6 External economies and learning by doing

"One for all! All for one!" [Alexandre Dumas]

Learning Goals:

- Distinction between internal and external economies of scale
- Acknowledge the different types of external economies related to capital accumulation
- Distinguish the implications of non-decreasing versus increasing returns to scale for economic growth and convergence
- Explain why increasing returns are a source of cumulative causation
- Discuss the role of external economies in shaping comparative advantages and the pattern of international trade

6.1 Introduction

The main limitation of the Solow model is that technological progress is assumed exogenous. As already explained in section 3.1, this is a natural implication of the model' assumptions: since there is perfect competition and technology is assumed to diffuse instantaneously at no cost, no single user will be willing to pay for it and no selfish agent will engage in a deliberate effort to invent a new technology. In the Solow model, all income is distributed to the owners of capital and labour, and no profit is left to reward successful research.

It is however possible to construct a model sticking with perfect competition and perfect technological diffusion where the level of technology is endogenous. The trick is to assume that the level of technology evolves over time as an *unintended* consequence of investment decisions by individual firms. As long as technological improvements arise unintendedly, there will be no need for reward. There is, however, an externality, in that each firm benefits from each other firm' investments. Each firm will consider the level of technology as exogenous, and hence it will not take into account the impact of its investment decisions in the aggregate. As firms invest, they will be contributing to the common technological change. With externalities, it is possible to stick with the assumptions of diminishing returns and perfect competition at the firm level with endogenous technological change. This idea was first developed by Alfred Marshall (1890), to explain the tendency for some industries to concentrate in phew areas[101].

In the growth literature, this avenue was first explored in the early 1960s by Marvin Frankel (1962) and Kenneth Arrow (1962)[102]. The purpose of Frankel was to reconcile the convenient properties of the new-classical production function regarding factor allocation and income distribution, with the hypothesis of constant returns to capital that delivers unceasing growth in the Harrod-Domar model. The purpose of Arrow was to model technological change as an unintended outcome of cumulative working experience, a phenomenon labelled "learning by doing". These ideas were rediscovered two decades later by Paul Romer (1986) and Robert Lucas Jr. (1988)[103]. Romer extended the Arrow model of learning by doing to the case with non-diminishing returns in reproducible factors, obtaining unceasing growth. Lucas emphasized the role of externalities associated to human capital as a source of non-diminishing returns. The contributions of Romer and Lucas marked the first wave of the so-called *new growth theory*.

This chapter shows how externalities associated to capital accumulation may overcome the limitation imposed by diminishing returns, delivering a production function for the economy as a whole with the required properties to generate unceasing growth, without the need to depart from perfect competition. Section 6.2 describes the general model introduced by Marvin Frankel, a particular specification of which delivers an AK production function in the aggregate. Section 6.3 explains why the competitive equilibrium with externalities is no longer efficient, and discusses the possible role of the government in addressing the market failure. Section 6.4 focus on specifications of the model with

[101] Marshall, A., 1890. *Principles of Economics*, London: Macmillan.

[102] Frankel, M., (1962). The production function in allocation and growth: a synthesis. The American Economic Review LII(5), 995-1022. Arrow, K., 1962. "The economic implications of Learning by Doing", Review of Economic Studies 29: 155-173

[103] Romer, P. 1986. "Increasing returns and long run growth". Journal of Political Economy 94, 1002-37. Lucas, R., 1988. "On the mechanics of economic development". Journal of Monetary Economics 22, 3-42.

externalities that deliver increasing returns in the aggregate, allowing the model to display both unceasing growth and agglomeration effects. Section 6.5 discusses the implications of external economies of scale for comparative advantages and for the benefits of international trade. Section 6.6 concludes.

6.2 Externalities on capital accumulation

6.2.1 A "development modifier"

In his 1962 paper, Frankel first observed that the neo-classical production function used in the Solow model is capable of describing factor allocation and income distribution but is not capable of generating sustained growth of per capita income. In turn, the linear production function used in Harrod-Domar model, is capable of generating long-run growth, but it does not offer a satisfactory theory for factor allocation and income distribution. Frankel then proposed a method to conciliate the two production functions, so that the desirable properties of each but none of the limitations are retained: the key for such conciliation was to assume a production externality whereby the "overall level of development of a region" impacts positively on the productivity of each private firm.

The author observed that firms in developed economies are able to produce more with given inputs of capital and labour than firms in underdeveloped regions. Frankel related this to "various external effects", such as "improvements in the organization and the quality of labour, technical change, external economies of scale, and better social overhead facilities in the form of transport and communication networks" (p 1001). These external effects are *bounded in space*, in the sense that they are specific to a given economy, acting therefore as a local public goods.

To model these externalities, Frankel assumed that each individual firm faces a Cobb-Douglas production function, where TFP is a positive function of the economy-wide capital stock. Formally, let the production function for each individual firm *i* be given by:

$$Y_i = BK_i^{\beta}N_i^{1-\beta} \quad , \qquad (6.1)$$

where Y_i, K_i and N_i denote, respectively, for output, capital and labour employed by firm i. The technological parameter, B - the "development modifier", as coined by Frankel - was assumed to depend positively on the aggregate levels of capital and labour. Its more general specification was as follows:

$$B = A\frac{K^{\varepsilon}}{N^{\varepsilon'}}, \text{ with } \varepsilon \geq 1-\beta \text{ and } 0 \leq \varepsilon' \leq \varepsilon, \qquad (6.2)$$

where $K = \sum_i K_i$ and $N = \sum_i N_i$ stand, respectively for the *aggregate* levels of capital (human, physical) and labour in the economy.

According to (6.2), an increase in the aggregate stock of capital impacts positively on the productivity of each firm. Thus, whenever a firm accumulates capital for private reasons, it will be "indirectly" contributing to the productivity of all other firms. The productivity term (6.2) also accounts for a negative externality on aggregate labour, in case $\varepsilon' > 0$. This effect captures the possibility of the positive externality related to capital accumulation being partially or totally *diluted* by the size of the labour force. When, for instance, $\varepsilon' = \varepsilon$, the firm productivity depends on the aggregate stock of capital per worker, rather than with the aggregate stock of capital in absolute terms. When instead $\varepsilon' = 0$, what matters is the absolute level of capital in the aggregate, not the capital labour ratio. In the following, we'll consider alternative cases regarding the relative weights of these external effects.

Production externalities specified in this manner are labelled "Marshallian" or "Technological" externalities". Comparing to the Solow model, the model with externalities retains the assumption that technology spills over instantaneously across firms at zero cost. Hence, like in the Solow model, there are no private incentives to improve the level of technology. But the level technology is now endogenous: even though there are no purposeful efforts to develop new technologies, the level of technology improves over time as a by-product of capital accumulation, which is driven by economic decisions.

