

# **A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation.**

**by**

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*CEPA Working Paper 96/07*

## **ABSTRACT**

This paper describes a computer program which has been written to provide maximum likelihood estimates of the parameters of a number of stochastic production and cost functions. The stochastic frontier models considered can accommodate (unbalanced) panel data and assume firm effects that are distributed as truncated normal random variables. The two primary model specifications considered in the program are an error components specification with time-varying efficiencies permitted (Battese and Coelli, 1992), which was estimated by FRONTIER Version 2.0, and a model specification in which the firm effects are directly influenced by a number of variables (Battese and Coelli, 1995). The computer program also permits the estimation of many other models which have appeared in the literature through the imposition of simple restrictions. Asymptotic estimates of standard errors are calculated along with individual and mean efficiency estimates.

## 1. INTRODUCTION

This paper describes the computer program, FRONTIER Version 4.1, which has been written to provide maximum likelihood estimates of a wide variety of stochastic frontier production and cost functions. The paper is divided into sections. Section 2 describes the stochastic frontier production functions of Battese and Coelli (1992, 1995) and notes the many special cases of these formulations which can be estimated (and tested for) using the program. Section 3 describes the program and Section 4 provides some illustrations of how to use the program. Some final points are made in Section 5. An appendix is added which summarises important aspects of program use and also provides a brief explanation of the purposes of each subroutine and function in the Fortran77 code.

## 2. MODEL SPECIFICATIONS

The stochastic frontier production function was independently proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). The original specification involved a production function specified for cross-sectional data which had an error term which had two components, one to account for random effects and another to account for technical inefficiency. This model can be expressed in the following form:

$$(1) \quad Y_i = x_i\beta + (V_i - U_i) \quad ,i=1,\dots,N,$$

where  $Y_i$  is the production (or the logarithm of the production) of the  $i$ -th firm;

$x_i$  is a  $k \times 1$  vector of (transformations of the) input quantities of the  $i$ -th firm;<sup>1</sup>

$\beta$  is a vector of unknown parameters;

the  $V_i$  are random variables which are assumed to be iid.  $N(0, \sigma_v^2)$ , and

independent of the

$U_i$  which are non-negative random variables which are assumed to account for technical inefficiency in production and are often assumed to be iid.

$|N(0, \sigma_u^2)|$ .

This original specification has been used in a vast number of empirical applications over the past two decades. The specification has also been altered and extended in a number of ways. These extensions include the specification of more general distributional assumptions for the  $U_i$ , such

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<sup>1</sup>For example, if  $Y_i$  is the log of output and  $x_i$  contains the logs of the input quantities, then the Cobb-Douglas production function is obtained.

as the truncated normal or two-parameter gamma distributions; the consideration of panel data and time-varying technical efficiencies; the extension of the methodology to cost functions and also to the estimation of systems of equations; and so on. A number of comprehensive reviews of this literature are available, such as Forsund, Lovell and Schmidt (1980), Schmidt (1986), Bauer (1990) and Greene (1993).

The computer program, FRONTIER Version 4.1, can be used to obtain maximum likelihood estimates of a subset of the stochastic frontier production and cost functions which have been proposed in the literature. The program can accommodate panel data; time-varying and invariant efficiencies; cost and production functions; half-normal and truncated normal distributions; and functional forms which have a dependent variable in logged or original units. The program cannot accommodate exponential or gamma distributions, nor can it estimate systems of equations. These lists of what the program can and cannot do are not exhaustive, but do provide an indication of the program's capabilities.

FRONTIER Version 4.1 was written to estimate the model specifications detailed in Battese and Coelli (1988, 1992 and 1995) and Battese, Coelli and Colby (1989). Since the specifications in Battese and Coelli (1988) and Battese, Coelli and Colby (1989) are special cases of the Battese and Coelli (1992) specification, we shall discuss the model specifications in the two most recent papers in detail, and then note the way in which these models encompass many other specifications that have appeared in the literature.

## 2.1 Model 1: The Battese and Coelli (1992) Specification

Battese and Coelli (1992) propose a stochastic frontier production function for (unbalanced) panel data which has firm effects which are assumed to be distributed as truncated normal random variables, which are also permitted to vary systematically with time. The model may be expressed as:

$$(2) \quad Y_{it} = x_{it}\beta + (V_{it} - U_{it}) \quad ,i=1,\dots,N, t=1,\dots,T,$$

where  $Y_{it}$  is (the logarithm of) the production of the  $i$ -th firm in the  $t$ -th time period;

$x_{it}$  is a  $k \times 1$  vector of (transformations of the) input quantities of the  $i$ -th firm in the  $t$ -th time period;

$\beta$  is as defined earlier;

the  $V_{it}$  are random variables which are assumed to be iid  $N(0, \sigma_v^2)$ , and independent of the

$U_{it} = (U_i \exp(-\eta(t-T)))$ , where

the  $U_i$  are non-negative random variables which are assumed to account for technical inefficiency in production and are assumed to be iid as truncations at zero of the  $N(\mu, \sigma_U^2)$  distribution;

$\eta$  is a parameter to be estimated;

and the panel of data need not be complete (i.e. unbalanced panel data).

We utilise the parameterization of Battese and Corra (1977) who replace  $\sigma_v^2$  and  $\sigma_U^2$  with  $\sigma^2 = \sigma_v^2 + \sigma_U^2$  and  $\gamma = \sigma_U^2 / (\sigma_v^2 + \sigma_U^2)$ . This is done with the calculation of the maximum likelihood estimates in mind. The parameter,  $\gamma$ , must lie between 0 and 1 and thus this range can be searched to provide a good starting value for use in an iterative maximization process such as the Davidon-Fletcher-Powell (DFP) algorithm. The log-likelihood function of this model is presented in the appendix in Battese and Coelli (1992).

The imposition of one or more restrictions upon this model formulation can provide a number of the special cases of this particular model which have appeared in the literature. Setting  $\eta$  to be zero provides the time-invariant model set out in Battese, Coelli and Colby (1989). Furthermore, restricting the formulation to a full (balanced) panel of data gives the production function assumed in Battese and Coelli (1988). The additional restriction of  $\mu$  equal to zero reduces the model to model One in Pitt and Lee (1981). One may add a fourth restriction of  $T=1$  to return to the original cross-sectional, half-normal formulation of Aigner, Lovell and Schmidt (1977). Obviously a large number of permutations exist. For example, if all these restrictions excepting  $\mu=0$  are imposed, the model suggested by Stevenson (1980) results. Furthermore, if the cost function option is selected, we can estimate the model specification in Hughes (1988) and the Schmidt and Lovell (1979) specification, which assumed allocative efficiency. These latter two specifications are the cost function analogues of the production functions in Battese and Coelli (1988) and Aigner, Lovell and Schmidt (1977), respectively.

There are obviously a large number of model choices that could be considered for any particular application. For example, does one assume a half-normal distribution for the inefficiency effects or the more general truncated normal distribution? If panel data is available,

should one assume time-invariant or time-varying efficiencies? When such decisions must be made, it is recommended that a number of the alternative models be estimated and that a preferred model be selected using likelihood ratio tests.

One can also test whether any form of stochastic frontier production function is required at all by testing the significance of the  $\gamma$  parameter.<sup>2</sup> If the null hypothesis, that  $\gamma$  equals zero, is accepted, this would indicate that  $\sigma_U^2$  is zero and hence that the  $U_{it}$  term should be removed from the model, leaving a specification with parameters that can be consistently estimated using ordinary least squares.

## 2.2 Model 2: The Battese and Coelli (1995) Specification

A number of empirical studies (e.g. Pitt and Lee, 1981) have estimated stochastic frontiers and predicted firm-level efficiencies using these estimated functions, and then regressed the predicted efficiencies upon firm-specific variables (such as managerial experience, ownership characteristics, etc) in an attempt to identify some of the reasons for differences in predicted efficiencies between firms in an industry. This has long been recognised as a useful exercise, but the two-stage estimation procedure has also been long recognised as one which is inconsistent in its assumptions regarding the independence of the inefficiency effects in the two estimation stages. The two-stage estimation procedure is unlikely to provide estimates which are as efficient as those that could be obtained using a single-stage estimation procedure.

