The Industrial Revolution: Past and Future

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February, 1998

This book was originally prepared for the Kuznets Lectures, given at Yale in November, 1996. I grateful to my hosts, T. Paul Schultz and T.N. Srinivasan, and to many other friends at Yale for the invitation, for the hospitality shown to me during my visit, and for many stimulating discussions.

Material taken from or related to these lectures was also presented in lectures at the University of Pennsylvania, the Technion, the Bank of Korea, Seoul National University, Korea University, the Argentine Center for Macroeconomic Studies in Buenos Aires, the Liberty Foundation in Rosario, the Center for Public Studies in Santiago, the University of California at Los Angeles, the 1997 Far Eastern Regional Meeting of the Econometric Society in Hong Kong, and the Federal Reserve Bank in Minneapolis. I thank everyone who offered comments, criticism, and encouragement on these occasions. In particular, I would like to thank Fernando Alvarez, Michael Bordo, Larry Epstein, Patrick Kehoe, Ellen McGrattan, Nancy Stokey, David Weil, and Robert Willis for helpful comments and discussion. I owe special thanks to Robert Tamura for many very useful discussions.

The Data Appendix to these lectures was prepared with the help of several very able Chicago students. Vimut Vanitcharearnthum and I first assembled long time series on population and production in regions of the world in 1994. These estimates were revised, refined, checked, and plotted in various ways with the help of Krishna Kumar and Enric Fernandez. Daniel Chin of the Minneapolis Fed also helped with the graphics. My thanks to all of them.

1. INTRODUCTION

From the earliest historical times until around the beginning of the 19th century, the number of people in the world and the volume of goods and services they produced grew at roughly equal, slowly increasing rates. The living standards of ordinary people in 18th century Europe were about the same as those of people in contemporary China or ancient Rome or, indeed, as those of people in the poorest countries in the world today. Then, during the last 200 years, both production and population growth have accelerated dramatically and production has begun to grow *much* more rapidly than population. For the first time in history, the living standards of masses of ordinary people have begun to undergo sustained growth. The novelty of the discovery that a human society has this potential for generating sustained improvement in the material aspects of the lives of all of its members, not just of a ruling elite, cannot be overstressed. We have entered an entirely new phase in our economic history.

The progress of the industrial revolution—the somewhat outmoded term for this new phase of sustained growth in living standards—has, of course, differed considerably across societies. It began as a northern European event, and has gradually diffused to other societies, a process that has accelerated dramatically since World War II and that is still very far from completion. The uneven pace of the industrial revolution has given rise to an enormous and unprecedented inequality in average living standards across economies. In 1800, incomes per capita differed by perhaps a factor of two between the richest and poorest countries. It was the problem of explaining differences in income *levels* of this size that Adam Smith undertook in the *Wealth of Nations.* Today, per capita incomes in the United States are about 25 times average income in India, while per capita incomes in both countries continue to grow at between one and two percent per year.

Nothing remotely like this economic behavior is mentioned by the classical economists, even as a theoretical possibility. And why should it have been, since nothing like it had yet been observed? The classical economists, or certainly Malthus and Ricardo, took it as a central problem for theory to account for the tendency of per capita incomes in any society to return to a roughly constant, stable level in the face of improvements in technology. Though they did not deny the possibility that this stable income level might differ from one economy to another, they sought a theory that would attribute such differences to differences in preferences or custom, not in technology or available resources. Differences in societies' *abilities* to produce would then induce differences in population only. Ricardo put the Malthusian theory of fertility that produces these consequences as at the center of his aggregate theory of production and distribution.

Modern theories of economic growth have similarly failed to deal adequately with the change in the human condition that the industrial revolution represents. These theories are built around a positive rate of technological change, either simply assumed or generated as an equilibrium outcome by the assumption of constant or increasing returns to the accumulation of knowledge.[1] They are made consistent with the sustained growth of per capita incomes by the even more central assumption of a given, constant rate of population growth. In contrast to the classical prediction that technical change induces *only* changes in population, modern theories typically assume that technical change affects only incomes, with *no* effect on population.

Both classical and modern theories of production thus succeed in accounting for the central features of the contemporary behavior of production and population. The predictions of both theories, however, conflict sharply with the data the other was designed to explain. To understand the industrial revolution, and hence to understand a world in which some economies have joined the industrial revolution and others have

[1]I will not attempt to list all the descendants of Solow (1956), but recent examples that I have in mind are Romer (1986), Lucas (1988), and Mankiw, Romer, and Weil (1992).

not, we will need to unify these conflicting theories of production. That is to say, we need to discover a more general theory of which the two we now have can be seen as special cases, a theory that lets us see the nature of the transition from the situation of stable incomes that has characterized most of history to the sustained growth that has emerged in the last two centuries.

This way of stating the central theoretical problem of growth theory represents a change in my thinking, much influenced by the work of Becker, Murphy, and Tamura (1990) and Tamura (1988), (1994). These papers consider models of equilibrium growth in which a household's choice of the number of children to bear and the amounts of different kinds of capital to accumulate are viewed as simultaneous decisions, thus building on the earlier work of Razin and Ben-Zion (1975) and Becker and Barro (1988). By introducing the fertility decision into growth theory, this research leads one to view the industrial revolution and the associated reduction in fertility levels—generally referred to as the *demographic transition*—as different aspects of a single economic event. In particular, Becker, Murphy, and Tamura (1990) exhibits a combination of production technology and preferences (over children as well as goods consumed) that is consistent with the existence of both a traditional steady state with Malthusian demography and a modern balanced growth path. A transition from the first kind of behavior to the second can be triggered, in the model, by an increase in the return on investment in human capital.[2]

[2]Wrigley's (1988) lectures emphasize the inconsistency of classical demography with sustained economic growth. The viewpoint advanced in Nerlove (1974) is also close to that taken here. There is, of course, an enormous literature on the industrial revolution and the associated demographic revolution. Even if one restricts attention to studies that involve explicit theoretical modeling, there are many contributions beyond those cited in the text.

Goodfriend and McDermott (1991) models the onset of the industrial revolution as a population-induced shift of activity between a stagnant household economy and a dynamic market sector. Murphy, Shleifer, and Vishny (1989) also put scale economies at the center of a theory of industri-

These ideas are at the center of the lectures that follow. Since their appeal, to me, is their close connection to the main facts of the industrial revolution, I will spend some time reviewing the evidence on which these introductory paragraphs are based. Recent research has filled in our factual knowledge of the economics of the postwar world and also of the more distant past, to the point where it is now possible to set out a fairly comprehensive history of population and production for the entire world. I will do this in Chapter 2.

Chapters 3 and 4 develop a modern—which is to say, mathematically explicit—version of Ricardo's theory of production and distribution for an economy with land and labor as factors of production. Chapter 5 introduces capital accumulation into this Ricardian economy—an exercise that was beyond the methods at Ricardo's command but well within the capabilities of modern dynamic theory—and reviews the reasons why a theory of the industrial revolution cannot be based on the accumulation of physical capital alone. Chapter 6 presents models of sustained growth generated by technological change and by human capital accumulation, in which the fertility decision is incorporated. We will see that theories in which sustained growth is generated by exogenous technological change cannot be made consistent with the demographic transition, while otherwise similar models in which growth rates are endogenously determined by human capital investment decisions can do so quite easily. Chapter 7 contains concluding comments.

alization. Neither theory associates a reduction in fertility with industrialization.

