2011/32

■

Crowdfunding: tapping the right crowd

Paul Belleflamme, Thomas Lambert
and Armin Schwienbacher

CORE

DISCUSSION PAPER

CORE DISCUSSION PAPER
2011/32

Crowdfunding: tapping the right crowd

Paul BELLEFLAMME [1], Thomas LAMBERT[2]
and Armin SCHWIENBACHER [3]

June 2011

Abstract

The basic idea of crowdfunding is to raise external finance from a large audience (the "crowd"), where each individual provides a very small amount, instead of soliciting a small group of sophisticated investors. The paper develops a model that associates crowdfunding with pre-ordering and price discrimination, and studies the conditions under which crowdfunding is preferred to traditional forms of external funding. Compared to traditional funding, crowdfunding has the advantage of offering an enhanced experience to some consumers and, thereby, of allowing the entrepreneur to practice menu pricing and extract a larger share of the consumer surplus; the disadvantage is that the entrepreneur is constrained in his/her choice of prices by the amount of capital that he/she needs to raise: the larger this amount, the more prices have to be twisted so as to attract a large number of "crowdfunders" who pre-order, and the less profitable the menu pricing scheme.

Keywords: crowdfunding, pre-ordering, menu pricing.

JEL Classification: G32, L11, L13, L15, L21, L31

---

1   Introduction

It is well recognized that new ventures face difficulties in attracting external finance at their very initial stage, be it through bank loans or equity capital (see, e.g., Berger and Udell, 1995, Cassar, 2004, and Cosh et al., 2009). While business angels and venture capital funds fill gaps for larger amounts, the smallest amounts are provided by entrepreneurs themselves and friends & family. Still, many ventures remain unfunded, partially because of a lack of sufficient value that can be pledged to investors, partially because of unsuccessful attempts to convince investors (Hellmann, 2007; Casamatta and Haritchabalet, 2010). To circumvent these problems, creative founders have recently made use of a new source of finance–so-called crowdfunding–by tapping the "crowd" instead of specialized investors.

The concept of crowdfunding finds its root in the broader concept of crowdsourcing, which refers to using the crowd to obtain ideas, feedback and solutions in order to develop corporate activities (Howe, 2008; Kleemann et al., 2008). In the case of crowdfunding, the objective is to collect money for investment; this is generally done by using social networks, in particular through the Internet (Twitter, Facebook, LinkedIn and different other specialized blogs). In other words, instead of raising the money from a very small group of sophisticated investors, the idea of crowdfunding is to obtain it from a large audience (the "crowd"), where each individual will provide a very small amount. This can take the form of equity purchase, loan, donation or pre-ordering of the product to be produced. The amounts that have been targeted through crowdfunding have continuously increased, with *Trampoline Systems* targeting more than £1 million for the financing of the commercialization stage of their new software. Recently, *TikTok+LunaTik* raised $941,718 from 13,512 individuals in the form of product pre-ordering of its multi-touch watch kit.

In this paper, we develop a model that associates crowdfunding with pre-ordering and price discrimination, and we study the conditions under which crowdfunding is preferred to traditional forms of external funding (bank loan or equity investor). In this framework, the funding is needed to finance up-front fixed costs of production. Since the remaining consumers will pay a different price, crowdfunding that takes the form of pre-ordering gives the opportunity to price discriminate between the first group (those who pre-order and thus constitute the investing crowd) and the second group (the

2

other consumers who wait that production takes place before purchasing directly). However, an entrepreneur is generally unable to identify these consumers. The entrepreneur must then use some self-selecting device so as to induce high-paying consumers to reveal themselves. In this sense, crowdfunding appears as a form of menu pricing.

The trade-off we explore in the model is thus the following: compared to traditional funding, crowdfunding has the advantage of offering an enhanced experience to some consumers and, thereby, of allowing the entrepreneur to practice second-degree price discrimination and extract a larger share of the consumer surplus; the disadvantage is that the entrepreneur is constrained in his/her choice of prices by the amount of capital that he/she needs to raise initially to fund production: the larger this amount, the more prices have to be twisted so as to attract a large number of "crowdfunders", and the less profitable the menu pricing scheme.

The model highlights the importance of community-based experience for crowdfunding to be a viable alternative to traditional funding. If the entrepreneur is not able to create such benefits, no consumer will find it worthwhile to pre-order the good, unless a discount is offered. However, crowdfunding then becomes less profitable compared to traditional funding. Also, the analysis shows that crowdfunding is the most profitable option only for lower levels of finance. Indeed, crowdfunding yields higher profits only for small amounts where the entrepreneur faces no (or limited) constraints in his/her price setting between the two types of consumers while still securing enough up-front financing. As the amount required becomes larger, the entrepreneur is forced to distort more the prices so that more consumers are willing to pre-order and thus the entrepreneur can collect up-front more money. This in turn reduces the gains from price discrimination. Our results are robust to the possibility that the entrepreneur may take the money collected from the crowdfunding initiative and run away with it.

Another important result is that crowdfunders eventually pay more than other, regular consumers. This outcome is consistent with findings from different case studies where community benefits are important. Indeed, our model shows that through price-discrimination, individuals with the highest willingness-to-pay become crowdfunders. As they are willing to pay the most and additionally enjoy non-monetary community benefits, it turns out that they end up paying more than consumers waiting for the product to

3

reach the market.

The remaining of this paper is structured as follows. The next section offers a definition of crowdfunding, discusses crowdfunding practices and provides a survey of related literature. Section 3 presents the theoretical model and discusses its results and implications. Finally, Section 4 concludes with suggested topics for future research.[1]

2   Crowdfunding as a distinct form of entrepreneurial finance

Our objective in this section is twofold. First, we aim at providing a general definition of crowdfunding. Second, we discuss how crowdfunding differs from other sources of entrepreneurial finance. To this end, we present selected crowdfunding initiatives and then provide a review of the related literature.

2.1   A definition

As mentioned, the concept of crowdfunding can be seen as part of the broader concept of crowdsourcing, which refers to using the "crowd" to obtain ideas, feedback and solutions in order to develop corporate activities. The term "crowdsourcing" has been first used by Jeff Howe and Mark Robinson in the June 2006 issue of Wired Magazine, an American magazine for high technology (Howe, 2008). Kleemann et al. (2008) point out that "crowdsourcing takes place when a profit oriented firm outsources specific tasks essential for the making or sale of its product to the general public (the crowd) in the form of an open call over the internet, with the intention of animating individuals to make a [voluntary] contribution to the firm's production process for free or for significantly less than that contribution is worth to the firm." Although this definition of crowdsourcing is a useful starting point, several caveats and clarifications need to be made in order to transpose it to crowdfunding.

Raising funds by tapping a general public (the crowd) is the most important element of crowdfunding. This means that consumers can volunteer to

---

[1]Section 5 is an appendix containing the details of the mathematical developments and the proofs of the propositions.

provide input to the development of the product, in this case in form of financial help. How the interaction with the crowd takes place may, however, differ from crowdsourcing. For instance, platforms have recently emerged to facilitate the interaction between entrepreneurial initiatives and potential crowdfunders. Entrepreneurs can post their project on the platform and benefit from the platform's visibility to reach potential investors. A platform of this sort, *Kickstarter*, has successfully intermediated more than 7,000 projects already.[2]

Yet, while the use of the Internet to make an "open call" may be very efficient for crowdsourcing in general, it can become more problematic for crowdfunding, especially if it involves the offering of equity to the crowd. Indeed, making a general solicitation for equity offering is limited to publicly listed equity. In many countries, there is also a limit as to how many private investors a company can have (see Larralde and Schwienbacher, 2010, for an extended discussion). This creates important legal limitations to crowdfunding initiatives, given that the input of the crowd is capital and not an idea or time. Therefore, most initiatives do not offer shares but provide other types of rewards such as a product or membership.

Besides, while the Web 2.0 has been a critical ingredient in the development of crowdfunding practices, it also differs from open-source practices (Brabham, 2008; Fershtman and Gandal, 2011). An important distinction is that in the case of open-source, the resource belongs to the community, which can then exploit it on an individual basis (there is no restriction on who can use it); in the case of crowdfunding (and also crowdsourcing), it ultimately belongs to the firm, which will be the only one to use it. This distinction with open-source practices becomes even more obvious when related to crowdfunding, since capital cannot be shared. Unlike an idea or a software code, capital is not a public good in the economic sense that assumes non-rivalness and non-excludability.

Based on this discussion and in the spirit of Kleemann et al. (2008), we offer the following, refined definition:

Definition 1 *Crowdfunding involves an open call, mostly through the Internet, for the provision of financial resources either in form of donation or*

---

[2]These platforms share some similarities with online lending markets (Everett, 2008; Freedman and Jin, 2010); while the latter more prominently target social entrepreneurship, crowdfunding platforms have a broader scope of entrepreneurial initiatives.

in exchange for the future product or some form of reward and/or voting rights.

As mentioned above, the promised reward can be monetary or non-monetary (such as recognition). This definition encompasses many forms of crowdfunding practices and has been discussed as such. However, in this paper, we focus on crowdfunding initiatives that take the form of pre-ordering of products. The following sections are restricted to the latter form.

2.2 Examples

Different reasons may explain recent successes of entrepreneurs who have relied on crowdfunding. Also, there exist many ways to "crowdfund" a project. However, crowdfunding initiatives share some common characteristics, which we stress below in the light of selected cases.

In 2005, the South African singer *Verity Price* launched the "Lucky Packet Project". To record her album without assistance of a record label, Verity Price needed to advance an up-front investment of ZAR400,000.[3] To this end, she set up a website where she asked people to pre-purchase her album at ZAR200 before she recorded it. In return from their contributions, people were compensated with some form of non-monetary rewards, such as their name credited on her website, the possibility to vote on which songs are recorded, and what artwork and photography are used. Also, 10% of sales would be transferred to charities. Verity Price managed to reach the threshold of ZAR400,000 with the contributions of 2061 individuals. Then, she used the money to record her album. Now, the album has been put on the market and is sold to everyone at ZAR100.

In the same vein, the *LINCH three Project* aims at making a documentary film about the artist David Lynch. The filmmakers ask to David Lynch's fans to donate $50 each to fund the film project. The fans' community are rewarded by having online access to exclusive content on the filmmaking process and by receiving limited edition of footage created by Lynch himself either into print, T-shirt, or bag. Once the money is raised, the documentary film will be produced and distributed via the regular distribution channels.[4]

---

[3]ZAR (South African Rands) 400,000 is approximately €35,000.

[4]*Buyacredit.com*'s initiative or the film "*The Age of Stupid*" are other examples of independent films for which financing is crowdfunded.

As exemplified by these two cases, crowdfunding seems more successful in the entertainment industry. However, entrepreneurial ventures in other industries have been financed in the same way and share similar characteristics. Initiatives have been undertaken in other industries such as journalism (*Spot.Us*), beer (*BeerBankroll*), software (*Blender Foundation, Trampoline Systems*) and tourism (*MediaNoMad*). For instance, by investing in the launch of the Lebanese restaurant *mybar*, people may enjoy financial and social benefits, such as voting rights, special privileges at *mybar*, and annual dividends. Also, members of *MyFootballClub* (who own the football club *Ebbsfleet United* in United Kingdom) are completely involved in the management of the club through their voting right. The contribution of fans (a membership fee of £35) allowed them to complete the takeover of the club and form a community with real decision power.

