Matrix Mathematics

Errata and Addenda for the Second Edition

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This document contains an updated list of errata for the second edition of *Matrix Mathematics*. Please email me if you discover additional errors, and I will include them in future updates.

1. Page *xxxv*), line -10: Replace "$x \leq y$ and $x \neq y$ do not imply that $x < y$" with "$x \leq\leq y$ and $x \neq y$ do not imply that $x << y$".
2. Page 3, line 20: Replace "$\cup$" and "$\cap$" with "$\cap$" and "$\cup$", respectively.
3. Page 13, Fact 1.7.12, statement *i*): Delete "$\subseteq f^{-1}[f(A)]$".
4. Page 24, Fact 1.10.6: Append ")" to the left hand side of the inequality, and "]" to the right hand side of the inequality.
5. Page 46, Fact 1.13.10: In the inequality below "In particular," multiply the right hand side by 27.
6. Page 49, Proof of Fact 1.13.21: Replace "obtuse" with "isosceles".
7. Page 51, Fact 1.14.5: Replace "$(y^2 + z^2)^2$" with "$(y^2 - z^2)^2$".
8. Page 55, Fact 1.17.11: Replace "complex" with "real".
9. Page 61, Fact 1.17.32: For $r = 0$, define M_r by $(\prod_{i=1}^n x_i^{\alpha_i})^{1/n}$.
10. Page 67, Fact 1.18.5: Delete the statement beginning with "Now," and the following statement.
11. Page 68, Fact 1.18.9: Replace the last sentence with: "Furthermore, equality holds if and only if there exists $\alpha \geq 0$ such that either $[x_1 \ \cdots \ x_n] = \alpha[y_1 \ \cdots \ y_n]$ or $[y_1 \ \cdots \ y_n] = \alpha[x_1 \ \cdots \ x_n]$.".
12. Page 75, Fact 1.20.1: In statements *xix*) and *xx*) change "$(\operatorname{Re} z)^2$" to "$2(\operatorname{Re} z)^2$".
13. Page 76, Fact 1.20.2: In *xxiii*), change "$\leq$" to "$=$".
14. Page 83, Fact 1.21.2, statement *xxviii*): Replace "$\tan^{-1} \frac{xy}{1-xy}$" with "$\tan^{-1} \frac{x+y}{1-xy}$".
15. Page 105, Fact 2.5.6: Replace "$\mathbb{F}^{n \times m}$" with "$\mathbb{F}^{n \times n}$".

16. Page 106, Proof of Proposition 2.5.9: Change the second "$\leq$" to "$=$".

17. Page 110, line -2: Change "$\mathbb{F}^m$" to "$\mathbb{F}^n$".

18. Page 123, Fact 2.9.27: In statement *i*), require "$\mathcal{S} \subseteq \mathbb{F}^m$".

19. Page 124, Fact 2.9.30: In statement i), delete "$\subseteq f^{-1}[f(\mathcal{S})]$".

20. Page 126, Fact 2.10.14, statement *iii*): Replace "$\mathcal{N}(A^*) \cap \mathcal{R}(B^*)$" with "$\mathcal{N}(B^*) \cap \mathcal{R}(A^*)$".

21. Page 128, Fact 2.10.24: For the two equalities, assume that both x and y are nonzero.

22. Page 131, Fact 2.11.4: In *iv*) replace the first "$=$" with "$= \{0\}$ and".

23. Page 137, Fact 2.12.8: In *ii*) change "$\mathcal{R}$" to "rank".

24. Page 138, Fact 2.12.18: In the definition of C replace $(A^{\mathrm{T}} + yx^{\mathrm{T}})^k y$ with $(A^{\mathrm{T}} + yx^{\mathrm{T}})^{k-1} y$.

25. Page 154, Fact 2.16.4: Replace "$\frac{1}{(\det A)(a - y^{\mathrm{T}} A^{-1} x)}$" with "$\frac{1}{a - y^{\mathrm{T}} A^{-1} x}$".