2. THE BASIC FACTS OF THE INDUSTRIAL REVOLUTION

In the Introduction I proposed using the term *industrial revolution* to refer to the onset of sustained growth in per capita incomes. I want next to provide a quantitative description of this event, and in particular to document the claims that it occurred in the late 18th or even early 19th century, that it was unprecedented, and that it has proceeded at an accelerating pace up to the present day.[3] Then I will turn to refinements of this description that will be helpful in understanding the origins and the course of the industrial revolution.

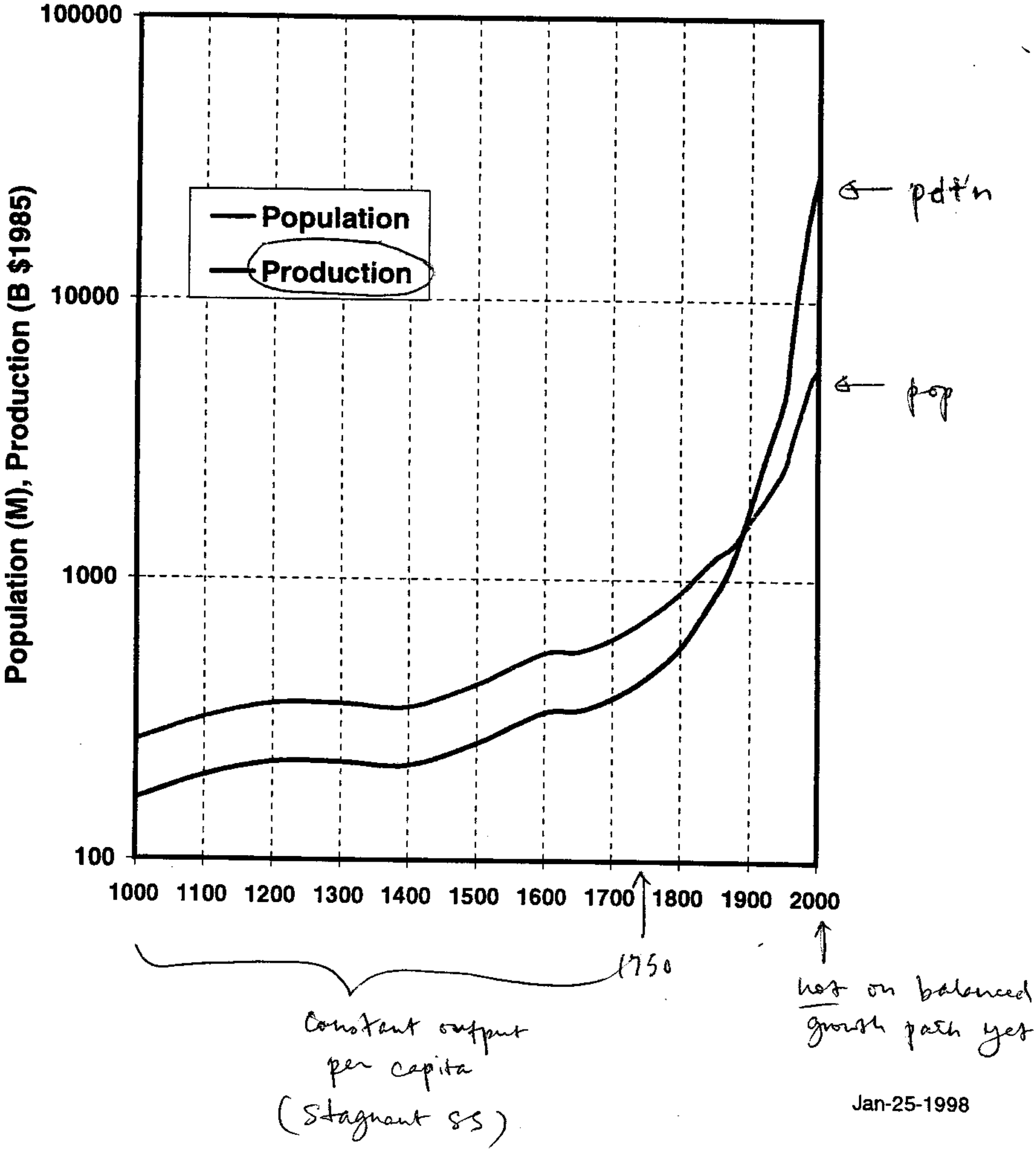
Figure 1 plots world population and the total world production of goods and services (GDP) from the year 1000 up to the present. I use a logarithmic scale rather than natural units in this figure so that one can see clearly the *acceleration* in both of these series (which shows up as a deviation from linearity). The scale on the left is millions of persons and billions of 1985 U.S. dollars. Population data for years since 1500 and production data since 1750 are taken from Tables 1 and 2 in the Data Appendix. Population from earlier years is from McEvedy and Jones (1978), p. 342. Earlier production estimates are extrapolations based on the assumption that *per capita* incomes were equal to their 1750 levels in all earlier years. (Thus prior to 1750 the total population and production series differ by a constant.)

[INSERT FIGURE 1 HERE]

The estimated population for the year 1000 is 265 million, slightly less than the population of the United States today. By 1960, world population had grown to 3

[3] I found Kremer's (1993) presentation of long term population trends extremely stimulating. So too were similarly motivated overviews of trends in production and living standards provided by Parente and Prescott (1993), Johnson (1997), and Pritchett (1997).

Figure 1: World Population and Production



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billion; by 1990, it was 5.2 billion. One can see the acceleration of population growth since 1800 on the figure, but as Michael Kremer (1993) has emphasized, there was accelerating growth in earlier years as well.[4] In a world with constant income per capita, population growth is itself a measure of production growth, and hence also of the rate of technological change. Thus whatever it was that was new to economic life in the 18th century, it was not technological change, for continued improvement in technology was needed to support the impressive population growth in all the centuries preceding.

Our knowledge of production and living standards at different places and times has grown enormously in the past few decades. The most recent empirical contribution, one of the very first importance, is the Penn World Table project conducted by Robert Summers and Alan Heston.[5] This readily available, conveniently organized data set contains population and production data on every country in the world, from about 1950 or 1960 (depending on the country) to the present time. Data on real production are converted to common units on the "purchasing power parity" basis that is consistent with index number theory. The availability of this marvelous body of data has given the recent revival of mathematical growth theory an explicitly empirical character that is quite different from the more purely theoretical investigations of the 1960s. It has also stimulated a more universal, ambitious style of theorizing, aimed at providing a unified account of the behavior of rich and poor societies alike.

[4]One can argue whether the pre-1800 acceleration of population can be "seen" on my Figure 1, but it is clearly visible in the much long series displayed by Kremer and McEvedy and Jones and in the curves fit to these data by Kremer.

[5]The basic descriptive article on the Penn World Table is Summers and Heston (1991), which contains data for the years 1950-1988. The data themselves are periodically extended and revised, and made available in a numbered series. All data cited in this paper as "Summers and Heston" or "S&H" are taken from the version PWT5.6, downloaded from the Website of the National Bureau of Economic Research (http://www.nber.org).

As a result of the Penn project, we now have for the first time a reliable picture of production in the entire world, both rich and poor countries. Let us review the main features of this picture, beginning with population estimates. Over the 30 year period from 1960 through 1990, world population grew from about 3 billion to 5.2 billion, or at an annual rate of 1.8 percent. These numbers are often cited with alarm, and obviously the number of people in the world cannot possibly grow at 2 percent per year forever. But the idea that population growth is outstripping available resources, advanced by many recent exponents of what a friend of mine calls "the economics of gloom," is simply nonsense, unrelated to the facts we observe.

