PHYSICAL-BASED MODEL OF INK DIFFUSION IN CHINESE INK PAINTINGS

Sheng-Wen Huang[1,3], Der-Lor Way[1,2] , Zen-Chung Shih[1]

[1]Department of Computer and Information Science, National Chiao Tung University
1001 Ta-Hsueh Rd, Hsinchu, Taiwan 30010,R.O.C.
Email: adler, samhuang, zcshih@cis.nctu.edu.tw
[2]Department of Information Management, Van Nung Institute of Technology.
Email: adler@cc.vit.edu.tw
[3]Multimedia Product Division of Silicon Integrated Systems Corp.
Email: samhuang@sis.com.tw

ABSTRACT

Chinese ink painting is a traditional art that is over three thousand years old. It is a type of non-photorealistic rendering. However, research on Chinese ink painting is scarce. Simulating the behavior of Chinese ink is challenging work because ink moves in a complex manner. This paper presents a new method for simulating ink diffusion based on observation and analysis. The proposed method can simulate various expressions of tones on different types of paper. The elucidation of the effect of mixing simulated strokes made by different kinds of brushes is an important contribution of the method. Finally, the simulated results are compared with real ink painting.

Keywords: Non-Photorealistic Rendering (NPR), Chinese Ink Painting, Ink Diffusion, Slashed Painting.

1. INTRODUCTION

In recent years, several studies have focused on Western painting, including watercolor, impressionistic, pencil sketching and hatching [Elber99, Freudenberg01, Hertzmann00, Markosian97, Sousa99]. These methods yield good results for Western painting. However, these methods are not suited to Chinese ink painting. Generally, Western painting is precise but Chinese ink painting is more imprecise. Restated, Chinese paintings typically consist of a few simple strokes to convey the artist's "deep feelings" of a painted object. Simulating Chinese ink painting is non-trivial. It relies on free brush strokes and ink diffusion on the paper. [Chiu, Liu]

Importantly, an artist can use thousands of styles to express his mental state while painting, using various brush storks and rich ink gradation. Ink diffusion is crucial in Chinese ink painting, and generates for example, the fluffy-edged effect, a variety of ink intensities, blurring the boundary of a stroke, the merging of two strokes, and other effects. Some particular techniques, such as, "dense brush following dilute brush" and the splashed painting technique are fairly important in Chinese ink painting. Figures 11(a) and 12(a) presents two pieces of painting by Chang Dai-chien [Gao].

Little research has addressed methods for simulating brush strokes and the behavior of ink. The brush has been modeled as a collection of bristles that move in the course of the stroke. Strassmann [Strassmann86] first described the hairy brush as a 1D array of bristles. Jintae Lee[Lee99] presented elastic bristles governed by Hooke's law. Der-Lor Way et al. [Way01] created the effects of brush strokes and brush model involves two mechanisms - stroke geometry and brush profile. The stroke geometry mechanism governs the path of a stroke, and the brush profile mechanism determines the various characteristics of ink deposition, such as darkness, wetness, and pressure, among others.

Guo and Kunii [Guo91] first addressed ink diffusion in 1991. The diffusion of the ink into the absorbent paper is one of the most notable features of black ink painting (called 'Sumie' in Japanese). Qing Zhang et al. [Zhang99] presented a 2D simple cellular automaton-based simulation of ink behavior. Jintae Lee [Lee01] diffusion-rendered black ink paintings using new paper and ink models.

These papers proposed several methods for simulating brush strokes and the diffusion of ink. Although they realistically reproduced the diffusion of a single stroke, no mechanism has been presented to simulate the blending of two or more different kinds of brush strokes. Some results are unreasonable for Chinese ink painting. Accordingly, the simulated results in the above papers were not compared to real ink paintings.

Watercolors have also been simulated. Small [Small90] proposed a parallel approach to predicting the action of the pigment and water on paper fibers. Curtis and Anderson [Curtis97] employed a more sophisticated paper model, a more complex shallow water simulation and more faithful rendering and optical composition of pigmented layers based on the Kubelka-Munk model, to simulate watercolors more realistically. Unfortunately, the properties of Chinese ink painting differ from those of watercolors. The physical behavior of watercolors differs from that of Chinese ink.

This paper proposes a new method for simulating the diffusion of ink in paper. The method is based on a physical mechanism and an observational model of the interaction among real drawing materials used in Chinese ink painting and the variations in the diffusion of ink in the real world. The goal is to capture the core physical properties and behaviors to produce a high-quality ink diffusion model that a painter can use to generate a Chinese ink painting, including brush strokes, in various styles.