6.2.2 Factor prices in the competitive equilibrium

Because each firm is small relative to the economy, its decisions will have a negligible impact on the aggregate. Hence, each firm will take parameter B as exogenous, and independent of its investment decisions. Each individual firm will perceive its own production function in the form (6.1), with the standard neoclassical properties of constant returns to scale and diminishing returns to capital.

Profit maximization by individual price taking firms will therefore deliver the usual conditions stating that firms employ labour and capital until their marginal products equal the respective factor prices:

$$r + \delta = \frac{\partial Y_i}{\partial K_i} = \beta \frac{Y_i}{K_i}, \qquad (6.3)$$

and

$$w_t = \frac{\partial Y_i}{\partial N_i} = (1-\beta)\frac{Y_i}{N_i}. \qquad (6.4)$$

Because all firms are equal, we have $Y_i/K_i = Y/K$ and $Y_i/N_i = Y/N$.

Thus, in the competitive equilibrium, the shares of capital and of labour on domestic income are given, respectively, by β and $1-\beta$. This is the very convenient result, as it implies that factor shares remain in accordance to the Kaldor stylized fact, regardless the presence of externalities.

6.2.3 The aggregate production function

In the presence of externalities, the aggregate production function differs from the individual production functions, even if all firms are alike. The aggregate production function is obtained substituting (6.2) in (6.1) and summing up across all firms. This gives:

$$Y = AK^{\varepsilon+\beta}N^{1-\beta-\varepsilon'} \qquad , \quad (6.5)$$

with $Y = \sum_i Y_i$. The *aggregate production function* (6.5) may deviate from the neoclassical assumptions of constant returns to scale and diminishing marginal returns. For instance, when $\varepsilon + \beta \geq 1$, returns to capital are non-decreasing. As we already know, this is the condition we need for a model to display *unceasing growth* through capital accumulation. On the other hand, whenever $\varepsilon > \varepsilon'$, the aggregate production function will exhibit *increasing returns to scale*: that is, rising capital and labour by a given proportion causes output to increase more than proportionally. As we will discuss next, this scale effect makes larger economies more attractive for location, acting therefore as a mechanism of economic divergence.

Note that at the individual level, production functions retain the neoclassical properties of constant returns and diminishing returns to capital. The aggregate production function departs from these properties because of the externality, which individual firms – being small - do not take into account.

6.2.4 The AK model again

Equation (6.5) is general enough to account for external effects of different magnitudes. Frankel was however concerned with the particular case where externalities are such that the aggregate production function becomes exactly linear in the economy' capital stock, to mimic the Harrod-Domar model. Therefore, the author focused on the case with $\varepsilon = \varepsilon'$ and $\varepsilon + \beta = 1$ [104]. In this case, the positive externality in K is exactly enough to overwhelm the normal process of diminishing returns to capital, and – at the same time - the negative externality on labour exactly matches the externality on capital, implying that returns to scale remain constant. The aggregate production function (6.5) becomes exactly linear in K:

$$Y_t = AK_t \ . \qquad (6.5a)$$

The aggregate production function (6.5a) takes the AK form, but firm level production functions (6.1) retains the neoclassical properties. Each firm perceives its production function as with diminishing returns to capital, so it will employ capital and labour according to (6.3) and (6.4). In the aggregate, the production function is linear in K, so the marginal product of capital will never decline and the economy will never stop growing. In this model, policies influencing the rate of capital accumulation will indirectly influence the level of technology, and by then the rate of economic growth.

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[104] In case $\varepsilon = \varepsilon'$ and $\beta + \varepsilon < 1$, the aggregate production function exhibits constant returns to scale and diminishing returns to capital. As you may easily check, in that case the steady state growth rate of output is equal to the growth rate of population, just like in the basic Solow model. The only difference is that, because of the externality, private returns to capital in laissez faire will be too low.

The advantage of this model when compared to the simple AK model, is that it does not rely on the peculiar assumption that labour plays no role in production. Like in the Solow model, both factors are used in production and factor income shares are in accordance to reality. Note that the model also accords to the other Kaldor stylized facts: the wage rate and per capita income grow steadily over time, the user cost of capital is constant and equal to $r+\delta=\beta A$ (equation 6.3), and growth rates may be different in different economies.

6.3 The market failure and optimal intervention

6.3.1 Growth accounting revisited

Conventional growth accounting (as exemplified in Section 2.7) typically uses the share of capital on national income as the proxy for the contribution of capital to production. The discussion in Chapter 4 revealed, however, that this procedure delivers an estimate for the contribution of capital (β), that is too small to account for the observed differences in per capita incomes across countries. In order to account for such large differences, one would need a contribution of capital to production much larger than that implied by the observed income shares.

The Frankel model offers an explanation for this puzzle: a larger contribution of capital to production than that implied by the observed shares in national incomes might be accounted for by externalities. Formally, equation (6.5) reveals that, as long as the externality parameter ε is positive, the *actual* contribution of capital to production ($\beta+\varepsilon$) is larger than that implied by its share in income (β). Log-differentiating (6.5), one obtains:

$$\hat{Y}=(\varepsilon+\beta)\hat{K}+(1-\beta-\varepsilon')n \qquad (6.6)$$

In (6.6), input changes have two effects, a private one and an external effect. The external effect may amplify or diminish the private effect, depending on the sign and magnitude of the respective parameter. For instance, when $\varepsilon=1-\beta$, a one-percentage point increase in the capital stock will result in a one-percentage point increase in output, a result that conforms with the AK model (and that Frankel argued to conform as well to the U.S. data).

Equation (6.6) suggests that conventional growth accounting, by underestimating the effective contribution of capital to production, overestimates the Solow residual.

6.3.2 The social return of capital

With externalities, the competitive equilibrium is not efficient. Each firm, being small relative to the economy, decides its capital stock taking into account its own profits, only. The positive contribution of its investment decisions to the overall capital stock are considered negligible and hence ignored. Still, investments made by all firms together impact positively on the profit of each individual firm. Thus, the competitive equilibrium delivers a suboptimal level of investment.

Formally, the marginal contribution of aggregate capital to aggregate production (i.e, taking into account the externalities) as stated in (6.5) is:

$$\left(\frac{\partial Y}{\partial K}\right)_{social} = (\beta + \varepsilon)\frac{Y}{K} \qquad (6.7)$$

In its profit maximization problem, the firm considers only the narrow private returns to capital (equation 6.3). As long as $\varepsilon \neq 0$, there will be a wedge between private returns and social returns. The wedge between private returns and social returns to capital implies that *incentives are misaligned*: in the decentralized economy, investment will be too low.