There is, to be sure, a lot of poverty and starvation in this world, but nothing could be further from the truth than the idea that poverty is increasing. Over the same period during which population has grown from 3 billion to 5.2 billion, total world production grew *much* faster than population, from $6.7 trillion in 1960 to $22.3 trillion in 1990. That is, world production more than tripled over this 30 year period, growing at an annual rate of 4 percent. Production per person—real income—thus grew at 2.2 percent per year, which is to say that the living standard of the average world citizen nearly doubled! Please understand: I am not quoting figures for the advanced economies or for a handful of economic miracles. I am not excluding Africa or the communist countries. These are numbers for the world *as a whole*. The entire human race is getting rich, at historically unprecedented rates.

Average figures like these mask a lot of diversity, of course. Figure 2 is one way to use the information in the Penn World Table to summarize the *distribution* of the levels and growth rates of population and per capita incomes in the post war world. It contains two histograms of per capita incomes, one for 1960 and the other for 1990. The horizontal axis is GDP in thousands of 1985 international dollars (the units used by Summers and Heston, for my purposes indistinguishable from 1985 U.S. dollars) on a logarithmic scale. The vertical axis is population. The volumes on the figures are proportional to the number of people in the world with average incomes in the indicated range, based on the assumption (though of course it is false) that everyone in a country has that country's average income. The total volume over all four income ranges shown is thus total world population in the indicated year, about 3 billion in 1960 and 5.2 billion in 1990. The means of the two distributions are $2200 (1960) and $4300 (1990).

[INSERT FIGURE 2 HERE]

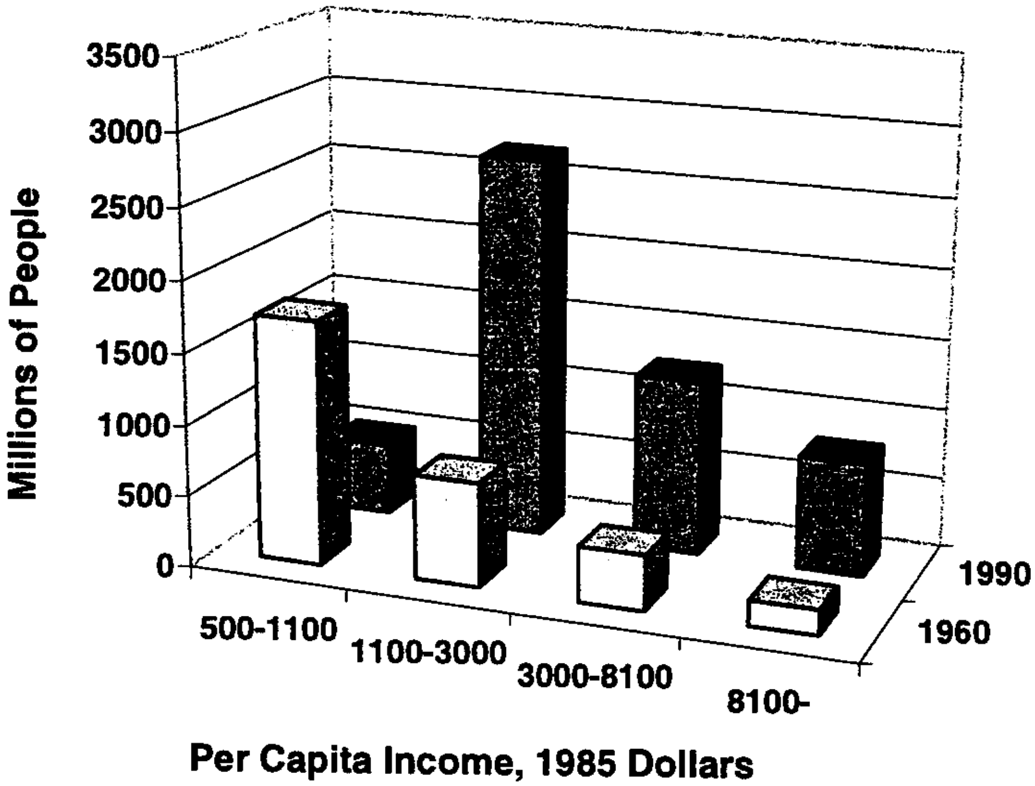
One can see from Figure 2 that the *number* of people (not just the fraction) in countries with mean incomes below $1100 has declined between 1960 and 1990. In general, the entire world income distribution has shifted to the right, without much change in the degree of income inequality since 1960. On the other hand, the degree of inequality is enormous. The poorest countries in 1990 have per capita incomes of around $600 1985 U.S., as compared to the U.S. average of $17,000. This is a factor of $17000/600 = 28$! This degree of inequality between the richest and poorest societies is without precedent in human history, just as is the growth in both population and living standards in the post war period.

A great deal of recent empirical work has been focused on the question of whether per capita incomes are converging to a common (growing) level, or possibly diverging.[6] From Figure 2 it is evident that this is a fairly subtle question, and one is not surprised that this literature has not converged on a definite conclusion. In any case, it seems obvious that we are not going to learn much about the future of our race by simple statistical extrapolation of events from 1960 to 1990, however it is carried out. Extrapolating the 2 percent population growth rate backward from 1960, one would conclude that Adam and Eve were expelled from the Garden of Eden in about

[6]See, for example, Barro and Sala-i-Martin (1992), (1997), Quah (1996), and Jones (1997). For an approach to the issue of convergence that is much closer to the one I take below, see also Baumol (1986) and Pritchett (1997). Ben-David's "convergence clubs"?

(conditional level conver

Figure 2: Income Distribution



Why use log scale here?
(not easy to see growth trend anyway)

the year 1000. Extrapolating the 2.2 rate of per capita income growth backward, one would infer that people in 1800 subsisted on less than $100 1985 U.S. Extrapolating forward leads to predictions that the human race will exhaust the earth's water supply (or supply of anything else) in a finite period. Such exercises make it clear that the years since 1960 are part of a period of *transition*, but from what to what?

History can answer half of this question. Well-documented estimates on population, for the world as a whole and for many sub-regions, are available in a variety of sources. My Table 1, covering population in 21 regions in various years since 1500 is given in the Data Appendix. It is based almost entirely on McEvedy and Jones's (1978) excellent and readable volume, at least for the years not covered by Summers and Heston. (Where these sources overlap, they generally agree.) My Table 2, containing per capita real incomes in these same 21 regions for various years since 1750, is based on Summers and Heston for post-1950 years. For earlier years, the construction of real GDP series is, of course, rather more complicated.

For years much before 1960 there is no single source for production data for *all* the world's countries, and indeed, most of the world's current nations did not even exist as independent states prior to World War II. But for a few economies—the rich countries that have roughly preserved their boundaries over the years—excellent national product data are available extending way back into the nineteenth century, collected in historical research projects for individual countries. Many of these estimates are collected in Maddison (1983, 1991), Bairoch (1981), and most recently Maddison (1995). To construct the per capita real GDP estimates given in Table 2 in the Data Appendix, I relied primarily on estimates provided for many relatively rich countries and regions in Bairoch (1981). The details are given in the Appendix.

