5 — Binary Systems (I) [*Revision* : 1.2]

- Binary Systems
  - A pair of stars orbiting around their common center of mass
  - (Can also have triple, quadruple systems, etc.)
  - Not uncommon: more stars in multiple systems than single
  - Only direct way to measure mass of star $\longrightarrow$ important
  - Various types
    * **Optical double** — not real binary
    * **Visual binary** — can resolve both stars
    * **Eclipsing binary** — stars pass in front of each other
    * **Spectrum binary** — can see signature of each star in combined spectrum
    * **Spectroscopic binary** — can see orbital motion of one/both stars from spectral Doppler shifts

- Kepler's Laws
  - Originally developed to describe orbit of planets around Sun
  - Three laws:
    1. Orbits are **ellipses**, with the Sun at one focus
       * Semi-major axis $a \leftrightarrow$ distance from center to long edge
       * Semi-minor axis $b \leftrightarrow$ distance from center to short edge
       * Eccentricity $\leftrightarrow$ distance from center to focus (also, $b^2 = a^2(1 - e^2)$)
    2. Area swept out per unit time is constant
    3. Square of orbital period proportional to cube of semi-major axis

$$P^2 = a^3,$$

       with P in years and a in AU.
  - Laws are direct consequence of Newton's laws of motion and gravitation
  - Apply equally to *any* pair of orbiting bodies — incl. stars
  - Updated versions: orbits of bodies are ellipses, with common **center of mass** at focus
  - Orbits expressed in terms of equivalent problem of reduced mass

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}$$

    in elliptical orbit around *fixed* central mass

$$M \equiv m_1 + m_2$$

  - Components have position vectors

$$\mathbf{r}_1 = -\frac{\mu}{m_1}\mathbf{r}$$

$$\mathbf{r}_2 = -\frac{\mu}{m_2}\mathbf{r}$$

    where $\mathbf{r}$ is position of reduced mass in equivalent system

– In polar coordinates, reduced mass located at

$$r = \frac{L^2/\mu^2}{GM(1 + e\cos\theta)}$$

(this is equation for ellipse). Total angular momentum of system:

$$L = \mu\sqrt{GMa(1 - e^2)}$$

where a is semi-major axis of equivalent system ($a = a_1 + a_2$),

– Note: eccentricity e and period P is same for each star and equivalent system

– Kepler's third law becomes

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3$$

(P and a now in 'ordinary' units, e.g. cgs or SI)