FREE FORM SHAPE REPRESENTATION USING NURBS MODELING

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ABSTRACT

The representation, matching and analysis of objects of interest are of prime importance in shape-based retrieval systems. Considering that these systems involve analysis of various complex shapes, an accurate representation of free form shape is required. A simple and accurate shape representation procedure would ensure meaningful results from the shape-based retrieval systems. Motivated by this factor, this paper presents a free form shape representation technique using Non-Uniform Rational B-Spline (NURBS) modelling. The free form shapes are modelled using control points and weights. NURBS posses attractive properties such as spatial uniqueness, bounded and continuous, local shape controllability and shape invariance under transformation. Furthermore, NURBS based shape descriptor allows accurate reconstruction of the shape boundary from the NURBS features used to describe it. This paper presents the details of deriving a set of NURBS features using the boundary of the object. The accuracy and data reduction properties using NURBS are examined by carrying out an experiment on two sets of images: geometric and free form. Accuracy of the representation is evaluated by using centroid-radii error function, which computes the cumulative distance between the intersection points by radii lines on the boundary of the original image and the reconstructed image. The data reduction property is shown by the ratio computation between the number of control points and the boundary points. The overall experiment results show that NURBS is an accurate shape descriptor and a potential candidate for use in shape-based image retrieval systems.

Keywords: Shape Representation, NURBS, Shape-Based Image Retrieval

1. INTRODUCTION

With the advances in digital imagery, large accessible data storage, internet repositories, and image applications, information conveyed through images is gaining in importance. A large database of images will be shared among different group of peoples, such as journalists, engineers, historians, designers, teachers, artists and advertising agencies. However, all the information in the image databases will be useless, if we cannot efficiently index in order to ease the task of searching and browsing through these collections. Due to the emergence of large scale of image databases, traditional methods of indexing images through text annotation is becoming outdated. An effective retrieval approach is needed, to retrieve images from a database that are relevant to a query.

To overcome these difficulties, content-based image retrieval was proposed [Vngud 95]. Content-based image retrieval relies on the characterization of primitive perceptual features such as colour, shape and texture that can be automatically extracted from the images themselves. Retrieval by shape is considered one of the most difficult and challenging aspects of content-based search [Rkamp00]. Many techniques in this research direction have been developed and many shape retrieval systems, both research and commercial, have been built. Perceiving a shape is to capture prominent elements of an object. Humans have the capability to determine only a few selected signs, which enable them to determine the impression of a complete and real representation of the object. On the other hand, this is not that simple a case for the computer vision research. For the purposes of retrieval by shape similarity, representations are preferred such that the salient perceptual aspects of a shape are captured

and are able to imitate the human perception on perceiving shapes. Therefore, several of representation schemes have been introduced to suite these criteria [Slonc98].

To retrieve images according to perceptual properties, the basic retrieval paradigm requires that, for each image, a set of distinguishing features are pre-computed. In the QBIC system [Wnibl93], a set of global features such as area, circularity, eccentricity and major-axis orientation have been used to represent the query shape. Such simple descriptors only allow discrimination between shapes of very different forms because a small change in the shape may sometimes cause a significant change in these descriptors. In the latter approach, algebraic moment invariants have also been introduced as additional shape descriptors [Makhu62]. However, it is difficult to correlate high order moments with shape features. An alternative transform approach is the 2-D Fourier transform of the shape [Yorui96]. The disadvantage of this method is that it is impossible to detect local shape features and it is also computationally intensive. To overcome this problem, Mallat[Smalt89] introduced a multi resolution signal decomposition to represent shape using discrete wavelet transform. However, most of the methods proposed to address these problems solve only one problem while creating new difficulties. For example, in wavelet representation method, the accuracy of representation is dependent on the filtering method.

In order to have a shape descriptor, which can be defined in mathematical form and have high accuracy, spline approximations for curves have been used. Ikebe[Yikeb82] presented an overview of spline applications for shape design, representation and restoration. Spline approximates a given function with a curve having the minimum average curvature. This makes them perfect candidates for the "natural" representation of curves. Spline has an initial problem because local function value modification changes the complete spline representation. In this case, Cohen [Fscoh95] proposed a technique for curve representation and matching using B-spline. B-spline is constructed so that the local function value change does not spread to the rest of the intervals.

