CORRECTING TOPOLOGICAL DEFECTS OF TESSELLATIONS

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Abstract

In automation of geometric modelling in industry, models of solids are often converted to collections of oriented triangles. Each triangle is described by the coordinates of three vertices and an outward normal vector. The topology is discarded. For many applications it is useful to reconstruct the topology of the boundary of a real solid as an oriented two-manifold. This poses three types of difficulties. First because of the finite precision, the coordinates of matching vertices in two triangles may not be equal. If a tolerance is assumed that is small compared to the shortest triangle edge, coordinates can be matched. A more serious problem is that many CAD systems produce collections of triangles that do not match. They often contain topological defects such as non-manifold edges. An algorithm is presented that has successfully transformed non-manifold edges into oriented two manifolds for most cases encountered to date.

1. Introduction

Geometric modeling technology, which has developed rapidly since 1970, plays a central role in industrial CAD[5]. Solid modelers have complex data structures that are usually proprietary to the computer code used to generate them and therefore are secret. There complexity and secrecy makes it difficult to exchange data between different solid modelers. One of the easiest ways to overcome the difficulties of data exchange between solid modelers is to create a simple boundary representation (BRep) of a tessellation that covers the surface of the solid with planar triangles. In this paper, an STL tessellation of a BRep approximates a partition of a 2-manifold into triangles. It has a simple data structure. Each triangle is described by an outward normal and the coordinates of three ordered points. Some industrial and research applications begin their solid modeling with such files that contain the information describing the boundary of the solid model. Typical industrial tessellations can have several hundreds of thousands of triangles.

One difficulty that arises with this representation is that most current solid models do not create topologically correct tessellations. In order to use these tessellations as the definition of the geometry of a real solid part for other processes, such as Finite Element Method (FEM) mesh generation, it may be necessary to have a mathematically correct oriented 2-manifold. In such cases it is necessary to correct the topological defects in the tessellation of solid models in these files. The automation of solid modeling from initial BRep to the final meshing of the solid model continues to be an important research goal.

Many research papers discuss how to build correct, complete and efficient BRep models[5][6]. However, because the solid modeling itself is complicated or because the defects of tessellation can vary in many ways, few papers study the issue of detecting and correcting topological defects. One of our aims is to help to fill this gap. The major areas of research covered in this paper include identifying topological defects, classifying topological defects and correcting topological defects automatically if feasible and with the help of a graphical user interface (GUI) when necessary. However, due to the length of the paper, only one of the most difficult aspects: resolving high degree non-manifold edges is discussed in detail.

2. Two-Manifold BRep

There are two mathematical theories that define the modeling space for solid modeling: **point-set topology** and **algebraic topology**[3][4]. Point-set topology stresses the three-dimensional solidity of a mathematical object while the algebraic topology stresses the bounding surface of a mathematical object.

In point-set topology, a solid is defined as a bounded, closed subset of E^3. The solid should remain invariant under a rigid transformation such as translation or rotation. All shapes that can be formed from stretching an infinitely elastic sphere without tearing or ripping form a topologically equivalent class. The topological transformation gives a hint that a solid model with some topological defects can be corrected mainly by topological information.

A bounded regular set is termed an r-set. This definition of regularization describes the interior of a point set, i.e., forms an open set, then covers it completely with a tight skin. This means that it will not contain isolated points, isolated lines, isolated faces or missing points, lines or faces. The boundary should also be sufficiently smooth, e.g., Lipschitz, in order to be modeled. Real solids do have quite smooth boundaries. Manifold (2-manifold) BRep based solid modeling topology representations are the basis of some of the most popular forms of solid modeling representations used today.

A 2-manifold M is a topological space where every point has a neighborhood topologically equivalent to an open disk of E^2, which is the same as saying that every point has a neighborhood which is homeomorphic to R^2. The BRep model in this paper is defined to be a 2-manifold with triangular tessellation.

There is an inherent theoretical mismatch between r-set models and 2-manifold models: not all r-sets are realizations of some 2-manifolds. The problem with such objects is that a surface can "touch" itself at a point or on a curve segment. The neighborhoods of such points are not homeomorphic to an open disk in E^2. These problems can be resolved by separating the touching surfaces. This is accomplished by duplicating the nodes with the same coordinates but a different node identity. Hence, the object would be represented as the rigid combination of two or more components that are separated topologically but still touch each other at a geometrical point. There are 2-manifolds that do not have physical counterparts in E^3, i.e., that cannot be constructed in three-dimensional space at all, and are hence not the boundaries of any r-set. The *Klein bottle* is an example. Also a correct 2-manifold should obey *Möbius' Rule* that requires the faces of the BRep to be consistently oriented in E^3.