26. Page 155, Fact 2.16.6: Replace "$1_{1\times n}^{\mathrm{T}} A^{\mathrm{A}} 1$" with "$1_{1\times n} A^{\mathrm{A}} 1_{n\times 1}$".

27. Page 166, line 6: Replace "nonsingular. Then," with "nonsingular, then"

28. Page 170, Fact 2.20.11: Replace "$18R^2 \leq a^2 + b^2 + c^2$" with "$18rR \leq a^2 + b^2 + c^2$".

29. Page 172, Fact 2.20.13, lines 6 and 12: Replace "$ac + bc$" with "$ac + bd$".

30. Page 175, Fact 2.20.21, line 2: Replace "$|[x \;\; y \;\; z]|$" with "$|\det [x \;\; y \;\; z]|$".

31. Page 185, Section 3.3, line 2: Replace "Section 11.5" with "Section 11.6".

32. Page 191, Fact 3.5.9: Change "$\operatorname{rank} C = r$" to "$\operatorname{rank} C = \operatorname{rank} CB = r$".

33. Page 193, Fact 3.7.7: Change "Let $A \in \mathbb{R}^{n\times n}$." to "Let $A \in \mathbb{R}^{n\times n}$, and assume that A is symmetric.".

34. Page 193, Fact 3.7.9: In *v*) and *vi*), change "if and only if" to "only if".

35. Page 194, Fact 3.7.11: Assume that A is skew-Hermitian.

36. Pages 197 and 198, Fact 3.7.28 and Fact 3.7.29: Change "then $\hat{B}$ and $\hat{C}$ are given by" to "then B and C are given by".

37. Page 203, Fact 3.10.1: In *xliii*) replace $(x^{\mathrm{T}} x)^{-1}$ with $-(x^{\mathrm{T}} x)^{-1}$.

38. Page 215, Fact 3.12.2: Replace "only if" with "only if A is semisimple and".

39. Page 219, Fact 3.12.25: Change "There exist $\alpha, \beta \in \mathbb{F}$" to "There exist nonzero $\alpha, \beta \in \mathbb{F}$".

40. Page 220, Fact 3.12.28: Change "there exist $\alpha, \beta \in \mathbb{F}$" to "there exist nonzero $\alpha, \beta \in \mathbb{F}$", and change "for all $\alpha, \beta \in \mathbb{F}$" to "for all nonzero $\alpha, \beta \in \mathbb{F}$".

41. Page 221, Fact 3.12.28: Replace *vi*) and *vii*) with the statements:

    vi) $\mathcal{R}(A)$ and $\mathcal{R}(B)$ are complementary subspaces, and $\mathcal{R}(A^*)$ and $\mathcal{R}(B^*)$ are complementary subspaces.

vii) $\mathcal{R}(A)$ and $\mathcal{R}(B)$ are complementary subspaces, and $\mathcal{N}(A)$ and $\mathcal{N}(B)$ are complementary subspaces.