In our approach, we propose Non-Uniform Rational B-spline (NURBS) model to represent query shape in the shape based retrieval system. The difference between NURBS and B-spline is that the non-uniform knot vector and additional parameter, which is the weight, have been used. These NURBS properties give more freedom and flexibility to represent complex and free form shapes compared to the B-spline.

This paper is organised as follows. In Section 2, an overview of the NURBS definition and the NURBS characteristic are presented. Section 3 introduces the method used to determine the control points and the weights. Section 4 presents the experiments to evaluate the proposed NURBS representation for use in shape-based image retrieval and reports the results. Section 5 concludes the approach presented in this paper and discusses the future directions of our research.

2. NURBS CURVE THEORY AND PROPERTIES

2.1 NURBS Curve Definition

A *pth*-degree NURBS curve $C(u)$ defines a point that traces a trajectory in 2D space as the scalar parameter value u varies within the range [0, 1].

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i B_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i}$$

(1)

where a set of n control points $\{B_i\}$ forms a control polygon and $\{w_i\}$ is the weights. An increase in the weight w_i pulls the curve closer to the control point B_i. $N_{i,p}(u)$ is the *ith* B-spline basis function of degree p (order $p+1$), defined recursively as

$$N_{i,p}(u) = \begin{cases} 1 & if \ u_i \leq u \leq u_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

(2)

A non–uniform knot vector, which is a nondecreasing sequence of real numbers is defined as

$$U = \left\{ \underbrace{a, \cdots a,}_{p+1} \cdots, u_{m-p-1}, \underbrace{b, \cdots b}_{p+1} \right\}$$

(3)

where $0 \leq u_i \leq u_{i+1} \leq 1$, $i = 0, \cdots, m-1$ and m = number of knots. Knots in a NURBS curve are the points in parameter space where rational polynomial curves are grafted together to form a multi segment curve. Detailed explanations of NURBS can be found in [Lpieg97].

2.2 NURBS Characteristics as a Shape Descriptor

For an accurate shape representation, free-form shape should be represented by a mathematical definition that exactly matches with what the user feels as a shape, so that the shape may be manipulated in parametric form. The choice of NURBS as a shape descriptor offers a common mathematical form for representing any arbitrary shape including standard analytic shapes such as conics and quadrics, as well as free form curves. Therefore, both analytic and free form shapes are represented precisely. Inclusion of weight $\left(w_i\right)$ as an additional parameter adds an extra degree of freedom to NURBS and facilitates the representation of a wide variety of shapes. Furthermore, the use of non-uniform knot vectors allows better shape control and the modelling of a much larger class of shapes than the uniform knot vector used in B-splines. On the other hand, the use of a B-spline curve with uniform knot vector to interpolate highly unevenly spaced data points can result in unwanted oscillations or loops. The non-uniform knot vector allows the endpoints of the curve coincide with the first and last control points.

NURBS representation is suitable for use in the shape-based image retrieval systems because NURBS features computation is reasonably fast and stable. This is because they can represent very complex shapes with remarkably little data and are well defined in the mathematical form. Furthermore, NURBS posses characteristic of shape invariance under affine transformation, which means that the affine transformed curve is still a NURBS curve whose control points and weights are related to the original object control points and weights through the transformation. In additional, a very important motivation for using NURBS representation is its ability to control smoothness and curvature continuity. The control points of the NURBS curve can be modified without altering the curve's continuity because this property is determined by its basis functions, which are independent of the control point modification. The NURBS model allows a curve to be defined with no sudden changes of direction or with precise control when kinks and bends occur. Another important property of NURBS features is that they offer local controllability, which implies that local changes in shape are confined to the NURBS parameters local to that change.

With all these attractive characteristics, we strongly believe that NURBS is a highly suitable shape descriptor for shape based similarity retrieval in the future.