For poor societies we often have no national product data, or data of very low reliability, even for the postwar years. This lack of data becomes more acute as we go back in time. As discussed in the Data Appendix, the estimates reported in Table 2 for Africa and much of Asia for the years before 1960 are in large part extrapolations back in time of the 1960 estimates provided by Summers and Heston. The idea is that incomes in, say, ancient China cannot have been much lower than incomes in 1960 China and still sustained stable or growing populations. And if incomes in any part of the world in any time period had been much larger than the levels of the poor countries of today—a factor of two, say—we would have heard about it. Such enormous percentage differences, had they ever existed, would have made some kind of appearance in the available accounts of the historically curious, from Herodotus to Marco Polo to Adam Smith.

In fact, there are many other sources that provide evidence on production in poor—primarily agricultural—economies. In the front hall of my apartment in Chicago, for example, there is a painting of an agricultural scene, the gift of a Korean student of mine. In the painting, a farmer is plowing his field behind an ox. Fruit trees are in flower, and mountains rise in the background. It is a peaceful scene, inspiring nostalgia for an older, simpler time. There is also much information for an economist in this picture. It is not difficult to estimate the income of this farmer, for we know about how much land one farmer and his ox can care for, about how much can be grown on this land, how much fruit the little orchard will yield, and how much the production would be worth at 1985 U.S. dollar prices. This income is about $2000. Moreover, we know that up until recent decades, almost all of the Korean workforce (way over 90%) was engaged in traditional agriculture, so this figure of $2000—$500 per capita for the farmer, his wife, and his 2 children—must be pretty close to the per capita income for the country as a whole. Though we do not have sophisticated national income and product accounts for Korea 100 years ago, we do not need them to arrive at fairly good estimates of living standards that prevailed back then. Traditional agricultural societies are very like one another, all over the world and over time, and the standard of living they yield is not hard to estimate reliably. Indeed, my Korean farm scene (though perhaps not the style of the painting) could be drawn from any century in this millenium or the last one.

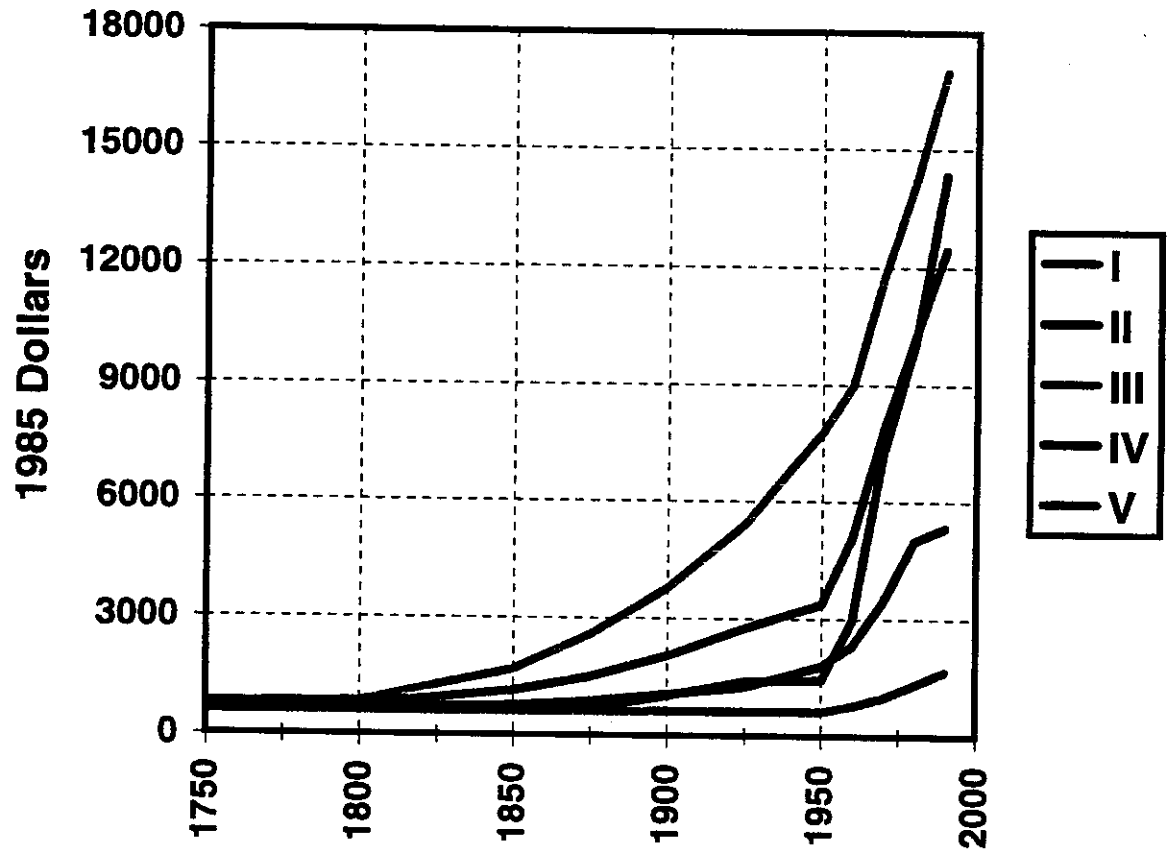
The income detail provided in Table 2 make it possible to illustrate the very uneven course of the industrial revolution in different parts of the world. This is done on Figure 3. To construct this figure, the 21 countries or regions of Table 2 were organized into five groups, based on the similarity in income growth histories and order by their current income levels. Group I—basically, the English-speaking countries—are those in which per capita incomes first exhibited sustained growth. Group III consists of the rest of northwest Europe, the countries that began sustained growth somewhat later. Group IV is the rest of Europe, together with the European-dominated economies of Latin America. Group II is Japan, isolated only because I want to highlight its remarkable economic history. Group V contains the rest of Asia and Africa.

[INSERT FIGURE 3 HERE]

As shown on Figure 3, per capita incomes were approximately constant, over space and time, over the period 1750-1800, at a level of something like \$600-700 U.S. 1985. Here and below, the modifier "approximately" must be taken to mean something like ±\$200. Following the reasoning I have advanced above, \$600 is taken as an estimate of living standards in all societies prior to 1750, so there would be no interest in my extending Figure 3 to the left. The inequality among societies that Figure 2 displays, then, is a recent event, emerging for the first time in the 19th century and reaching current levels only in the 20th century.

The numbers at the bottom of Figure 3 indicate the 1990 populations, in millions of people, for the five groups of countries. One sees in particular that about two thirds of the world's people live in Group V, which contains all of Africa and Asia except for Japan. Notice that per capita income in this group of countries is constant at

Figure 3: GDP Per Capita, Five Regions



		1990 Population in millions
I	UK, USA, Canada, Australia, New Zealand	354
II	Japan	124
III	France, Germany, Netherlands, Scandinavia	323
IV	Rest of Western Europe, Latin America, Eastern Europe, Soviet Union	847
V	Asia (except Japan), Africa	3590

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$600 1985 U.S. for the entire period up to 1950. The colonial era was a period of impressive population growth, but also of stagnation in the living standards of masses of people. Growth in per capita incomes in Africa and Asia is entirely a post-colonial phenomenon, and this growth is, of course, the main reason that post war growth rates for the world as a whole have attained such unprecedented levels.