This issue was addressed by Kumbhakar, Ghosh and McGukin (1991) and Reifschneider and Stevenson (1991) who propose stochastic frontier models in which the inefficiency effects ( $U_i$ ) are expressed as an explicit function of a vector of firm-specific variables and a random error. Battese and Coelli (1995) propose a model which is equivalent to the Kumbhakar, Ghosh and McGukin (1991) specification, with the exceptions that allocative efficiency is imposed, the first-order profit maximising conditions removed, and panel data is permitted. The Battese and Coelli (1995) model specification may be expressed as:

$$(3) \quad Y_{it} = x_{it}\beta + (V_{it} - U_{it}) \quad ,i=1,\dots,N, t=1,\dots,T,$$

where  $Y_{it}$ ,  $x_{it}$ , and  $\beta$  are as defined earlier;

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<sup>2</sup>It should be noted that any likelihood ratio test statistic involving a null hypothesis which includes the restriction that  $\gamma$  is zero does not have a chi-square distribution because the restriction defines a point on the boundary of the parameter space. In this case the likelihood ratio statistic has been shown to have a mixed chi-square distribution. For more on this point see Lee (1993) and Coelli (1993, 1994).

the  $V_{it}$  are random variables which are assumed to be iid.  $N(0, \sigma_v^2)$ , and independent of the

$U_{it}$  which are non-negative random variables which are assumed to account for technical inefficiency in production and are assumed to be independently distributed as truncations at zero of the  $N(m_{it}, \sigma_U^2)$  distribution; where:

$$(4) \quad m_{it} = z_{it}\delta,$$

where  $z_{it}$  is a  $p \times 1$  vector of variables which may influence the efficiency of a firm; and  $\delta$  is an  $1 \times p$  vector of parameters to be estimated.

We once again use the parameterisation from Battese and Corra (1977), replacing  $\sigma_v^2$  and  $\sigma_U^2$  with  $\sigma^2 = \sigma_v^2 + \sigma_U^2$  and  $\gamma = \sigma_U^2 / (\sigma_v^2 + \sigma_U^2)$ . The log-likelihood function of this model is presented in the appendix in the working paper Battese and Coelli (1993).

This model specification also encompasses a number of other model specifications as special cases. If we set  $T=1$  and  $z_{it}$  contains the value one and no other variables (i.e. only a constant term), then the model reduces to the truncated normal specification in Stevenson (1980), where  $\delta_0$  (the only element in  $\delta$ ) will have the same interpretation as the  $\mu$  parameter in Stevenson (1980). It should be noted, however, that the model defined by (3) and (4) does not have the model defined by (2) as a special case, and neither does the converse apply. Thus these two model specifications are non-nested and hence no set of restrictions can be defined to permit a test of one specification versus the other.

### 2.3 Cost Functions<sup>3</sup>

All of the above specifications have been expressed in terms of a production function, with the  $U_i$  interpreted as technical inefficiency effects, which cause the firm to operate below the stochastic production frontier. If we wish to specify a stochastic frontier cost function, we simply alter the error term specification from  $(V_i - U_i)$  to  $(V_i + U_i)$ . For example, this substitution would transform the production function defined by (1) into the cost function:

$$(5) \quad Y_i = x_i\beta + (V_i + U_i) \quad , i=1, \dots, N,$$

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<sup>3</sup> The discussion here will be in terms of the cross-sectional model. The extension to the panel data cases are straightforward.

where  $Y_i$  is the (logarithm of the) cost of production of the  $i$ -th firm;

$x_i$  is a  $k \times 1$  vector of (transformations of the) input prices and output of the  $i$ -th firm;

$\beta$  is an vector of unknown parameters;

the  $V_i$  are random variables which are assumed to be iid  $N(0, \sigma_v^2)$ , and independent of the

$U_i$  which are non-negative random variables which are assumed to account for the cost of inefficiency in production, which are often assumed to be iid

$|N(0, \sigma_u^2)|$ .

In this cost function the  $U_i$  now defines how far the firm operates above the cost frontier. If allocative efficiency is assumed, the  $U_i$  is closely related to the cost of technical inefficiency. If this assumption is not made, the interpretation of the  $U_i$  in a cost function is less clear, with both technical and allocative inefficiencies possibly involved. Thus we shall refer to efficiencies measured relative to a cost frontier as “cost” efficiencies in the remainder of this document. The exact interpretation of these cost efficiencies will depend upon the particular application.

The cost frontier (5) is identical one proposed in Schmidt and Lovell (1979). The log-likelihood function of this model is presented in the appendix of that paper (using a slightly different parameterisation to that used here). Schmidt and Lovell note that the log-likelihood of the cost frontier is the same as that of the production frontier except for a few sign changes. The log-likelihood functions for the cost function analogues of the Battese and Coelli (1992, 1995) models were also found to be obtained by making a few simple sign changes, and hence have not reproduced here.

## 2.4 Efficiency Predictions<sup>4</sup>

The computer program calculates predictions of individual firm technical efficiencies from estimated stochastic production frontiers, and predictions of individual firm cost efficiencies from estimated stochastic cost frontiers. The measures of technical efficiency relative to the production frontier (1) and of cost efficiency relative to the cost frontier (5) are both defined as:

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<sup>4</sup>The discussion here will again be in terms of the cross-sectional models. The extension to the panel data cases are straightforward.

$$(6) \quad \text{EFF}_i = E(Y_i^* | U_i, X_i) / E(Y_i^* | U_i=0, X_i),$$

where  $Y_i^*$  is the production (or cost) of the  $i$ -th firm, which will be equal to  $Y_i$  when the dependent variable is in original units and will be equal to  $\exp(Y_i)$  when the dependent variable is in logs. In the case of a production frontier,  $\text{EFF}_i$  will take a value between zero and one, while it will take a value between one and infinity in the cost function case. The efficiency measures can be shown to be defined as:

| <b>Cost or<br/>Production</b> | <b>Logged Dependent<br/>Variable.</b> | <b>Efficiency (EFF<sub>i</sub>)</b> |
|-------------------------------|---------------------------------------|-------------------------------------|
| production                    | yes                                   | $\exp(-U_i)$                        |
| cost                          | yes                                   | $\exp(U_i)$                         |
| production                    | no                                    | $(x_i\beta - U_i)/(x_i\beta)$       |
| cost                          | no                                    | $(x_i\beta + U_i)/(x_i\beta)$       |

The above four expressions for  $\text{EFF}_i$  all rely upon the value of the unobservable  $U_i$  being predicted. This is achieved by deriving expressions for the conditional expectation of these functions of the  $U_i$ , conditional upon the observed value of  $(V_i - U_i)$ . The resulting expressions are generalizations of the results in Jondrow et al (1982) and Battese and Coelli (1988). The relevant expressions for the production function cases are provided in Battese and Coelli (1992) and in Battese and Coelli (1993, 1995), and the expressions for the cost efficiencies relative to a cost frontier, have been obtained by minor alterations of the technical efficiency expressions in these papers.

### 3. THE FRONTIER PROGRAM

FRONTIER Version 4.1 differs in a number of ways from FRONTIER Version 2.0 (Coelli, 1992), which was the last fully documented version. People familiar with previous versions of FRONTIER should assume that nothing remains the same, and carefully read this document before using Version 4.1. You will, however, find that a number of things are the same, but that many minor, and some not so minor things, have changed. For example, Version 4.1 assumes a linear functional form. Thus if you wish to estimate a Cobb-Douglas production function, you must log all of your input and output data before creating the data file for the



program to use. Version 2.0 users will recall that the Cobb-Douglas was assumed in that version, and that data had to be supplied in original units, since the program obtained the logs of the data supplied to it. A listing of the major differences between Versions 2.0 and 4.1 is provided at the end of this section.

### 3.1 Files Needed

The execution of FRONTIER Version 4.1 on an IBM PC generally involves five files:

- 1) The executable file FRONT41.EXE
- 2) The start-up file FRONT41.000
- 3) A data file (for example, called TEST.DTA)
- 4) An instruction file (for example, called TEST.INS)
- 5) An output file (for example, called TEST.OUT).

The start-up file, FRONT41.000, contains values for a number of key variables such as the convergence criterion, printing flags and so on. This text file may be edited if the user wishes to alter any values. This file is discussed further in Appendix A. The data and instruction files must be created by the user prior to execution. The output file is created by FRONTIER during execution.<sup>5</sup> Examples of a data, instruction and output files are listed in Section 4.

The program requires that the data be listed in a text file and is quite particular about the format. The data must be listed by observation. There must be  $3+k[+p]$  columns presented in the following order:

- |           |  |
|-----------|--|
| 1)        | Firm number (an integer in the range 1 to N)   |
| 2)        | Period number (an integer in the range 1 to T) |
| 3)        | $Y_{it}$                                       |
| 4)        | $x_{1it}$                                      |
|           | :  |
| $3+k)$    | $x_{kit}$                                      |
| $[3+k+1)$ | $z_{1it}$                                      |
|           | :  |
| $3+k+p)$  | $z_{pit}$ ].                                   |

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<sup>5</sup>Note that a model can be estimated without an instruction file if the program is used interactively.