Another characteristic of a BRep of a solid is the Euler characteristic. It states that: for a surface S of an r-set, the sum $v - e + f$, where v, e, f stand for the numbers of vertices, edges and faces respectively, is a constant independent of the manner in which the surface S is partitioned and independent of any operation on the surface, provided that the surfaces before and after the operation are topologically equivalent. Usually this does not help in correcting topological defects.

Based on the discussion above, we can defined a topologically correct 2-manifold BRep model using planar triangles to be one that satisfies:

1. **Each edge's degree is two, which means that each edge is only adjacent to two polygon faces.**
2. **Each vertex has only one open disk.**
3. **The face normal of each polygon is directed outward from the r-set.**
4. **A solid could be a union of one or more 2-manifolds.**
5. **Faces of the model do not intersect each other except at common vertices or edges.**

3. Identification and Classification of Topological Defects

It is assumed that topological defects in tessellations can be arbitrarily complex. Therefore our objective is to correct automatically those defects that occur most frequently in a set of real STL files that have been provided by industry[2].

As an r-set can be a union of several sub-r-sets, the checking can be carried out on each separate set of connected face elements, each of which is adjacent to other face elements in the same set at least at one edge. The sets of connected triangles can be easily found by face flood filling[1]. Geometric information can also be calculated for each separate r-set, such as the bounding-box, the volume of the set, the average area of the face, the average edge length for certain sets of edges. All the information including that pertaining to the topological defects will be used to correct the topological defects in the BRep. The GUI should be used to check for connections between separate sets since global information about the BRep is needed. If the BRep is reasonably behaved, a single r-set will be one set of connected face elements. Our attention will focus on checking each set of connected face elements.

It is possible to identify most topological defects by checking if every edge in the model is degree two (the degree of an edge in this paper represents the number of faces that are adjacent to this edge). This test will detect objects with missing surfaces or that are joined together. Missing surfaces will introduce **open loops edges** which are odd degree edges caused by missing faces adjacent to these edges. The open loop edges form loops and the loops can further form loops. In most cases, open loop edges are degree one edges. The open loop edges can be fixed by separating open loops into single open loops and adding the missing faces or by merging the nodes on each of the single open loops. **High degree edges** are introduced by touching of the surfaces at the edges or by dangling undesired faces or bodies at these edges. They can be further classified into two categories: odd high degree edges and even high degree edges. High degree edges cannot be simply resolved by duplicating edges since it is also necessary to check which faces should be adjacent to which edges. Otherwise the correct adjacency of the faces may not be guaranteed which could exacerbate face intersection or even result in a *Klein Bottle*, which has no physical counterpart in 3D.

The criteria that each edge should be adjacent to two faces will miss at least three kinds of topological defects: **wrong orientation edges** exist when there are some patches – sets of faces – that are oriented differently, such that for each pair of faces along the boundaries of these sets, one is facing outward while the other is facing inward from the solid body. This problem can be identified by checking the paired faces adjacent to each edge in a tessellation to see if they have opposite orientation along the edge. For a correct edge, the edge adjacent faces should have the opposite orientation relative to the edge; while for an incorrect edge, the edge adjacent faces have the same orientation with respect to the edge. Wrong orientation edges will form loops, so that the incorrectly oriented faces can be collected by face flood filling bounded by the loops.

The second case of topologically defects that cannot be detected using the criteria above is the one where the surfaces touch at points. Often, the points that are touching can be separated without effort: reproduce the joining nodes two or more times and separate them along with their open disks such that after separation, each node will only have one open disk. For some cases, this kind of topological defect will not have any negative impact on other processes such as mesh generation which is based on the assumption that the surface of the part is topologically correct.

The third and perhaps the worst type of topological defect that cannot be detected by the above criteria above is exemplified by the *Klein Bottle*. This kind of topological defect cannot be corrected unless the associated geometric information is also corrected. This involves the very costly check of face intersection which is usually several orders of magnitude more expensive than pure topological checking[1]. It may be necessary for the user to rebuild the local geometry and topology in the GUI. The detection of the intersecting triangles is necessary before the user can correct the defects.

From the above analysis, we can see that the topological defects can be identified mainly by identifying the non-manifold edges in the part. If the part passes this test, further tests will include checking the number of open disks for each vertex on the part and face intersections in the tessellation. There are maybe other defects, such as duplicate faces and ill-shaped faces, such as two nodes in a triangle element have the same coordinates.