Although I am using sustained growth in per capita incomes as the defining characteristic of the industrial revolution, it is clear enough from Figure 1 that it is hopeless to try to account for income growth since 1800 as a purely technological event. Technological change occurred rapidly after 1800, but it occurred at an accelerating rate for centuries prior to 1800 as well. What occurred around 1800 that *is* new, that differentiates the modern age from all previous periods, is not technological change by itself but the fact that fertility increases ceased to translate improvements in technology into increases in population. It is not that the increasing pace of technological change made it impossible to population to keep up, to lift "beyond visible limits the ceiling of Mathus's positive checks."[7] Such a development might be consistent with a Malthusian model, but only if accelerating technology were associated with extremely *high* fertility levels. In fact, the industrial revolution is invariably associated with the *reduction* in fertility known as the demographic transition.

Figure 4 provides a rough description of the demographic transitions since 1750 that have occurred and are still occurring. The figure exhibits five plotted curves, one for each of the country groups defined in Figure 3. Each curve connects 10 points, corresponding to the time periods beginning and ending in the dates indicated at the bottom of the figure. (Note that the periods are *not* of equal length.) Each point plots the group's average rate of population growth for that period against its per capita income at the beginning of the period. The per capita GDP figures in 1750 can just be read off Figure 3; that they are about $600 for all five groups. Population

[7] Landes, (1969), p. 41.

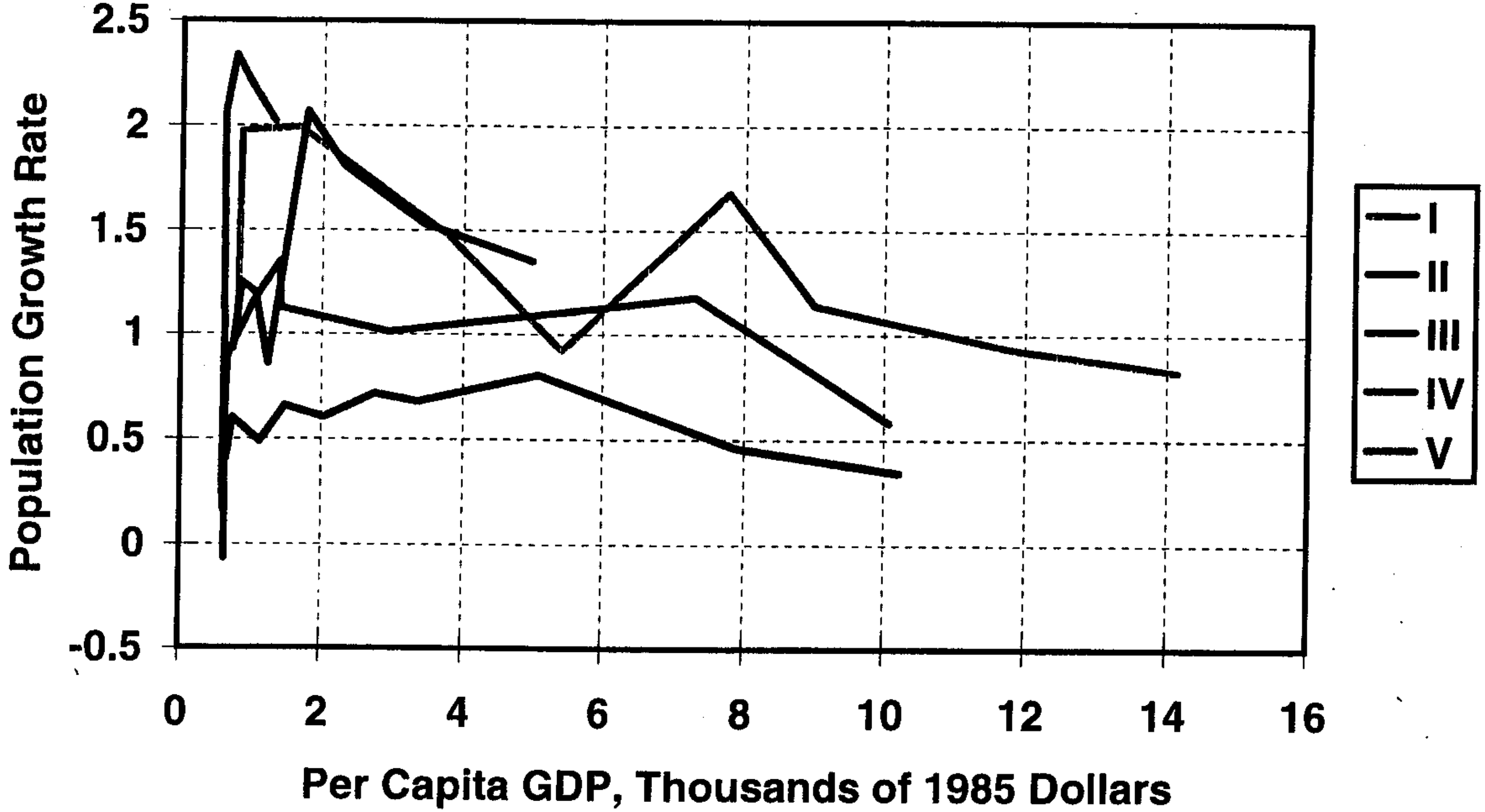
growth rates in 1750 average about 0.4 percent and are well below one percent for all five groups. For each group, one can see a nearly vertical increase in population growth rates with little increase in GDP per capita, corresponding to the onset of industrialization. This, of course, is precisely the response to technological advance that Malthus and Ricardo told us to expect. Then in groups I-IV a maximum is reached, and as incomes continue to rise, population growth rates decline. In group V—most of Asia and Africa—the curve has only leveled off, but does anyone doubt that these regions will fail to follow the path that the rest of us have already worn? The facts shown in Figure 4 can be refined, not only by distinguishing among more than five groups of societies but also by treating changes in birth and death rates and changes in immigration rates separately. I found it striking that the inverted U pattern is so clearly documented even without making such refinements. The demographic transition obviously requires a different view of fertility from that taken by Malthus.

[INSERT FIGURE 4 HERE]

The most striking feature of the data reviewed in this chapter is the fact that only two types of aggregative behavior are observed, one with per capita income roughly constant at a level in the range of $400 to $800 1985 U.S., and another with per capita income growing at rates in excess—sometimes far in excess—of one percent per year. This observation is the point of departure for Becker, Murphy, and Tamura (1990). It will also be the basis of the theoretical work described in these lectures.

Stagnant vs. persistent growth eqi

Figure 4: Demographic Transitions



3. THE CLASSICAL THEORY OF PRODUCTION

The methods and results of national income and product accounting were not available to Adam Smith and David Ricardo, but they and the other classical economists had at their disposal and actively debated a vast amount of historical information bearing on levels and rates of growth of living standards. There is no mention of sustained economic growth in the writings of the classical economists, nor would one expect there to be any in view of the facts as displayed, for example, in Figures 1 and 3 in the last chapter. For Smith and Ricardo, the wealth of nations meant their income *levels*, not their growth rates, and it was the stability of these levels over time, not growth rates, that called for explanation. When Thomas Malthus (1798) proposed a fertility model that accounted for the observed stability in living standards, his contemporaries quickly accepted it.

Ricardo's *Principles of Political Economy and Taxation* (1817) set out an aggregative theory of income and its distribution, almost explicit enough for a modern economist to call it a *model.* Ricardo put the Malthusian view of fertility at the center of his theory of long run income determination, and emphasized that a central implication of the theory was the prediction that an improvement in technology would result in an increase in population but no change in real income per person. The theory is *classical:* It makes no use of utility theory. One of the aims of this chapter is to restate the classical theory in *neoclassical* terms, which is to say in terms that make use of mathematically explicit descriptions of agents' preferences and the technology available to them. Once the classical theory of production and distribution is so translated into modern terms, it becomes a serviceable empirical model of income determination in economies that predate the industrial revolution, which is to say, in all economies known to Smith, Ricardo, and their contemporaries.