The z entries are listed in square brackets to indicate that they are not always needed. They are only used when Model 2 is being estimated. The observations can be listed in any order but the columns must be in the stated order. There must be at least one observation on each of the N firms and there must be at least one observation in time period 1 and in time period T. If you are using a single cross-section of data, then column 2 (the time period column) should contain the value "1" throughout. Note that the data must be suitably transformed if a functional form other than a linear function is required. The Cobb-Douglas and Translog functional forms are the most often used functional forms in stochastic frontier analyses. Examples involving these two forms will be provided in Section 4.

The program can receive instructions either from a file or from a terminal. After typing "FRONT41" to begin execution, the user is asked whether instructions will come from a file or the terminal. The structure of the instruction file is listed in the next section. If the interactive (terminal) option is selected, questions will be asked in the same order as they appear in the instruction file.

## 3.2 The Three-Step Estimation Method

The program will follow a three-step procedure in estimating the maximum likelihood estimates of the parameters of a stochastic frontier production function.<sup>6</sup> The three steps are:

- 1) Ordinary Least Squares (OLS) estimates of the function are obtained. All  $\beta$  estimators with the exception of the intercept will be unbiased.
- 2) A two-phase grid search of  $\gamma$  is conducted, with the  $\beta$  parameters (excepting  $\beta_0$ ) set to the OLS values and the  $\beta_0$  and  $\sigma^2$  parameters adjusted according to the corrected ordinary least squares formula presented in Coelli (1995). Any other parameters ( $\mu$ ,  $\eta$  or  $\delta$ 's) are set to zero in this grid search.
- 3) The values selected in the grid search are used as starting values in an iterative procedure (using the Davidon-Fletcher-Powell Quasi-Newton method) to obtain the final maximum likelihood estimates.

### 3.2.1 Grid Search

As mentioned earlier, a grid search is conducted across the parameter space of  $\gamma$ . Values of  $\gamma$  are considered from 0.1 to 0.9 in increments of size 0.1. The size of this increment can be altered by changing the value of the GRIDNO variable which is set to the value of 0.1 in the start-up file FRONT41.000.

Furthermore, if the variable, IGRID2, in FRONT41.000, is set to 1 (instead of 0) then a second phase grid search will be conducted around the values obtained in the first phase. The width of this grid search is GRIDNO/2 either side of the phase one estimates in steps of GRIDNO/10. Thus a starting value for  $\gamma$  will be obtained to an accuracy of two decimal places instead of the one decimal place obtained in the single phase grid search (when a value of GRIDNO=0.1 is assumed).

### 3.2.2 Iterative Maximization Procedure

The first-order partial derivatives of the log-likelihood functions of Models 1 and 2 are lengthy expressions. These are derived in appendices in Battese and Coelli (1992) and Battese and Coelli (1993), respectively. Many of the gradient methods used to obtain maximum

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<sup>6</sup>If starting values are specified in the instruction file, the program will skip the first two steps of the procedure.

likelihood estimates, such as the Newton-Raphson method, require the matrix of second partial derivatives to be calculated. It was decided that this task was probably best avoided, hence we turned our attention to Quasi-Newton methods which only require the vector of first partial derivatives be derived. The Davidon-Fletcher-Powell Quasi-Newton method was selected as it appears to have been used successfully in a wide range of econometric applications and was also recommended by Pitt and Lee (1981) for stochastic frontier production function estimation. For a general discussion of the relative merits of a number of Newton and Quasi-Newton methods see Himmelblau (1972), which also provides a description of the mechanics (along with Fortran code) of a number of the more popular methods. The general structure of the subroutines, MINI, SEARCH, ETA and CONVRG, used in FRONTIER are taken from the appendix in Himmelblau (1972).

The iterative procedure takes the parameter values supplied by the grid search as starting values (unless starting values are supplied by the user). The program then updates the vector of parameter estimates by the Davidon-Fletcher-Powell method until either of the following occurs:

a) The convergence criterion is satisfied. The convergence criterion is set in the start-up file FRONT41.000 by the parameter TOL. Presently it is set such that, if the proportional change in the likelihood function and each of the parameters is less than 0.00001, then the iterative procedure terminates.

b) The maximum number of iterations permitted is completed. This is presently set in FRONT41.000 to 100.

Both of these parameters may be altered by the user.

### **3.3 Program Output**

The ordinary least-squares estimates, the estimates after the grid search and the final maximum likelihood estimates are all presented in the output file. Approximate standard errors are taken from the direction matrix used in the final iteration of the Davidon-Fletcher-Powell procedure. This estimate of the covariance matrix is also listed in the output.

Estimates of individual technical or cost efficiencies are calculated using the expressions presented in Battese and Coelli (1991, 1995). When any estimates of mean efficiencies are reported, these are simply the arithmetic averages of the individual efficiencies. The ITE

variable in FRONT41.000 can be used to suppress the listing of individual efficiencies in the output file, by changing it's value from 1 to 0.

### 3.4 Differences Between Versions 2.0 and 4.1

The main differences are as follows:

- 1) The Battese and Coelli (1995) model (Model 2) can now be estimated.
- 2) The old size limits on N, T and K have been removed. The size limits of 100, 20 and 20, respectively, were found by many users to be too restrictive. The removal of the size limits have been achieved by compiling the program using a Lahey F77L-EM/32 compiler with a DOS extender. The size of model that can now be estimated by the program is only limited by the amount of the available RAM available on your PC. This action does come at some cost though, since the program had to be re-written using *dynamically allocatable arrays*, which are not standard Fortran constructs. Thus the code cannot now be transferred to another computing platform (such as a mainframe computer) without substantial modification.
- 3) Cost functions can now be estimated.
- 4) Efficiency estimates can now be calculated when the dependent variable is expressed in original units. The previous version of the program assumed the dependent variable was in logs, and calculated efficiencies accordingly. The user can now indicate whether the dependent variable is logged or not, and the program will then calculate the appropriate efficiency estimates.
- 5) Version 2.0 was written to estimate a Cobb-Douglas function. Data was supplied in original units and the program calculated the logs before estimation. Version 4.1 assumes that all necessary transformations have already been done to the data before it receives it. The program estimates a linear function using the data supplied to it. Examples of how to estimate Cobb-Douglas and Translog functional forms are provided in Section 4.
- 6) Bounds have now been placed upon the range of values that  $\mu$  can take in Model 1. It is now restricted to the range between  $\pm 2\sigma_U$ . This has been done because a number of users (including the author) found that in some applications a large (insignificant) negative value of  $\mu$  was obtained. This value was large in the sense that it was many standard deviations from zero (e.g. four or more). The numerical accuracy of calculations of areas in the tail of the standard normal

distribution which are this far from zero must be questioned.<sup>7</sup> It was thus decided that the above bounds be imposed. This was not viewed as being too restrictive, given the range of truncated normal distribution shapes which are still permitted. This is evident in Figure 1 which plots truncated normal density functions for values of  $\mu$  of -2, -1, 0, 1 and 2

7) Information from each iteration is now sent to the output file (instead of to the screen). The user can also now specify how often (if at all) this information is reported, using the IPRINT variable in FRONT41.000.

8) The grid search has now been reduced to only consider  $\gamma$  and now uses the corrected ordinary least squares expressions derived in Coelli (1995) to adjust  $\sigma^2$  and  $\beta_0$  during this process.