To sum up, these cases highlight three recurrent characteristics. First, crowdfunding initiatives often rely on advance purchase of a product, which is not yet on the market in its finished form. At the pre-ordering stage, the entrepreneur offers just a description and promise on what the final product will be, and also commits that the product will indeed be put on the market. Second, in most of the cases, consumers who pre-order the product pay more than the regular consumers, who wait that production takes place before purchasing directly. In the Verity Price's experience, the regular consumers pay ZAR100 whereas the pre-ordering consumers pay ZAR200. The crowdfunders are therefore willing to pay more for the product. Third, the crowd must identify themselves as such. Crowdfunders must feel that they are being part of a community of privileged consumers. This community can take many forms, starting with receiving rewards up to direct involvement in the project. In the case of *LINCH three Project*, they gain access to exclusive footage of David Lynch. Hence, consumers may self-select into this community and entrepreneurs ensure that consumers enjoy such community benefits and trust in the project.

2.3   Related literature

As crowdfunding is a relatively new phenomenon, it is no surprise that the literature specifically devoted to crowdfunding is only nascent. It is however worthwhile making parallels with other sources of entrepreneurial finance. This allows us to better understand the specificities of crowdfunding as a

distinct form of finance.

First, looking at crowdfunding from a pure financial perspective, connections can be made with bootstrap finance. This form of financing consists of using internal financing ways rather than traditional sources of external finance (e.g., bank loan, angel capital and venture capital). Several studies provide evidence of the different forms of internal sources used by bootstrapping entrepreneurs (see Bhidé, 1992, Winborg and Landstrom, 2001, and Ebben and Johnson, 2006, just to cite a few). Bhidé (1992) shows that even among the Inc. 500 companies in the US, most of them started by bootstrapping the company. Further financing methods for startups companies are analyzed, for instance, by Cosh et al. (2009), who examine a broader range of financing alternatives. None of these studies however consider the "crowd" as possible alternative (regardless of whether it constitutes potential consumers or simply profit-driven individuals).

A few studies have recently focused on crowdfunding more specifically. One is by Agrawal, Catalini and Goldfarb (2011) that examines the geographic origin of consumers who invest on the *SellaBand* platform.[5] The authors observe that "the average distance between artist-entrepreneurs and investors is about 3,000 miles, suggesting a reduced role for spatial proximity." However, they establish that distance still plays a role insofar as "local investors are more likely than distant ones to invest in the very early stages of a single round of financing and appear less responsive to decisions by other investors."

The idea that investors may be responsive to other investors' decisions is also present in Ward and Ramachandran (2010). The goal of this paper is to estimate the extent to which demand for crowdfunding projects is driven by peer effects. Like in our model, it is assumed that consumption cannot happen until projects successfully complete their funding. What differs is the

---

[5]*SellaBand* is an online platform based in Amsterdam that enables musicians to raise money to produce their album. *SellaBand*'s business model is as follows. Artists can post a number of songs (demos) on the platform; visitors to the site can then listen to the music for free and choose the artists they want to invest in; artists seek to raise $50,000 by selling "Parts" at $10 each; during the fundraising stage, money is held in an escrow until the threshold of $50,000 is reached. The $50,000 will be used to fund the artist's recording project; finally investors (the "Believers") are compensated by receiving 10% of revenue from the album. *SellaBand* has been one of the first website of this kind; followers are, e;g., *MyMajorCompany* in France, *Akamusic* in Belgium, and *ArtistShare* in the United States.

link that the authors make between crowdfunding and information. While we assume that crowdfunding allows the firm to gain information about its consumers, they posit that crowdfunding allows consumers to refine their information about the quality of an experience good. In their model, crowdfunders may update their prior based on information from their investor social network. Analyzing also data from *SellaBand*, they find that crowdfunders are influenced by the success or failure of related projects and use the actions of other crowdfunders as a source of information in their funding decisions.

Finally, when crowdfunding is associated with pre-ordering and price discrimination, some strand of literature in the realm of industrial organization provides useful insight. Nocke et al. (2011) have recently linked product pre-ordering to price discrimination, however in a context of information asymmetry.[6] There, the true quality of the product is revealed later so that the firm faces consumers with different expected valuations for its forthcoming product. This induces consumers with highest expected valuation to pre-order before the quality is known. Advance-purchase then leads to price discounts, in contrast to our setting that abstracts from information asymmetry. Also, their setting does not induce any network effect like in our setting.

Such network effects and similar coordination issues among consumers are present in another strand of literature that can be related to our work, namely the literature on threshold (or discrete) public goods. These goods can be provided only after a sufficient amount of contributions is reached so as to cover their cost of provision (think, e.g., of a lighthouse or a bridge). The parallel with crowdfunding is clear as the entrepreneur also needs to collect private contributions before producing her good (which is private rather than public in the present case). When contributing, an individual exerts a positive externality on other individuals by raising the likelihood that the good (public or private) will be produced. Free-riding may then be observed. In our setting, as will be explained below, we treat this problem by requiring that consumers' expectations (about the contribution behavior of other consumers) be fulfilled at equilibrium and by assuming that the entrepreneur is able to somehow coordinate consumers' decisions so as to

---

[6]Other studies have shown that advance-purchase discounts may arise in environments where production capacity is limited or the aggregate level of demand is uncertain (Gale and Holmes, 1992, 1993; Dana, 1998, 1999, 2001).

avoid free-riding. This simplification allows us to focus on other important issues of crowdfunding. In contrast, the literature on threshold public goods explicitly addresses the possibility of free-riding and examines the design of optimal mechanisms for the provision of those goods when, for instance, each individual has private information about the cost or benefit associated with his or her participation in the provision of the good (Gradstein, 1994), or when permitting continuous rather than binary "all-or-nothing" contributions (Cadby and Maynes, 1999), or under different rules of allocating excess contributions if the total amount collected exceeds the required threshold (Spencer et al., 2009).

3 Crowdfunding, pre-ordering and menu pricing

In this section, we focus on crowdfunding experiences where consumers are invited to pre-order the product. For the entrepreneur to be able to launch production, the amount collected through pre-ordering must cover the fixed cost of production. Since the remaining consumers will pay a different price, crowdfunding that takes the form of pre-ordering gives the opportunity to price discriminate between the first group (those who pre-order and thus constitute the investing "crowd") and the second group (the other consumers who wait that production takes place before purchasing directly).

Since the consumers who pre-order are those with a high willingness to pay for the product, these will generally constitute the bulk of the "crowd". However, an entrepreneur is generally unable to identify these consumers. The entrepreneur must then use some self-selecting device so as to induce high-paying consumers to reveal themselves. The sort of 'community experience' that web-based crowdfunding offers may be a means by which the entrepreneur enhances the perceived quality of the product for the consumers who agree to pre-order it. In this sense, crowdfunding appears as a form of menu pricing (i.e., of second-degree price discrimination).

The trade-off we explore in our model is thus the following: compared to external funding, crowdfunding has the advantage of offering an enhanced experience to some consumers and, thereby, of allowing the entrepreneur to practice second-degree price discrimination and extract a larger share of the consumer surplus; the disadvantage is that the entrepreneur is constrained in the first period by the amount of capital that she needs to raise. The

larger this amount, the larger the number of consumers that have to be attracted to cover it, which eventually reduces the profitability of the menu pricing scheme.

In what follows, we first present our model; we then derive, in turn, the outcome under traditional sources of financing (such as debt) and under crowdfunding; finally, we derive the optimal funding choice.

3.1 Model

Suppose a unit mass of consumers identified by θ, with θ uniformly distributed on $[0, 1]$. The parameter θ denotes a consumer's taste for an increase in product's quality. Consumers have unit demand (they buy one or zero unit of the product). All consumers have a reservation utility $r > 0$ for the product; any increase from the basic quality is valued in proportion to the taste parameter θ. Normalizing basic quality to zero, we have that if consumer θ buys one unit of product of increased quality s sold at price p, her net utility is $r + \theta s - p$.[7] To ensure interior solutions at the pricing stage, we assume:

Assumption 1. $r < s < 2r$.

The product is marketed by a monopolist. In our simple model, we consider the quality of the product, s, as exogenous and known by the consumers before purchase.[8] For simplicity, we set to zero the marginal cost of production. There is, however, a fixed cost of production $K > 0$. The timing of the game is as follows. In period zero, the entrepreneur chooses her funding mechanism—traditional funding or crowdfunding—with the following implications. If the entrepreneur chooses traditional funding, then, in period 1, it incurs the fixed cost K, which is financed through, e.g., a bank loan; in period 2, the entrepreneur sets a price p for her product, and consumers decide to buy or not.

On the other hand, if the entrepreneur chooses crowdfunding, then it is able to set a menu pricing scheme. In period 1, the entrepreneur sets

---

[7]This problem was initially examined by Mussa and Rosen (1978). We use here the results of the extended analysis of Bhargava and Choudary (2001).

[8]A natural extension of this framework would be to assume that the quality of the product is unknown to the consumers before purchase. As observed in some crowdfunding experiences, the entrepreneur may then use web-based crowdfunding to reveal information about the product and, thereby, alleviate the experience good problem. We discuss this possibility in the concluding section.

p_1, the price for consumers who pre-order the product; the total revenue collected through pre-orders is meant to cover the fixed cost of production. In period 2, the entrepreneur sets two prices: p_c, the price to be paid by those consumers who have contributed to the financing of the venture (the so-called "crowdfunders"), and p_r, the price to be paid by those consumers who have not (the so-called "regular consumers").[9] As for consumers, they choose in period 1 whether to pre-order or not; in period 2, they decide whether to purchase the product or not (as long as the product has been put on the market, i.e., if total contributions in period 1 are at least as large as K).

The key feature of crowdfunding from the point of view of the consumers is that the participation to the mechanism may provide consumers with an increase in the product quality. We have indeed observed in Section 2 that entrepreneurs resorting to crowdfunding use the Internet to maintain an interaction with their funders so as to provide them with so-called 'community benefits'. Various forms of rewards are sometimes offered but it appears that these rewards are mostly symbolic and that crowdfunders essentially value the feeling of belonging to a group of 'special' or 'privileged' consumers. It is therefore important for the entrepreneur to attract a sufficient number of regular (i.e., 'non-privileged') consumers to whom crowdfunders can feel somehow 'superior'. Denoting the number of regular consumers by n_r and the minimal number of them by ρ (with $\rho \geq 0$), we capture these findings in the model through the following assumption:

Assumption 2. *Crowdfunders perceive the quality of the product to be equal to $s + \sigma$, with $\sigma > 0$ if $n_r \geq \rho$ and $\sigma = 0$ otherwise.*

Assumption 2 translates the idea that crowdfunders enjoy some additional utility from the product compared to regular consumers as long as the number of those regular consumers is above some threshold. To eliminate two sub-cases of very little interest, we put an upper bound on the parameters σ and ρ:

Assumption 3. $\sigma < s\,(r + s)\,/\,(2r)$.

Assumption 4. $\rho < r/\,(2s)$.

---

[9]The entrepreneur is able to recognize consumers who pre-ordered in period 1 and therefore to tell them apart from regular consumers; no personal arbitrage is thus possible in period 2.

The entrepreneur maximizes the present discounted value of her profits over the two periods. Consumers maximize the present discounted value of their net utility over the two periods. We assume that the entrepreneur and the consumers have the same discount factor and we let $0 < \delta \leq 1$ denote it.