To begin, consider an animal species, endowed with behavior implying that when food consumption per individual rises above some biologically given subsistence level, numbers increase, while if consumption is below this critical level, numbers decrease. Then if food availability in total is held fixed in the region occupied by this species, the onset of a disease that kills many individuals will induce an increase in consumption per individual, an increase in population growth, and ultimately the restoration of the original consumption level and population. Such a "Malthusian" model has the important virtue of predicting the existence of a stable subsistence level of consumption in the face of shocks to population and technology.

There are other facts about human populations, however, that do not at all resemble the behavior of animal populations. In most human societies, some individuals, or individual families, accumulate vast wealth and consumption far above the consumption level at which most people subsist. In most societies, even prior to the 18th century, average consumption is too large to be viewed as a purely biological survival level. Ricardo dealt with this observation crudely, by applying Malthusian assumptions to "workers," referred to a distinct "race," leaving other "classes," created by systems of property rights, free to reproduce less, consume more, and accumulate the achievements that we call civilizations.

To restate Ricardo's theory in neoclassical terms, and to move beyond it in other respects as well, I will begin by formulating the preferences of a single family or "dynasty" over the number of its children as well as over the quantity of goods it consumes. This will permit the derivation of Malthusian fertility theory as a matter of conscious choice, not merely biology. Later on, I will situate families with these dynastic preferences in a variety of different social arrangements and apply equilibrium reasoning to determine how the interaction of individual and social forces determines fertility, production, and population.

Dynastic Preferences

The Malthusian hypothesis of a desired living standard for children is evidently an assumption about preferences, one that a modern economist naturally formalizes in terms of a utility function. This useful construct was not available to Malthus and Ricardo, but it will not be hard to translate their thinking on demographic questions into neoclassical language. In this section, I introduce the assumptions about preferences, adapted from those of many earlier, similarly motivated writers, that I will use throughout these lectures.

The typical household whose preferences I will describe can be thought of as a single agent, consuming c units of a single consumption good in its lifetime and producing n children. Thus $n - 1$ is the rate of growth of the population of the household, and a choice of $n = 1$ means that the household is just maintaining its size. Each child then becomes the head of its own household, and so the process continues. The household's preferences are assumed to be described by a utility function $W(c, n, u)$ that depends on its own consumption, the number of children it has, and the lifetime utility u that each child will enjoy. These preferences are about the simplest one can think of that are general enough to accomodate a "quantity-quality trade-off" in the fertility decision. People are assumed to value both the number of children n (quantity) and the lifetime utility u (quality) that each child is expected to enjoy.[8]

[8]The particular formulation of dynastic preferences that I am using here was introduced in Razin and Ben-Zion (1975). In my formulation, as in theirs, parents' utility depends on goods consumption, the number of children, and utility per child. See also Becker (1960) (where a quantity-quality tradeoff was introduced into the theory of fertility), Willis (1973), Becker and Barro (1988), Barro and Becker (1989), and Alvarez (1995).

Other writers have constructed integrated fertility and growth theories without recourse to the "quantity-quality" tradeoff, stressing the value of children as a parental investment. Ehrlich and

Using the subscript t to denote a generation, parent's and child's utilities are linked by the difference equation

$$u_t = W(c_t, n_t, u_{t+1}). \tag{1}$$

If we solve (1) backwards through time by repeated substitution we get an expression

$$u_t = W(c_t, n_t, W(c_{t+1}, n_{t+1}, W(c_{t+2}, n_{t+2}, \dots)))$$

for the utility u_t of the infinite sequence $\{(c_s, n_s)\}_{s=t}^{\infty}$ of this dynasty's consumption and children per generation at each date from t on. For example, if we assume the familiar additive log utility,

$$W(c, n, u) = (1-\beta)\ln(c) + \eta\ln(n) + \beta u,$$

where the discount factor β is between 0 and 1, the difference equation (1) has the solution

$$u_t = \sum_{s=0}^{\infty} \beta^t[(1-\beta)ln(c_{t+s}) + \eta\ln(n_{t+s})].$$

In general, I will always require of a utility function W that W_u, the derivative of parents' lifetime utility with respect to the lifetime utility of their children, be between 0 and 1. This is a discounting assumption that is essential to the analysis to follow. Other assumptions that will be maintained below will be that W is strictly increasing in both c and n and strictly quasi-concave in (c, n, u), that the "goods" c and n are both non-inferior, and that n and u are complements.[9]

Lui (1991) and Raut and Srinivasan (1994) are leading examples.

Loury (1981) does not treat the fertility decision, but his analysis of the intergenerational aspects of investment in human capital is evidently complementary to theories that do.

[9]The last two properties are defined in terms of the problem $\max_{c,n} W(c, n, u)$ s.t. $c + kn \leq I$. By non-inferiority of c and n, I mean that the maximizing values of c and n are both increasing functions of I for all values of k. This is true if and only if $W_n W_{cn} - W_c W_{nn} > 0$ and $W_c W_{cn} - W_n W_{cc} > 0$. By the assumption that n and u are complementary, I mean that the optimal value of n in this problem is a non-decreasing function of u. This is true if and only if $W_c W_{nu} - W_n W_{cu} \geq 0$.

A Hunter-Gatherer Society

In order to get a sense of the implications of these recursive preferences for observed behavior it will be instructive to situate some of these hypothetical dynasties in a specific setting and see how they behave. I begin with an example of what we might call, today, a hunter-gatherer society. Both Smith and Ricardo make expository use of a similar situation, which they describe as existing "before the appropriation of land." It is not clear whether they intended this as a historical example or, as in my exposition, merely an illustrative one.

Consider, then, a population of N families that supports itself and its progeny on a fixed amount L of land (or territory or resources more generally conceived). Let total production be $Y = F(L, N)$, where F is a constant returns to scale function, so production per household is $y = Y/N = F(L/N, 1) = f(x)$, (say) where $x = L/N$ is the land-population ratio. Maintain the competitive assumption that each household takes this value y as given to it, as beyond its own control and the control of its children, though it may be changing through time in a predictable way. This assumption that there is nothing a parent can do that affects the opportunities available to its children (or, for that matter, of its own future self) is just what we mean by the absence of private property.

In such a setting, consider the decision problem of a single dynasty with preferences $W(c, n, u)$ and the current goods income y_t. There is a cost (in terms of goods) of k per child. The parents must choose the number of children, n_t, and their own consumption, $c_t = y_t - kn_t$. In the society we have postulated, this household has nothing to leave to its offspring that could affect their utility levels. Nonetheless, expectations about the utility level u_{t+1} that each child will enjoy may affect the parents' attitudes over its own consumption and number of children. Hence the household solves

$$\max_n W(y_t - kn_t, n_t, u_{t+1}).$$

The first-order condition for this problem is

$$kW_c(y_t - kn_t, n_t, u_{t+1}) = W_n(y_t - kn_t, n_t, u_{t+1}),$$

which implies one equilibrium condition. Three others are provided by (1), by the fact that n_t is a growth rate, and by the technology. The full system that must obtain in equilibrium is thus:

$$u_t = W(c_t, n_t, u_{t+1}), \tag{2}$$

$$kW_c(c_t, n_t, u_{t+1}) = W_n(c_t, n_t, u_{t+1}), \tag{3}$$

$$x_{t+1} = x_t/n_t, \tag{4}$$

$$c_t + kn_t = f(x_t). \tag{5}$$

Think of (2)-(5) as four equations in the state-like variables x_t and u_t and the flow-like variables c_t and n_t.