4. Resolving High Degree Non-Manifold Edges

Fixing open loops involves more geometric information and can include high degree edges[1]. Therefore, a quite logical and practical strategy is to resolve the high degree edges first which will leave only non-manifold edges of degree one to be fixed later. Resolving the high degree edges will be carried out sequentially in each set of connected non-manifold edges. From a topological point of view, there is sufficient information in each set of non-manifold edges and their connected face elements to resolve the high degree edges for a real solid.

The main steps to resolve the high degree edges include:

1. **Finding all of the non-manifold edges in the part;**
2. **Do edge flood filling by the adjacency of connecting nodes to find connected sets of non-manifold edges;**
3. **Checking each set of connected non-manifold edges to see if there are high degree non-manifold edges in the set;**
4. **If there are, resolve the high degree edges in the set.**

This divides the tasks into manageable portions and also makes recursive operations more efficient. As in some cases, some of the geometric information or topological information is missing or incorrect, resolving the high degree edges in a set of connected non-manifold edges is one of the most difficult challenges in this research. Solving this creates non-manifold degree one edges which can be much easier to fix. All the available topological and geometric information may have to be used to perform this task correctly. Often, cases of high degree edges are very complex. The degrees of high degree edges can be odd or even.

In the odd high degree cases, there must exist missing or additional face elements adjacent to the edge. In the even degree cases, there might also exist missing or additional faces, but the number of missing or additional faces should be an even number, which makes these cases rare. The high degree edges may not all be separate edges. They can form links, loops, trees and more general graphs.

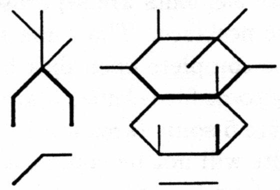

Figure 1: High degree edges can form links, loops, trees and more general graphs. The thickness represents the edge degrees.

To resolve a high degree edge will depend strongly upon the local topological and geometric information. In a valid tessellation, both kinds of information are consistent. In a invalid tessellation, one or both kinds of information might be incorrect or incomplete. In the following steps, a complex problem is broken down into sub-problems and each subproblem is dealt with separately.

For a connected set of connected non-manifold edges that include high degree edges, all the high degree edges can be collected. As the degree of a non-manifold edge is directly associated with the number of open disks that its end nodes have, it can be seen that for a high degree edge, each of its end nodes will have at least two open disks. To resolve a high degree edge, either or both of the end nodes should be separated from the original nodes. In order to separate a node from the original node, the new node must find all the face elements in its open disk. Separating the face elements without regard for this requirement can result in an incorrect topology (see Figure 2).

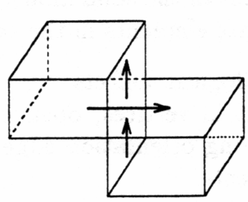

Figure 2: An incorrectly separated open disk.

As it has been discussed earlier, the high degree edges will form links, loops, trees or more general graphs and they cut the open disks into several pieces. It is easier to solve if the separation begins at the ends of branches of a tree (graph) of high degree edges, but it is also possible to begin with any node on high degree edges.

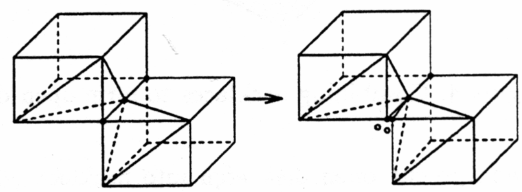

Figure 3: Separating open disks of the high degree edges.

For a set of non-manifold edges with high degree edges, the high degree edges in the set can be resolved recursively. This can begin by separating the open disks from the easiest node — the node that has the fewest high degree edges in the set such that the open disks of this node are being cut the least by the high degree edges. Often the node will be connected to only one high degree edge. As the separation proceeds, the number of high degree edges in the set and also the degrees of the high degree edges will decline. At the same time, the data structure of the set of non-manifold edges and the data structure of the part should be updated. When all the high degree edges in the set have been resolved the processing of this set will terminate.

Now the key problem is to resolve a node with several open disks. How to find the complete open disks for the node is the main problem, since as soon as a complete open disk is extracted from the face elements of the node, a new node can be generated with the same coordinates (but with a different topological neighborhood) and then the open disk with the new node can be separated from the original node. By doing so, the related high degree edges will reduce their degrees by one (see Figure 3). Since the high degree edges will cut the open disks into several pieces, a face element flood filling by edge adjacency but not adjacency of the non-manifold edge, in the connected set of non-degree two non-manifold edges to form connected sets of elements called necklaces of the node. These necklaces are parts of open disks or complete open disks bounded by the high degree edges (see Figure 4).