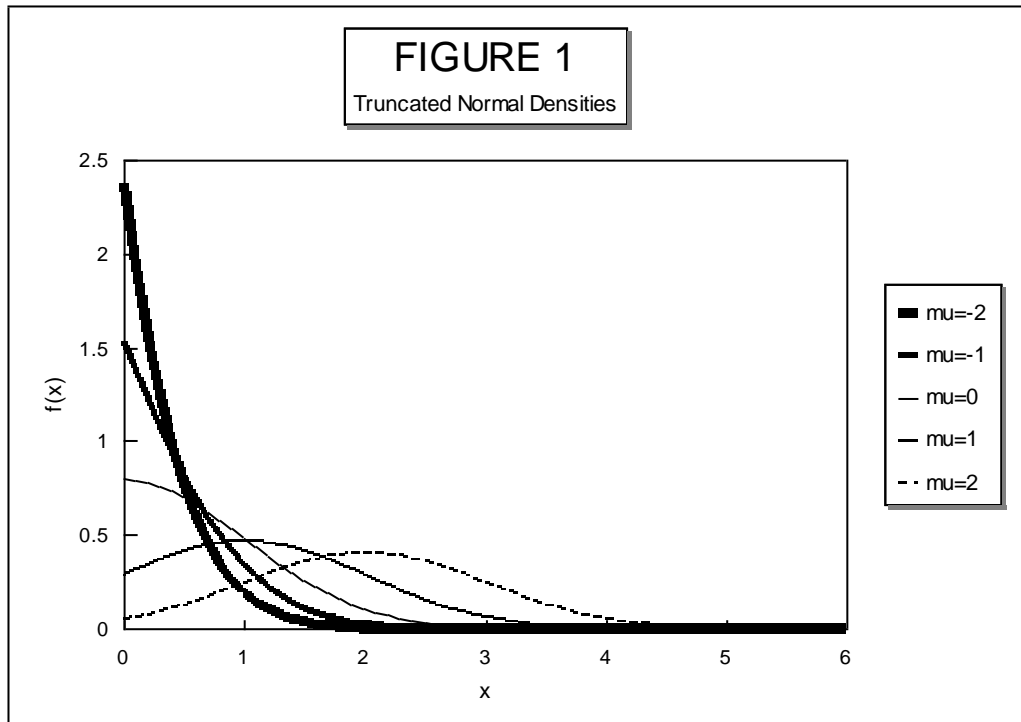
9) A small error was detected in the first partial derivative with respect to  $\eta$  in Version 2.0 of the program. This error would have only affected results when  $\eta$  was assumed to be non-zero. The error has been corrected in Version 4.1, and the change does not appear to have a large influence upon estimates.

10) As a result of the use of the new compiler (detailed under point 2), the following minimum machine configuration is needed: an IBM compatible 386 (or higher) PC with a math co-processor. The program will run when there is only 4 mb RAM but in some cases will require 8 mb RAM.

11) There have also been a large number of small alterations made to the program, many of which were suggested by users of Version 2.0. For example, the names of the data and instruction files are now listed in the output file.

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<sup>7</sup>A monte carlo experiment was conducted in which  $\mu$  was set to zero when generating samples, but was unrestricted in estimation. Large negative (insignificant) values of  $\mu$  were obtained in roughly 10% of samples. A 3D plot of the log-likelihood function in one of these samples indicated a long flat ridge in the log-likelihood when plotted against  $\mu$  and  $\sigma^2$ . This phenomenon is being further investigated at present.



#### 4. A FEW SHORT EXAMPLES

The best way to describe how to use the program is to provide some examples. In this section we shall consider the estimation of:

- 1) A Cobb-Douglas production frontier using cross-sectional data and assuming a half-normal distribution.
- 2) A Translog production frontier using cross-sectional data and assuming a truncated normal distribution.
- 3) A Cobb-Douglas cost frontier using cross-sectional data and assuming a half-normal distribution.
- 4) The Battese and Coelli (1992) specification (Model 1).
- 5) The Battese and Coelli (1995) specification (Model 2).

To keep the examples brief, we shall assume two production inputs in all cases. In the cross-sectional examples we shall have 60 firms, while in the panel data examples 15 firms and 4 time periods will be used.

##### 4.1 A Cobb-Douglas production frontier using cross-sectional data and assuming a half-normal distribution.

In this first example we wish to estimate the Cobb-Douglas production frontier:

$$(7) \quad \ln(Q_i) = \beta_0 + \beta_1 \ln(K_i) + \beta_2 \ln(L_i) + (V_i - U_i),$$

where  $Q_i$ ,  $K_i$  and  $L_i$  are output, capital and labour, respectively, and  $V_i$  and  $U_i$  are assumed normal and half-normal distributed, respectively. The text file<sup>8</sup> EG1.DAT contains 60 observations on firm-id, time-period, Q, K and L, in that order (refer to Table 1a). Note that the time-period column contains only 1's because this is cross-sectional data. To estimate (7) we first must construct a data file which contains the logs of the the inputs and output. This can be done using any number of computer packages. The SHAZAM program (see White, 1993) has been used in this document. The SHAZAM instruction file EG1.SHA (refer Table 1b) reads in data from EG1.DAT, obtains the logs of the relevant variables and writes these out to the file EG1.DTA<sup>9</sup> (refer Table 1c). This file has a similar format to the original data file, except that the inputs and output have been logged.

We then create an instruction file for the FRONTIER program (named EG1.INS) by first making a copy of the BLANK.INS file which is supplied with the program. We then edit this file (using a text editor such as DOS EDIT) and type in the relevant information. The EG1.INS file is listed in Table 1d. The purpose of the majority of entries in the file should be self explanatory, due to the comments on the right-hand side of the file.<sup>10</sup> The first line allows you to indicate whether Model 1 or 2 is required. Because of the simple form of the model this first example (and the next two examples) it does not matter whether "1" or "2" is entered. On the next two lines of the file the name of the data file (EG1.DTA) and an output file name (here we have used EG1.OUT) are specified. On line 4 a "1" is entered to indicate we are estimating a production function, and on line 5 a "y" is entered to indicate that the dependent variable has been logged (this is used by the program to select the correct formula for efficiency estimates). Then on the next four lines we specify the number of firms (60); time periods (1); total number of observations (60) and number of regressors (2). On the next three lines we have answered no

---

<sup>8</sup> All data, instruction and output files are (ASCII) text files.

<sup>9</sup>Note the DOS restriction that a file name cannot contain any more than 12 characters - 8 before the period and 3 following it.

<sup>10</sup>It should be mentioned that the comments in BLANK.INS and FRONT41.000 are not read by FRONTIER. Hence users may have instruction files which are made from scratch with a text editor and which contain no comments. This is not recommended, however, as it would be too easy to lose track of which input value belongs on which line.



(n) to each question. We have said no to  $\mu$  because we are assuming the half normal distribution.<sup>11</sup> We have answered no to  $\eta$  because we have only one cross-section of data and hence cannot consider time-varying efficiencies.<sup>12</sup> Lastly, we have answered no to specifying starting values because we wish them to be selected using a grid search.<sup>13</sup>

Finally we type FRONT41 at the DOS prompt, select the instruction file option (f)<sup>14</sup> and then type in the name of the instruction file (EG1.INS). The program will then take somewhere between a few seconds and a few minutes to estimate the frontier model and send the output to the file you have named (EG1.OUT). This file is reproduced in Table 1e.

**Table 1a - Listing of Data File EG1.DAT**

---

|     |    |        |       |        |
|-----|----|--------|-------|--------|
| 1.  | 1. | 12.778 | 9.416 | 35.134 |
| 2.  | 1. | 24.285 | 4.643 | 77.297 |
| 3.  | 1. | 20.855 | 5.095 | 89.799 |
|     | .  |        |       |        |
|     | .  |        |       |        |
| 58. | 1. | 21.358 | 9.329 | 87.124 |
| 59. | 1. | 27.124 | 7.834 | 60.340 |
| 60. | 1. | 14.105 | 5.621 | 44.218 |

---

**Table 1b - Listing of Shazam Instruction File EG1.SHA**

---

```
read(eg1.dat) n t y x1 x2
genr ly=log(y)
genr lx1=log(x1)
genr lx2=log(x2)
file 33 eg1.dta
write(33) n t ly lx1 lx2
stop
```

---

<sup>11</sup> We would answer yes if we wished to assume the more general truncated normal distribution in which  $\mu$  can be non-zero.

<sup>12</sup> Note that if we had selected Model 2 on the first line of the instruction file, we would need to answer the questions in the square brackets on lines 10 and 11 of the instruction file instead. For the simple model in this example we would answer “n” and “0”, respectively.

<sup>13</sup> If we had answered yes to starting values, we would then need to type starting values for each of the parameters, typing one on each line, in the order specified.

<sup>14</sup> If you do not wish to create an instruction file, these same instructions can be sent to FRONTIER by selecting the terminal (t) option and answering a series of questions.