42. Page 227, Fact 3.13.19: In statement $iii)$ replace "$=$" with "$\subseteq$".

43. Page 227, Fact 3.13.20: The solution to the problem is yes.

44. Page 231, Fact 3.16.5: Change "p. 114" to "p. 215".

45. Page 236, Fact 3.18.10: Delete "Toeplitz".

46. Page 237, Fact 3.19.1: In statement *ii*), change A to A^{-1}.

47. Page 248: Replace "$c^2 + c^2$" with "$c^2 + d^2$".

48. Page 272, line -4: Change "$n = 1$" to "$i = 1$".

49. Page 276, Fact 4.8.2, line -5: Change "β_3" to "β_{n-3}".

50. Page 276, Fact 4.8.2, line -2: Change "Fact 1.17.11" to "Fact 1.17.10".

51. Page 300, Fact 4.11.5: Change "perturbation" to "permutation" twice.

52. Page 304, line 8: Replace "$\frac{L_{k+1}}{L_k}$" with "$\lim_{k\to\infty} \frac{L_{k+1}}{L_k}$".

53. Page 306, Fact 4.11.19: Change "[174, p. 27]" to "[186, p. 27]".

54. Page 306, Fact 4.11.22: Change "$x > 0$ and $y > 0$" to "$x >> 0$ and $y >> 0$".

55. Page 320, proof of Proposition 5.4.6: Replace S by $\hat{S}$, $\operatorname{diag}(B_1, 0, -B_2)$ by $\operatorname{diag}(-B_1, 0, B_2)$, $I_{\nu_-}(A)$ by $-I_{\nu_-}(A)$, and $-I_{\nu_+}(A)$ by $I_{\nu_+}(A)$.

56. Page 328, replace (5.6.9) by: $\{\sigma_1, \ldots, \sigma_n\}_{\mathrm{ms}} = \{|\lambda_1(A)|, \ldots, |\lambda_n(A)|\}_{\mathrm{ms}}$.

57. Page 333, Proposition 5.7.4: Change "hold" to "are equivalent".

58. Page 337, Fact 5.8.18: Delete ", and assume that S is nonsingular".

59. Page 338, Fact 5.9.2: Change "$\min\{|a|, |b|\}$" to "$\sqrt{a^2 + b^2}$".

60. Page 342, Fact 5.9.26: Change "$\operatorname{mspec}(I_n) = -\operatorname{mspec}(I_n)$" to "$\operatorname{mspec}(\hat{I}_n) = -\operatorname{mspec}(\hat{I}_n)$".

61. Page 342, Fact 5.9.27: Change the (1,2) block of S to I.

62. Page 342, Fact 5.9.30: Change "$\mathbb{F}^{n\times m}$" to "$\mathbb{F}^{n\times n}$".

63. Page 347, Fact 5.10.13: Change "an orthogonal matrix" to "a projector".

64. Page 355, Fact 5.11.24: Delete the last sentence.

65. Page 355, Fact 5.11.25: Delete the left-hand inequalities in the first two strings.

66. Page 365, Fact 5.12.13: Assume $B \in \mathbb{F}^{m\times m}$ and replace "$\sigma_i(B)$" by "$\sigma_i^2(B)$"

67. Page 393, Fact 5.18.5: Change "$S = (AA^*)^{-1/2}A$" to "$S = A(A^*A)^{-1/2}$".

68. Page 394, Fact 5.18.8: Can omit "assume that A is nonsingular"

69. Page 399, Fact 6.1.6: In *xxx*), change "orthogonal" to " unitary".

70. Page 404, Fact 6.3.6: Change "Let $A \in \mathbb{F}^{n\times m}$" to "Let $n \geq 2$, let $A \in \mathbb{F}^{n\times n}$.