In Figure 4, let's assume that open disk *abcd* belongs to a part of a surface, open disk *efgh* belongs to another part of a surface, face *aoe* and face *goi* are topological defects. Because of face *aoe* and face *goi*, edge *ao*, *eo* and *ggo* are degree three edges, edge *gi* and edge io are degree one edges. The high degrees

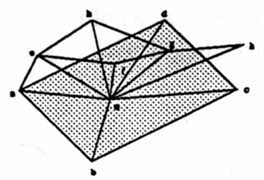

Figure 4: Combining necklaces to form open disks.

eo and *go* cut open disk *efgh* into necklace *efg* and *ghe* while on the other hand, the even high degree edge *ao* cuts the open disk *abcd*. Open disk *abcd* is still complete. It can be separated without effort. Separating open disk *abcd* can simplify the situation: after open disk *abcd* has been separated, face *aoe* and *goi* can be considered as **dangling faces**, they can be removed easily. In some complicated cases, no open disks can be extracted easily. For example, if there is a topological defect face *goc*, then open disk *abcd* is separated into two necklaces. The following steps can combine the paired faces to form complete open disks.

So far, no geometric information has been used to resolve the high degree edges. From the definition of a necklace we can know that its boundaries are high degree non-manifold edges. If the two boundaries of a necklace are actually one high degree edge, the necklace is a complete open disk and can be separated. In most cases, the necklaces are not complete open disks at all. In such cases, two or more necklaces should be connected to form a complete open disk. Then one must find the correct pair of face elements on the boundaries of the necklaces.

The problem of pairing can be solved by searching for the face element that will form a valid solid with another face element, both of which are adjacent to the boundary high degree edge. Geometric information plays a very important role in this case.

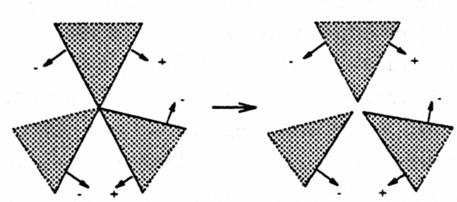

Figure 5: Radially ordering the faces around a hight degree edge and then separate the pairs.

Figure 5 shows the pairing process. To combine the necklaces of a node, first the high degree edges adjacent to the node are found, and then the triangles around each of these high degree edges will be radially ordered according to their face normals and their node orientations. If the ordering result is a circular list (+,-,+,-,+,-, ..., +, -) then the order is defined to be valid (see Figure 5). If the order is valid, then each triangle can find its paired triangle by searching in the opposite direction of its face normal and the necklaces can be combined in pairs. If the two face elements in pair are in one necklace already, they are already combined. Otherwise the necklaces in which the paired face elements are separated will be combined into one necklace. The same process can continue until one complete open disk has been formed. In cases where some face elements are missing, (which means in Figure 5 some faces would be missing,) the ordering results will not be complete, but valid pairs might still be found provided there are no face elements inside the solid body of other paired face elements. The ordering results will be combined with the local topological information to judge if the pairing is valid. If the pairing is valid, the paired face elements can be combined to further form complete open disks. In our research, no cases have not been resolved by this procedure. Often, success at one node can lead to resolving the whole set of connected high degree edges adjacent to this node.

If high degree edges remains that cannot be resolved by the above procedure, the set of non-manifold edges that contains these high degree edges will be sent to the GUI for correction by the user. After the high degree non-manifold edges have been resolved, the remaining non-manifold edges are much easier to be fixed[1].

5. Conclusions

Based on the results of the study of identifying and correcting topological defects described in this paper, the following conclusions can be drawn:

- The topological defects can be identified and classified by examination of non-manifold edges and vertices in the solid models and the intersections of face elements in the tessellation.
- The topological defects can be classified into non-manifold vertices, open loops, high degree edges, wrong orientation edges and intersecting face elements.
- Most of the topological defects can be corrected by a systematic approach which can include several clearly isolated components.
- A GUI can be very useful to enable the user to check and correct topological defects that cannot be corrected automatically.

The method described in this paper mainly depends on topological and geometric information on the boundary of solid models. During the course of study it has been found that certain aspects of topological defects which maybe difficult to correct by the algorithms given in this paper may be easier to correct by meshing the solid model first which may supply information not only about the topology of the boundary but also the topology of the volume of the solid model. Further research should be carried out on this aspect. As the scope of this paper dealt mainly with topological defects, geometric defects such as removing zero area faces and improving the tessellation are not discussed in depth in this paper.

6. References

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