to find out how the phase portrait of this system will behave when changing the above input parameters, when there is no increment.

$$\begin{cases} f(x, y, z, \bar{a}, \bar{b}, \bar{q}) = 0; \\ f(x, y, z, \bar{p}, \bar{c}, \bar{\gamma}) = 0; \\ f(x, y, z, \bar{\mu}, \bar{\tau}, \bar{\delta}) = 0. \end{cases} \quad (29)$$

3 Results and Discussion

This system of nonlinear differential equations does not have a purely analytical solution, it is possible only by methods of numerical integration, for example, such as Adams, Euler, Runge-Kutta, which allow you to build solutions in the form of smooth curves. The obvious drawback for the practical application of the method is the difficulty of perception for analysis. Since the values may be the same or in absolute value be unsuitable for viewing. The solution is to construction phase portraits in Python,

which sufficiently fully and succinctly reflect the properties of the function under consideration. Phase trajectory-the trace of the movement of the image point. A phase portrait is a complete set of different phase paths. It well illustrates the behavior of the system and its main properties, such as equilibrium points. Using phase portraits, you can synthesize regulators (the phase plane Method) or analyze the stability positions and the nature of the system's movements.

Given that the state of three system-forming indicators is considered, the most complete is the consideration of phase portraits in three-dimensional space, for a more detailed analysis, it is advisable to compare the phase portraits of three-dimensional and two-dimensional spaces. An example of such an analysis, with conditionally arbitrary input parameters, is shown in figure 1, where considering that three systems are considered, the most complete is the consideration of phase portraits in three-dimensional space, where the solution is represented as a corresponding spiral.

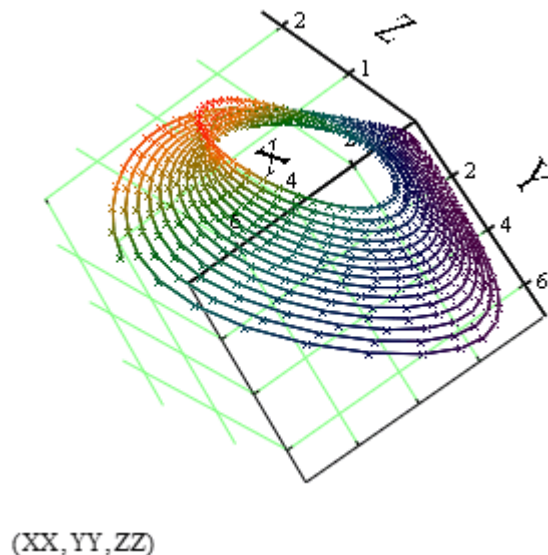


Fig. 1. Phase portrait of the system in three-dimensional space ("x" -number of population; "y" - number of jobs; "z" - energy supply in the Saint- Petersburg в 2010-2018 yeas).

The above synthesized model, formalized as a system of three differential equations, the solution and the analysis of which is proposed through numerical integration, which allows to evaluate the behavior of the main characteristics of the long time interval and to generate proposals for the adjustment of certain parameters.

With synthesis there is a set of output characteristics of the projected system and it is required to define the quantitative and qualitative makeup of the system. That is, with analysis a task is solved "from the beginning" and the result is analyzed, whereas with synthesis a task is solved "from the end", from the desired result, and the system with the required output characteristics is formed. The methods of decomposition,

abstraction (mathematical interpretation) and aggregation take a central place in system modeling.

4 Conclusions

The analysis of the properties and parameters of the management of TS of the region makes it possible to characterize it as a dynamic system consisting of a set of elements for which a functional relationship is established between the time and state of each element of the system. The methodological approach is developed, which allows by modeling the interaction of social, economic and technical-technological systems. Such mathematical dependencies make it possible to study and describe the change of the management of TS of the region in time, taking into account external and internal influences, and solve the inverse task of management and to construct TS in the region with the given parameters, considering the possibilities of functioning social and economic region's systems. The proposed method of control and management makes it possible to make the process of forming the control effect more efficient.

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