The proposed method has the following advantages. First, it simulates the physical behavior of ink diffusion, and can thus generate strokes that exhibit a feathery effect; second, it can blend two strokes with different thickness. Using this method to render a Chinese ink painting can generate highly realistic blending effect

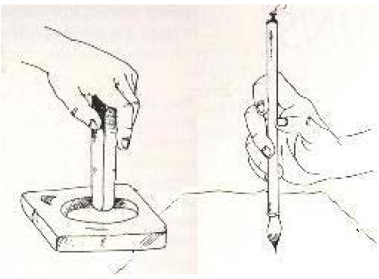


Figure 1: the four treasures of Chinese ink painting.

2. PROPERTIES OF CHINESE INK

Chinese ink painting uses four tools, commonly called the "four treasures ". Figure 1 depicts The Four Treasures - brush, ink stick, ink stone and paper. They are all used in calligraphy, writing and painting in China. The bristles of the brush touch the surface of the paper, and the ink in the bristles seeps into the highly absorbent paper, creating a stroke whose edge is fluffy and blurred. These characteristics of diffusion represent complex physical phenomena that cannot be accurately simulated by conventional graphical techniques such as texture mapping or degradation functions, since a purely mathematical method generally results in flatly blurred images that are unlike realistic diffusion images.

The ink is a kind of colloidal liquid [Theo] and diffusion phenomena can be considered as typical instances of the diffusion of a colloidal liquid in a highly absorbent paper. The capillary effect importantly causes ink to diffuse into the structure of the paper. Typical paper consists of fibers in random positions and directions; small holes and spaces among the fibers act as thin capillary tubes that carry water away from the area in which it is initially applied, causing diffusion, as shown in Fig. 2.

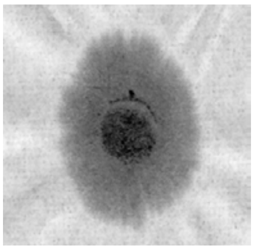


Figure 2: An example of ink diffusion in Hsuan paper.

Besides the capillarity, the forces that move the ink include interactions among water molecules, water and carbons, and the force due to gravity, among others. The black ink is a dilute mixture of water and colloidal black carbon particles, which diffuse into paper in the absorbed water. Water and carbon are the two main constituents of Chinese ink and the motion of ink in the fibers as simulated as chaotic will be discussed in Sections 2.2 and 2.3

2.1 Paper Cell

Several kinds of paper are used in Chinese ink painting. Basically, papers have one of two types of fiber mesh. The first kind is a regular fiber mesh, such as in silk paper, whose fibers are uniformly aligned as woven. The second kind is an irregularly distributed fiber mesh, such as in Hsuan paper that consists of a mesh of randomly positioned fibers.

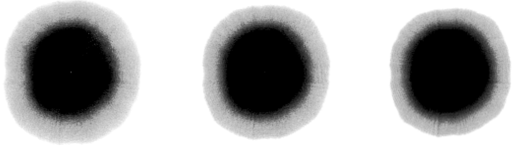
Constructing a mesh like Hsuan paper requires an appropriate data format in which to represent a mesh structure. Traditionally, a network format is used to represent paper with a random fiber network [Guo91]. The continuous interaction between water and fiber is discretely simulated by computers; a two-dimensional array, whose entries specify the attributes of the structure of the mesh, is used. The entire mesh in the paper is separated out into many layers, each of which are divided into [X x Y] cells called papels (paper element) [Lee01]. A papel is a basic unit of paper structure and corresponds to a pixel.

Capillarity is evident in paper modeled as interlaced fibers. The ink seeps into the paper and is then pulled away from the area of application by capillary attraction; it then travels through the fibers. Some of the diffused ink is deposited in the holes or spaces between the fiber through which it passes; the remaining ink continuously flows along the fibers until it is completely absorbed.

Let Absorbency(p) of papel p be defined as follows. When the moving ink passes through p with N fibers, the amount of water deposited in p is Q. The relationship between Q and N can be expressed as $Absorbency(p) \propto N \propto Q$. Based on this relationship, several models of paper can be defined with various absorbency, by fibers with various densities. An equation for the absorbency of each papel is,

$$Absorbency(p) = Base + Var \times rand() \quad \ldots.\ (1)$$

Figure 3 depicts the resulting ink diffusion in three types of paper with different absorbencies. The coefficient of absorbency is a real number between zero and one.