At any steady state of this economy, the typical dynasty is just reproducing itself, $n_t = 1$, and parents and children enjoy the same lifetime utility levels, $u_t = u_{t+1}$. The steady state goods consumption and utility levels (c, u) must then be the solutions to the two equations

$$u = W(c, 1, u) \tag{6}$$

and

$$kW_c(c, 1, u) = W_n(c, 1, u) \tag{7}$$

that are implied by (2) and (3).

By the discounting assumption $W_u \in (0, 1)$, (6) can always be uniquely solved for steady state utility as an increasing function of net consumption: $u = g(c)$, say. Now define the steady state marginal rate of substitution function $m(c)$ by

$$m(c) = \frac{W_n(c, 1, g(c))}{W_c(c, 1, g(c))}.$$

Then (7) implies that steady state consumption levels coincide with solutions to

$$m(c) = k. \tag{8}$$

Assume that this marginal rate of substitution function satisfies the restrictions $m(0) = 0$ and $m(\infty) = +\infty$. which together ensure that a solution c to (8) exists. At any such solution, the derivative $m'(c)$ of the marginal rate of substitution function has the sign of

$$W_{cn} - kW_{cc} + [W_{nu} - kW_{cu}]g'(c),$$

which is strictly positive from the assumptions that fertility is a normal good, complementary to future utility. Thus as illustrated on Figure 5, there exists exactly one steady state consumption level, which I denote c_m (m for Malthus).

[INSERT FIGURE 5 HERE]

In the case of the logarithmic preferences, the function m is given by

$$m(c) = \frac{\eta}{1-\beta}c,$$

from which it follows that steady state consumption is $c_m = [(1-\beta)/\eta]k$. Notice that in general as in the logarithmic special case, the steady state level of consumption c_m is determined entirely independently of the goods-producing technology F, or of the levels of population and resources, N and L. It depends only on the goods cost of raising children, k, and on parental attitudes toward children and child raising, as summarized in the function W.

The off-steady-state population dynamics of this economy, given only implicitly in (2)-(5), can be complicated. The logarithmic case offers a simple point of departure. In this case, future utility u_{t+1} does not figure in equation (3), which then specializes to:

$$k\frac{1-\beta}{c_t} = \frac{\eta}{n_t}.$$

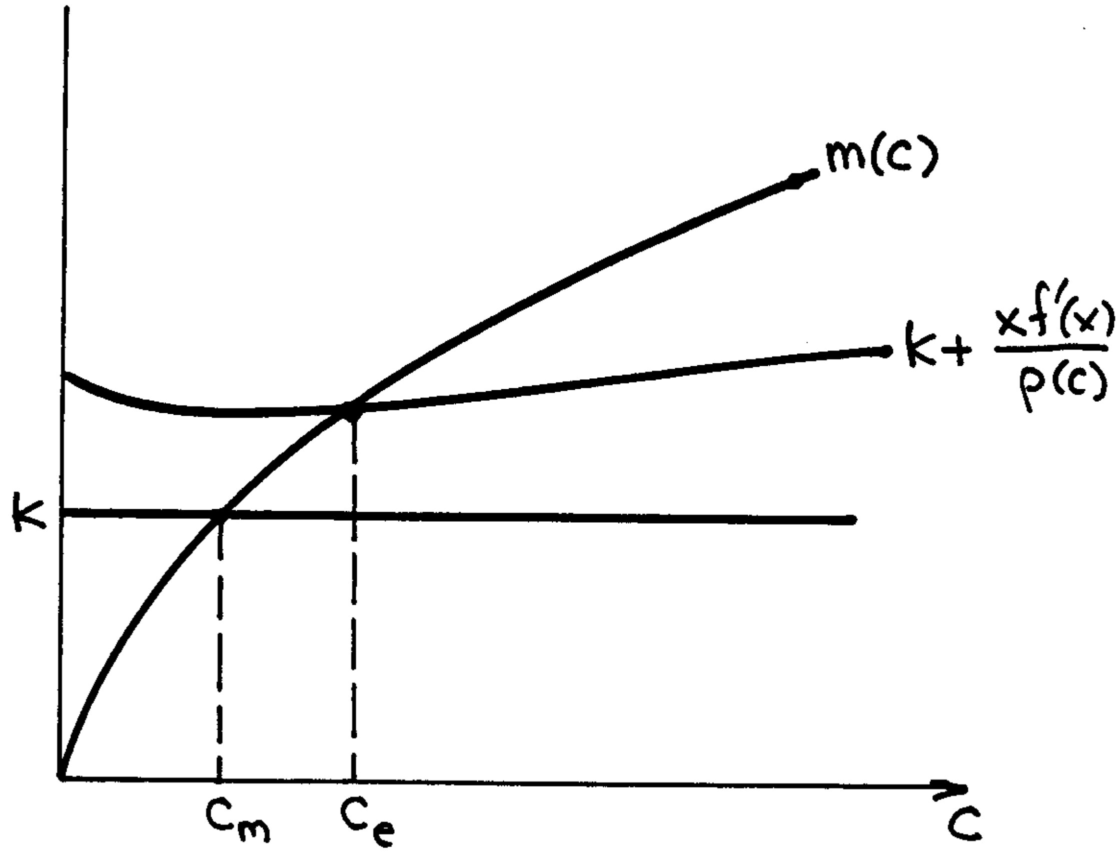


FIGURE 5

Then eliminating c_t and n_t from (3), (4), and (5) gives:

$$x_{t+1} = k(\frac{1-\beta+\eta}{\eta})\,[x_t/f(x_t)]\,.$$

If we assume, in addition, that the production function f is Cobb-Douglas, $f(x) = Ax^{\alpha}$, $\alpha \in (0,1)$, we have

$$x_{t+1} = \frac{k}{A}(\frac{1-\beta+\eta}{\eta})x_t^{1-\alpha}. \tag{9}$$

In this case, it is clear that the unique steady state

$$x_m = [(\frac{1-\beta+\eta}{\eta})\frac{k}{A}]^{1/\alpha}$$

is globally stable.[10]

An Egalitarian Equilibrium with Private Property

Where is it in the previous model?

The empirical attraction of the Malthusian dynamics I have just described is that they capture the rough constancy of living standards over many centuries, and reconcile the occurrence of important technological advances with stagnation in per capita incomes and with accelerating population growth. But the assumption that land is held in common obviously does not fit historical societies. In this section, I replace this assumption with the assumption of private ownership, including heritability, of land (or of natural resources more generally), under the assumption that land is initially equally distributed over households of equal size. In this case, and given identical preferences, there will be an equilibrium in which land remains equally distributed and we can characterize this equilibrium by analyzing a society of identical agents.

[10]To study local stability under more general assumptions on W and f, we need to examine the two roots of the system (2)-(5) at the steady state. Under the assumptions I have imposed (see note 9), these roots are both real, one is between 0 and 1, and one exceeds 1.