3. OVERVIEW OF THE APPROACH

Shape representation of a curve using NURBS involves determining the NURBS parameters such as control points and weights. This procedure is referred to as NURBS fitting and is a very challenging task. One of the difficulties in this procedure is the determination of the weights. Farin [Gfari92] suggested allocating weights according to the curvature, with points of high curvature being assigned larger weight. Hoschek[Jhosc94] investigated non-linear approaches for NURBS curve approximation. In this method, both control points and the weights of a NURBS curve are identified simultaneously by minimising the sum of the square of the distances from the original boundary to the corresponding fitted curve points.

In this paper, a new method for defining NURBS parameters is introduced. In this, allocation of a set of positive weights is determined according to the frequency distribution of the boundary points. This allocation method ensures a better weight fitting solution than the methods mentioned earlier. An accurate and stable weight would ensure good control point determination. Parameterization techniques for both the boundary points and knot parameters have been derived in order to perform least squares fitting of the NURBS curve. After the knot vector is determined, a two-step linear approach for fitting the NURBS curve is introduced. During the first step, the weights are identified from a homogeneous system through Singular Value Decomposition (SVD) procedure [Weima94]. In SVD, an orthogonal matrix and a diagonal matrix are decomposed. A set of positive weights is generated from the combination of the eigen vectors from the orthogonal matrix. At the second step, the control points are solved with the identified weights as known parameters.

3.1 Determination Of Knot Vector

The determination of the knot vector from a set of boundary points involves two parameterization steps. The first step involves the parameterization of boundary points. Prior to the least squares fitting, each of the boundary points is parameterized by allocating a location parameter, u_i. The second step involves the parameterization of knots. After the boundary points are parameterized, the complete knot sequence is defined. The information of a complete knot sequence includes the order, the number of control points and the knot parameters. For practical applications, there are three parameterization methods that are commonly used to assign the location parameters [Weima95a]. These are the uniform, cumulative chord length and centripetal model parameterization methods. In our

approach, we choose the centripetal model parameterization because the extracted boundary points in our samples are more or less evenly spaced.

For the parameterization of knots, a proper knot selection method consistent with the parameterization method of the boundary points is needed in order to achieve a good fitting result in the NURBS curve fitting process. Average Knot method is chosen because this method samples the boundary with high frequency in order to allocate more knots at places where the curve changes rapidly. When the parameter values of the boundary points are fixed, an optimal set of knots for a fixed order and number of control points will be derived. The details of these two parameterization methods can be found in [Weima95a].

3.2 Numerical Approach: Determination Of Weights And Control Points

In this approach, a homogeneous system has been derived so that the weight parameter can be computed independently. The homogeneous system defined in [Weima95b] is as follows:

$$M.w = [0]_{nx1}$$

(4)

where $M = M_x + M_y$ is a n x n symmetric and non-negative matrix with

$$M_x = N^T \overline{X}^2 N - \left(N^T \overline{X} N\right)\left(N^T N\right)^{-1}\left(N^T \overline{X} N\right)$$
$$M_y = N^T \overline{Y}^2 N - \left(N^T \overline{Y} N\right)\left(N^T N\right)^{-1}\left(N^T \overline{Y} N\right)$$

(5)

If $w \in R^n$ is a solution of Eq. 4, for all $\alpha \in R$ and $\alpha \neq 0$, so αw is also a solution of Eq. 4. Owing to this property, one can simply use the following criterion for solving the equation:

$$\min_{\|w\|_2 = 1} \|M.w\|_2^2$$

This equation is equivalent to Rayleigh quotient function for w, which is defined as

$$\min_{\|w\|_2 \neq 0} R(w) = \frac{w^T Q w}{w^T w} \quad \text{where } Q = M^T M$$

(6)

As an application, both the general solution and the solution with positive weights of Eq. 4 can be represented as a linear combination of some

eigenvectors of Q corresponding to smaller eigen values. According to the properties of $R(w)$, there is an important relationship between the singular value decomposition (SVD) of M and the singular eigen value decomposition (SEVD) of Q. Owing to this relationship, one can use the SVD of M for NURBS identification [Weima94].