(a) 0.1 (b) 0.4 (c) 0.7

Figure 3: Three simulated ink diffusion image represent different kinds paper with different degree of absorbency value.

2.2 Water Particles

Water is a liquid which can move to anywhere in the paper under the forces associated with capillary action. All water particles are defined as objects with the same volume, mass, color and *respond* to forces. They only differ in position, recorded as coordinates in the papel. The quantity of water accordingly governs the span of the diffusion image or the number of diffusion steps. When the water in a certain papel flows out, its quantity and direction must be determined.

Based on above description, the approximate equation for K(p), the ratio of the quantity of out-flowing water to the quantity of water in the papel p is represented as,

$$K(p) = F_{base} + F_{diff} \times \left(1 - \left(1 - Absorbency\ (p)^2\right)\right), \quad (2)$$

where F_{base} is a real number between zero and one that represents the basic flow rate p, and F_{diff} is a real number between zero and one that represents the difference between the highest flow rate and the lowest.

The quantity of water that flows in all the directions to neighboring papels is determined by associated probabilities. Section 3 discusses the estimation of the probability associated with each direction of water flow.

2.3 Carbon Particles

A carbon particle is a black and solid grain. It cannot move by itself. When water particles move, carbon particles move with them. Carbon particles are suspended and move in this liquid since they collide with water particles. Suspended carbon particles undergo Brownian motion, buffeted water particles [Theo].

The carbon particles can be most simply simulated like water particles. They have mass, position, diameter and color. These attributes all vary among particles. The diameter and mass of a carbon particle are determined by the fineness to which the ink is initially ground. If the ink is initially ground coarsely, it contains small and large particles that produce observably different color intensities at the border of the initially brushed area. However, most homogeneous, small and uniform carbon particles move in water unhindered by the fibers, such the intensity changes smoothly over across the diffusion area.

Only carbon particles that are smaller than the space between the fibers can seep into the mesh in the water. Particles larger than the space remain in their initial positions, as shown in Fig. 4. This phenomenon is referred to as the "filtering effect" of the fiber mesh, and can be represented as follows, where p is the papel in which the carbon particle is located.

```
if ( Carbon_Diameter > Hole_Diameter(p)) then
    Carbon_Position ← p
else
Carbon_Position← Water_OutFlow_Direction(p)
```

In Fig. 4, two adjacent cubes represent two neighboring papels. Black grains in papels are carbon particles with different sizes. It is chaotic between two papels represent fibers. The arrow represents the direction in which the water flows; the carbon particles move in this direction. Larger carbon particles cannot pass through holes in the paper.

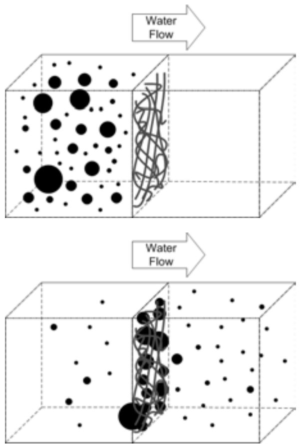


Figure 4: An illustration to explain the phenomenon called "*filter effect*".

As well as the diameter-filtering mechanism, a mass-filtering mechanism is proposed. Suppose V_c is the velocity of a carbon particle c, suspended in water in papel p, and W_p is the quantity of out-flowing water from p. The relationship between V_c, W_p and the diameter of holes in p, Hole_Diameter(p), is given by $V_c \propto W_p$, $V_c \propto \frac{1}{(Hole\_Diameter(p))^2}$. If carbon particle c is too heavy to exit papel p and is deposited in p, then $V_c \leftarrow 0$. Accordingly [Theo], an upper-bounded threshold T_p for papel p determines whether the carbon particle can. If the mass of carbon particle c exceeds T_p then $V_c \leftarrow 0$. The value of T_p is determined by depending V_c. The relationship between T_p and V_c is represented as,

$$T_p = T(V_c) = T(W_p \times \frac{1}{(Hole\_Diameter(p))^2}), \quad \ldots\ldots (3)$$

where T a transformation from V_c to T_p.

3. MOTION OF WATER

Water may flow from one papel to some of its neighboring eight papels. The directions of this motion are determined by considering the following factors that dominate the flow of water.

1. Gradient of water between neighboring papels, based on *Brownian* motion. []
2. Absorbency of neighboring papels for water.
3. Paper texture of neighboring papels.
4. Inertia of water.