**Table 1c - Listing of Data File EG1.DTA**


---

|          |          |          |          |          |
|----------|----------|----------|----------|----------|
| 1.000000 | 1.000000 | 2.547725 | 2.242410 | 3.559169 |
| 2.000000 | 1.000000 | 3.189859 | 1.535361 | 4.347655 |
| 3.000000 | 1.000000 | 3.037594 | 1.628260 | 4.497574 |
| .        |          |          |          |          |
| .        |          |          |          |          |
| 58.00000 | 1.000000 | 3.061426 | 2.233128 | 4.467332 |
| 59.00000 | 1.000000 | 3.300419 | 2.058473 | 4.099995 |
| 60.00000 | 1.000000 | 2.646529 | 1.726510 | 3.789132 |

---

**Table 1d - Listing of Instruction File EG1.INS**


---

|         |  |
|---------|--|
| 1       | 1=ERROR COMPONENTS MODEL, 2=TE EFFECTS MODEL         |
| eg1.dta | DATA FILE NAME                                       |
| eg1.out | OUTPUT FILE NAME                                     |
| 1       | 1=PRODUCTION FUNCTION, 2=COST FUNCTION               |
| y       | LOGGED DEPENDENT VARIABLE (Y/N)                      |
| 60      | NUMBER OF CROSS-SECTIONS                             |
| 1       | NUMBER OF TIME PERIODS                               |
| 60      | NUMBER OF OBSERVATIONS IN TOTAL                      |
| 2       | NUMBER OF REGRESSOR VARIABLES (Xs)                   |
| n       | MU (Y/N) [OR DELTA0 (Y/N) IF USING TE EFFECTS MODEL] |
| n       | ETA (Y/N) [OR NUMBER OF TE EFFECTS REGRESSORS (Zs)]  |
| n       | STARTING VALUES (Y/N)                                |
|         | IF YES THEN BETA0                                    |
|         | BETA1 TO   |
|         | BETAK  |
|         | SIGMA SQUARED  |
|         | GAMMA  |
|         | MU [OR DELTA0  |
|         | ETA DELTA1 TO  |
|         | DELTAK]  |
|         |  |
|         | NOTE: IF YOU ARE SUPPLYING STARTING VALUES           |
|         | AND YOU HAVE RESTRICTED MU [OR DELTA0] TO BE         |
|         | ZERO THEN YOU SHOULD NOT SUPPLY A STARTING           |
|         | VALUE FOR THIS PARAMETER.                            |

---

**Table 1e - Listing of Output File EG1.OUT**


---

Output from the program FRONTIER (Version 4.1)

instruction file = eg1.ins  
data file = eg1.dta

Error Components Frontier (see B&C 1992)  
The model is a production function  
The dependent variable is logged

the ols estimates are :

|        | coefficient    | standard-error | t-ratio        |
|--------|----------------|----------------|----------------|
| beta 0 | 0.24489834E+00 | 0.21360307E+00 | 0.11465114E+01 |
| beta 1 | 0.28049246E+00 | 0.48066617E-01 | 0.58354940E+01 |

beta 2 0.53330637E+00 0.51498586E-01 0.10355748E+02  
sigma-squared 0.11398496E+00

log likelihood function = -0.18446849E+02

the estimates after the grid search were :

beta 0 0.58014216E+00  
beta 1 0.28049246E+00  
beta 2 0.53330637E+00  
sigma-squared 0.22067413E+00  
gamma 0.80000000E+00  
mu is restricted to be zero  
eta is restricted to be zero

iteration = 0 func evals = 19 llf = -0.17034854E+02  
0.58014216E+00 0.28049246E+00 0.53330637E+00 0.22067413E+00 0.80000000E+00  
gradient step  
iteration = 5 func evals = 41 llf = -0.17027230E+02  
0.56160697E+00 0.28108701E+00 0.53647803E+00 0.21694170E+00 0.79718731E+00  
iteration = 7 func evals = 63 llf = -0.17027229E+02  
0.56161963E+00 0.28110205E+00 0.53647981E+00 0.21700046E+00 0.79720730E+00

the final mle estimates are :

|                              | coefficient    | standard-error | t-ratio        |
|------------------------------|----------------|----------------|----------------|
| beta 0                       | 0.56161963E+00 | 0.20261668E+00 | 0.27718331E+01 |
| beta 1                       | 0.28110205E+00 | 0.47643365E-01 | 0.59001301E+01 |
| beta 2                       | 0.53647981E+00 | 0.45251553E-01 | 0.11855501E+02 |
| sigma-squared                | 0.21700046E+00 | 0.63909106E-01 | 0.33954545E+01 |
| gamma                        | 0.79720730E+00 | 0.13642399E+00 | 0.58436004E+01 |
| mu is restricted to be zero  |                |                |                |
| eta is restricted to be zero |                |                |                |

log likelihood function = -0.17027229E+02

LR test of the one-sided error = 0.28392402E+01  
with number of restrictions = 1  
[note that this statistic has a mixed chi-square distribution]

number of iterations = 7

(maximum number of iterations set at : 100)

number of cross-sections = 60

number of time periods = 1

total number of observations = 60

thus there are: 0 obsns not in the panel

covariance matrix :

|                 |                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.41053521E-01  | -0.31446721E-02 | -0.80030279E-02 | 0.40456494E-02  | 0.92519362E-02  |
| -0.31446721E-02 | 0.22698902E-02  | 0.40106205E-04  | -0.29528845E-04 | -0.91550467E-04 |
| -0.80030279E-02 | 0.40106205E-04  | 0.20477030E-02  | -0.47190308E-04 | -0.16404645E-03 |
| 0.40456494E-02  | -0.29528845E-04 | -0.47190308E-04 | 0.40843738E-02  | 0.67450773E-02  |
| 0.92519362E-02  | -0.91550467E-04 | -0.16404645E-03 | 0.67450773E-02  | 0.18611506E-01  |

technical efficiency estimates :

| firm | eff.-est.      |
|------|----------------|
| 1    | 0.65068880E+00 |

|    |                |
|----|----------------|
| 2  | 0.82889151E+00 |
| 3  | 0.72642592E+00 |
| .  | .              |
| .  | .              |
| 58 | 0.66471456E+00 |
| 59 | 0.85670448E+00 |
| 60 | 0.70842786E+00 |

mean efficiency = 0.74056772E+00

---

## 4.2 A Translog production frontier using cross-sectional data and assuming a truncated normal distribution.

In this example we wish to estimate the Translog production frontier:

$$(8) \quad \ln(Q_i) = \beta_0 + \beta_1 \ln(K_i) + \beta_2 \ln(L_i) + \beta_3 \ln(K_i)^2 + \beta_4 \ln(L_i)^2 + \beta_5 \ln(K_i) \ln(L_i) \\ + (V_i - U_i),$$

where  $Q_i$ ,  $K_i$ ,  $L_i$  and  $V_i$  are as defined earlier, and  $U_i$  has truncated normal distribution. We follow a similar presentation to that in Section 4.2, but only list 4 tables: 2a to 2d. We suppress the listing of the output file to conserve space. The main differences to note between the procedure in Section 4.1 and here are that: the squared and interaction terms have to be generated in the SHAZAM instruction file (refer to Table 2b); because of this the file EG2.DTA contains three more columns<sup>15</sup> than EG1.DTA; and in EG2.INS we have made the number of regressors equal to 5 and answered yes (y) to the  $\mu$  question (because we wish  $U_i$  to be truncated normal).

**Table 2a - Listing of Data File EG2.DAT**

---

|     |    |        |       |        |
|-----|----|--------|-------|--------|
| 1.  | 1. | 12.778 | 9.416 | 35.134 |
| 2.  | 1. | 24.285 | 4.643 | 77.297 |
| 3.  | 1. | 20.855 | 5.095 | 89.799 |
| .   | .  | .      | .     | .      |
| .   | .  | .      | .     | .      |
| 58. | 1. | 21.358 | 9.329 | 87.124 |
| 59. | 1. | 27.124 | 7.834 | 60.340 |
| 60. | 1. | 14.105 | 5.621 | 44.218 |

---

**Table 2b - Listing of Shazam Instruction File EG2.SHA**

---

```
read(eg2.dat) n t y x1 x2
genr ly=log(y)
genr lx1=log(x1)
genr lx2=log(x2)
genr lx1s=log(x1)*log(x1)
genr lx2s=log(x2)*log(x2)
genr lx12=log(x1)*log(x2)
file 33 eg2.dta
write(33) n t ly lx1 lx2 lx1s lx2s lx12
stop
```

---

<sup>15</sup> Note that the SHAZAM WRITE command will only list five numbers on each line. If you have more than five columns, the extra numbers will appear on a new line. FRONTIER has no problems reading this form of data file.