In general, the weights can be computed from the singular value decomposition of M, in which M is factorized as

$$M = PDP^T$$

(7)

where $D = diag[d_1, d_2, \cdots, d_n]$ is a diagonal matrix whose diagonal elements are the eigen values of M in decreasing order with $d_i \geq d_{i+1} \geq 0.0$ and P is an orthogonal matrix whose columns p_i for $i = 1, 2, \cdots, n$ are eigen vectors of M corresponding to d_i.

The general solution of the positive weights $w \in R^n$ with $\|w\|_2 = 0$ are given by

$$w = \sum_{i=n-r+1}^{n} \alpha_i p_i$$

(8)

where r is the increment step achieved from the minimization of an objective function to obtain the best subspace of vector p, which contains positive weights, and a set of feasible solutions in this subspace. A constrained minimization algorithm is applied to the objective function to determine a set of vector α in order to find a set of best fitting solutions in this subspace. The objective function, which is derived by introducing w into Eq. 6 is:

$$\begin{cases} R(\alpha) = \min_\alpha \dfrac{\displaystyle\sum_{j=n-r+1}^{n} \alpha_j^2 d_j^2}{\displaystyle\sum_{j=n-r+1}^{n} \alpha_j^2} \\ subject\ to: w_l \leq w \leq w_u \end{cases}$$

(9)

where the value $r \in [1, n]$ will be increased until best fitting subspace can be determined and $w_u \geq w_l > 0.0$ are positive upper and lower bounds for the weights. By having a set of vector α, the weight of the corresponding control points is derived from Eq. 8.

By taking the identified weights as known parameters, the corresponding control points are obtained. When the weights are available, a non-

negative least square optimization method is applied to achieve an accurate solution for control point determination. The control points in R^3 can be recovered from the control points in homogeneous space divided by the related weight.

4. EXPERIMENTAL RESULTS

Fig. 1 summarizes all the steps taken to represent shapes with NURBS features using the proposed method as described above. These have been implemented using the MATLAB software, which is a powerful mathematical software offering a high performance language for technical computing, visualization and programming. In order to validate the proposed method, two sets of silhouette images have been used. These images consist of geometrical shapes and free form shapes. Both sets of images are converted into an array of 128 x 128 pixels.

The experiments in this work are aimed at proving that NURBS is a powerful shape descriptor with an ability to represent various shapes that include free form shapes and have high data reduction and accuracy. The experiments are carried out on five images that include geometric as well as free form shapes. In these experiments, the number of control points of the reconstructed image is iteratively incremented until the matching value is equal to or less than a desired value. In these experiments, the desired matching value used is 10 pixels. The initial number of control points is equivalent to the number of corner points. Corner points are used as a basis because they coarsely approximate the optimum number of control points to represent the original shape. The centroid-radii method is used as an error function to determine the matching value between the original shape and the reconstructed shape using NURBS representation parameters. In this method, radii lines are projected from the centroid to the boundary point of the original shape and the reconstructed shape at regular interval. For each radii lines, distance between the intersection points by radii lines on the boundary of the original shape and the reconstructed shape is accumulated. In this experiment, a sampling interval of 10^0 is used. The matching value is the cumulative distance difference between the original shape and the reconstructed shape at the sampled points. This method is illustrated in Fig. 2. In this figure, the red curve is the boundary of the original shape and the blue curve is the boundary of the reconstructed shape. The radii lines in magenta are projected from the centroid to these boundaries to find the interpolation points, which are coloured with yellow and cyan. The results of the experiments conducted with shapes shown in Fig.3 are presented in Table 1.

In Fig. 3, Table (a) shows the original geometric shapes at the left column and the reconstructed geometric shapes from the NURBS features at the right column. Meanwhile, Table (b) in Fig. 3 shows the original free form shapes at the left column and the reconstructed free form shapes from the NURBS features at the right column. Based on the right columns of Table (a) and Table (b), they show the reconstructed geometric as well as free form shapes that are very similar to the original shapes.