71. Page 411, Fact 6.3.33: Change "$(A-I)^* = (A-I)^+$" to "$(A-I)^\# = (A-I)^+$".

72. Page 412, Fact 6.4.2: Change "Finally," to "Finally, assume that A does not have full rank. Then,".

73. Page 419, Fact 6.4.38: Change "A^*B" to "AB^*."

74. Page 423, Fact 6.5.5: Change "and assume" to "and assume that $\mathcal{A}$ is Hermitian and".

75. Page 424, Fact 6.5.8: Change "$\mathrm{rank}(D - B^*A^+B)$" to "$\mathrm{rank}(C - B^*A^+B)$".

76. Page 431, Fact 6.6.3: Delete "$AB = 0$".

77. Page 435, Fact 6.6.13: Change "BA is nonsingular" to "CB is nonsingular".

78. Page 447, Fact 7.4.23: Change "$I_{l\times l}$" to I_l, and change "$I_{n\times n}$ to I_n".

79. Page 452, Fact 7.5.13: Change "A, D" to "A, C" and change "C, B" to "B, D"

80. Page 452, Fact 7.5.17: Change "$A^{(k)}B^{(k)}$" to '$A^{(n)}B^{(n)}$".

81. Page 456, Fact 7.6.11: Change "$x_1, \ldots, x_n \in \mathbb{R}^n$" to "$x_1, \ldots, x_n \in \mathbb{R}$".

82. Page 463, Proposition 8.2.7: In v) change "$\beta_0, \ldots, \beta_{n-1} \geq 0$" to "$(-1)^{n-i}\beta_i \geq 0$."

83. Page 466, proof of Corollary 8.3.7, line 2: Replace "$\mathbb{R}$" by "$\mathbb{F}$".

84. Page 473, Lemma 8.5.1: Change "λ_{n_r}" to "λ_r"; change "μ_{n_r}" to "μ_r"; and change "$\lambda_r \in \mathbb{R}$" to "$\mu_r \in \mathbb{R}$".

85. Page 488, Fact 8.7.8: Delete "and for $\mathcal{S} \subseteq \{1, \ldots, n\}$" and change "inclusions" to "inequalities".

86. Page 502, Fact 8.10.13: Assume that $A \circ I$ is positive definite.

87. Page 505, Fact 8.10.29: Assume that S is nonsingular.

88. Page 505, Fact 8.10.30 and Fact 8.10.31: Change "$\mathbf{H}^{n\times n}$" to "$\mathbf{H}^n$".

89. Page 520, Fact 8.11.23: Change "$A^* = \tilde{I}A\tilde{I}$" to "$B^* = \tilde{I}B\tilde{I}$".

90. Page 520, Fact 8.11.24: Change "$A^*\tilde{I}A = \tilde{I}$" to "$B^*\tilde{I}B = \tilde{I}$".

91. Page 522, Fact 8.11.31: Change "A^*_{32}" to "A^*_{23}".

92. Page 524, Fact 8.12.8: Delete "$\mathrm{tr}\, AB \leq$".

93. Page 534, Fact 8.13.4: Change "$A + B$" on the left hand side to "B"

94. Page 526, Fact 8.12.15: The upper left inequality of the Problem is false.

95. Page 534, Fact 8.13.4: Change "and assume that A is positive semidefinite" to "assume that A is positive semidefinite, and assume that B is skew Hermitian." Also, on the left hand side of the first inequality, replace "$A + B$" with "B".

96. Page 535, Fact 8.13.7: Change "$\det(A + \jmath B)$" to "$|\det(A + \jmath B)|$".

97. Page 541, Fact 8.13.38: Change "and let $\mathcal{S} \subseteq \{1, \ldots, n\}$" to "and assume that A is positive semidefinite"

98. Page 541, Fact 8.13.39: Reverse the second inequality in the first string.

99. Page 543, Fact 8.13.43: Change "$\det A \leq \det \tilde{A}$" to "$\det A \leq \det \tilde{A}_n$".

100. Page 547, Fact 8.14.8: Change the second instance of "$\lambda_{\min}$" to "$\lambda_{\max}$".

101. Page 553, Fact 8.15.21: Change "$\mathbb{R}^n$" to "$\mathbb{F}^n$" and change "min" to "max".

102. Page 559, Fact 8.18.1: Append "where $\lambda_2 \leq \lambda_1$" to the first sentence.

103. Page 561, Fact 8.18.6: Change "$\sigma_i(A)$" to "$r\sigma_i(A)$"

104. Page 563, Fact 8.18.15: Change "λ_i" to "$\lambda_i(A)$" twice.

105. Page 565, Fact 8.19.5: Change "$\lambda_i[f(\alpha A + (1-\alpha)B)] \leq \alpha\lambda_i[f(A)] + (1-\alpha)\lambda_i[f(B)]$." to "$\lambda_i[f(\alpha A + (1-\alpha)B)] \leq \lambda_i[\alpha f(A) + (1-\alpha)f(B)]$.".