Figure 5 shows that a papel C_0 has eight neighboring cells p_k defining eight directions, d_k (k = 1, 2,, 8). The probabilities of motion in each direction are calculated according to the above four factors. The amount of water particles that flow into a neighboring papel is proportional to the calculated probability.

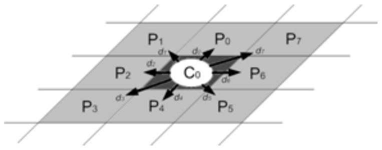


Figure 5: Determine directions of water flowing into neighboring papels, p_k (k = 1, 2, , 8), according to the probabilities in eight directions calculated based on four factors.

3.1 Gradient

The motion of water particles in paper is assumed to obey Brownian motion. A mixture of two sets of different numbers of water particles will produce irreversible diffusion in which water particles are transferred from the set with more to that with fewer particles. This movement continues until the difference between the numbers of particles in the two sets reaches a value that expresses the balance of forces on these two sets. Gradient represents the difference between the numbers of water particles in the two sets.

The number of water particles in c_0 and p_k are assumed to be W_c and W_k, respectively. The probability, based on Brownian motion, is determined by the equation,

$$G_k = \frac{u(W_c - W_k)}{G_{sum}}, \quad G_{sum} = \sum_{i=1}^{8} u(W_c - W_i) \quad \ldots\ldots\ldots (4)$$

where G_k is a probability determined by gradient. $u(x)$ is the unit function; that is, if $x \geq 0$, then $u(x) = x$, otherwise $u(x) = 0$.

3.2 Absorbency

Attraction to each neighboring papel causes different amounts of water to flow into each. Newton's Second Law of Motions is: $f_d = M \times a$ (5)

The dynamic friction f_d is ideally a constant force between the flowing water and the fibers. The term a is the acceleration of the flowing water particles. Based on the theorem of Theo, a is usually much smaller than g, the acceleration due to gravity. Therefore, it can be regarded as constant. M is the mass of the flowing water particles. The pre-defined uniformity of the mass of water particles is such that the amount of flowing water is proportional to M. Assume N_w is the number of water particles in the flowing water.

From Eqs. (4) and (5), $f_d \propto N_w \times a$.This important deduction indicates that increases with f_d. Based on the relationship in Eq. (3),

$f_d \propto a_p$ ➔ $N_w \propto a_p$, where a_p is the absorbency of papel p. Assume that the eight neighbors of the central papel c_0 are p_k, and the absorbency of c_0 and p_k are $Absorbency(c_0)$ and $Absorbency(p_k)$, respectively. Probabilities, based on absorbency, are attributed to the eight directions according to,

$$A_k = \frac{Absorbency(p_k)}{A_{sum}}, \quad A_{sum} = \sum_{i=1}^{8} Absorbency(p_i) \quad \text{…….(6)}$$

where A_k (k = 1, 2,, 8) is a probability based on absorbency.

3.3 Paper Texture

The texture of the paper also determines the directions in which the water flows. When water undergoes capillary action and flows in the holes among the fibers, fibers in the trajectory of the flowing water become saturated. Variously aligned of fibers promote different trajectories of the flowing water. Two kinds of paper exist – one with regular and the other with an irregular fiber mesh, such as silk paper and Hsuan paper, respectively.

Papers with differently distributed fibers have different textures. When the water particles in a papel c_0 flow out, c_0 is the center of a 3×3 texture mask, M_{direct}, with a central element m_0 at c_0. The eight elements at the periphery of M_{direct}, m_k (k = 1, 2, …, 8), are assigned weights to represent the alignment of the fibers.

3.4 Inertia

Besides the three aforementioned factors, inertia is involved in another important physical mechanism. According Newton's First Law of Motion, inertia increases with the mass of an object. During painting, water is treated as a moving object. Assume that water in papel p_0^t in the (t)-th time interval originates from papels p_k^{t-1} in the (t-1)-th time interval, and the quantity of water particles flowing in the direction d_i^{t-1}, from p_k^{t-1} to p_0 is w_k^{t-1}. Based on the relationship between inertia and the mass of an object, water in p_0^t will flow out in the same direction as w_k^{t-1}. The probabilities associated with the eight directions of flow from p_0^t, I_k^t, are proportional to the w_k^{t-1} in direction d_i^{t-1}.