Two comments are warranted on the pricing schedule, which is meant to be very general. First, contributors pay $p_1 + \delta p_c$ in total, other consumers pay δp_r. This framework encompasses several, more restrictive schemes, including full pre-payments (where contributors pay one single amount up-front, equal to $p_1 + \delta p_c$) as well as ex post price discrimination (where each type pays a different price). This means that we do not exogenously impose p_c and p_r to be identical, although the framework here allows for this. Second, by enabling the participating crowd to pay up-front only part of their contribution, we avoid any price setting in which the entrepreneur would raise funds well beyond what she really needs, namely K. Here, contributors provide in period 1 merely what is needed for starting production, the rest being paid in period 2. As we will see, this implies that the first-period price charged to crowdfunders (p_1) is not directly set by the entrepreneur but is rather indirectly determined by the interplay between the capital requirement K and the number of crowdfunders (noted n_c), which depends itself on the two second-period prices, p_c and p_r. These two prices are therefore the two actual choice variables of the entrepreneur.

We now consider the choice of prices under the two funding mechanisms. We then compare optimal profits in the two cases and address the choice of funding mechanism.

3.2 Traditional funding

The case of traditional funding is straightforward. In period 1, the entrepreneur gathers funds and in period 2, she sets a uniform price p. All consumers perceive that the product has quality s. Hence, the indifferent consumer is such that $r + \theta s - p \geq 0$, or $\theta \geq (p - r)/s \equiv \hat{\theta}$. As we assume a unit mass of consumers uniformly distributed on the unit interval, we have that the quantity demanded is equal to $q(p) = 1 - \hat{\theta} = 1 - (p - r)/s$. From the first-order condition for profit-maximization, we easily find that the optimal price is $p^* = (r + s)/2$. It follows that $\hat{\theta}^* = (s - r)/2s$, which is positive according to Assumption 1. We can then compute the optimal

gross profit as $p^* \left(1 - \hat{\theta}^*\right) = (r + s)^2 / (4s)$. The net profit under traditional funding is thus equal to

$$
\pi_T = \begin{cases} \delta \frac{(r+s)^2}{4s} - K & \text{for } K < \delta \frac{(r+s)^2}{4s}, \\ 0 & \text{otherwise.} \end{cases} \tag{1}
$$

3.3 Crowdfunding

The crowdfunding case is more complex to analyze for two reasons. First, the entrepreneur tries to achieve a form of second-degree price discrimination; profit is thus maximized under a set of incentive compatibility and participation constraints. Second, in period 1 consumers who contemplate pre-ordering the product must form expectations regarding the number of consumers who will do likewise: the larger this number, the lower the pre-ordering price as the fixed cost will be spread over more consumers, which generates a form of network effects. We look for a subgame-perfect equilibrium of the game played by the entrepreneur and the consumers over the two periods.

3.3.1 Consumer choices

Suppose that each consumer expects that a mass n_c^e of consumers will choose to pre-order and pay the price p_1 set by the entrepreneur in period 1. We adopt the fulfilled-expectations approach: consumers base their decision on their expectation on the mass of contributors, and attention is restricted on equilibria in which these expectations turn out to be correct (i.e., are rational; see Katz and Shapiro, 1985). Two cases have to be distinguished. First, if $n_c^e = 0$, then it is optimal for each consumer not to contribute.[10] As the initial expectation is realized, we have a fulfilled expectations equilibrium. Naturally, crowdfunding is doomed to failure under such equilibrium. As some successful crowdfunding experiences exist in reality, it seems natural to assume that entrepreneurs can find some ways to coordinate consumers so that this 'bad' equilibrium is not selected.[11]

The second case is the case of interest. For any $n_c^e > 0$, the entrepreneur can set p_1 such as $p_1 n_c^e \geq K$. As there is no need to gather more capital

---

[10]This is so because each consumer is infinitesimal and thus cannot on his own make sure that the product will be put on the market.

[11]Another reason to select the 'good' equilibrium is that it clearly Pareto-dominates the 'bad' equilibrium.

than needed, we have $p_1 = K/n_c^e$. So, if consumers expect a positive mass of contributors, they can be sure that the good will be produced.[12] They also realize that the lower their expectation, the larger the value of p_1, i.e., the contribution that will be asked by the entrepreneur.

To decide whether to pre-order or not, consumer θ compares his expected utility in the two options. Let us consider for the moment that the entrepreneur attracts a sufficient number of regular consumers at equilibrium: $n_r \geq \rho$. (Naturally, we will have to check below if this condition is actually met.) If the consumer pre-orders, he pays $p_1 = K/n_c^e$ today and gets tomorrow a product of enhanced quality $(s + \sigma)$. We can thus express the expected utility of a crowdfunder as

$$U_c^e = -\frac{K}{n_c^e} + \delta \left(r + \theta \left(s + \sigma \right) - p_c \right).$$

If the consumer decides not to pre-order, he does not pay anything today and he gets tomorrow a product of quality s at price p_r. Hence, his expected utility as regular consumer is

$$U_r^e = \delta \left(r + \theta s - p_r \right).$$

So, for a consumer to contribute, we must have

$$
\begin{aligned}
U_c^e \; &\geq \; U_r^e \Leftrightarrow \delta \left(\theta \sigma + p_r - p_c \right) \geq \frac{K}{n_c^e} \\
&\Leftrightarrow \; \theta \geq \frac{K}{\delta \sigma n_c^e} - \frac{p_r - p_c}{\sigma} \equiv \bar{\theta} \left(n_c^e \right).
\end{aligned}
$$

All consumers with a value of θ larger than $\bar{\theta} \left(n_c^e \right)$ prefer to pre-order. We observe logically that the mass of crowdfunders increases as (i) the expected number of contributors (n_c^e) increases, (ii) the capital requirement (K) decreases, (iii) the enhancement in quality (σ) resulting from pre-ordering increases, (iv) the difference between the price for regular consumers and for crowdfunders $(p_r - p_c)$ increases.

To ease the exposition, we define $\Delta \equiv p_r - p_c$; that is, Δ is the difference between the second period prices for regular consumers and crowdfunders. For a given expected mass of crowdfunders n_c^e, the actual mass of crowdfunders is equal to $n_c = 1 - \bar{\theta} \left(n_c^e \right)$. We require fulfilled expectations at

---

[12]Provided that the entrepreneur does not find it profitable to run away with the contributions at the start of period 2. We consider this issue in Subsection 3.5. For the moment, we assume that the entrepreneur is able to credibly commit that she will not run away.

equilibrium: $n_c = n_c^e$. We must thus solve

$$n_c = 1 + \frac{\Delta}{\sigma} - \frac{K}{\delta \sigma n_c}.$$

This equation is represented in Figure 1: solutions are the intersection between the 45° line (n_c) and the function $1 + \frac{\Delta}{\sigma} - \frac{K}{\delta \sigma n_c}$, which is increasing and concave in n_c. Figure 1 depicts the latter function for different values of Δ. We observe that an intersection exists as long as

$$\Delta \geq \sigma \left(2\sqrt{K/(\delta \sigma)} - 1 \right) \equiv \underline{\Delta}. \tag{2}$$

To understand the meaning of this condition, let us describe what happens when it is violated. For $\Delta < \underline{\Delta}$, the price charged to crowdfunders is not sufficiently smaller than the price charged to regular consumers, so that a large value of n_c^e is needed to convince consumers to pre-order the product; indeed, the larger the expected number of crowdfunders, the lower the price p_1 each crowdfunder has to pay in period 1, which increases the attractiveness of pre-ordering, other things being equal. Yet, the number of consumers who actually decide to pre-order always remains smaller than the expected number, meaning that expectations cannot be fulfilled (i.e., there is no solution to the above equation). Note that the threshold $\underline{\Delta}$ logically increases with K: the higher the capital requirement, the more difficult it becomes for expectations to be fulfilled. Note also that at $\Delta = \underline{\Delta}$, there is a unique solution, which is easily computed as $n_c = \sqrt{K/(\delta \sigma)}$. This value is strictly lower than unity as long as $K < \delta \sigma$, which we assume for the moment.[13] For $\Delta > \underline{\Delta}$, there are two intersections. As we expect the mass of crowdfunders to increase with Δ, we select the largest value of n_c, which is computed as

$$n_c = \frac{1}{2\sigma} \left(\sigma + \Delta + \sqrt{(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2} \right). \tag{3}$$

As shown in Figure 1, this value is strictly smaller than unity for $\Delta < K/\delta$.

3.3.2 Optimal prices

Suppose for now that $n_c < 1$. We have then that n_c consumers pre-order the product at price p_1 and buy it in period 2 at price p_c. As for the

---

[13]For $K \geq \delta \sigma$, the fulfilled-expectations equilibrium is such that all consumers become crowdfunders. This implies that there can be no regular consumers and that σ inevitably falls to zero. This possibility is examined under Case 2b below.
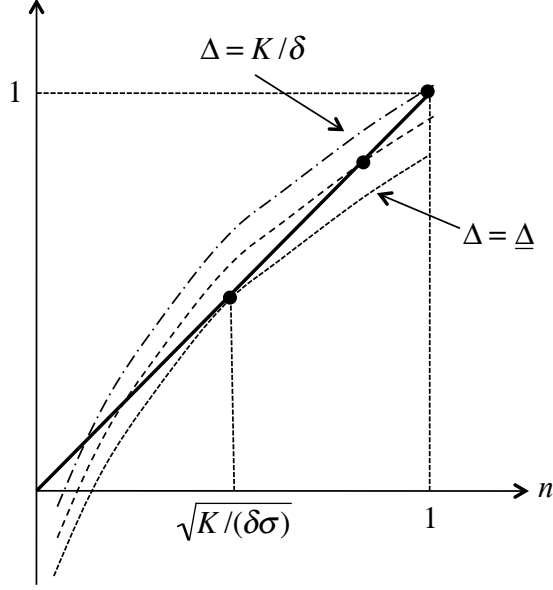
Figure 1: Fulfilled expectations equilibrium

other consumers, they buy the product as long as $r + \theta s - p_r \geq 0$, or $\theta \geq (p_r - r)/s \equiv \hat{\theta}$. If $0 < (p_r - r)/s < 1 - n$, the number of regular consumers is equal to $n_r = 1 - n - (p_r - r)/s$. As long as $n_r > \rho$, the entrepreneur's profit can be written as

$$\pi = \underbrace{p_1 n_c - K}_{=0} + \delta p_c n_c + \delta p_r n_r$$

$$= \delta p_r \left(1 - \frac{p_r - r}{s}\right) - \delta \Delta \frac{1}{2\sigma}\left(\sigma + \Delta + \sqrt{(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2}\right),$$

where the second line is obtained by substituting expression (3) for n_c, and Δ for $p_r - p_c$. It is equivalent to maximize π over p_r and p_c, or over p_r and Δ; we choose the latter option.

It is easily found that the first-order condition with respect to p_r yields the optimal value $p_r^* = (r + s)/2$, which implies that $\hat{\theta} = (p_r^* - r)/s = (s - r)/2s$.

The derivative of profit with respect to Δ is

$$\frac{d\pi}{d\Delta} = -\frac{\delta}{2\sigma}\left[\sigma + 2\Delta + \sqrt{(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2} + \frac{\Delta(\sigma+\Delta)}{\sqrt{(\sigma+\Delta)^2 - (\sigma+\underline{\Delta})^2}}\right]. \quad (4)$$

It is clear that the bracketed term is strictly positive for positive values

of Δ. Hence, any interior solution must be such that $\Delta < 0$ (i.e., that $p_r < p_c$, meaning that crowdfunders pay more than other consumers in period 2). Because of the constraint imposed by (2), this is only possible if $\underline{\Delta}$ is negative, which is equivalent to $K < (\delta\sigma)/4$. We therefore have to distinguish between two cases (we sketch the results here and refer the reader to the appendix for the detailed computations).