106. Page 566, Fact 8.19.7: Multiply the left hand sides of the third and fourth strings by $1/2$

107. Page 568, Fact 8.19.14: The answer to the problem is "yes".

108. Page 572, Fact 8.19.24: Change "$\operatorname{spec}(A)$" to "$\operatorname{spec}(AB)$".

109. Page 578, Fact 8.21.4: Delete "$1/2$" on the right-hand side of the equation.

110. Page 578, Fact 8.21.5: In the definition of S, change "$\langle A \rangle$" to "$\langle A^* \rangle$". Also, change "$\langle A \rangle AA^+ \langle A \rangle$" to "$\langle A^* \rangle AA^+ \langle A^* \rangle$".

111. Page 585, Fact 8.22.4: Change "$1 \in \operatorname{spec}(A \circ A^{-1})$" to "$1 \in \operatorname{spec}(A \circ A^{-\mathrm{T}})$".

112. Page 590, Fact 8.22.29: Change "Hermitian" to "positive semidefinite".

113. Page 611, Corollary 9.4.12: Change "$A \in \mathbb{F}^{m \times l}$" to "$B \in \mathbb{F}^{m \times l}$".

114. Page 617, Corollary 9.6.9: Assume that $n \leq m$.

115. Page 618, Fact 9.7.4: In *v*), change the second "$+$" to "$-$".

116. Page 622, Fact 9.7.11: Change "$\alpha = 2$" to "$\alpha = 1$".

117. Page 625, Fact 9.7.21: In the second string, change the fifth "$+$" to "$-$". In the last string, change the last "$+$" to "$-$".

118. Page 636, Fact 9.9.5: Change "$A \leq B$" to "$-B \leq A \leq B$".

119. Page 642, Fact 9.9.31: Change "$\lambda_i(A)$" to "λ_i", and change "$\lambda_j(A)$" to "λ_j"

120. Page 643, Fact 9.9.35: Change all $\|\cdot\|$ to $\|\cdot\|_{\sigma p}$

121. Page 658, Fact 9.12.8: Change λ_j to μ_j

122. Page 659, Fact 9.12.10: Delete the second remark.

123. Page 663, Fact 9.13.18: Change the upper limit of the last summation to n

124. Page 665, Fact 9.14.3: Define $r \triangleq \min\{m, n\}$.

125. Page 667, Fact 9.14.11: In statement *iii*), change "H" to "A".

126. Page 671, Fact 9.14.27: Replace "$A, B \in \mathbb{F}^{n\times m}$" by "$A \in \mathbb{F}^{n\times m}$ and $B \in \mathbb{F}^{m\times n}$"

127. Page 702, Fact 10.12.6: Change "$2sB$" to "$2sB^2$".

128. Page 731, first displayed equation: Change "$x^*[(\jmath\omega I$" to "$-x^*[(\jmath\omega I$".

129. Page 742, Fact 11.12.1: In *vii*), change "-" to "+".

130. Page 773, Fact 11.18.34: Replace the equations with

$$\int_{-\infty}^{\infty}(\jmath\omega I - A)^{-1}\,\mathrm{d}\omega = \pi I, \quad \int_{-\infty}^{\infty}(\omega^2 I + A^2)^{-1}\,\mathrm{d}\omega = -\pi A^{-1},$$

$$(A \oplus A)^{-1} = \frac{-1}{2\pi}\int_{-\infty}^{\infty}(-\jmath\omega I - A)^{-1} \otimes (\jmath\omega I - A)^{-1}\,\mathrm{d}\omega.$$

131. Page 779, Fact 11.20.8: In the 4th inequality, replace "$\max_{i=1,\ldots,n-1}$" with "$\max_{i=0,\ldots,n-1}$".

132. Page 798, line 3: Change "(12.1.20)" to "(12.1.19)".

133. Page 809, 6 lines above (12.6.5): Change "η^{T}" to "η".

134. Page 809, 2 lines above (12.6.5): Change "$m \times nm$" to "$n \times nm$"

135. Page 813, Proposition 12.6.14: Change "$K \in \mathbb{R}^{n\times m}$" to "$K \in \mathbb{R}^{m\times n}$". Also, the proof is missing a change of basis transformation.