The probabilities are used to determine the directions of water flow. A higher probability of a neighboring cell corresponds to more water's flowing into it. The probabilities are,

$$R_k = \frac{\alpha_1 G_k + \alpha_2 A_k + \alpha_3 m_k + \alpha_4 I_k^t}{R_{sum}},$$

$$R_{sum} = \sum_{i=1}^{8} (\alpha_1 G_i + \alpha_2 A_i + \alpha_3 m_i + \alpha_4 I_i^t) \quad (7)$$

where R_k (k = 1, 2, …, 8) is the probability governed by four main factors for each neighboring papel and α_1, α_2, α_3, and α_4 are weights that control the behavior and movement of water, resulting in different kinds of effects.

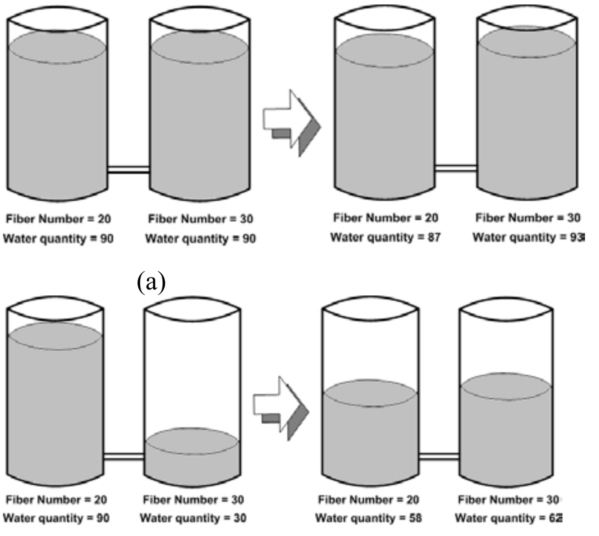
The directions of the water particles, the points in the mesh to which water will flow are determined using these probabilities. The quantity of water that flows to neighboring papel p_k (k = 1, 2, …, 8) is proportional to the probability R_k.

4. INK DIFFUSION

The diffusion of ink in paper is complex. It can be regarded as a continuous and time-dependent. The evaporation of water and the absorption of any ink left on the surface of the paper are also considered.

4.1 Ink Diffusion Schema

Ink diffusion is caused by the capillary action of water between fibers and the gradient of the quantity of water in paper cells. Given two neighboring papels saturated with water, as shown in Fig. 6(a), as a rough estimate, only the strength of the capillary forces in these two papels influences the direction of flow of the water. In contrast, in Fig.6(b), only one papel is saturated with water and the other is absolutely dry. The water gradient between these two papels is maximal, resulting in an obvious propagation of water from the papel with much to the other with little.



(a)

(b)

Figure 6: Two illustrations are given to describe the water propagation influenced by capillary force and gradient of quantity of water, relatively.

Besides, three issues were addressed in simulating ink diffusion - brush strokes, initial area and propagation. The area on the surface of the paper touched by the brush is approximately circular, because the profile in the horizontal direction of the brush used in Chinese Ink Painting is round. The stroke is a sequence of circular segments, as proposed by Der-Lor Way et al. [Way01, Way02] and shown in Fig. 7. Ink in old segments starts to diffuse earlier than in new segments, and ink within old segments may even dry up. Stroke segmentation makes the simulation of the painting processes more realistic. The skeleton of a brush stroke area of the application of ink is just one line, and can be simply used to describe a brush stroke as a trajectory of the center of a circle.

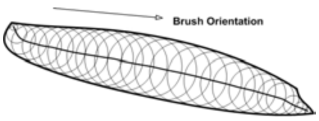


Figure 7: An example of Stroke segmentation. The stroke is divided into circular segments with their center positions on a given curve.

Now the basic initial area has been defined, pipelinizing is described as follows. First, the concentric reference point c is approximated. The median of the initial area is calculated as $c_x = \frac{x_{\max} + x_{\min}}{2}$, $c_y = \frac{y_{\max} + y_{\min}}{2}$, where ($c_x$, c_y) is the coordinate of c, and $x_{\min}$, $x_{\max}$, $y_{\min}$ and $y_{\max}$ are the minimum and maximum distances along the x-axis and y-axis of all the papels in the area of initial application, respectively. After the concentric reference point is determined, all the papels in the initial area are clustered into concentric –rings, as shown in Fig. 8. After pipelinizing, each papel is given a number that determines the concentric ring to which it belongs; the number is called the ring index. Many clustering methods are applied to group the papels into rings, and the best one is determined by comparison simulated result. The distance between the concentric reference point and the papel is calculated, and its ring index assigned a distance:

$$r = \sqrt{(p_x - c_x)^2 + (p_y - c_y)^2} \quad \cdots\cdots \quad (8)$$

where r is the ring index of p, which is a certain papel in the initial area. (c_x , c_y) is the coordinate of the reference point in this initial area.