Case 1: $K < (\delta\sigma)/4$. In this case, we solve $d\pi/d\Delta = 0$ for Δ and find:

$$\Delta^* = \frac{4K - \delta\sigma}{2\delta}.$$

We verify that $K < (\delta\sigma)/4$ implies that $\Delta^* < 0$, i.e. that $p_c^* > p_r^*$: crowdfunders pay more than other consumers in period 2 (we will return to this below). We also compute that the number of crowdfunders is given by $n_c^* = 1/2$. Hence, at (p_r^*, Δ^*), consumers split into three groups: those with $\theta \in [0, (s-r)/2s]$ do not consume, those with $\theta \in [(s-r)/2s, 1/2]$ buy in period 2, and those with $\theta \in [1/2, 1]$ pre-order in period 1. We compute the equilibrium number of regular consumers as $n_r^* = 1/2 - (s-r)/2s = r/2s$. From Assumption 4, we have that $n_r^* > \rho$, which implies that $\sigma > 0$ as initially assumed. We then compute the optimal profit as

$$\pi = \delta\frac{(r+s)^2}{4s} + \frac{\delta\sigma}{4} - K. \tag{5}$$

In the present case, the capital requirement imposes no constraint whatsoever on the entrepreneur. To see this, let us first compute the total price paid by crowdfunders. It is equal to $p_1 + \delta p_c = \frac{\delta}{2}(r+s+\sigma)$. Next, we observe that this is exactly the price that the entrepreneur would set if it was only selling in period 1 a product of quality $(s+\sigma)$ to be delivered in period 2. Indeed, the indifferent consumer would be identified by θ_0 such that $-p + \delta(r + \theta_0(s+\sigma)) = 0$, which is equivalent to $\theta_0 = \frac{1}{s+\sigma}\left(\frac{1}{\delta}p - r\right)$. The entrepreneur would then maximize $\pi = \delta(p(1-\theta_0))$. It is easy to check that the optimal price is indeed $p = \frac{1}{2}\delta(r+s+\sigma)$.

Case 2: $K \geq (\delta\sigma)/4$. Here, $\Delta \geq 0$ under condition (2). Then, expression (4) is clearly negative, meaning that the optimal choice is the lowest admissible value of Δ, i.e., $\Delta = \underline{\Delta} > 0$. The intuition goes as follows: the higher capital requirement, combined to the fulfilled expectations requirement, forces the entrepreneur to give a discount to crowdfunders ($p_c < p_r$)

but the entrepreneur prefers to keep this discount as small as possible. The number of crowdfunders is then given by

$$\underline{n}_c = \frac{1}{2} + \frac{\underline{\Delta}}{2\sigma} = \sqrt{K/(\delta\sigma)} \geq \frac{1}{2}.$$

We see thus that the entrepreneur has to attract a larger number of crowd-funders than in the previous, unconstrained, case; this number grows with K (and remains smaller than unity as long as $K < \delta\sigma$).

As the number of crowdfunders grows, it is not clear whether the entrepreneur still finds it optimal to attract a sufficient number of regular consumers in period 2. It does so as long as $1 - \underline{n}_c - (p_r^* - r)/s \geq \rho$. As $p_r^* = (r + s)/2$, the latter condition is equivalent to

$$K \leq \delta\sigma \left(\frac{r + s}{2s} - \rho\right)^2 \equiv K_1(\sigma, \rho).$$

It can be checked that Assumptions 1 and 4 ensure that $K_1(\sigma, \rho)$ is comprised between $(\delta\sigma)/4$ and $\delta\sigma$; it is also clear that K_1 increases with σ and decreases with ρ. There are thus two subcases to consider.

Case 2a: $(\delta\sigma)/4 \leq K \leq K_1(\sigma, \rho)$. Here, as $n_r \geq \rho$, we have that $\sigma > 0$ for crowdfunders and we can compute the equilibrium value of p_c as $p_c^* = p_r^* - \underline{\Delta} = (r + s)/2 - 2\sigma\sqrt{K/(\delta\sigma)} + \sigma$ (which is positive under Assumption 3). The equilibrium profit is then equal to

$$\pi = \delta\frac{(r + s)^2}{4s} + \sqrt{\delta\sigma K} - 2K. \tag{6}$$

Case 2b: $K > K_1(\sigma, \rho)$. In this case, given $\underline{n}_c$ and p_r^*, the number of regular consumers is too low for crowdfunders to enjoy any additional utility through 'community benefits'. The entrepreneur faces then the following alternative: she either adjusts p_r so as to keep n_r above ρ and, thereby, $\sigma > 0$, or she accepts $n_r < \rho$, $\sigma = 0$ and maximizes a different objective function. We show in the appendix that the latter option is equivalent, in terms of equilibrium profits, to traditional funding. As for the former option, the entrepreneur sets the price p_r such that $n_r = \rho$, which is equivalent to

$$p_r = r + s - s\left(\sqrt{K/(\delta\sigma)} + \rho\right).$$

The price for crowdfunders is then computed as $p_c = p_r - \underline{\Delta} = r + s + \sigma - s\rho - (s + 2\sigma)\sqrt{K/(\delta\sigma)}$. It seems reasonable to exclude negative prices. We

have thus that this option is feasible as long as $p_c \geq 0$, which is equivalent to

$$K \leq \delta\sigma \left(\frac{r + s + \sigma - s\rho}{s + 2\sigma} \right)^2 \equiv K_2\left(\sigma, \rho\right).$$

We show in the appendix that Assumption 3 implies that $K_2\left(\sigma, \rho\right) > K_1\left(\sigma, \rho\right)$. Hence, for $K_1\left(\sigma, \rho\right) < K \leq K_2\left(\sigma, \rho\right)$, the option of setting p_r so that $n_r = \rho$ yields the following profit:

$$\pi = \delta \left(r + s - s \left(\sqrt{K/\left(\delta\sigma\right)} + \rho \right) \right) \left(\sqrt{K/\left(\delta\sigma\right)} + \rho \right) + \sqrt{\delta\sigma K} - 2K. \quad (7)$$

We will show below that the latter function decreases with K. At $K = K_2$, we have $p_c = 0$, $p_r > 0$ and $n_r = \rho > 0$; it follows that the entrepreneur still earns positive profits in this extreme case. The question remains, however, whether this option is more profitable than traditional funding or not. We will examine this issue in the next section but before, we collect our results and perform some comparative statics exercises.

Summary. Combining expressions (5) to (7), we can express equilibrium profits in the crowdfunding case as

$$\pi_C = \begin{cases} \delta \frac{(r+s)^2}{4s} + \frac{\delta\sigma}{4} - K & \text{for } K < \frac{\delta\sigma}{4}, \\ \delta \frac{(r+s)^2}{4s} + \sqrt{\delta\sigma K} - 2K & \text{for } \frac{\delta\sigma}{4} \leq K \leq K_1, \\ \delta \left(r + s - s\rho - s\sqrt{\frac{K}{\delta\sigma}} \right) \left(\sqrt{\frac{K}{\delta\sigma}} + \rho \right) \\ + \sqrt{\delta\sigma K} - 2K & \text{for } K_1 < K \leq K_2. \end{cases} \quad (8)$$

We show in the appendix that each segment of this profit function is a decreasing function of K and an increasing function of σ; moreover, the third segment is a decreasing function of ρ. We record these results in the following proposition.

Proposition 1 *The equilibrium profit under crowdfunding decreases with the capital requirement (K) and with the minimal number of regular consumers required to generate community benefits (ρ); it increases with the magnitude of community benefits (σ).*

The intuition behind these comparative static results is clear. The capital requirement has a twofold negative impact on profit: on the one hand, it makes production more expensive and on the other hand, it reduces the possibility to implement the optimal menu pricing scheme. An increase in

the minimal number of regular consumers required to generate community benefits also constraints price discrimination and thereby negatively affects profits. Indeed, when ρ increases, communuty benefits cannot be shared as broadly as before. Finally, if crowdfunders have a higher valuation for the community benefits, their willingness to pay increases and the entrepreneur is able to increase her margins.

3.4   Choice of funding method

Comparing expressions (8) and (1), we observe first that for small values of K $(K \leq (\delta\sigma)/4)$, crowdfunding clearly yields larger profits than traditional funding. The intuition is obvious: in this region of parameters, crowdfunding allows the entrepreneur to optimally price discriminate between the high-valuation crowdfunders and the remaining consumers. As the 'enhanced quality' σ comes at no cost for the entrepreneur, menu pricing performs better than the uniform pricing that prevails under traditional funding.

For larger values of K, however, the entrepreneur is constrained to implement corner solutions under crowdfunding. Here, σ is no longer a sort of 'manna from heaven' for the entrepreneur: several requirements constrain the prices that the entrepreneur can choose, which inevitably reduces her profits. The first constraining requirements are the network effects among crowdfunders and the imposition of fulfilled expectations. Nevertheless, crowdfunding still dominates for values of K comprised between $(\delta\sigma)/4$ and $K_1(\sigma,\rho)$: $\pi_C - \pi_T = \sqrt{\delta\sigma K} - K$, which is positive as $K \leq K_1(\sigma,\rho) < \delta\sigma$.

Yet, an additional constraint bites for $K_1(\sigma,\rho) < K \leq K_2(\sigma,\rho)$, namely the minimal number of regular consumers necessary to generate the community benefits for crowdfunders (combined with the non-negativity of prices). We show in the appendix that the profit under crowdfunding may fall under the profit under traditional funding when the capital requirement becomes large enough. More precisely, we find that for $K_1(\sigma,\rho) < K \leq K_2(\sigma,\rho)$, $\pi_C - \pi_T$ if and only if $K \leq K_3(\sigma,\rho)$, with

$$K_3(\sigma,\rho) \equiv \delta\sigma \left(\frac{s(r+s+\sigma-2s\rho) + \sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s + s^2 - r^2)}}{2s(s+\sigma)} \right)^2.$$

We show in the appendix that $K_3(\sigma,\rho) > K_1(\sigma,\rho)$ and that, for given values of the other parameters, there exists a value $\hat{\sigma}$ such that $K_2(\sigma,\rho) > K_3(\sigma,\rho)$ for $\sigma < \hat{\sigma}$ and $K_2(\sigma,\rho) < K_3(\sigma,\rho)$ for $\sigma > \hat{\sigma}$. Moreover, it is intuitive that the three thresholds increase with σ and decrease with ρ.
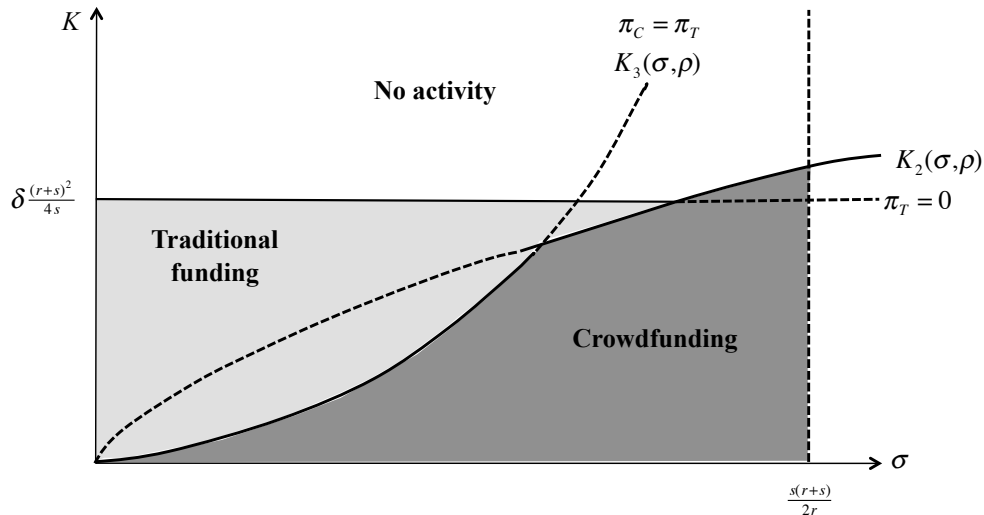
Figure 2: Choice of funding method

We collect our results in the following proposition and we depict them in Figure 2.