**Table 2c - Listing of Data File EG2.DTA**


---

|          |          |          |          |          |
|----------|----------|----------|----------|----------|
| 1.000000 | 1.000000 | 2.547725 | 2.242410 | 3.559169 |
| 5.028404 | 12.66769 | 7.981118 |          |          |
| 2.000000 | 1.000000 | 3.189859 | 1.535361 | 4.347655 |
| 2.357333 | 18.90211 | 6.675219 |          |          |
| 3.000000 | 1.000000 | 3.037594 | 1.628260 | 4.497574 |
| 2.651230 | 20.22817 | 7.323218 |          |          |
| .        |          |          |          |          |
| .        |          |          |          |          |
| 58.00000 | 1.000000 | 3.061426 | 2.233128 | 4.467332 |
| 4.986860 | 19.95706 | 9.976124 |          |          |
| 59.00000 | 1.000000 | 3.300419 | 2.058473 | 4.099995 |
| 4.237312 | 16.80996 | 8.439730 |          |          |
| 60.00000 | 1.000000 | 2.646529 | 1.726510 | 3.789132 |
| 2.980835 | 14.35752 | 6.541973 |          |          |

---

**Table 2d - Listing of Instruction File EG2.INS**


---

|         |  |
|---------|--|
| 1       | 1=ERROR COMPONENTS MODEL, 2=TE EFFECTS MODEL         |
| eg2.dta | DATA FILE NAME                                       |
| eg2.out | OUTPUT FILE NAME                                     |
| 1       | 1=PRODUCTION FUNCTION, 2=COST FUNCTION               |
| y       | LOGGED DEPENDENT VARIABLE (Y/N)                      |
| 60      | NUMBER OF CROSS-SECTIONS                             |
| 1       | NUMBER OF TIME PERIODS                               |
| 60      | NUMBER OF OBSERVATIONS IN TOTAL                      |
| 5       | NUMBER OF REGRESSOR VARIABLES (Xs)                   |
| y       | MU (Y/N) [OR DELTA0 (Y/N) IF USING TE EFFECTS MODEL] |
| n       | ETA (Y/N) [OR NUMBER OF TE EFFECTS REGRESSORS (Zs)]  |
| n       | STARTING VALUES (Y/N)                                |
|         | IF YES THEN BETA0                                    |
|         | BETA1 TO   |
|         | BETAK  |
|         | SIGMA SQUARED  |
|         | GAMMA  |
|         | MU [OR DELTA0  |
|         | ETA DELTA1 TO  |
|         | DELTAK]  |
|         |  |
|         | NOTE: IF YOU ARE SUPPLYING STARTING VALUES           |
|         | AND YOU HAVE RESTRICTED MU [OR DELTA0] TO BE         |
|         | ZERO THEN YOU SHOULD NOT SUPPLY A STARTING           |
|         | VALUE FOR THIS PARAMETER.                            |

---

### 4.3 A Cobb-Douglas cost frontier using cross-sectional data and assuming a half-normal distribution.

In this example we wish to estimate the Cobb-Douglas cost frontier:

$$(9) \quad \ln(C_i/W_i) = \beta_0 + \beta_1 \ln(Q_i) + \beta_2 \ln(R_i/W_i) + (V_i + U_i),$$

where  $C_i$ ,  $Q_i$ ,  $R_i$  and  $W_i$  are cost, output, capital price and labour price, respectively, and  $V_i$  and  $U_i$  are assumed normal and half-normal distributed, respectively. The file EG3.DAT contains 60

observations on firm-id, time-period, C, Q, R and W, in that order (refer to Table 3a). The SHAZAM code in Table 3b generates the required transformed variables and places them in EG3.DTA (refer Table 3c). The main point to note regarding the instruction file in Table 3d is that we have entered a “2” on line 4 to indicate a cost function is required.

**Table 3a - Listing of Data File EG3.DAT**

---

|     |    |          |        |        |        |
|-----|----|----------|--------|--------|--------|
| 1.  | 1. | 783.469  | 35.893 | 11.925 | 28.591 |
| 2.  | 1. | 439.742  | 24.322 | 12.857 | 23.098 |
| 3.  | 1. | 445.813  | 34.838 | 14.368 | 16.564 |
|     |    | .        |        |        |        |
|     |    | .        |        |        |        |
| 58. | 1. | 216.558  | 26.888 | 7.853  | 10.882 |
| 59. | 1. | 408.234  | 20.848 | 9.411  | 23.281 |
| 60. | 1. | 1114.369 | 32.514 | 14.919 | 29.672 |

---

**Table 3b - Listing of Shazam Instruction File EG3.SHA**

---

```
read(eg3.dat) n t c q r w
genr lcw=log(c/w)
genr lq=log(q)
genr lrw=log(r/w)
file 33 eg3.dta
write(33) n t lcw lq lrw
stop
```

---

**Table 3c - Listing of Data File EG3.DTA**

---

|          |          |          |          |            |
|----------|----------|----------|----------|------------|
| 1.000000 | 1.000000 | 3.310640 | 3.580542 | -0.8744549 |
| 2.000000 | 1.000000 | 2.946442 | 3.191381 | -0.5858576 |
| 3.000000 | 1.000000 | 3.292668 | 3.550709 | -0.1422282 |
|          |          | .        |          |            |
|          |          | .        |          |            |
| 58.00000 | 1.000000 | 2.990748 | 3.291680 | -0.3262144 |
| 59.00000 | 1.000000 | 2.864203 | 3.037258 | -0.9057584 |
| 60.00000 | 1.000000 | 3.625840 | 3.481671 | -0.6875683 |

---

**Table 3d - Listing of Instruction File EG3.INS**

---

```
1      1=ERROR COMPONENTS MODEL, 2=TE EFFECTS MODEL
eg3.dta  DATA FILE NAME
eg3.out  OUTPUT FILE NAME
2      1=PRODUCTION FUNCTION, 2=COST FUNCTION
y      LOGGED DEPENDENT VARIABLE (Y/N)
60     NUMBER OF CROSS-SECTIONS
1      NUMBER OF TIME PERIODS
60     NUMBER OF OBSERVATIONS IN TOTAL
2      NUMBER OF REGRESSOR VARIABLES (Xs)
n      MU (Y/N) [OR DELTA0 (Y/N) IF USING TE EFFECTS MODEL]
n      ETA (Y/N) [OR NUMBER OF TE EFFECTS REGRESSORS (Zs)]
n      STARTING VALUES (Y/N)
      IF YES THEN  BETA0
              BETA1 TO
              BETAK
              SIGMA SQUARED
              GAMMA
              MU      [OR DELTA0
              ETA      DELTA1 TO
                      DELTAK]
```

NOTE: IF YOU ARE SUPPLYING STARTING VALUES  
AND YOU HAVE RESTRICTED MU [OR DELTA0] TO BE  
ZERO THEN YOU SHOULD NOT SUPPLY A STARTING  
VALUE FOR THIS PARAMETER.

---

#### **4.4 The Battese and Coelli (1992) specification (Model 1).**

In this example we estimate the full model defined by (2). We are using data on 15 firms observed over 4 time periods. The data has been reproduced in full in Table 4a to make clear the form of the firm-id and time-period columns (columns 1 and 2). The SHAZAM instructions (refer Table 4b) are no different to the first example. The FRONTIER instruction file (refer Table 4d) does differ in a number of ways from the first example: the number of firms has been set to 15 and the number of time periods to 4; and the  $\mu$  and  $\eta$  questions have been answered by a yes (y).