After the ink saturates the initial area of the paper, it propagates into the concentric rings. The propagation proceeds stepwise. Although each of the processes of water diffusion into concentric rings is treated discretely, it is actually continuous diffusion. Figure 9 displays four diffused ink drops at different stages of diffusion.

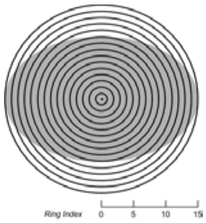


Figure 8: Initial area pipelinizing. The area with gray color represents the initial area. Concentric rings are indexed by the distance between the concentric center and the papel in the ring.

4.2 Evaporation

In the real world, water evaporates continuously from paper. The evaporation of water is a complex process governed by many parameters. One important parameter is the area of contact with the atmosphere. When other parameters are fixed, a larger contact area increases the rate of water evaporation. Assume that the contact area of each papel with atmosphere is the same. The rate of evaporation from each papel is then approximately equal.

Another important parameter humidity resists the evaporation of water. For simplicity, assume that the number of water particles evaporated from papel p ion the t-th step, E_p^t, depends on the humidity H ($0 \le H \le 1$) according to the equation $E_p^t = h(1-H) \times Water_p$, where $Water_p$ is the number of water particles in papel p, and function $h(x)$ yields a coefficient for the evaporation of water, where $0 \le x \le 1$.

4.3 Refilling Ink

When the brush's bristles first touch the initial area, the paper does not completely absorb the ink pulled out by capillary action from the bristles in a single time step. The rates of absorbency and capillary action are not sufficiently high to prevent ink from remaining on the surface of paper. Some of this remaining ink saturates a papel in the next time step. This phenomenon of saturation by remaining ink, called ink refilling, occurs continuously in the subsequent time steps. Ink refilling is promoted by adding ink to the papels in the initial area stepwise at a certain rate until the remaining ink on the surface of paper has been exhausted.

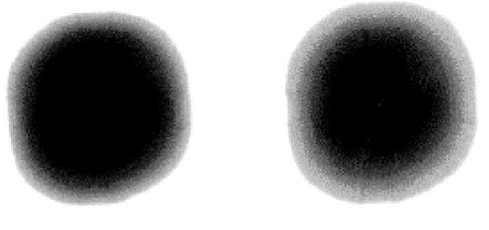
4.4 Intensity of Paper Cells

A papel contains water particles and carbon particles. Water particles are achromatic. Carbon particles are defined as absolutely black. On the gray scale, each carbon particle is zero. Accordingly, the color intensity of the paper is determined by the color of the fibers of the paper and the density of carbon particles.

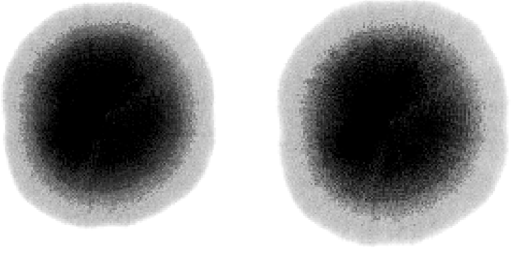
Assume c is the number of carbon particles in papel p, and the maximum capacity of the carbon particles of p is $c_{\max}$. The color intensity CI_p in p can be calculated from $CI_p = 255 \times \left(1 - \frac{c}{c_{\max}}\right) - P$, $P = 255 - PI_p$ where PI_p is the original color intensity of paper.

5. RESULTS

The proposed method yields several results. Figure 9 shows the simulated dropping of ink onto Hsuan paper. The time required for the simulation is also given. Figure 10 depicts an example of the basic blending of two brush strokes. The process of “dense brush following dilute brush” is described as follows. After a dilute brush stroke is applied to the paper, a dense brush stroke is immediately applied to the same area, before the first stroke has dried. Figure 11 shows the simulated result using the proposed ink diffusion method, according to ”Yellow Buds of The Water Lily” by Chang Dai-chien. The strokes of the resulting image are similar to those of an artist’s painting on real Hsuan paper. The time taken for the whole process, involving 66 strokes, is 1256 seconds. Figure 12 simulates the painting called, “The Dawn of Huang Mountain”. Twenty-eight strokes are applied. This simulation takes 847 seconds. All results are rendered on a PC with a PⅢ 1000 MHz CPU and 256 MB of RAM; the presented algorithm is implemented in C++ language.