6.3.3 Optimal intervention

The wedge between social returns and private returns to capital constitutes a market distortion: investment in physical capital is too low relative to the efficient allocation. Given this sort of diagnosis, a benevolent planner may find appropriate to subsidize capital accumulation.

To find out the optimal subsidy, let's rewrite the individual firm profit function, but allowing for subsidy $\tau_K < 0$ on capital incomes:

$$\pi_i = BK_i^{\beta} N_i^{1-\beta} - (r+\delta)(1+\tau_K)K_i - wN_i \qquad (6.8)$$

In light of this specification, the cost of one unit of capital –the cost to firms - is $(r+\delta)(1+\tau_K)$, while capital owners receive as net income $(r+\delta) > (r+\delta)(1+\tau_K)$. Regarding how this subsidy will be financed, assume that lump sum taxation is available, so that the policy will not imply further distortions.

From the first order conditions of profit maximization, one obtains (instead of 6.3):

$$\frac{\partial Y_i}{\partial K_i} = \beta \frac{Y}{K} = (r+\delta)(1+\tau_K) \qquad (6.9)$$

To *get the incentives right*, the government must set the subsidy such that the (net) rental price of capital, $(r+\delta)$, fully reflects the social return of capital (6.7). That is, τ_K should be set such that:

$$r+\delta = \beta \frac{Y/K}{(1+\tau_K)} = (\beta+\varepsilon)\frac{Y}{K} \qquad (6.10)$$

Solving for τ_K , the optimal (first best) tax rate will be:

$$\tau_K = \frac{-\varepsilon}{\varepsilon+\beta} \leq 0 \qquad (6.11)$$

This result is intuitive: if the contribution of capital to production is given by (6.7) and private firms only perceive it to be equal to (6.3), then a subsidy filling the gap will achieve the aim of getting private returns aligned with the social interest.

In the particular case in which $\varepsilon = 1-\beta$ (the AK model), the optimal subsidy will be such that $\tau_K = \beta - 1$ (note however, that in this extreme case *all* income in the economy would be devoted to capital owners and nothing would be left to raw labour; this would be only possible if K referred to a broad concept of capital, including human capital).

6.3.4 Growth effects

In this model, removing the distortion leads to a greater efficiency and, by then, to faster economic growth. To see this, consider again the optimal consumption rule $\gamma = r - \rho$ and let's focus in the particular case in which $\varepsilon = \varepsilon' = 1-\beta$ (the AK model).

In the competitive equilibrium, the interest rate is determined according to (6.3). Substituting r in the optimal consumption rule, one obtains the growth rate of per capita income under laissez faire:

$$\gamma = \beta \frac{Y}{K} - \rho - \delta = \beta A - \rho - \delta \qquad (6.12)$$

With intervention, the user cost of capital would become equal to (6.7) and the growth rate of the economy will be:

$$\gamma^* = (\beta + \varepsilon)\frac{Y}{K} - \rho - \delta = A - \rho - \delta \qquad (6.13)$$

Comparing, we see that the growth rate of this economy with an appropriate intervention will be higher than in the laissez fare.

This example illustrates how judicious government intervention might be used to establish the “right” prices and thereby stimulate growth. Note however that such a “perfect” intervention requires a high level of confidence by the government regarding the size of the external effect, as well as availability of non-distortionary taxation. Whenever these conditions are not met, it may well be the case that the government may fail to do better than the market.

6.4 The case with increasing returns

6.4.1 External economies of scale

So far, the analysis focused on the case with $\varepsilon = \varepsilon'$. This is however a very special case. It requires the positive effect arising from a larger stock of physical capital to be exactly offset by the “dilution” effect resulting from a larger number of workers. In this version of the model the aggregate production function exhibits constant returns to scale, even though returns to capital are non-decreasing.

A quite distinct case occurs when $\varepsilon > \varepsilon'$. In that case, expanding the use of capital and labour by a given proportion has a more than proportional impact on output: the aggregate production function exhibits *increasing returns to scale*. Remember that increasing returns do not arise at the individual firm level, but instead at the aggregate level. Because of this, increasing returns are said to be *external* to the firm (Box 6.1).

When the aggregate production function displays increasing returns, there will be a tendency for the region to become larger and larger. To see this, just note that the average product of labour in (6.5) becomes equal to:

$$y = Y/N = Ak^{\varepsilon+\beta}N^{\varepsilon-\varepsilon'} \qquad (6.14)$$

This means that that, in a competitive equilibrium, the wage rate – determined according to (6.4) - will also be an increasing function of the size of the workforce.

$$w = (1-\beta)Ak^{\varepsilon+\beta}N^{\varepsilon-\varepsilon'} \qquad (6.4a)$$

Thus, a larger region will be a more attractive place to work than a smaller region. This will generate a tendency for employment to move to the larger region, further expanding the larger region and depressing the smaller region.

This illustrates why increasing returns are a source of divergence: if for whatever reason, a region starts out bigger, increasing returns will assure that it will become a more attractive place to work and invest. With free factor movements, labour will move from depressed areas to the more dynamic region and the later will get bigger and richer, absorbing resources from the rest of the world.

The idea that development brings more development in a virtuous cycle was coined "cumulative causation" by Thorstein Veblen, in 1898. This concept was popularised by Gunnar Myrdal in the 1950s[105]. This author contended that labour migration, capital movements and trade may lead to cumulative expansion of the favoured regions and retard the development of backward regions, leading to persistent or even divergent cross-country differences in per capita income.

[105] Veblen, T., 1898, Why is economics not an evolutionary science? *Quarterly Journal of Economics*, 12, 373-97. Myrdal, G., 1957. *Economic theory and underdeveloped regions*, Duckworth, London.

Box 6.1. Internal and external economies of scale

The distinction between "internal" and "external" economies of scale dates back from Scitovsky (1954)[106]. "Internal" economies of scale refer to the case in which a single firm faces a downward sloping average cost curve when increasing *its own* level of output. In this case, there is a tendency for the larger form to outprice its competitors, becoming larger and larger, until becoming monopolist in the market. Internal economies of scale are inherently linked to imperfect competition.

The concept of "external" economies of scale refers to the case in which scale economies arise at the aggregate (spatial or industry) level. In that case, average costs for the individual firm decline with aggregate output, but not with the individual firm output. "External economies of scale" in the aggregate may co-exist with constant returns to scale and declining marginal productivities at the firm level. Hence, one does not need to abandon the assumption of perfect competition.

6.4.2 Alfred Marshall and the theory of economic geography

The theory of external economies of scale was pioneered by one of the founders of modern economics, the British economist Alfred Marshall. In his book "Principles of Economics" (1920, first published in 1890), Marshall was concerned with the question as to why there is a tendency for some industries to concentrate in few areas within a country ("industrial districts"). Examples at that time included cutlery manufactures in Sheffield, and hosiery firms in Northampton. In our days, similar examples include the Silicon Valley, Hollywood and Las Vegas. This type of spatial concentration of economic activities cannot be explained by proximity to natural resources.