**Table 4a - Listing of Data File EG4.DAT**

---

|     |    |        |       |        |
|-----|----|--------|-------|--------|
| 1.  | 1. | 15.131 | 9.416 | 35.134 |
| 2.  | 1. | 26.309 | 4.643 | 77.297 |
| 3.  | 1. | 6.886  | 5.095 | 89.799 |
| 4.  | 1. | 11.168 | 4.935 | 35.698 |
| 5.  | 1. | 16.605 | 8.717 | 27.878 |
| 6.  | 1. | 10.897 | 1.066 | 92.174 |
| 7.  | 1. | 8.239  | 0.258 | 97.907 |
| 8.  | 1. | 19.203 | 6.334 | 82.084 |
| 9.  | 1. | 16.032 | 2.350 | 38.876 |
| 10. | 1. | 12.434 | 1.076 | 81.761 |
| 11. | 1. | 2.676  | 3.432 | 9.476  |
| 12. | 1. | 29.232 | 4.033 | 55.096 |
| 13. | 1. | 16.580 | 7.975 | 73.130 |
| 14. | 1. | 12.903 | 7.604 | 24.350 |
| 15. | 1. | 10.618 | 0.344 | 65.380 |
| 1.  | 2. | 13.936 | 2.440 | 63.839 |
| 2.  | 2. | 23.104 | 7.891 | 59.241 |
| 3.  | 2. | 8.314  | 2.906 | 72.574 |
| 4.  | 2. | 17.688 | 2.668 | 68.916 |
| 5.  | 2. | 24.459 | 4.220 | 57.424 |
| 6.  | 2. | 15.490 | 2.661 | 87.843 |
| 7.  | 2. | 13.023 | 2.455 | 30.789 |
| 8.  | 2. | 20.548 | 2.827 | 93.734 |
| 9.  | 2. | 10.708 | 0.439 | 35.961 |
| 10. | 2. | 7.921  | 0.312 | 94.264 |
| 11. | 2. | 14.966 | 3.265 | 95.773 |
| 12. | 2. | 25.989 | 6.752 | 80.275 |
| 13. | 2. | 14.264 | 4.425 | 49.886 |
| 14. | 2. | 9.690  | 1.583 | 22.072 |
| 15. | 2. | 9.034  | 0.907 | 38.727 |
| 1.  | 3. | 5.379  | 6.149 | 5.322  |
| 2.  | 3. | 2.498  | 0.479 | 2.520  |
| 3.  | 3. | 7.884  | 1.955 | 41.545 |
| 4.  | 3. | 24.334 | 8.169 | 68.389 |
| 5.  | 3. | 19.668 | 4.055 | 77.556 |
| 6.  | 3. | 22.337 | 5.029 | 77.812 |
| 7.  | 3. | 38.323 | 6.937 | 98.904 |
| 8.  | 3. | 17.388 | 8.405 | 42.740 |
| 9.  | 3. | 21.160 | 2.503 | 59.741 |
| 10. | 3. | 10.069 | 6.590 | 18.085 |
| 11. | 3. | 7.964  | 7.149 | 26.651 |
| 12. | 3. | 20.535 | 8.040 | 50.488 |
| 13. | 3. | 24.019 | 4.896 | 88.182 |
| 14. | 3. | 18.820 | 6.722 | 30.451 |
| 15. | 3. | 23.563 | 4.195 | 95.834 |
| 1.  | 4. | 11.583 | 4.551 | 36.704 |
| 2.  | 4. | 31.612 | 7.223 | 89.312 |
| 3.  | 4. | 12.088 | 9.561 | 29.055 |
| 4.  | 4. | 13.736 | 4.871 | 50.018 |
| 5.  | 4. | 19.274 | 9.312 | 40.996 |
| 6.  | 4. | 15.471 | 2.895 | 63.051 |
| 7.  | 4. | 23.190 | 8.085 | 60.992 |
| 8.  | 4. | 30.192 | 8.656 | 94.159 |
| 9.  | 4. | 23.627 | 3.427 | 39.312 |
| 10. | 4. | 14.128 | 1.918 | 78.628 |
| 11. | 4. | 11.433 | 6.177 | 64.377 |
| 12. | 4. | 4.074  | 7.188 | 1.073  |
| 13. | 4. | 23.314 | 9.329 | 87.124 |
| 14. | 4. | 22.737 | 7.834 | 60.340 |
| 15. | 4. | 22.639 | 5.621 | 44.218 |

---

**Table 4b - Listing of Shazam Instruction File EG4.SHA**


---

```

read(eg4.dat) n t y x1 x2
genr ly=log(y)
genr lx1=log(x1)
genr lx2=log(x2)
file 33 eg4.dta
write(33) n t ly lx1 lx2
stop

```

---

**Table 4c - Listing of Data File EG4.DTA**


---

|          |          |          |          |          |
|----------|----------|----------|----------|----------|
| 1.000000 | 1.000000 | 2.716746 | 2.242410 | 3.559169 |
| 2.000000 | 1.000000 | 3.269911 | 1.535361 | 4.347655 |
| 3.000000 | 1.000000 | 1.929490 | 1.628260 | 4.497574 |
| .        | .        | .        | .        | .        |
| 13.00000 | 4.000000 | 3.149054 | 2.233128 | 4.467332 |
| 14.00000 | 4.000000 | 3.123994 | 2.058473 | 4.099995 |
| 15.00000 | 4.000000 | 3.119674 | 1.726510 | 3.789132 |

---

**Table 4d - Listing of Instruction File EG4.INS**


---

```

1      1=ERROR COMPONENTS MODEL, 2=TE EFFECTS MODEL
eg4.dta  DATA FILE NAME
eg4.out  OUTPUT FILE NAME
1      1=PRODUCTION FUNCTION, 2=COST FUNCTION
y      LOGGED DEPENDENT VARIABLE (Y/N)
15     NUMBER OF CROSS-SECTIONS
4      NUMBER OF TIME PERIODS
60     NUMBER OF OBSERVATIONS IN TOTAL
2      NUMBER OF REGRESSOR VARIABLES (Xs)
y      MU (Y/N) [OR DELTA0 (Y/N) IF USING TE EFFECTS MODEL]
y      ETA (Y/N) [OR NUMBER OF TE EFFECTS REGRESSORS (Zs)]
n      STARTING VALUES (Y/N)
      IF YES THEN  BETA0
                BETA1 TO
                BETAK
                SIGMA SQUARED
                GAMMA
                MU      [OR DELTA0
                ETA      DELTA1 TO
                DELTAK]

      NOTE: IF YOU ARE SUPPLYING STARTING VALUES
      AND YOU HAVE RESTRICTED MU [OR DELTA0] TO BE
      ZERO THEN YOU SHOULD NOT SUPPLY A STARTING
      VALUE FOR THIS PARAMETER.

```

---

#### 4.5 The Battese and Coelli (1995) specification (Model 2).

In this example we estimate the full model defined by (3) and (4) with the z vector containing a constant and one other variable (which incidently is a time trend in this simple

example). Thus the data file EG5.DAT (refer Table 5a) contains one more column (the z variable), than the data file in the previous example. The SHAZAM instructions (refer Table 5b) are similar to those in first example, except that data on the z variable must be read in and read out. The FRONTIER instruction file (EG5.INS) differs in a number of ways from the previous example: the model number on line one has been set to “2”; the question regarding  $\delta_0$  has been answered by a yes (line 10) and the number of z variables has been set to 1 (line 11).

**Table 5a - Listing of Data File EG5.DAT**

---

|     |    |        |       |        |       |
|-----|----|--------|-------|--------|-------|
| 1.  | 1. | 15.131 | 9.416 | 35.134 | 1.000 |
| 2.  | 1. | 26.309 | 4.643 | 77.297 | 1.000 |
| 3.  | 1. | 6.886  | 5.095 | 89.799 | 1.000 |
|     | .  |        |       |        |       |
|     | .  |        |       |        |       |
| 13. | 4. | 23.314 | 9.329 | 87.124 | 4.000 |
| 14. | 4. | 22.737 | 7.834 | 60.340 | 4.000 |
| 15. | 4. | 22.639 | 5.621 | 44.218 | 4.000 |

---

**Table 5b - Listing of Shazam Instruction File EG5.SHA**

---

```
read(eg5.dat) n t y x1 x2 z1
genr ly=log(y)
genr lx1=log(x1)
genr lx2=log(x2)
file 33 eg5.dta
write(33) n t ly lx1 lx2 z1
stop
```

---

**Table 5c - Listing of Data File EG5.DTA**

---

|           |          |          |          |          |
|-----------|----------|----------|----------|----------|
| 1.000000  | 1.000000 | 2.716746 | 2.242410 | 3.559169 |
| 1.000000  |          |          |          |          |
| 2.000000  | 1.000000 | 3.269911 | 1.535361 | 4.347655 |
| 1.000000  |          |          |          |          |
| 3.000000  | 1.000000 | 1.929490 | 1.628260 | 4.497574 |
| 1.000000  |          |          |          |          |
|           | .        |          |          |          |
|           | .        |          |          |          |
| 13.000000 | 4.000000 | 3.149054 | 2.233128 | 4.467332 |
| 4.000000  |          |          |          |          |
| 14.000000 | 4.000000 | 3.123994 | 2.058473 | 4.099995 |
| 4.000000  |          |          |          |          |
| 15.000000 | 4.000000 | 3.119674 | 1.726510 | 3.789132 |
| 4.000000  |          |          |          |          |

---

**Table 5d - Listing of Instruction File EG5.INS**

---

|         |  |
|---------|--|
| 2       | 1=ERROR COMPONENTS MODEL, 2=TE EFFECTS MODEL         |
| eg5.dta | DATA FILE NAME                                       |
| eg5.out | OUTPUT FILE NAME                                     |
| 1       | 1=PRODUCTION FUNCTION, 2=COST FUNCTION               |
| y       | LOGGED DEPENDENT VARIABLE (Y/N)                      |
| 15      | NUMBER OF CROSS-SECTIONS                             |
| 4       | NUMBER OF TIME PERIODS                               |
| 60      | NUMBER OF OBSERVATIONS IN TOTAL                      |
| 2       | NUMBER OF REGRESSOR VARIABLES (Xs)                   |
| y       | MU (Y/N) [OR DELTA0 (Y/N) IF USING TE EFFECTS MODEL] |
| 1       | ETA (Y/N) [OR NUMBER OF TE EFFECTS REGRESSORS (Zs)]  |
| n       | STARTING VALUES (Y/N)                                |
|         | IF YES THEN BETA0                                    |
|         | BETA1 TO   |
|         | BETAK  |
|         | SIGMA SQUARED  |
|         | GAMMA  |
|         | MU [OR DELTA0  |
|         | ETA DELTA1 TO  |
|         | DELTAK]  |

NOTE: IF YOU ARE SUPPLYING STARTING VALUES AND YOU HAVE RESTRICTED MU [OR DELTA0] TO BE ZERO THEN YOU SHOULD NOT SUPPLY A STARTING VALUE FOR THIS PARAMETER.