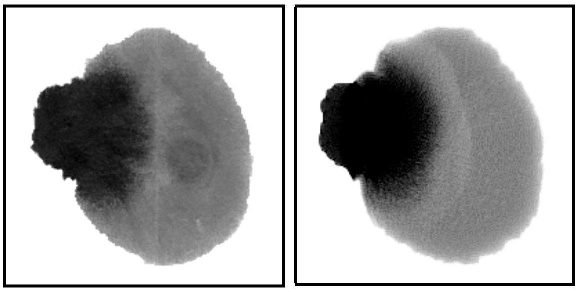


(a) 10 steps, 8 sec (b) 20 steps, 22sec



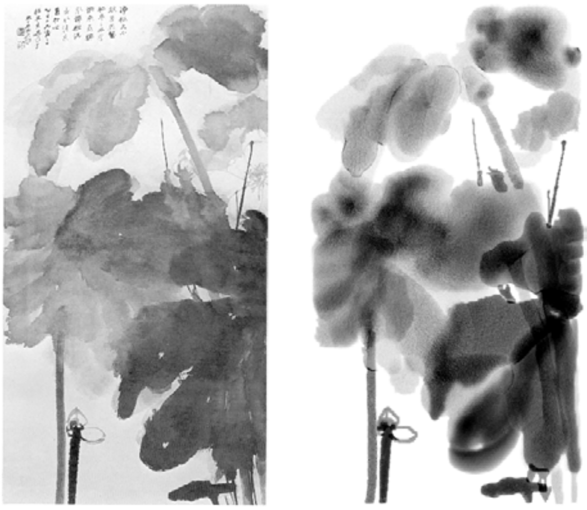
(c) 30 steps, 39 sec (d) 40 steps, 63 sec

Figure 9: Generated image of diffused ink drop in different step.



(a) Actual ink diffusion (b) Simulated ink diffusion

Figure 10: (a) Actual ink diffusion image on Hsuan paper, (b) Simulated ink diffusion image.



(a) Original painting (b) Simulated image

Figure 11: (a) Original painting “Yellow Buds of The Water Lily” by Chang Dai-Chien, (b) Simulated image generated by us.

6. CONCLUSIONS AND FUTURE WORK

This investigation presents a new method to generate the effects of ink diffusion in Chinese ink painting. Paper is modeled as several X-Y plane layers, each divided into paper cells. Water particles flow into holes or spaces among fibers in the paper mesh by capillarity action. The carbon particles float and move in this liquid since they collide with water particles. The direction and amount of flowing water are determined from absorbency, the alignment of fibers and the inertia due to each paper cell. The proposed ink diffusion algorithm can be used to paint many subjects in the style of Chinese ink painting with ink diffusion effects. The most important contributions of this works are as follows.

(1) The proposed algorithms are based on physical theory and analysis of observations. The resulting images are very realistic.

(2) The most important contribution of this proposed method is the expression of a mixture of different kinds of brush strokes, such as those of two wet brushes. “Dense

brush following dilute brush", is a typical example in Chinese ink painting.

(3) The diffusion of brush strokes can be easily controlled, according to experimental data, by specifying parameters. Users can easily use these parameters to control local and global variation and achieve their desired effects.

This work has opened up some topics for future research. Several unknown factors affect the diffusion of ink and should be be addressed. According to the proposed method, ink diffusion yields strokes that are uniform and regular on the boundary. For example, glue is a common ingredient in paper, and reduces paper's ability to absorb water. The issue of the quantity of glue added to paper should be addressed in future work. Other future work should simulate styles of Chinese ink painting with various colors, instead of just using gray-scale intensity. The ink diffusion schema must be extended to a colored-diffusion model.