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[106] Scitovsky, T., 1954. Two concepts of external economies. Journal of political economy, 62, 143-151.

To explain the tendency for firms of the same industry to cluster together, Marshal conjectured that the productivity of each firm in a given location may depend positively on the general progress of the corresponding industry *in the same location*, via three types of external effects: availability of specialized suppliers, labour market pooling, and knowledge spillovers.

First, the availability of specialized suppliers: in many industries, production requires the use of specialized inputs, such as intermediate products and specific supporting activities, that cannot be acquired at distance because of high transportation costs. For instance, the production of a motion picture requires a variety of services, such as casting services, sound effects, costume design, choreography, catering, etc. Many of these services are better purchased to specialized firms, because specialized firms can split the fixed costs of investing in technology through different costumers. If, in a given region, there is only one film producer, it will not pay for upstream suppliers to locate in that region. Instead, they will prefer locations where there are already many moviemakers, ensuring that the market is large enough to break even. By the same token, moviemakers will find it more profitable to join locations where other moviemakers are already located, because this will imply a higher market for – and hence a higher availability of - specialized services, competing with each other.

Second, labour market pooling: when many firms and specialized workers locate in the same region, both sides of the market will be less exposed to events affecting a small number of firms or workers. For instance, the failure of one firm will be less problematic for a specialized worker located in a region with many firms than if located in a region with one employer only. The same holds for firms. By clustering together, both firms and workers will benefit from the law of large numbers, being therefore less exposed to specific shocks affecting individual agents.

Third, technological spillovers: workers engaged in a given production process have incentive to observe what similar others are doing, so as to imitate the best practices. Arguably this process of learning takes place more effectively when various firms of the same industry are concentrated in a given location, so that workers belonging to different firms have the opportunity to meet together and discuss technical problems, face-to-face. Personal contacts are essential for the diffusion of Tacit Knowledge (see box 6.2). Similarly, the knowledge embodied in skills may diffuse easily through workers mobility across neighbourhood firms.

All in all, these three types of external effects (often called “Marshallian externalities”) act as a *local* public good, implying that each firm becomes more productive, the more firms of the same industry are located nearby. The local nature of these externalities is essential to explain why economic activities tend to cluster together.

Formally, this theory is modelled assuming that the technological parameter of an individual firm’ production function depends positively on an variable measuring the size of the industry in that location (as done in equation 6.2). In that case, the aggregate production function may display increasing returns to scale, creating the incentive for firms to cluster together in particular regions.

The Marshall model of technological externalities launched the theory of Economic Geography. This theory is concerned with the question of how economic activities are located across the space. The theory had however to be refined to account for the fact that there is no tendency for all activities to be concentrated in a single location. That would be wholly unrealistic. In fact, location decisions are also influenced by *centrifugal* (dispersion) forces, such as congestion effects, whereby the cost of a firm adopting a given location increases with the number of firms that are already in that location. For instance, the concentration of activities in a small area leads to higher land prices, high commuting costs, pollution and other sociological factors. Another dispersion force arises due to transport costs: to the extent that some activities have to be undertaken in the periphery (for instance, agriculture, exploitation of natural resources), being close to the centre implies higher transport costs in transacting with the periphery. The location of economic activities must therefore obey to a balance between centripetal forces and centrifugal forces. Since the weights of these two forces differ from industry to industry, there is scope for diversity in the spatial organisation of economic activities[107].

Box 6.2. Tacit knowledge

Knowledge is not all alike. Some knowledge is suitable for codification, for instance in manuals, or in textbooks. When this is so, it has the potential to be transmitted at distance. Not all knowledge, however, is suitable for codification. In many technologies, only the broad guidelines are codified. The remainder pieces are embodied in the skills of practitioners. This component of knowledge is dubbed "tacit knowledge".

The main feature of tacit knowledge is that it can only be transmitted through face-to-face contacts. This concept was first proposed by a Doctor of Medicine and of Physical Science, Michael Polanyi. It his words, "Tacit knowledge can be passed only by example from master to apprentice"(p.53)[108]. Since the diffusion of tacit knowledge is mediated through personal contacts, it does not propagate easily across the space.

A natural mechanism through which tacit knowledge may diffuse across the space is via labour mobility. Tacit knowledge is embodied with people and can migrate with people. Similarly, tacit knowledge can be transmitted at distance, via training actions. Multinational firms typically spend considerable amounts of resources in organizing meetings, workshops, demonstrations and seminars for their employees, to overcome the problem of communicating knowledge at distance.

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[107] Classical contributions accounting for these centrifugal forces in the context of "Marshallian externalities" include Henderson (1974) and Fujita and Ogawa (1982). The theory was later refined with the works of Paul Krugman and Anthony Venables [Henderson, 1974. "The sizes and types of cities". American Economic Review 64, 640-656. Fujita, M., Ogawa, H., 1982. "Multiple equilibrium and structural transition on non-monocentric urban configurations". Regional Science and Urban Economics 12, 161-196. Krugman, P., 1991. "Increasing returns and economic geography", Journal of Political Economy 99, 483-499. Krugman, P. and Venables, A., 1995. "Globalization and the Inequality of Nations". Quarterly Journal of Economics 60, 857-880].

[108] Polanyi, M., 1958. Personal knowledge: towards a post-critical philosophy. Chicago: U. Chicago Press.

6.4.3 The Arrow model

Kenneth Arrow proposed a model of endogenous technological change, whereby investment by individual firms increase the firm' stock of knowledge through learning by doing (see Box 6.3). Arrow modelled learning-by-doing at the individual firm level, assuming that investment in physical capital impacts in the firm' stock of knowledge[109]. This assumption was then combined with the assumption that knowledge leaks. Arguably, firms tend to imitate the improvements achieved by fellow firms, so they all end up benefiting from the accumulated experience of each other. Thus, when one firm invests in new capital, it adds to its own stock of knowledge and at the same time to the common stock of knowledge. In the Arrow' model, the externality on capital accumulation arises because the learning acquired through investment by each one firm leaks out to other firms, giving rise to increasing returns to scale in the aggregate.

Like in the Frankel model, total factor productivity at the firm level is an increasing function of the economy-wide accumulated stock of capital. The main difference in is that in the Arrow model there is no negative externality associated to the number of workers. In terms of equation (6.2), Arrow restricted attention to the case with $\varepsilon' = 0$. This assumption captures the non-rival nature of knowledge: many workers and firms can use the same piece of knowledge without reducing its effectiveness. The implication is that the learning by doing model unequivocally displays increasing returns. With all firms identical, the aggregate production function becomes:

$$Y = AK^{\varepsilon+\beta}N^{1-\beta} \qquad (6.5b)$$

Thus, as long as $\varepsilon > 0$, the production function will exhibit increasing returns to scale on capital and labour altogether, giving rise to scale effects and agglomeration economies.