Proposition 2 *In situations where an entrepreneur can use crowdfunding and pre-sales to induce self-selection of high paying consumers, crowdfunding is preferred over traditional funding if the capital requirement is below some threshold value (i.e., $K \leq \min\{K_2(\sigma, \rho), K_3(\sigma, \rho)\}$). This condition becomes less stringent as community benefits become more important, either because their magnitude (σ) increases or because they are generated more easily (ρ decreases).*

One important implication of Proposition 1 is that the level of additional benefits accruing to the pre-ordering crowd (i.e., σ) must be sufficiently large. If the crowd does not enjoy any of such benefits or utility, crowdfunding does not yield any benefits over traditional funding for the entrepreneur. The parameter σ can be seen as additional utility or benefits from a community-based experience. Then, the lack of a community would result in a value of σ equal to zero. An important implication is the need for the entrepreneur to identify and target this community. While consumers with a high willingness to pay for the product may self-select into the community, the entrepreneur still needs to ensure that the "crowd" can

22

generate these additional benefits. The following managerial lesson can thus be drawn from our analysis: *entrepreneurs who cannot identify or create a community around their products so that this community enjoys additional benefits, will hardly ever opt for crowdfunding.* Indeed, we observe on Figure 2 that as σ decreases, the range of values of K for which crowdfunding is preferred narrows down.

The previous finding is consistent with many observed crowdfunding initiatives. Indeed, while some offer monetary rewards, an important other form of reward is recognition or credits offered to crowdfunders. The importance of non-monetary benefits for crowdfunders is further stressed by the observation that *at equilibrium, crowdfunders always end up paying a larger total price than regular consumers.* To see this, we compute the difference $(p_1^* + \delta p_c^*) - \delta p_r^*$ in the three regimes. We find that this difference is equal to $\delta\sigma/2$ for $K < \delta\sigma/4$, and to $\delta\sigma \left(1 - \sqrt{K/(\delta\sigma)} \right)$ for $K \geq \delta\sigma/4$. As crowdfunding is not feasible for $K > \delta\sigma$, we check that $(p_1^* + \delta p_c^*) > \delta p_r^*$ for all admissible values of the parameters. This result is not surprising insofar as crowdfunders are high-valuation consumers and that their willingness to pay is further enhanced by the community benefits.

3.5 Take the money and run

In the previous analysis, we have abstracted away the possibility that the entrepreneur could "take the money and run" at the start of period 2, i.e., to collect p_1 from the crowdfunders without incurring the fixed cost and thus, without producing the product. That is, we implicitly assumed that the entrepreneur had some form of commitment at her disposal to guarantee her second-period activity. Absent such commitment device, consumers would only be convinced that production will take place if it is indeed in the entrepreneur's best interest; otherwise, no consumer would agree to pre-order the product and crowdfunding would fail. The entrepreneur's net profit when producing must then be at least as large as the total amount that is collected at the end of period 1, i.e., K. We show in the appendix that

$$\pi_C \geq K \Leftrightarrow K \leq K_4 (\sigma, \rho)$$

23

with[14]

$$K_4\left(\sigma,\rho\right) \equiv \begin{cases} \delta\sigma\left(\frac{(r+s(1-2\rho)+\sigma)+\sqrt{-12s\sigma\rho^2+4\sigma(3r+2s)\rho+(r+s+\sigma)^2}}{2(s+3\sigma)}\right)^2 \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{for } 0 < \sigma \leq \sigma_1, \\[2ex] \frac{\delta\sigma}{36}\left(1+\sqrt{1+3\frac{(r+s)^2}{\sigma s}}\right)^2 \quad \text{for } \sigma_1 \leq \sigma \leq \frac{s(r+s)}{2r}, \end{cases}$$

$$\sigma_1 \equiv \frac{s(r+s)^2}{(r+s-2s\rho)(3r+s-6s\rho)}.$$

As depicted on Figure 3, $K_4\left(\sigma,\rho\right)$ lies below the minimum of $K_2\left(\sigma,\rho\right)$ and $K_3\left(\sigma,\rho\right)$ for sufficiently large values of σ and sufficiently large values of K.[15] This implies that when consumers may fear that the entrepreneur could take the money and run, crowdfunding becomes harder to implement: there exists indeed a region of parameters (characterized by high values of σ and K) where consumers will not agree to pre-order the product as they rightfully anticipate that the entrepreneur will not put the product on the market. For these parameters, traditional funding appears as the only option (assuming, of course, that banks are better equipped than crowdfunders to prevent the entrepreneur's default). We record this result in the following proposition.

Proposition 3 *When the entrepreneur has no credible way to commit that she will not run away with the money collected in period 1, crowdfunding is less likely to be preferred to traditional funding.*

4   Concluding remarks

This paper sheds light on crowdfunding practices of entrepreneurial activities. It stresses the need for building a community that ultimately enjoys additional private benefits from their participation to make crowdfunding a viable alternative to investor- or creditor-based funding such as through banks, business angels or even venture capital. In setting up the initiative, the entrepreneur potentially faces the following trade-off. Crowdfunding allows for price discrimination if pre-ordering is used. The capacity to optimally implement price-discrimination between pre-ordering consumers (the

---

[14]The second branch of K_4 only obtains for sufficiently small values of ρ.

[15]Logically, $K_4\left(\sigma,\rho\right)$ (where $\pi_C = K$) intersects with $K_3\left(\sigma,\rho\right)$ (where $\pi_C = \pi_T$) at $K = \delta\left(r+s\right)^2/\left(8s\right)$, which is the value of K such that $\pi_T = K$. Above this threshold, we have that for any σ, $K_4\left(\sigma,\rho\right) < K_3\left(\sigma,\rho\right)$.
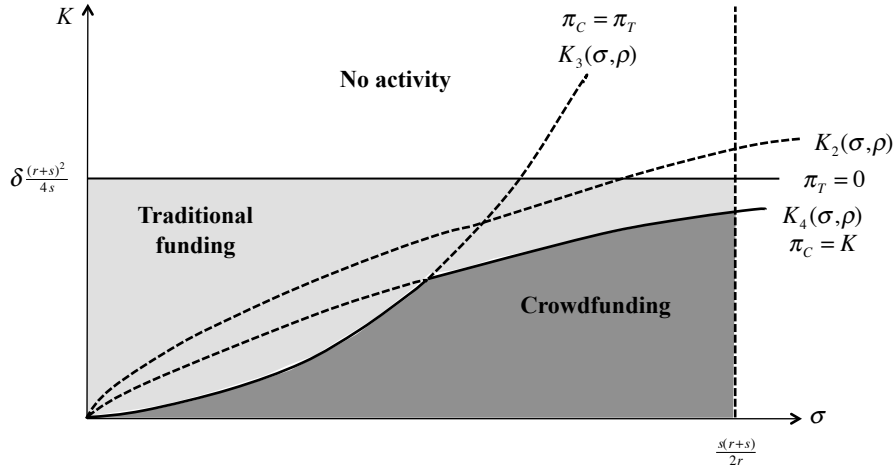
Figure 3: Funding method if the entrepreneur can take the money and run

crowdfunders) and other consumers may however be constrained by the amount of capital that the entrepreneur needs to raise to cover the up-front (fixed) costs. Whenever this amount exceeds some threshold, the distortion in the price discrimination becomes excessive, in which case the profitability of the crowdfunding initiative is reduced and the entrepreneur may be better off approaching a single, larger investor (a bank or a large equity investor) who can cover the full costs on its own.

To our knowledge, this is the very first study offering a theoretical analysis of crowdfunding. It also highlights new follow-up research questions on the topic. For instance, an interesting avenue for future research is to incorporate the fact that the crowdfunders can at times also participate in strategic decisions or even have voting rights. In this case, control rights and voting power become an additional benefit for the participating crowd. Crowdfunding through pre-ordering will have a very different effect on information and voting results than if the crowd purchases equity in the entrepreneurial firm. Also, outcomes of votes can provide valuable insights into the optimal design of products if the voting community is representative for the overall population of end-consumers.

Future works may further explore information motivations of entrepreneurs. Indeed, while the primary goal of crowdfunding is certainly to raise money, it may also help firms in testing, promoting and marketing their products,

in gaining a better knowledge of their consumers' tastes, or in creating new products or services altogether. In this sense, crowdfunding can be used as a promotion device, as a means to support mass customization or user-based innovation, or as a way for the producer to gain a better knowledge of the preferences of its consumer. Crowdfunding seems thus to have implications that go beyond the financial sphere of an organization: it also affects the flow of information between the organization and its customers.

In any case, a strong advantage of this form of financing is the attention that the entrepreneur may attract on his/her project or company. This can become a vital asset for many of them, especially for artists or entrepreneurs in need to present their talent and product to the crowd (as potential customers). In other cases, it is a unique way to validate original ideas in front of a specifically targeted audience. This may in turn provide insights into market potential of the product or service offered. From this perspective, crowdfunding may be viewed as a broader concept than purely raising funds: it is a way to develop corporate activities through the process of fundraising.

Also, several platforms have emerged recently, such as *IndieGoGo*, *Kickstarter*, *Sandawe*, *SellaBand*, *MyMajorCompany* and *Artistshare*. These platforms intermediate between entrepreneurs and potential crowdfunders. Therefore, a distinction can be made between direct and indirect fundraising because at times entrepreneurs make use of such crowdfunding platforms instead of seeking direct contact with the crowd. These platforms share some similarities with online lending markets (Everett, 2008; Freedman and Jin, 2010); while the latter more prominently target social entrepreneurship, crowdfunding platforms have a broader scope of entrepreneurial initiatives. Our understanding of the role played by platforms is still limited; it is worth investigating the extent to which platforms increase the chances of success of crowdfunding initiatives or solve asymmetric information issues. As an example, for crowdfunders, platforms may facilitate in learning the quality of the product through the possible interaction between crowdfunders (e.g., via other crowdfunder comments on a forum) or by observing the contributions of other crowdfunders. More research could be done along the line of peer effects as it relates to crowdfunding platforms, as suggested by Ward and Ramachandran (2010).

From a more general perspective, crowdfunding practices raise questions with respect to corporate governance and investor protection issues if most

individuals only invest tiny amounts. Crowdfunders are most likely to be offered very little investor protection. This may lead to corporate governance issues, which in turn may entail reputation concerns if some cases of fraud or bad governance are uncovered. Crowdfunders have very little scope to intervene to protect their interests as stakeholders. Moreover, the fact that their investment is small is likely to create a lack of incentive to intervene. Therefore, trust-building is an essential ingredient for any successful crowdfunding initiative. It is also not a surprise that many of the observed crowdfunded initiatives are either project-based or based on donations. In many cases, the financial return seems to be of secondary concern for those who provide funds. This suggests that crowdfunders care about social reputation or enjoy private benefits from participating in the success of the initiative (Glaeser and Shleifer, 2001; Ghatak and Mueller, 2011).