---

## 5. FINAL POINTS

Various versions of FRONTIER are now in use at over 150 locations around the world. This new version of FRONTIER has benefited significantly from feedback from you the users. Hopefully many of you will see that some of your suggestions have been adopted in this new version. If you have any suggestions regarding how the program could be improved or if you think you may have found a bug, then you are urged to contact the author either by email at: **tcoelli@metz.une.edu.au** or by writing to the address on the front of this paper. If you have not been supplied a copy of the program directly by the author and wish to be notified of any major bugs or new versions please contact the author so that you may be put on the mailing list.

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## **APPENDIX - PROGRAMMER'S GUIDE**

### **A.1 The FRONT41.000 File**

The start-up file FRONT41.000 is listed in Table A1. Ten values may be altered in FRONT41.000. A brief description of each value is provided below.

**Table A1 - The start-up file FRONT41.000**

---

KEY VALUES USED IN FRONTIER PROGRAM (VERSION 4.1)

NUMBER: DESCRIPTION:

|         |  |
|---------|--|
| 5       | IPRINT - PRINT INFO EVERY "N" ITERATIONS, 0=DO NOT PRINT     |
| 1       | INDIC - USED IN UNIDIMENSIONAL SEARCH PROCEDURE - SEE BELOW  |
| 0.00001 | TOL - CONVERGENCE TOLERANCE (PROPORTIONAL)                   |
| 0.001   | TOL2 - TOLERANCE USED IN UNI-DIMENSIONAL SEARCH PROCEDURE    |
| 1.0D+16 | BIGNUM - USED TO SET BOUNDS ON DEN & DIST                    |
| 0.00001 | STEP1 - SIZE OF 1ST STEP IN SEARCH PROCEDURE                 |
| 1       | IGRID2 - 1=DOUBLE ACCURACY GRID SEARCH, 0=SINGLE             |
| 0.1     | GRIDNO - STEPS TAKEN IN SINGLE ACCURACY GRID SEARCH ON GAMMA |
| 100     | MAXIT - MAXIMUM NUMBER OF ITERATIONS PERMITTED               |
| 1       | ITE - 1=PRINT ALL THE ESTIMATES, 0=PRINT ONLY MEAN THE       |

THE NUMBERS IN THIS FILE ARE READ BY THE FRONTIER PROGRAM WHEN IT BEGINS EXECUTION. YOU MAY CHANGE THE NUMBERS IN THIS FILE IF YOU WISH. IT IS ADVISED THAT A BACKUP OF THIS FILE IS MADE PRIOR TO ALTERATION.

FOR MORE INFORMATION ON THESE VARIABLES SEE: COELLI (1996), CEPA WORKING PAPER 96/07, UNIVERSITY OF NEW ENGLAND, ARMIDALE, NSW, 2351, AUSTRALIA.

INDIC VALUES:

indic=2 says do not scale step length in unidimensional search

indic=1 says scale (to length of last step) only if last step was smaller

indic= any other number says scale (to length of last step)

---

- 1) IPRINT - specifies how often information on the likelihood function value and the vector of parameter estimates should be recorded during the iterative process. It is initially set to 5, hence information is printed every 5 iterations. It can be set to any non-negative integer value. A 0 will result in no reporting of intermediate information.
- 2) INDIC - relates to the Coggin uni-dimensional search which is conducted before each iteration to determine the optimal step length. It may be used as follows: indic=2 says do not scale step length in uni-dimensional search; indic=1 says scale (to length of last step) only if last step was smaller; and indic=any other number says scale (to length of last step) For more information see Himmelblau (1972).
- 3) TOL - sets the convergence tolerance on the iterative process. If this value is say set to 0.00001 then the iterative procedure would terminate when the proportional change in the log-likelihood function and in each of the estimated parameters are all less than 0.00001.
- 4) TOL2 - sets the required tolerance on the Coggin uni- dimensional search done each iteration to determine the step length. For more information see Himmelblau (1972).
- 5) BIGNUM - bounds the size of the largest number that the program should deal with. Its primary use is to place bounds upon what the smallest number can be in the subroutines DEN (which evaluates the standard normal density function) and DIS (which evaluates the standard normal distribution function). Errors with numerical underflows and overflows were the

problems most frequently encountered by people attempting to install earlier versions of this program on various mainframe computers. This number has been set to  $1.0e+16$  for the IBM PC. If you plan to mount this program on a mainframe computer it is advised that you consult computer support staff on the correct setting of this number. It generally would be safe to leave it as it is, however, greater precision may be gained if larger numbers are permitted.

6) STEP1 - sets the size of the first step in the iterative process. This should be set carefully as too large a value may result in the program 'stepping' right out of the sensible parameter space.

7) IGRID2 - a flag which if set to 1 will cause the grid search to complete a second phase grid search around the estimate obtained in the first phase of the grid search. If set to zero only the first phase of the grid search will be conducted. For more information refer to the description of the grid search in Section 3.

8) GRIDNO - sets the width of the steps taken in the grid search between zero and one on the  $\gamma$  parameter. For more information refer to the description of the grid search in Section 3.

9) MAXIT - sets the maximum number of iterations that will be conducted. This is especially a useful option when batch files are written for monte carlo simulation.

10) ITE - specifies whether individual efficiency estimates should be listed in the output file. A value of 1 will cause them to be listed, and a 0 will suppress them.

## A.2 Subroutine Descriptions

EXEC: This is the main calling program. It firstly reads the start-up file FRONT2.000 before calling INFO.

INFO: This subroutine reads instructions either from a file or from the terminal then reads the data file. It then calls MINI.

MINI: This is the main subroutine of the program. It firstly calls GRID to do the grid search (assuming starting values are not specified by the user). MINI then conducts the main iterative loop of FRONTIER, calling SEARCH, ETA and CONVRG repeatedly until the convergence criteria are satisfied (or the maximum number of iterations is achieved). The Davidon-Fletcher-Powell method is used.

RESULT: Sends all final results to the output file. These include parameter estimates, approximate standard errors, t-ratios, and the individual and mean technical efficiency estimates.

GRID: Does a grid search over  $\gamma$ .

SEARCH: Performs a uni-dimensional search to determine the optimal step length for the next iteration. The Coggin method is used (see Himmelblau, 1972).

ETA: This subroutine updates the direction matrix according to the Davidon-Fletcher-Powell method at each iteration. For more information refer to Himmelblau (1972).

CONVRG: Tests the convergence critereon at the end of each iteration. If the proportion change in the log-likelihood function and each of the parameters is no greater than the value of TOL (initially set to 0.00001) the iterative process will terminate.

FUN1: Calculates the negative of the log-likelihood function (LLF) of Model 1. Note that FRONTIER minimizes the negative of the LLF which is equivalent to maximizing the LLF.

DER1: Calculates the first partial derivatives of the negative of the LLF of Model 1.

FUN2: Calculates the negative of the log-likelihood function (LLF) of Model 2.

DER2: Calculates the first partial derivatives of the negative of the LLF of Model 2.

CHECK: Ensures that the estimated parameters do not venture outside the theoretical bounds (i.e.  $0 < \gamma < 1$ ,  $\sigma^2 > 0$  and  $-2\sigma_U < \mu < 2\sigma_U$ ).



OLS: Calculates the Ordinary Least Squares estimates of the model to be used as starting values. It also calculates OLS standard errors which are presented in the final output.

INVERT: Inverts a given matrix.

FUNCTIONS:

DEN: Evaluates the density function of a standard normal random variable.

DIS: Evaluates the distribution function of a standard normal random variable.