[109] Arrow (1962), p. 157: "each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning takes place with continuous new stimuli".

In this model, the assumption of knowledge spillovers is critical for the model to be consistent with perfect competition: if the knowledge created did not leak out, the individual firm accumulating capital would become more productive than its competitors; its returns would be higher and higher and the conditions would exist for this firm to grow alone and capture the entire market, becoming monopolist.

With the assumption of perfect technological diffusion, the model follows in an intuitive manner: each firm, perceiving its production function as a CRS, buys new capital until the *private* marginal product of capital equals the user cost of capital (eq. 6.3). Buying the state-of-the-art capital, the firm inadvertently increases its own stock of knowledge, but this effect is small. Since knowledge leaks, the acquisition of physical capital by each one firm adds to the common stock of knowledge, which impacts positively on the productivity of all firms. Thus, each firm will be more productive, the higher the productive experience (measured by the stock of capital) in the economy as a whole (eq. 6.2). Box 6.4 presents a well-known case-study where learning by doing and knowledge spillovers triggered a process of unceasing growth.

Note that increasing returns on capital and labour altogether is not a sufficient condition for the model to display *endogenous* growth. For this, one would need to assume as well that returns to capital are non-diminishing, $\beta+\varepsilon\geq 1$. The problem with that case is that the model would display a problematic (strong) scale effect, whereby the growth rate of per capita income would be a positive function of the size of population: that is, a larger economy should grow faster than a smaller economy (see Box 6.5). Arrow ruled out that possibility, sticking to the case with $\beta+\varepsilon<1$.

In the Arrow model, there are increasing returns to capital and labour altogether (and hence agglomeration effects), but diminishing returns to capital alone, ensuring that the capital-output ratio converges to a constant in the steady state, just like in the Solow model. To see this, let's log-differentiate (6.5b), obtaining:

$$\hat{Y}=[\beta+\varepsilon]\hat{K}+(1-\beta)n \qquad (6.15)$$

In the steady state, capital and output grow at the same rate. Imposing this restriction in the equation above, one obtains $\hat{Y}(1-\beta-\varepsilon)=(1-\beta)n$. Now using $\gamma=\hat{Y}-n$, we obtain the growth rate of per capita income in the Arrow model:

$$\gamma = \left[\frac{\varepsilon}{1-\beta-\varepsilon}\right]n. \qquad (6.16)$$

Equation (6.16) shows that, as long as population growth is positive, per capita income in this economy will grow over time, without the need to assume an exogenous rate of technological progress. Since the growth rate of per capita income in the steady state is determined by the growth rate of population, which is an exogenous parameter, this model displays *exogenous growth:* changes in policy influencing the saving rate or the efficiency parameter A will alter the steady state level of per capita income, but not its growth rate. Only *level effects* will be achieved.

The model displays a kind of scale effect, in that the long run growth of per capita income depends on how fast population is growing. This is a direct implication of the non-rival nature of knowledge: since sharing knowledge does not involve loss of its effectiveness, the larger the population being served with each given piece of knowledge, the better. Since this "scale effect" is of a second order, it is categorized as "weak".

Box 6.3. Learning by doing

A possible link between investment in physical capital and technology is that new investments force users to adapt and learn. When a firm buys a new capital good, it is also acquiring a new production technique. Learning how to operate with the new equipment and organizing the production process so as to better benefit from the opportunity opened up constitutes a technological improvement. The full materialization of this learning process may however take time. Workers benefit from experience. With the passage of time, workers get more accustomed to the new capital good. By undertaking similar actions repeatedly, workers perfection their routines and learn how to solve minor problems, becoming more productive as time goes by. This benefit occurs through practice - hence the label "learning by doing" - and is often summarized by a "learning curve", that relates the average cost of producing a given good to the cumulative experience in producing that good[110].

Box 6.4. Desh Garment Ltd

In 1980, a major world textile producer from South Korea, Daewoo Corporation, launched a joint venture with a local partner in Bagladesh, the Desh Garment Ldt, to produce garments. At that time, Bangladesh had no productive experience in garment production. And yet this investment triggered a process of learning by doing and technological diffusion that transformed the economy of Bangladesh into a major exporter of textiles.

At the outset, Desh Garment sent 130 Bangladeshi workers to Korea, for training. This training allowed young workers to acquire tacit knowledge, becoming familiar with the process of producing garments. This knowledge was not exactly a rocket science, but simply it had not been transmitted before, because there was no garment production in Bangladesh. After operations started, however, the knowledge acquired in practice started leaking, opening up an enormous potential. With no surprise, in the years that followed, 115 workers trained by Daewoo left the company to start their own businesses. The new firms not only produced garments, but also gloves, coats and trousers. Rapidly, an entire exporting sector emerged, through learning by doing and knowledge spillovers[111], and Bagladesh became a world player in textiles and related industries.

[110] The "learning curve" was first described by a German psychologist Herman Ebbinghaus (1850-1909), in a series of tests consisting in memorizing nonsense syllables. The relationship between experience and costs of production was first documented by an aeronautical engineer called Theodore Wright [Ebbinghaus, H., 1885. Memory: A contribution to experimental psychology. Wright, T., 1936. "Factors affecting the cost of airplanes". Journal of the Aeronautical Science 3, 122-128].

[111] Rhee, Y., W., 1990. The catalyst model of development: lessons from Bangladesh' success in garment exports. World Development, 18, 333-346.

Box 6.5. Strong scale effects in the LBD model

In the Arrow model there are diminishing returns to capital, so the ratio Y/K tends to a constant in the steady state. This ensures a constant interest rate and compliance with the Kaldor facts. A different case occurs when $\beta+\varepsilon\geq 1$: when returns to capital are increasing, the marginal product of capital becomes a positive function of the capital-labour ratio. This means that the interest rate will be itself increases over time.

To see this, let's substituting (6.5b) in (6.3) to obtain an expression for the user cost of capital:

$$r+\delta=\beta Ak^{\varepsilon+\beta-1}N^{\varepsilon} \qquad (6.17)$$

From (6.12), the endogenous growth rate of per capita income will be:

$$\gamma=\beta Ak^{\varepsilon+\beta-1}N^{\varepsilon}-\rho-\delta \qquad (6.18)$$

In case $\beta+\varepsilon=1$, the growth rate of per capita income is a positive function of the size of the labour force, N, displaying a "strong" scale effect: if the workforce grows at a constant rate, n, the interest rate will be ever-increasing interest rate, and so will do the rate of economic growth. In case $\beta+\varepsilon>1$, the growth rate of per capita income will be ever-accelerating even if population remains constant. These predictions do not square well with the real world facts: in general, there is no systematic tendency for large countries to grow faster than smaller countries, nor for growth rates to be increasing over time[112].