5 Appendix

We give here the details of the mathematical developments that we sketched in the text.

5.1 Optimal prices under crowdfunding

Case 1. If $K < (\delta\sigma)/4$, then there may exist a value of $\Delta \geq \underline{\Delta}$ that solves $d\pi/d\Delta = 0$, or

$$\sigma + 2\Delta + \sqrt{(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2} + \frac{\Delta(\sigma + \Delta)}{\sqrt{(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2}} = 0 \Leftrightarrow$$

$$\sqrt{(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2} = \frac{(\sigma + \underline{\Delta})^2}{\sigma + 2\Delta} - (\sigma + \Delta) \tag{9}$$

As long as the RHS is positive, we can take the square of the two sides of the equality:

$$(\sigma + \Delta)^2 - (\sigma + \underline{\Delta})^2 = \frac{(\sigma + \underline{\Delta})^4}{(\sigma + 2\Delta)^2} + (\sigma + \Delta)^2 - 2\frac{(\sigma + \Delta)(\sigma + \underline{\Delta})^2}{\sigma + 2\Delta},$$

which, after simplification, yields

$$\Delta^* = \frac{1}{2\sigma}\left[(\sigma + \underline{\Delta})^2 - \sigma^2\right] = \frac{1}{2\delta}(4K - \delta\sigma).$$

We still need to check whether condition (2) is satisfied:

$$\frac{1}{2\sigma}\left[(\sigma + \underline{\Delta})^2 - \sigma^2\right] > \underline{\Delta} \Leftrightarrow 2\sigma\underline{\Delta} + \underline{\Delta}^2 > 2\sigma\underline{\Delta},$$

which is true. We also need to check that the RHS of expression (9) is positive, as we assumed it. We compute

$$\frac{(\sigma + \underline{\Delta})^2}{\sigma + 2\underline{\Delta}} > (\sigma + \underline{\Delta}) \Leftrightarrow \frac{4\sigma K/\delta}{4K/\delta} > \frac{4K + \delta\sigma}{2\delta} \Leftrightarrow 2\delta\sigma > 4K + \delta\sigma \Leftrightarrow K < \delta\sigma/4$$

which is true.

To proceed, we compute

$$
\begin{aligned}
\sigma + \Delta^* &= \frac{1}{2\sigma}\left[(\sigma + \underline{\Delta})^2 + \sigma^2\right], \\
\sqrt{(\sigma + \Delta^*)^2 - (\sigma + \underline{\Delta})^2} &= \frac{1}{2\sigma}\left[\sigma^2 - (\sigma + \underline{\Delta})^2\right].
\end{aligned}
$$

It follows that

$$n^* = \frac{1}{2\sigma}\left(\frac{1}{2\sigma}\left[(\sigma + \underline{\Delta})^2 + \sigma^2\right] + \frac{1}{2\sigma}\left[\sigma^2 - (\sigma + \underline{\Delta})^2\right]\right) = \frac{1}{2}.$$

Recall that we need

$$\frac{p_r - r}{s} < 1 - n \Leftrightarrow \frac{s - r}{s} < \frac{1}{2} \Leftrightarrow s < 2r,$$

which is guaranteed by Assumption 1.

We can now compute the optimal profit:

$$
\begin{aligned}
\pi &= \delta\frac{(r + s)^2}{4s} - \delta\Delta^*\frac{1}{2\sigma\delta}\left(\delta(\sigma + \Delta^*) - \delta\Delta^*\right) \\
&= \delta\frac{(r + s)^2}{4s} + \frac{\delta\sigma}{4} - K.
\end{aligned}
$$

Case 2. We first have to establish under which condition there is a sufficient number of regular consumers. We need $1 - \underline{n}_c - (p_r^* - r)/s \geq \rho$. As $\underline{n}_c = \sqrt{K/(\delta\sigma)}$ and $p_r^* = (r + s)/2$, the condition can be rewritten as

$$1 - \sqrt{\frac{K}{\delta\sigma}} - \frac{s - r}{2s} \geq \rho \Leftrightarrow K \leq \delta\sigma\left(\frac{r + s}{2s} - \rho\right)^2 \equiv K_1(\sigma, \rho).$$

We compute

$$
\begin{aligned}
K_1(\sigma, \rho) - \frac{\delta\sigma}{4} &= \frac{\sigma\delta(r + 2s - 2s\rho)(r - 2s\rho)}{4s^2} > 0, \\
\delta\sigma - K_1(\sigma, \rho) &= \frac{\sigma\delta(r + 3s - 2s\rho)(s + 2s\rho - r)}{4s^2} > 0,
\end{aligned}
$$

which are both positive because of Assumptions 1 ($s > r$) and 4 ($\rho < r/(2s)$).

Case 2a: $(\delta\sigma)/4 \le K \le K_1(\sigma,\rho)$. We need to show that $p_c^* = p_r^* - \underline{\Delta} = (r+s)/2 - 2\sigma\sqrt{K/(\delta\sigma)} + \sigma \ge 0$. This condition is equivalent to

$$K \le \delta\sigma\left(\frac{r+s+2\sigma}{4\sigma}\right)^2 \equiv K_5(\sigma,\rho).$$

We have $K_5(\sigma,\rho) > K_1(\sigma,\rho)$ if and only if

$$\frac{r+s+2\sigma}{4\sigma} > \frac{r+s}{2s} - \rho \Leftrightarrow \rho > -\frac{s(r+s)-2r\sigma}{4s\sigma} \equiv \tilde{\rho}.$$

Assumption 3, i.e., $\sigma < s(r+s)/(2r)$, implies that $\tilde{\rho} < 0$ and hence, that the inequality is satisfied, meaning that $p_c^* > 0$ when $K \le K_1(\sigma,\rho)$.

Case 2b: $K > K_1(\sigma,\rho)$. We showed in the text that $p_c \ge 0$ as long as

$$K \le \delta\sigma\left(\frac{r+s+\sigma-s\rho}{s+2\sigma}\right)^2 \equiv K_2(\sigma,\rho).$$

We have that $K_2(\sigma,\rho) > K_1(\sigma,\rho)$ is equivalent to

$$\frac{r+s+\sigma-s\rho}{s+2\sigma} > \frac{r+s}{2s} - \rho \Leftrightarrow \frac{s(r+s)-2r\sigma+4s\sigma\rho}{2s(s+2\sigma)} > 0,$$

which is true under Assumption 3 as $2r\sigma < s(r+s)$.

We also need to show that when the entrepreneur accepts $n_r < \rho$ (which implies $\sigma = 0$), she achieves the same profit as under traditional funding. Suppose first that the entrepreneur decides to attract no regular consumer whatsoever. She can do so by setting $p > r + s$. In that case, a consumer decides to pre-order the product (which is the only option) if

$$-\frac{K}{n_c^e} + \delta(r+\theta s - p_c) \ge 0 \Leftrightarrow \theta \ge \frac{K}{\delta s n_c^e} - \frac{r-p_c}{s}.$$

As we require fulfilled expectations, we must have

$$n_c = 1 - \frac{K}{\delta s n_c} + \frac{r-p_c}{s}.$$

Solving the latter equation and keeping the largest root, we have

$$n = \frac{\delta(r+s-p_c) + \sqrt{\delta^2(r+s-p_c)^2 - 4\delta s K}}{2\delta s},$$

which is valid as long as $p_c \le r + s - 2\sqrt{sK/\delta}$.

The entrepreneur choose p_c to maximize

$$\pi = p_c \frac{\delta (r + s - p_c) + \sqrt{\delta^2 (r + s - p_c)^2 - 4\delta s K}}{2\delta s}.$$

Solving for the first-oder condition, we find the optimal price as

$$p_c = \frac{r + s}{2} - \frac{2s}{\delta (r + s)} K$$

and we check that it is indeed no larger than $r + s - 2\sqrt{sK/\delta}$ as required. We can then compute the number of consumers as $n = (r + s) / (2s)$ and the optimal profit as:

$$\pi = \delta \left(\frac{r + s}{2} - \frac{2s}{\delta (r + s)} K \right) \frac{r + s}{2s} = \delta \frac{(r + s)^2}{4s} - K = \pi_T.$$

Alternatively, the entrepreneur could still attract some regular consumers (but less than ρ). To do so, she must set $p_r < r + s$. A consumer would then choose to pre-order the product if

$$-\frac{K}{n_c^e} + \delta (r + \theta s - p_c) \geq \delta (r + \theta s - p_r) \Leftrightarrow \frac{K}{n_c^e} \leq \delta (p_r - p_c),$$

which is clearly impossible if $p_c \geq p_r$. Suppose then $p_c < p_r$. The latter condition can we rewritten as

$$n_c^e \geq \frac{K}{\delta (p_r - p_c)}.$$

Expectations are no longer an issue in the present case. Taking $n_c = K/(\delta (p_r - p_c))$, we express the entrepreneur's profit as

$$\pi = \delta p_c \frac{K}{\delta (p_r - p_c)} + \delta p_r \left(1 - \frac{K}{\delta (p_r - p_c)} - \frac{p_r - r}{s} \right) = \delta p_r \left(1 - \frac{p_r - r}{s} \right) - K,$$

which is exactly the same profit function as under traditional funding; this completes our proof.

5.2 Proof of Proposition 1

We have that profits under crowdfunding are equal to π_{C1} for $K < \frac{\delta \sigma}{4}$, to π_{C2} for $\frac{\delta \sigma}{4} \leq K \leq K_1$, and to π_{C3} for $K_1 < K \leq K_2$, with

$$\begin{aligned}
\pi_{C1} &= \delta \frac{(r+s)^2}{4s} + \frac{\delta \sigma}{4} - K, \\
\pi_{C2} &= \delta \frac{(r+s)^2}{4s} + \sqrt{\delta \sigma K} - 2K, \\
\pi_{C3} &= \delta \left(r + s - s\rho - s\sqrt{\frac{K}{\delta \sigma}} \right) \left(\sqrt{\frac{K}{\delta \sigma}} + \rho \right) + \sqrt{\delta \sigma K} - 2K.
\end{aligned}$$

It is obvious that π_{C1} increases with σ and decreases with K. As for π_{C2}, it clearly increases with σ while its derivative with respect to K is equal to

$$\frac{\partial \pi_{C2}}{\partial K} = \sqrt{\delta\sigma}\frac{1}{2\sqrt{K}} - 2.$$

The latter expression is negative if $K > \delta\sigma/16$, which is the case here. Regarding π_{C3}, we first compute:

$$\frac{\partial \pi_{C3}}{\partial K} = \frac{(r + s + \sigma - 2s\rho)\sqrt{\sigma\delta K} - 2(s + 2\sigma)K}{2\sigma K}.$$

The latter expression is negative if and only if

$$K > \delta\sigma \left(\frac{r + s + \sigma - 2s\rho}{2(s + 2\sigma)}\right)^2 \equiv K_6(\sigma, \rho).$$

We consider here values of $K > K_1(\sigma, \rho)$. Let us show that Assumption 4, i.e., $\rho < r/(2s)$, makes sure that $K_1(\sigma, \rho) > K_6(\sigma, \rho)$. The latter condition is equivalent to

$$\frac{r + s}{2s} - \rho - \frac{r + s + \sigma - 2s\rho}{2(s + 2\sigma)} = \frac{1}{2}\sigma\frac{2(r - 2s\rho) + s}{s(s + 2\sigma)} > 0.$$

Second, we have

$$\frac{\partial \pi_{C3}}{\partial \sigma} = \frac{2sK - (r - 2s\rho + s - \sigma)\sqrt{\sigma\delta K}}{2\sigma^2}.$$

If $\sigma > r - 2s\rho + s$, then the latter expression is clearly positive. Otherwise, it is positive as long as

$$K > \delta\sigma \left(\frac{r - 2s\rho + s - \sigma}{2s}\right)^2 \equiv K_7(\sigma, \rho).$$

We show again that $K_1(\sigma, \rho) > K_7(\sigma, \rho)$, which is equivalent to

$$\frac{r + s}{2s} - \rho - \frac{r - 2s\rho + s - \sigma}{2s} = \frac{\sigma}{2s} > 0.$$

Finally, we compute

$$\frac{\partial \pi_{C3}}{\partial \rho} = \delta\left(r + s - 2s\rho - 2s\sqrt{\frac{K}{\sigma\delta}}\right).$$

This expression is negative for

$$K > \delta\sigma \left(\frac{r + s - 2s\rho}{2s}\right)^2 = K_1(\sigma, \rho),$$

which completes the proof.