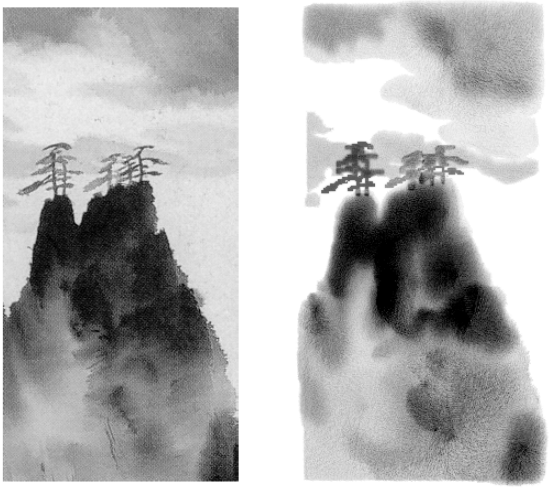

(a) Original painting (b) Simulated image

Figure 12: (a) Original painting "The Dawn of Huang Mountain", (b) Simulated image generated by us.

Acknowledgement

The authors would like to thank the National Science Council of the Republic of China for financially supporting this research under Contract No. NSC_ 90-2213-E-009-128

7. REFERENCE

[Chiu] Chow Chian Chiu and Chow Leung Chen Ying, "Chinese Painting: A Comprehensive Guide," Published in 1979 by Art Book Co., Ltd.

[Curtis97] Cassidy J. Curtis, Sean E. Anderson, Jonathon. E. Seims, and Kurt W. Fleischer, and David H. Salesin, "Computer-generated watercolor", Proceedings of SIGGRAPH 97, in Computer Graphics Proceedings, Annual Conference Series, pp. 421-430, August 1997.

[Elber99] Elberg, G. Interactive line art rendering of freeform surfaces. Computer Graphics Forum 18, 3 (September 1999), pp. 1–12.

[Freudenberg2001] Freudenberg, B. Real-Time Stroke Textures. (Technical Sketch) SIGGRAPH 2001 Conference Abstracts and Applications, p. 252.

[Gao] Gao Ling Mei, "Chinese Painting by Chang Da-Chien," Published in 1988 by Art Book Co., Ltd.

[Guo91] Qinglian Guo, Tosiyasu L. Kunii, "Modeling the Diffuse Paintings of 'Sumie'". In: Kunii T (Ed.), Modeling in Computer Graphics (Proceedings of the IFIP WG5.10). Berlin: Springer, 1991. Page 329~338.

[Hertzmann2000] Hertzmann, A. and Zorin, D. Illustrating Smooth Surfaces. Proceedings of SIGGRAPH 2000, Computer Graphics, Annual Conference Series, pp. 517–526.

[Lee99] Jintae Lee, "Simulating oriental black-ink painting", IEEE Computer Graphics & Applications 1999; 19(3): 74-81.

[Lee2001] Jintae Lee, "Diffusion rendering of black ink paintings using new paper and ink models", Computer & Graphics 25 (2001), Page 295~308.

[Liu] Yung Liu, "Ten Thousand Mountains," Published in the United States in 1984 by Shui-Yun-Chai Studio, pages 56–73 East Hampton Boulevard, Bayside, New York 11364.

[Markosian97] Markosian, L., Kowalski, M. A., Trychin, S. J. , Bourdev, L.D., Goldstein, D. and Hughes, J. F. "Real-time nonphotorealistic rendering". Proceedings of SIGGRAPH 97, pp. 415–420.

[Small90] David L Small, "Simulating Watercolor by Modeling Diffusion , Pigment and Paper Fibers", SPIE Proceedings, Vol. 1460, No. 15. San Jose, CA, 1990.

[Sousa99] Sousa, M. C., And Buchanan, J. W. "Computer-generated graphite pencil rendering of 3d polygonal models". Computer Graphics Forum 18, 3 (September 1999), pp. 195–208.

[Strassmann86] Steve Strassmann, "Hairy Brushes", Proceedings of ACM SIGGRAPH 86, page 225-232.

[Theo] Theo G. M., Van De Ven, "Colloidal Hydrodynamics", Colloid Science – A Series of Monographs, Page 51~77.

[Way2001] Der-Lor Way, Zen-Chung Shih, "The synthesis of rock textures in Chinese landscape painting". Computer Graphics Forum Volume 20, Number 3, pp. C123-C131, 2001.

[Way2002] Der-Lor Way, Yu-Ru Lin, Zen-Chung Shih, "The synthesis of trees in Chinese landscape painting using silhouette and texture strokes", Journal of WSCG Volume 10, Number3,2002, pp. 499-507.

[Zhang1999] Qing Zhang, Y. Sato, J. Takahashi, K. Muraoka and N. Chiba. "Simple cellular automaton-based simulation of ink behaviour and its application to suibokuga-like 3D rendering of trees". The Journal of Visualization and Computer Animation. Volume 10, 1999, pp. 27-37.