6.4.4 Lucas: externalities on Human Capital

An alternative way of modelling externalities in capital accumulation is focusing on human capital. This avenue was explored by Robert Lucas Jr. in its 1988' article. The main

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[112] Romer (1986) proposes a model to overcome this problem where the growth rate of knowledge is assumed bounded up, due to diminishing returns in knowledge accumulation. With such assumption, the interest rate becomes bounded up too, delivering a constant growth rate for per capita income.

argument is that people who get educated benefit more in a society where people are educated than in a society with low education levels.

To understand this, ask yourself why the best graduate economists prefer to work at the City of London or at Wall Street – where economics graduates are plentiful – rather than in, say, Mongolia where they are in very short supply. The economist working at City earn his high income in part because of the manner in which its own knowledge is enhanced by those of fellow well-educated economists. This happens because individuals benefit from *interacting* with each other. Exchange of ideas with other professionals enhances individual capabilities. Thus, just like in an assembly line, where the value of each worker's effort depends on the other worker's efforts, complementarities in human capital create the incentive for the best workers to match up with each other: if the best economists are *assembled* together, they will have better ideas and will get a higher payoff from their skills. If, instead, they are partnered with lazy or incompetent economists, they will have a lower reward for any effort that they might individually provide.

Note that this is exactly the opposite of the LDR: with diminishing returns, skills substitute for each other, so they become more valuable where they are scarcer – in Mongolia and not at City. Under diminishing returns, skilled labour would move from rich countries to poor countries. By contrast when externalities in human capital are strong enough to overwhelm diminishing returns, then skilled labour will be more valuable where it is more abundant: returns to skills for each individual are an increasing function of the existing skill level in the society. This mechanism helps explain why we observe skilled workers migrating from poor countries to wealthy ones, and not the other way around[113]. It also explains why wages for similar skills and education levels are higher in cities than in rural areas: skilled

[113] Carrington and Detragiache (1998) provide evidence that the stock of immigrant workers in the U.S. is, on average, better educated that the average worker in the home countries. The authors also showed that, for most developing countries, the highest migration rates are observed in the group of individuals with tertiary education [Carrington, W. Detragiache, E. 1998. "How Big Is the Brain Drain?" IMF Working Paper 98/102 (Washington)].

workers tend to move to where skilled workers are, because staying close to each other makes each one of them more productive.

Like in the case with physical capital, complementarities in human capital imply cumulative causation and vicious cycles: in a nation where skill levels are already deep and well established, people in that nation will find strong incentives to invest in their human skills. But in poorer economies where the skill base is thin, the incentive for individuals to invest in human skills is low. Thus, a country will be rich if it started out rich, a country will be poor if it started out poor.

These vicious and virtuous cycles in human capital accumulation also apply to ethnic groups within societies. If any sort of social segregation delivers people of the same ethnic group a higher probability to match and work with each other than with people from other ethnic groups, then there will be a tendency for education levels to converge within each group: people belonging to the low education ethnic group will invest less in education because working with people with low education implies a low return to education. On the contrary, people belonging to the highly educated ethnic group will have an incentive to invest in education, because the chances of being matched with well-educated people are high.

The same mechanism applies to the other component of human capita, health: an healthy society impacts positively on individual health through lower contagion of diseases. Thus, an individual' health will be a positive function of the average health in the society. This, in turn, impacts positively on individual productivity, because healthy individuals are likely to be more productive.

6.4.5 Localized versus global technological spillovers

Along this section, we argued that technological externalities favour the spatial concentration of economic activities, acting as a source of economic divergence. This is in sharp contrast with a basic assumption of the Solow model, that knowledge has the potential to diffuse *globally,* acting as a source of (conditional) convergence. A question therefore arises on how to conciliate these two perspectives.

The idea that knowledge travels well across borders linking the growth rates of interdependent economies in the long-run has to be taken seriously. However, it also has to be qualified. A well-documented fact in our days is that technological levels are not uniform across countries. Despite all progresses in telecommunications and the internet, we are far from the neoclassical assumption that knowledge spills over instantaneously at any distance at no cost.

On one hand, the empirical evidence gives supports to the idea that proximity matters for technological diffusion (see Box 6.6). On the other hand – and most importantly - one must recognize that other factors apart from geographical distance influence the pace of technological diffusion. Arguably, the same piece of knowledge will diffuses at different speeds across different countries, depending on the recipient country infrastructure, human capital, culture, as well as economic and political circumstances. Thus, while keeping an eye on the idea that technology has the potential to diffuse across the space, one must take into account that technology diffuses via specific mechanisms of human interaction, and that this diffusion may be retarded by *human-devised barriers*. These barriers give rise to persistent disparities in per capita incomes and in growth rates. This discussion suggests that, one needs to deepen our understanding on the mechanics of knowledge diffusion and on the role of economic policies in overcoming the existing barriers, so as to increase a country' permeability to the innovations occurring elsewhere.

Box 6.6. Proximity and technological diffusion: the evidence

The question as to whether knowledge spillovers tend to be bounded in space or global is of crucial importance for our understanding of the forces that promote economic growth and convergence: if most knowledge spillovers are localized, companies operating nearby benefit more from each other innovations than companies located elsewhere. In this case, there will be an incentive for firms to operate in the same location, giving rise to cumulative causation and divergence. If, in contrast, knowledge spillovers are mostly global, there will be a tendency for laggard economies to catch up and to converge with the more advanced economies.

A strand in the literature has examined technological diffusion in its geographical dimension. An interesting study by Wolfgang Keller using intra-industry data, found that with every additional 1200 kilometres distance, there is a 50-percent drop in technological diffusion, irrespectively of country borders. Other researchers found that technological diffusion tends to be stronger within countries than across countries, that agglomeration advantages are more prevalent in R&D than in operations, and that the benefits of proximity might have declined with improvements in telecommunications. Still, the main conclusion is that literature is proximity still matters for knowledge diffusion (see Keller, 2004, for a survey)[114].

6.5 Learning by doing and international trade

6.5.1 Comparative advantages locked in

External economies and learning by doing weigh on comparative advantages. If cumulative experience makes workers progressively more productive and regions progressively more attractive to industries they are already specialized in, then a process of cumulative causation arises whereby comparative advantages are reinforced over time. Home firms become progressively more productive in each of the goods initially produced at home, while foreign firms become progressively more productive in each of the goods initially produced abroad. Thus, once a pattern of specialization is established, changes in relative productivity will act to further lock the pattern in.