5.3   Proof of Proposition 2

We first derive the value of K such that $\pi_{3C} = \pi_T$.

$$\delta \left(r + s - s\rho - s\sqrt{\frac{K}{\delta\sigma}}\right)\left(\sqrt{\frac{K}{\delta\sigma}} + \rho\right) + \sqrt{\delta\sigma K} - 2K = \delta\frac{(r+s)^2}{4s} - K \Leftrightarrow$$

$$-(s+\sigma)K + (r+s+\sigma-2s\rho)\sqrt{\delta\sigma}\sqrt{K} + \delta\sigma(r+s-s\rho)\rho - \delta\sigma\frac{(r+s)^2}{4s} = 0.$$

This second-degree polynomial in $\sqrt{K}$ has real roots as long as

$$(r+s+\sigma-2s\rho)^2\delta\sigma + 4(s+\sigma)\left(\delta\sigma(r+s-s\rho)\rho - \delta\sigma\frac{(r+s)^2}{4s}\right) =$$
$$\frac{\sigma^2\delta}{s}\left(4s\rho(r-s\rho) + s\sigma + s^2 - r^2\right) > 0,$$

which is satisfied under Assumptions 1 and 4. The roots are then given by

$$\sqrt{K} = \sqrt{\delta\sigma}\frac{s(r+s+\sigma-2s\rho) \pm \sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)}}{2s(s+\sigma)}.$$

Some lines of computations establish that both roots are positive and that $\delta\sigma(r+s-s\rho)\rho < \delta\sigma\frac{(r+s)^2}{4s}$ under our assumptions. It follows that $\pi_{C3} > \pi_T$ if and only if

$$\delta\sigma\left(\frac{s(r+s+\sigma-2s\rho) - \sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)}}{2s(s+\sigma)}\right)^2 < K$$
$$< \delta\sigma\left(\frac{s(r+s+\sigma-2s\rho) + \sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)}}{2s(s+\sigma)}\right)^2 = K_3(\sigma,\rho).$$

We establish now that $K_1(\sigma,\rho)$ lies between these two bounds. First, the upper bound (i.e., K_3) being larger than $K_1(\sigma,\rho)$ is equivalent to

$$\frac{s(r+s(1-2\rho)+\sigma) + \sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)}}{2s(s+\sigma)} > \frac{r+s}{2s} - \rho \Leftrightarrow$$
$$\sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)} > \sigma(r-2s\rho) \Leftrightarrow$$
$$\sigma(s-r+2s\rho)(r-2s\rho+s)(s+\sigma) > 0,$$

which is satisfied under Assumptions 1 ($s > r$) and 4 ($\rho < r/(2s)$). As for the lower bound being smaller than $K_1(\sigma,\rho)$, this is so if

$$\frac{s(r+s(1-2\rho)+\sigma) - \sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)}}{2s(s+\sigma)} < \frac{r+s}{2s} - \rho \Leftrightarrow$$
$$\sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)} > -\sigma(r-2s\rho),$$

which is clearly satisfied under Assumption 4. We can thus state that when $K > K_1\left(\sigma, \rho\right)$,

$$\pi_C > \pi_T \Leftrightarrow K \leq \delta\sigma \left(\frac{s(r+s+\sigma-2s\rho)+\sqrt{\sigma s(4s\rho(r-s\rho)+\sigma s+s^2-r^2)}}{2s(s+\sigma)}\right)^2 = K_3\left(\sigma, \delta\right).$$

We show next that for given values of the other parameters, there exists a value $\hat{\sigma}$ such that $K_2\left(\sigma, \rho\right) > K_3\left(\sigma, \rho\right)$ for $\sigma < \hat{\sigma}$ and $K_2\left(\sigma, \rho\right) < K_3\left(\sigma, \rho\right)$ for $\sigma > \hat{\sigma}$. For $\delta\sigma > 0$, we have

$$K_2\left(\sigma, \rho\right) - K_3\left(\sigma, \rho\right) \propto \frac{r+s+\sigma-s\rho}{s+2\sigma} - \frac{s(r+s(1-2\rho)+\sigma)+\sqrt{\sigma s(-4s^2\rho^2+4rs\rho+s^2-r^2+\sigma s)}}{2s(s+\sigma)}.$$

The latter expression tends to $\left(r + s\right)/\left(2s\right)$ when σ tends to zero, and to $-1/2$ when σ tends to infinity. As both K_2 and K_3 are increasing functions of σ, there exists a value of σ below which $K_2 > K_3$ and above which $K_3 > K_2$. Numerical simulations suggest that this cutoff value of σ is smaller than $s\left(r + s\right)/\left(2r\right)$, i.e., the upper bound we have assumed for σ.

5.4   Proof of Proposition 3

Recall that profits under crowdfunding are equal to π_{C1} for $K < \frac{\delta\sigma}{4}$, to π_{C2} for $\frac{\delta\sigma}{4} \leq K \leq K_1$, and to π_{C3} for $K_1 < K \leq K_2$, with

$$\begin{aligned}
\pi_{C1} &= \delta\frac{(r+s)^2}{4s} + \frac{\delta\sigma}{4} - K, \\
\pi_{C2} &= \delta\frac{(r+s)^2}{4s} + \sqrt{\delta\sigma K} - 2K, \\
\pi_{C3} &= \delta\left(r + s - s\rho - s\sqrt{\frac{K}{\delta\sigma}}\right)\left(\sqrt{\frac{K}{\delta\sigma}} + \rho\right) + \sqrt{\delta\sigma K} - 2K.
\end{aligned}$$

We need to check under which conditions $\pi_C \geq K$. We first have that $\pi_{C1} \geq K$ if and only if

$$\delta\frac{(r+s)^2}{4s} + \frac{\delta\sigma}{4} > 2K \iff K < \frac{1}{2}\left(\delta\frac{(r+s)^2}{4s} + \frac{\delta\sigma}{4}\right).$$

The latter threshold is larger than $\frac{\delta\sigma}{4}$ provided that

$$\frac{1}{2}\left(\delta\frac{(r+s)^2}{4s} + \frac{\delta\sigma}{4}\right) - \frac{\delta\sigma}{4} = \frac{\delta}{8s}\left((r+s)^2 - \sigma s\right) > 0.$$

The latter condition is satisfied because of Assumptions 1 and 3; we have indeed that $s\left(r + s\right)/\left(2r\right) < \left(r + s\right)^2/s$. It follows that in the space of parameters that we consider, $\pi_{C1} > K$.

Next, we find that $\pi_{C2} \geq K$ if and only if

$$-3K + \sqrt{\delta\sigma}\sqrt{K} + \delta\frac{(r+s)^2}{4s} > 0.$$

This polynomial has one negative and one positive real root; as the polynomial is positive for $K = 0$, we have that the condition for $\pi_{C2} \geq K$ is

$$\sqrt{K} \leq \tfrac{1}{6}\left(\sqrt{\delta\sigma} + \sqrt{\delta\sigma + 3\delta\tfrac{(r+s)^2}{s}}\right)$$

or equivalently

$$K \leq \tfrac{\delta\sigma}{36}\left(1 + \sqrt{1 + 3\tfrac{(r+s)^2}{\sigma s}}\right)^2 \equiv K_8(\sigma, \rho).$$

Is the latter condition more or less stringent than $K \leq K_1(\sigma, \rho)$? We compute

$$K_1 > K_8 \Leftrightarrow \tfrac{r+s}{2s} - \rho > \tfrac{1}{6}\left(1 + \sqrt{1 + 3\tfrac{(r+s)^2}{\sigma s}}\right)$$

$$\Leftrightarrow \sigma > \tfrac{s(r+s)^2}{(3r+s-6s\rho)(r+s-2s\rho)} = \sigma_1(\rho).$$

Whether $\sigma_1(\rho)$ is above or below $s(r + s)/(2r)$ depends on the value of ρ. It can be checked that $\sigma_1(\rho)$ increases with ρ and $\sigma_1(0) = \tfrac{s(r+s)}{3r+s} < \tfrac{s(r+s)}{2r}$ while $\sigma_1\left(\tfrac{r}{2s}\right) = \tfrac{(r+s)^2}{s} > \tfrac{s(r+s)}{2r}$. The exact cutoff value is $0 < \hat{\rho} < \tfrac{r}{2s}$, with

$$\hat{\rho} = \frac{3r + 2s - \sqrt{6r^2 + 6rs + s^2}}{6s}.$$

Finally, we find that $\pi_{C3} \geq K$ if and only if

$$\delta\left(r + s - s\rho - s\sqrt{\tfrac{K}{\delta\sigma}}\right)\left(\sqrt{\tfrac{K}{\delta\sigma}} + \rho\right) + \sqrt{\delta\sigma K} \geq 3K.$$

Developing this inequality, we find that it is equivalent to

$$K < \delta\sigma\left(\frac{(r+s(1-2\rho)+\sigma)+\sqrt{-12s\sigma\rho^2+4\sigma(3r+2s)\rho+(r+s+\sigma)^2}}{2(s+3\sigma)}\right)^2 = K_9(\sigma, \rho).$$

Unsurprisingly, we find that $K_9 > K_1$ for $\sigma < \sigma_1(\rho)$. We have thus that $K_8(\sigma, \rho)$ and $K_9(\sigma, \rho)$ constitute the two branches of $K_4(\sigma, \rho)$ as described in the text.

References

[1] Agrawal, A., C. Catalini, and A. Goldfarb, 2011. The Geography of Crowdfunding. NBER Working Paper, No. 16820.

[2] Berger, A., and G. Udell, 1995. Relationship Lending and Lines of Credit in Small Firm Finance. *Journal of Business* 68, 351-381.

[3] Bhargava, H.K. and Choudhary, V., 2001. Second-degree price discrimination for information goods under nonlinear utility functions. In *Proceedings of the 34th Hawaii International Conference on System Sciences.*

[4] Bhidé, A., 1992. Bootstrap Finance: The Art of Start-ups. *Harvard Business Review* 70 (66), 109-117.

[5] Brabham D.C., 2008. Crowdsourcing as a Model for Problem Solving: An Introduction and Cases. *Convergence: The International Journal of Research into New Media Technologies* 14, 75- 90.

[6] Cadsby, C.B. and E. Maynes, 1999. Voluntary Provision of Threshold Public Goods with Continuous Contributions: Experimental Evidence. *Journal of Public Economics* 71, 53–73.