It may be seen from Table 1 that the accuracy of reconstruction for both geometric and free form shapes is very high with a maximum difference of 10 pixels. In Table 1, the number of boundary points of the original shapes and the number of control points needed to represent the geometric and free form shapes are shown respectively. The data reduction ratio between the number of boundary points and control points show that NURBS able to reduce the representation data, which is less than the quarter of the original amount of the boundary points.

In our experiments, we tested the NURBS descriptor for use in shape-based image retrieval on a database of 100 images of fishes and tools, which is displayed in the Fig. 4. In the experiments, 8 query images from the database are selected as query images. In Fig. 5, we represent some of our experimental results by showing the response of the system to these queries.

Shapes	No. Boundary Points, bp	No. Control Points, cp	Data Reduction Ratio =cp/bp(%)	Matching Value (pixel)
Geometric Shapes				
Circle	160	10	6.25	4
Ellipse	212	12	5.66	0
Rectangle	224	22	9.82	0
Square	278	30	10.79	0
U Shape	296	60	20.27	0
Free Form Shapes				
Fish 1	262	29	11.06	9
Fish 2	301	49	16.28	8
Fish 3	326	40	12.27	5
Fish 4	227	36	15.86	10
Fish 5	228	36	15.79	9

Table 1**:** This table shows the experiment results with ratio between the number of control point and number of boundary point (%) and matching value (pixel)

5. CONCLUSION AND FUTURE WORK

From the results presented in Section 4, it is clear that NURBS is powerful shape descriptor for free form shapes as well as geometric shapes. We are presently working on the further development of using NURBS-descriptor in shape-based image retrieval in order to improve the reliability and accuracy of the retrieval results from a larger database.
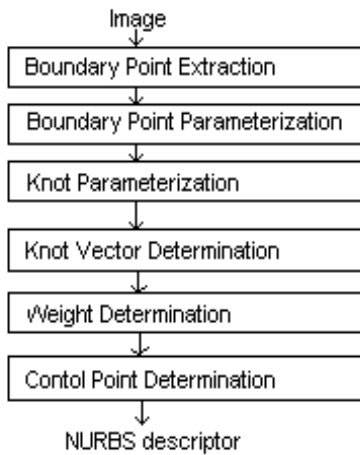

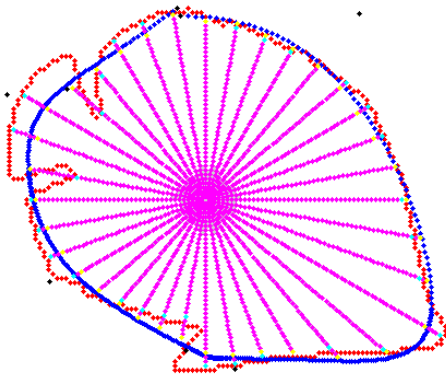Figure 1: Step by step procedures for NURBS parameters determination

Figure 2: The centroid radii method to determine the matching value

Original Shapes	Reconstructed Shapes

(a)

Original Shapes	Reconstructed Shapes

(b)

Figure 3: The difference between the original shapes and the reconstructed shapes by NURBS descriptor has been shown; (a) Geometric Shapes, (b) Free Form Shapes
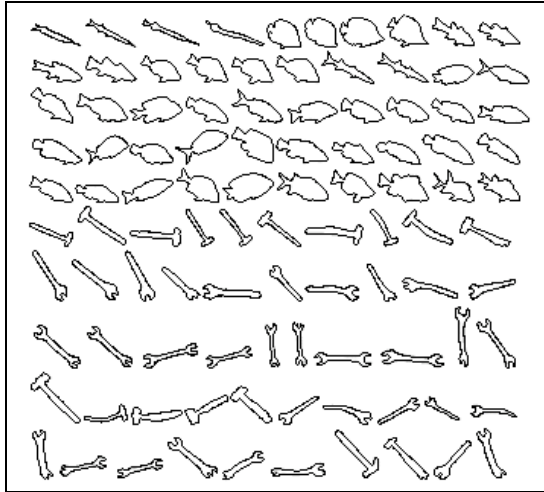
Figure 4: The database contains 100 images

Query Images	1st rank	2nd rank	3rd rank	4th rank

Figure 5: The results of the relevant image databases requested by the selected query images

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