To illustrate this, consider a World with two goods, say agriculture (Z) and manufactures (Y). Both goods are produced using labour, only, and the home country is small relative to the rest of the World. Wages are assumed flexible. The home country production functions are:

$$Y = AN_Y \qquad (6.19)$$

$$Z = BN_Z \qquad (6.20)$$

So far, this model is similar to the one analysed in Section 1.5. But instead of assuming that the productivity parameters A and B are exogenous, we now assume that

[114] Keller, W., 2004. "International Technological Diffusion". *Journal of Economic Literature* 42: 752-782.

technology in each sector evolves over time as a positive function of the country cumulative experience in that sector:

$$\dot{A} = \varepsilon_Y A N_Y \quad \text{with } \varepsilon_Y \geq 0 \tag{6.21}$$

$$\dot{B} = \varepsilon_Z B N_Z \quad \text{with } \varepsilon_Z \geq 0 \tag{6.22}$$

To abstract from scale effects, it is assumed that the total workforce in the economy is equal to 1, $N_Y + N_Z = 1$. To stick perfect competition, we retain the assumption that learning-by-doing takes place as a pure external effect: each producer ignores the effect of its decisions in the aggregate.

Now, let p be the relative price of the agriculture good in terms of manufactures in the world economy. If, at the time of trade openness, $p > A/B$ (that is, if the opportunity cost of producing the agriculture good at home is lower than the relative price of agriculture goods in the world economy), then the home country has comparative advantage in agriculture. Under free trade, the home country will specialize in agriculture.

Since the production possibilities frontier is linear, eventually the home economy will become fully specialized in agriculture. In that case, the only relevant production function after openness will be (6.20), with knowledge accumulating according to (6.22). This mechanism will then intensify the existing comparative advantages: home firms will be progressively more productive in agriculture goods, while domestic productivity in manufactures is stagnant due to lack of productive experience. The initial specialization pattern. once established, becomes entrenched over time.

6.5.2 Static versus dynamic benefits of trade

Whether the specialization pattern achieved under free trade is good or bad for the home economy, it depends on how fast technology in agriculture evolves in that sector, compared to manufactures. If the learning potential of both industries was the same (i.e, if $\varepsilon_Y = \varepsilon_Z$), then the growth rate of per capita incomes at home and abroad would be independent of the specialization pattern. Comparative advantages would still be reinforced with accumulated experience, but there would be no consequences for the international distribution of per capita income.

If however the two industries differ in terms of learning opportunities, growth rates will be dependent on which good the country specializes in the first place. As an extreme example, assume that there is no learning-by-doing in agriculture, that is $\varepsilon_Z = 0$. In that case, openness to trade dooms the home country to a constant level of output, while the foreign economy is expanding over time. If in alternative the country remained in autarky, there would be production in both sectors, and some growth would be achieved through learning by doing in manufactures.

This model suggests that the pattern of specialization implied by comparative advantages is not necessarily the one that delivers faster economic growth. As long as different goods differ in terms of their learning potential, trade openness may involve a trade-off between static efficiency and dynamic efficiency.[115]

Under this reasoning, many economists along time have been arguing that nations, instead of engaging in free trade *tout court*, should instead rely on temporary import protection in particular sectors, so as to develop economies of scale before opening up to the world markets. This idea, known as the *infant industry* argument, was first codified by the U.S. Secretary Alexandre Hamilton (1755-1784), who in turn was inspired by the mercantilist thinking of the 17th century[116].

In the context of the learning by doing model, Paul Krugman showed that, in a world with many goods, a policy of selective restriction of imports, protecting first an industry with high growth potential until that industry becomes strong enough to survive in the open

[115] The idea that, in the presence of Marshallian externalities, the static gains from trade and the dynamic gains from trade may not go along was first formulated by Graham (1923) [Graham, F., 1923, Some aspects of protection further considered. *Quarterly Journal of Economics* 37, 199-227].

[116] Hamilton, A., 1791. Report on manufactures. Reprinted in Syrett H et al (eds.) The Papers of Alexander Hamilton. New York and London: Columbia University Press, 1961.

market, and then move protection to another industry with high learning potential, could alter permanently the pattern of comparative advantages in the protecting country' favour[117].

Box 6.7. Learning by doing and the European fears of globalization

In light of the conventional theory of international trade, countries should specialize according to their comparative advantages. In the real world, however, many economists and think tanks believe that policymakers should not overlook the learning potential generated by a significant productive experience in manufactures. This argument gained importance in the last decades, as trade openness and globalization have generated a worldwide reallocation of manufactures away from industrial countries towards emerging economies, namely in Southeast Asia.

In the European Union, the European Commissioner, Jacques Barrot was one of the opposers to open globalization. He argued that giving up the European industrial base, allowing the low skill labour intensive components of the production chain to migrate to emerging economies, taking opportunity of the lower labour costs there, may benefit the European consumer in the short run, but will imply sooner or later the loss of the accumulated knowledge, making it impossible for Europe to explore the potential synergies between universities, research centres and firms, as envisaged by the European leaders: "it is not possible to maintain the knowledge accumulated through learning by doing if not supported by a production activity", he claimed[118].

6.5.3 Terms of trade effects

[117] Krugman (1987) argued that such strategy was followed with success by Japan, during its industrialization process [Krugman, P., 1987. "The narrow moving band, the Dutch disease and the competitive consequences of Mrs. Tatcher", Journal of Development Economics 27, 41-55].

[118] Barrot, J., 2008. Les illusions d'une Europe sans industries", Les Echos, 28/4/2008.

The model so far has assumed that world prices remain unchanged. However, if even if each country alone is small, it will be affected by shifts in terms of trade, as determined by the world demand and world supply of agriculture and manufacture goods.

To see how terms of trade effects may change our earlier conclusions, assume that the World is divided in two regions, say, North and South. Assume that the respective populations are constant. The North is initially specialized in manufactures and the South is specialized in agriculture. If learning by doing opportunities only occur in manufactures, then manufactures production will be expanding over time, while production in agriculture remains stagnant.

If both goods are normal in consumption, the increasing supply of manufactures will determine a fall in its relative price. This means that one unit of agricultural good will be traded by more and more units of the manufactured good as time goes by. Thus, even if the South is unable to expand its production, it will benefit by an increasing purchasing power in terms of manufacture goods. For consumption and utility, what matters is real income expressed in terms of both goods, and this will be increasing through a terms of trade effect. By the same token, the North will be growing faster in terms of its own good (with output measured at domestic prices), but will faces an adverse terms of trade effect weighing negatively on real income. On balance, what will happen to real incomes in both regions, it depends on preferences.