[7] Casamatta, C., and C. Haritchabalet, 2010. Dealing with Venture Capitalists: Shopping Around or Exclusive Negotiation. Working paper.

[8] Cassar, G., 2004. The Financing of Business Start-ups. *Journal of Business Venturing* 19, 261-283.

[9] Cosh, A., D. Cumming and A. Hughes, 2009. Outside Entrepreneurial Capital. *Economic Journal* 119, 1494-1533.

[10] Dana, J., 1998. Advance-Purchase Discounts and Price Discrimination in Competitive Markets, *Journal of Political Economy* 106, 395-422.

[11] Dana, J., 1999. Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure, *Rand Journal of Economics* 30, 632-660.

[12] Dana, J., 2001. Monopoly Price Dispersion Under Demand Uncertainty, *International Economic Review* 42, 649-670.

[13] Ebben, J., and A. Johnson, 2006. Bootstrapping in Small Firms: An Empirical Analysis of Change over Time. *Journal of Business Venturing* 21, 851-865.

[14] Everett, C.R., 2008. Group Membership, Relationship Banking and Loan Default Risk: The Case of Online Social Lending. Mimeo.

[15] Fershtman, C., and N. Gandal, 2011. A Brief Survey of the Economics of Open Source Software. CEPR Discussion Paper No. DP8434.

[16] Freedman, S., and G.Z. Jin, 2011. Learning by Doing with Asymmetric Information: Evidence from Prosper.com. NBER Working Paper, No. 16855.

[17] Gale, I., and T. Holmes, 1992. The Efficiency of Advance-Purchase Discounts in the Presence of Aggregate Demand Uncertainty, *International Journal of Industrial Organization* 10, 413-37.

[18] Gale, I., and T. Holmes, 1993. Advance-Purchase Discounts and Monopoly Allocation of Capacity, *American Economic Review* 83, 135-146.

[19] Ghatak, M. and H. Mueller, 2011. Thanks for nothing? Not-for-profits and motivated agents, *Journal of Public Economics*, 95, pp. 94-105.

[20] Glaeser, E.L., and A. Shleifer, 2001. Not-for-Profit Entrepreneurs. *Journal of Public Economics* 81, 99-115.

[21] Gradstein, M., 1994. Efficient Provision of a Discrete Public Good. *International Economic Review* 35, 877-897.

[22] Hellmann, T., 2007. Entrepreneurs and the Process of Obtaining Resources. *Journal of Economics and Management Strategy* 16, 81-109.

[23] Howe, J., 2008. *Crowdsourcing. Why the Power of the Crowd is Driving Future of Business*. New York: Three Rivers Press.

[24] Katz, M. and Shapiro, C., 1985. Network Externalities, Competition and Compatibility. *American Economic Review* 75, 424-440.

[25] Kleemann, F., G.G. Voß and K. Rieder, 2008. Un(der)paid Innovators: The Commercial Utilization of Consumer Work through Crowdsourcing. *Science, Technology & Innovation Studies* 4, 5-26.

[26] Larralde, B., and A. Schwienbacher, 2010. Crowdfunding of Small Entrepreneurial Ventures. Book chapter for "Entrepreneurial Finance" (Ed. D.J. Cumming), forthcoming at Oxford University Press.

[27] Mussa, M., and Rosen, S., 1978. Monopoly and Product Quality. *Journal of Economic Theory* 18, 301-317.

[28] Nocke, V., M. Peitz and F. Rosar, 2011. Advance-Purchase Discounts as a Price Discrimination Device. *Journal of Economic Theory* 146, 141-162.

[29] Spencer, M.A., S.K. Swallow, J.F. Shogren and J.A. List, 2009. Rebate Rules in Threshold Public Good Provision. *Journal of Public Economics* 93, 798–806.

[30] Ward, C., and Ramachandran, V., 2010. Crowdfunding the Next Hit: Microfunding Online Experience Goods. Mimeo.

[31] Winborg, J., and H. Landstrom, 2001. Financial Bootstrapping in Small Businesses: Examining Small Business Managers' Resource Acquisition Behaviors. *Journal of Business Venturing* 16, 235-254.

Recent titles

CORE Discussion Papers

2010/77.   Ana MAULEON, Vincent VANNETELBOSCH and Cecilia VERGARI. Bargaining and delay in patent licensing.

2010/78.   Jean J. GABSZEWICZ and Ornella TAROLA. Product innovation and market acquisition of firms.

2010/79.   Michel LE BRETON, Juan D. MORENO-TERNERO, Alexei SAVVATEEV and Shlomo WEBER. Stability and fairness in models with a multiple membership.

2010/80.   Juan D. MORENO-TERNERO. Voting over piece-wise linear tax methods.

2010/81.   Jean HINDRIKS, Marijn VERSCHELDE, Glenn RAYP and Koen SCHOORS. School tracking, social segregation and educational opportunity: evidence from Belgium.

2010/82.   Jean HINDRIKS, Marijn VERSCHELDE, Glenn RAYP and Koen SCHOORS. School autonomy and educational performance: within-country evidence.

2010/83.   Dunia LOPEZ-PINTADO. Influence networks.

2010/84.   Per AGRELL and Axel GAUTIER. A theory of soft capture.

2010/85.   Per AGRELL and Roman KASPERZEC. Dynamic joint investments in supply chains under information asymmetry.

2010/86.   Thierry BRECHET and Pierre M. PICARD. The economics of airport noise: how to manage markets for noise licenses.

2010/87.   Eve RAMAEKERS. Fair allocation of indivisible goods among two agents.

2011/1.   Yu. NESTEROV. Random gradient-free minimization of convex functions.

2011/2.   Olivier DEVOLDER, François GLINEUR and Yu. NESTEROV. First-order methods of smooth convex optimization with inexact oracle.

2011/3.   Luc BAUWENS, Gary KOOP, Dimitris KOROBILIS and Jeroen V.K. ROMBOUTS. A comparison of forecasting procedures for macroeconomic series: the contribution of structural break models.

2011/4.   Taoufik BOUEZMARNI and Sébastien VAN BELLEGEM. Nonparametric Beta kernel estimator for long memory time series.

2011/5.   Filippo L. CALCIANO. The complementarity foundations of industrial organization.

2011/6.   Vincent BODART, Bertrand CANDELON and Jean-François CARPANTIER. Real exchanges rates in commodity producing countries: a reappraisal.

2011/7.   Georg KIRCHSTEIGER, Marco MANTOVANI, Ana MAULEON and Vincent VANNETELBOSCH. Myopic or farsighted? An experiment on network formation.

2011/8.   Florian MAYNERIS and Sandra PONCET. Export performance of Chinese domestic firms: the role of foreign export spillovers.

2011/9.   Hiroshi UNO. Nested potentials and robust equilibria.

2011/10.   Evgeny ZHELOBODKO, Sergey KOKOVIN, Mathieu PARENTI and Jacques-François THISSE. Monopolistic competition in general equilibrium: beyond the CES.

2011/11.   Luc BAUWENS, Christian HAFNER and Diane PIERRET. Multivariate volatility modeling of electricity futures.

2011/12.   Jacques-François THISSE. Geographical economics: a historical perspective.

2011/13.   Luc BAUWENS, Arnaud DUFAYS and Jeroen V.K. ROMBOUTS. Marginal likelihood for Markov-switching and change-point GARCH models.

2011/14.   Gilles GRANDJEAN. Risk-sharing networks and farsighted stability.

2011/15.   Pedro CANTOS-SANCHEZ, Rafael MONER-COLONQUES, José J. SEMPERE-MONERRIS and Oscar ALVAREZ-SANJAIME. Vertical integration and exclusivities in maritime freight transport.

2011/16.   Géraldine STRACK, Bernard FORTZ, Fouad RIANE and Mathieu VAN VYVE. Comparison of heuristic procedures for an integrated model for production and distribution planning in an environment of shared resources.

2011/17.   Juan A. MAÑEZ, Rafael MONER-COLONQUES, José J. SEMPERE-MONERRIS and Amparo URBANO Price differentials among brands in retail distribution: product quality and service quality.

Recent titles

CORE Discussion Papers - continued

2011/18. Pierre M. PICARD and Bruno VAN POTTELSBERGHE DE LA POTTERIE. Patent office governance and patent system quality.

2011/19. Emmanuelle AURIOL and Pierre M. PICARD. A theory of BOT concession contracts.

2011/20. Fred SCHROYEN. Attitudes towards income risk in the presence of quantity constraints.

2011/21. Dimitris KOROBILIS. Hierarchical shrinkage priors for dynamic regressions with many predictors.

2011/22. Dimitris KOROBILIS. VAR forecasting using Bayesian variable selection.

2011/23. Marc FLEURBAEY and Stéphane ZUBER. Inequality aversion and separability in social risk evaluation.

2011/24. Helmuth CREMER and Pierre PESTIEAU. Social long term care insurance and redistribution.

2011/25. Natali HRITONENKO and Yuri YATSENKO. Sustainable growth and modernization under environmental hazard and adaptation.

2011/26. Marc FLEURBAEY and Erik SCHOKKAERT. Equity in health and health care.

2011/27. David DE LA CROIX and Axel GOSSERIES. The natalist bias of pollution control.

2011/28. Olivier DURAND-LASSERVE, Axel PIERRU and Yves SMEERS. Effects of the uncertainty about global economic recovery on energy transition and CO_2 price.

2011/29. Ana MAULEON, Elena MOLIS, Vincent J. VANNETELBOSCH and Wouter VERGOTE. Absolutely stable roommate problems.

2011/30. Nicolas GILLIS and François GLINEUR. Accelerated multiplicative updates and hierarchical als algorithms for nonnegative matrix factorization.

2011/31. Nguyen Thang DAO and Julio DAVILA. Implementing steady state efficiency in overlapping generations economies with environmental externalities.

2011/32. Paul BELLEFLAMME, Thomas LAMBERT and Armin SCHWIENBACHER. Crowdfunding: tapping the right crowd.

Books

P. VAN HENTENRYCKE and L. WOLSEY (eds.) (2007), *Integration of AI and OR techniques in constraint programming for combinatorial optimization problems*. Berlin, Springer.

P-P. COMBES, Th. MAYER and J-F. THISSE (eds.) (2008), *Economic geography: the integration of regions and nations*. Princeton, Princeton University Press.

J. HINDRIKS (ed.) (2008), *Au-delà de Copernic: de la confusion au consensus* ? Brussels, Academic and Scientific Publishers.

J-M. HURIOT and J-F. THISSE (eds) (2009), *Economics of cities*. Cambridge, Cambridge University Press.

P. BELLEFLAMME and M. PEITZ (eds) (2010), *Industrial organization: markets and strategies*. Cambridge University Press.

M. JUNGER, Th. LIEBLING, D. NADDEF, G. NEMHAUSER, W. PULLEYBLANK, G. REINELT, G. RINALDI and L. WOLSEY (eds) (2010), *50 years of integer programming, 1958-2008: from the early years to the state-of-the-art*. Berlin Springer.

G. DURANTON, Ph. MARTIN, Th. MAYER and F. MAYNERIS (eds) (2010), *The economics of clusters – Lessons from the French experience*. Oxford University Press.

J. HINDRIKS and I. VAN DE CLOOT (eds) (2011), *Notre pension en heritage*. Itinera Institute.

CORE Lecture Series

D. BIENSTOCK (2001), Potential function methods for approximately solving linear programming problems: theory and practice.

R. AMIR (2002), Supermodularity and complementarity in economics.

R. WEISMANTEL (2006), Lectures on mixed nonlinear programming.