136. Page 813: $A_1 \in \mathbb{R}^{q\times q}$ and $B_k \in \mathbb{F}^{q\times m}$. In line -4, replace A by A^{T}.

137. Page 821, Line 6 of the proof of Proposition 12.8.5: Change "Proposition 12.5.3" to "Proposition 12.8.3".

138. Page 823, Proof of Proposition 12.9.3: In the matrix at the bottom of the page, the entries below the sub-anti-diagonal are incorrect.

139. Page 824, Line 2: Multiply the 2×2 matrix with 1's on the diagonal by -1.

140. Page 824: Proposition 12.9.7: Change "$A \in \mathbb{R}^{\hat{n}\times\hat{n}}$" to "$\hat{A} \in \mathbb{R}^{\hat{n}\times\hat{n}}$".

141. Page 825: Corollary 12.9.9: Define $S_{\mathrm{c}} \triangleq [\mathcal{O}(A, C)]^{-1}\mathcal{O}(A_{\mathrm{c}}, C_{\mathrm{c}})$ and $S_{\mathrm{o}} \triangleq [\mathcal{O}(A, C)]^{-1}\mathcal{O}(A_{\mathrm{o}}, C_{\mathrm{o}})$.

142. Page 831, line 2: Change "Szeros" to "mSzeros".

143. Page 840, line -3: Change "$G_2 + G_2$" to "$G_1 + G_2$".

144. Page 840, lines 6 and 7: Change to

$$\begin{aligned}
&\leq \|G_1\|_{\mathrm{H}_2}^2 + \|G_2\|_{\mathrm{H}_2}^2 + 2[\mathrm{tr}(C_1 Q_{12} Q_2^{-1} Q_{12}^{\mathrm{T}} C_1^{\mathrm{T}})\,\mathrm{tr}(C_2 Q_2 C_2^{\mathrm{T}})]^{1/2} \\
&\leq \|G_1\|_{\mathrm{H}_2}^2 + \|G_2\|_{\mathrm{H}_2}^2 + 2[\mathrm{tr}(C_1 Q_1 C_1^{\mathrm{T}})\,\mathrm{tr}(C_2 Q_2 C_2^{\mathrm{T}})]^{1/2}
\end{aligned} \tag{1}$$

145. Page 842, line above (12.12.7): Change "$+\jmath \cos \phi_0$" to "$-\jmath \cos \phi_0$".

146. Page 842, one line above (12.12.8): Delete "Re".

147. Page 851, lines -1 and -2: Interchange -1 and 1.

148. Page 852, Example 12.16.8: Change the 4th sentence to: "The corresponding closed-loop dynamics matrices are $\tilde{A} = 1/\sqrt{R_2} > 0$ and $\tilde{A} = -1/\sqrt{R_2} < 0$."

149. Page 852, line -2: Change "P_4" to "P_1".

150. Page 855, proof of Theorem 12.17.2: Change "Corollary 12.8.6" to "Corollary 12.5.6".

151. Page 858, line 8: Change "$Z \in \mathbb{R}^{n \times n}$" to "$Z \in \mathbb{R}^{2n \times 2n}$".

152. Page 858, line -6: Change "$x^{\mathrm{T}} M_{21}$" to "$x^{\mathrm{T}} M_{21}^{\mathrm{T}}$".

153. Page 859, line 4: Change "λZ" to "λx".

154. Page 859, Section 12.18: Change the first paragraph to "In this section we consider the existence of the maximal solution of (12.16.4). Example 12.6.3 shows that (12.16.4) may not have a maximal solution."

155. Page 873, Fact 12.22.8: In the realization of $[I + GM]^{-1}$, change "B" to "BM".