To examine this question, let’s sticking with our example of agriculture versus manufactures. As we already know, the demand for agriculture goods tends to increase less than proportionally than income, due to the Engel Law holds (this reasoning was already explored in chapter 1.5). If that is so, then as the world output increases induced by an expansion in manufactures, one will expect the relative demand to be progressively tilted towards manufactures, implying that terms of trade do not improve the enough in the South. In the South, real income in terms of the two goods will be increasing, but less than in the North, and there will be economic divergence.

More generally, for any two goods, whether the terms of trade effect is enough to compensate for the diverging production, it depends on the elasticity of substitution between the two goods in consumption[119]. When the elasticity of substitution is equal to one (as with Cob-Douglas preferences), the terms of trade effect exactly offset the differential productivity growth, and relative real incomes remain unchanged. If, in alternative, the two goods were high substitutes, the fall in manufactures prices would lead to a more than proportional increase in the world desired demand for manufactures, and terms of trade in the South would not be improving fast enough. Finally, if the substitutability between the two goods was lower than one, then the terms of trade effect would dominate the learning by doing effect and real income in the North would grow at slower pace, despite its faster technological progress. That would be an (unlikely) case of immiserating growth in the North.

6.6 Key ideas of Chapter 6

- An avenue that some authors have followed to make technological change endogenous without departing from perfect competition is to assume that technological change arises unintentionally, as a by- product of investments in physical or human capital. When this is so, technological change arises as an externality.
- In the presence of positive externalities on capital, there will be a larger role for capital in production than that implied by the share of capital in national incomes. In that case, the conventional growth accounting overestimates the Solow residual.
- With externalities, the competitive equilibrium is no longer a social optimum. There is scope for government intervention.
- If externalities are large enough to generate non-decreasing returns to capital, the model will display unceasing growth, even with a constant population.
- Aggregate externalities may be a source of cumulative causation. In that case, there will be a tendency for agglomeration of economic activities and to the self-reinforcement of economic disparities, with the richer economies becoming more attractive and getting richer, and the poorer economies remaining poor. To some extent, this was what happened across the World after the Industrial Revolution.

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[119] This point was made in Lucas (1988).

- In reality, economic activities do not move all to a single point because there are also centrifugal forces.
- Knowledge spillovers may act as a source of convergence or as a source of divergence depending on whether they are global or bounded in space. While some knowledge travels well, some other remains mostly embodied in people, requiring face to face contacts to be transmitted. There are also human-devised barriers to technological diffusion that prevent poorer economies to catch up. Economic polices may have a role in mitigating these barriers.
- Learning by doing reinforces comparative advantages, locking-in trade patterns. To the extent that industries differ in terms of learning opportunities, a trade-off may arise between the static gains from trade and the dynamic gains. Along this reasoning, it has been argued that countries should impose temporary protection in selected sectors, so as to achieve a critical mass before opening.

6.7 Problems and Exercises

Key concepts

- External vs. internal economies of scale
- Technological externalities
- Tacit knowledge
- Agglomeration effects
- Cumulative causation
- Learning by doing
- Infant industry

Essay questions:

- Comment: "Perfect competition and increasing returns cannot hold together"
- Explain why the competitive equilibrium fails to deliver the first best allocation in the presence of externalities.
- To which extent do external economies help explain why capital doesn't flow from rich countries to poor countries?
- What are the implications of knowledge spillovers being localized or global in scope?
- Explain why with learning by doing, the static gains from trade may not go along with the dynamic gains.

Exercises

6.1. Consider an economy which production function is given by $Y = A_t K_i^{1/3} N_i^{2/3}$. In this economy, 16% of income is saved, the population is constant and capital does not depreciate.

a) Consider for a moment that $A_t = 16e^{(0.04/3)t}$. (a1) Describe the behaviour of per-capita income, Y/N, wages and of the interest rate in the steady state, as well as the capital and labour shares on national income. (a2) Are these results consistent with the empirical evidence?

b) Assume now that $A_t = 0.125(K/N)^{2/3}$. (b1) Explain the theory that fits in this specification. (b2) Does this aggregated production function verify the neoclassical properties? Explain. (b3) Find out the dynamics of per capita income in this model and represent it in a graph. (b4) will this economy display conditional convergence?

6.2. Consider an economy where the production function of the representative firm is given by $Y_t = B_t K_t^{1/2} N_t^{1/2}$. In this economy, 16% of income is saved, the population is constant and capital does not depreciate.

a) Assuming perfect competition, find out the capital share on income, as well as the expression for the interest rate as a function of the average product of capital.

b) Assume first that $B_t = 0.1(K/N)^{0.5}$. b1) Explain the theory underlying this specification. b2) Find out the expression of the economy-wide production function. b3) Find out the dynamics of per capita income in this model and represent it in a graph. b4) How much will be the interest rate in this economy? Is this interest rate socially optimal? Why?

c) Consider now the case in which $B_t = 0.1K^{0.5}$. c1) Explain the theory underlying this specification. c2) Find out the expression of the economy-wide production function. c3) How much will be the interest rate in this economy? c4) Discuss the implications of this specification in terms of per capita income convergence.

6.3. Consider as economy where the production function of the representative firm is given by $Y_t = B_t K_t^{1/3} N_t^{2/3}$. In this economy, savings amount to 16% of income, population is constant and capital does not depreciate.

a) Assuming perfect competition, what will be interest rate as a function of the capital-output ratio?

b) Assume that $B_t = 0.15K^{2/3}$. b1) Explain the theory. b2) Find out the aggregate production function in this economy. b2) Describe, quantifying, the dynamics of per capita income in this economy.

c) Find out the interest rate in this economy. What are the implications of this model in terms of per capita income convergence?

6.4. Consider a World where the production function of each individual firm is given by: $Y_i = AK_i^{0.4}L_i^{0.6}$, where A=0.2 and $L_i = \lambda N_i$ measures labour in efficiency units. In this world, capital depreciates 4% each year and the population in each country is time invariant. In that world, consider a particular country where the saving rate is s=25% and population is N=1.

a) Assume for a moment that $\lambda = (K/N)$. In this case: (i) per capita income will expand at 1% per year; (ii) the interest rate is socially optimal; (iii) there will be conditional convergence (iv) all the above.

b) In the model above, an increase in the efficiency parameter to A=0.25 will deliver: (i) a temporary expansion of per capita output; (ii) the same interest rate in the long run; (iii) the same level of per capita output in the long run; (iv) none of the above.

c) Assume now that $\lambda = K$. In this case: (i) the long run real interest rate will be 4%; (ii) the externality is non-rival; (iii) a larger economy should grow faster than a smaller economy; (iv) all the above.

d) Finally, consider the case in which $\lambda = K^{1/3}$. In this case: (i) there will be sustained growth of per capita income; (ii) the long run real interest rate will be 4%; (iii) capital will flow out to the largest economy; (iv); all the above.