Consider again, then, the society studied in the last section, in which there is a fixed amount of land L and a variable population N. A typical household has the same preferences $W(c, n, u)$ over consumption c, number of children n, and utility per child u that we assumed in the last section. If this household's initial land holdings are x and it bears n children, it produces $f(x)$ goods, is obligated to devote kn to child raising, and has $f(x) - kn$ left for parental consumption. We assume that land holdings are bequeathed equally to all children, leaving x/n per child. Denote the lifetime utility of this person by $v(x)$. Then if the number of children is chosen optimally, the function v satisfies the Bellman equation:

$$v(x) = \max_{c,n} W\left(c, n, v(\frac{x}{n})\right), \tag{10}$$

subject to:

$$c + kn \leq f(x). \tag{11}$$

The first-order and envelope conditions for this problem are:

$$W_n(c, n, v(\frac{x}{n})) - W_u(c, n, v(\frac{x}{n}))v'(\frac{x}{n})(\frac{x}{n^2}) = kW_c(c, n, v(\frac{x}{n})),$$

and

$$v'(x) = W_c(c, n, v(\frac{x}{n}))f'(x) + W_u(c, n, v(\frac{x}{n}))v'(\frac{x}{n})(\frac{1}{n}).$$

Suppose we write $u_t = v(x_t)$, and define $q_t = v'(x_t)$. Then the full dynamic system in this private property equilibrium is:

$$u_t = W(c_t, n_t, u_{t+1}), \tag{12}$$

$$kW_c(c_t, n_t, u_{t+1}) = W_n(c_t, n_t, u_{t+1}) - W_u(c_t, n_t, u_{t+1})q_{t+1}x_{t+1}(\frac{1}{n_t^2}), \tag{13}$$

$$q_t = W_c(c_t, n_t, u_{t+1})f'(x_t) + W_u(c_t, n_t, u_{t+1})q_{t+1}(\frac{1}{n_t}), \tag{14}$$

$$x_{t+1} = x_t/n_t, \tag{15}$$

$$c_t + kn_t = f(x_t). \tag{16}$$

Think of (12)-(16) as five equations in the three state-like variables u_t, q_t, x_t and the two flow variables c_t and n_t.

Consider possible steady states for such dynasties, in which $n = 1$ and the other four variables are constant. In this situation, the equilibrium conditions (12)-(16) become:

$$u = W(c, 1, u), \tag{17}$$

$$kW_c(c, 1, u) = W_n(c, 1, u) - W_u(c, 1, u)qx, \tag{18}$$

$$q = W_c(c, 1, u)f'(x) + W_u(c, 1, u)q, \tag{19}$$

$$c + k = f(x). \tag{20}$$

We can think of solving (17)-(20) by first using (17) to solve for u as a function $g(c)$ of net consumption c, as we did with equation (6) in the analysis of the hunter-gatherer economy. Then in a steady state the three derivatives of W can be viewed as known functions of c. We use (19) to obtain q in terms of these derivatives:

$$q = (1 - W_u)^{-1}W_c f'(x).$$

We then substitute back into (18) to get:

$$f'(x)x = \frac{(W_n - kW_c)(1 - W_u)}{W_c W_u}.$$

Now W_u has the dimensions of a discount factor, so $(1 - W_u)/W_u$ has the dimensions of a discount *rate*. Call this ratio $\rho = \rho(c)$, where c is steady state consumption. Similarly, denote the steady state marginal rate of substitution W_n/W_c by $m(c)$. Then steady state land holdings and consumption levels must satisfy

$$f'(x)x = [m(c) - k]\rho(c)$$

or, rearranging to facilitate comparison to (8),

$$m(c) = k + \frac{f'(x)x}{\rho(c)}. \tag{21}$$

Solutions (c_e, x_e) to (20)-(21) are the possible steady states for the individual land holding dynasty.

The product $f'(x)x$ has the dimension of a per capita land rent flow; division by the discount rate $\rho(c)$ gives the ratio on the right of (21) the dimension of a present value of land rent. Thus (21) equates the marginal rate of substitution between children and goods consumption to the sum of the direct child-raising cost k, as in (8), and the present value of the income flow that a second child would need to have to be as well off as the first. It is this second term, of course, that is absent when land is not privately owned.

Since the terms $xf'(x)$ obviously depend on the production technology, the solution (c_e, x_e) to (20) and (21) need not have the property that the steady state consumption level is determined by preferences only, as was the case in the last example. But suppose that f is Cobb-Douglas: $f(x) = Ax^\alpha$. Then using (20), $xf'(x) = \alpha f(x) = \alpha(c+k)$, and (21) becomes:

$$m(c) = k + \frac{\alpha(c+k)}{\rho(c)}. \tag{22}$$

Refer again to Figure 5. The only feature of the production technology that influences steady state consumption c_e is the share parameter α; the intercept A does not appear in (22).

In the logarithmic utility example, $m(c) = [\eta/(1-\beta)]c$ and $\rho(c) = (1-\beta)/\beta$. In this case, with Cobb-Douglas production, (22) can be solved explicitly for

$$c_e = \frac{1-\beta+\alpha\beta}{\eta-\alpha\beta}k,$$

provided that $\eta > \alpha\beta$. (As an example in the next chapter will show, only preference parameters satisfying $\eta > \beta$ are consistent with reasonable behavior, so I will impose this assumption in the sequel. Since $\alpha \in (0,1)$, we must then have $\eta > \alpha\beta$.) The case $\alpha = 0$ corresponds to the equilibrium c_m calculated in the last section, so one sees that $c_e > c_m$ for all $\alpha > 0$.

The logarithmic utility, Cobb-Douglas production case is again a convenient context for thinking about the full dynamics of the system (12)-(16). In this case, one can verify that the Bellman equation (10) has a solution of the form

$$v(x) = C + \alpha\frac{1-\beta+\eta}{1-\beta+\alpha\beta}\ln(x),$$

and an optimal fertility function given by:

$$n(x) = \frac{\eta-\alpha\beta}{1-\beta+\eta}\frac{Ax^{\alpha}}{k}. \tag{23}$$

Setting $\alpha\beta = 0$ in the numerator on the right of (23) amounts to assuming either that people do not value their children's utility ($\beta = 0$) or that they are unable to affect their children's utility ($\alpha = 0$).

Combining (23) and (15), the dynamics of land per household (and hence of population) are described in

$$x_{t+1} = \frac{k}{A}\left(\frac{1-\beta+\eta}{\eta-\alpha\beta}\right)x_t^{1-\alpha}. \tag{24}$$

The difference equation (24) implies monotonic convergence to the steady state population level

$$N_e = L\left[\left(\frac{\eta-\alpha\beta}{1-\beta+\eta}\right)\frac{A}{k}\right]^{1/\alpha}.$$

and the long run steady state income

$$y_e = \frac{1-\beta+\eta}{\eta-\alpha\beta}k.$$

These dynamics differ from those for the hunter-gatherer society described in (9) by the term $\alpha\beta$ in the denominator on the right of (24). As one would expect, then, establishing property rights in land gives each family the incentive to reduce fertility. Here is perhaps the most basic instance of the importance of the quality-quantity trade-off in fertility behavior.

The Cobb-Douglas-log-utility example is a borderline case. The general analysis of the effects of changes in technology on equilibrium living standards is complicated. Every dynasty in this economy is both a landowner and a worker. With a Cobb-Douglas technology, factor shares are constant with respect to changes in the production intercept parameter A, and the relative importance of these two roles in unaffected by changes in A. In general, though, increases in A can increase the share of land-rental income in the representative household's total income, strengthening the quality dimension in his quality-quantity tradeoff, reducing fertility, and increasing steady state consumption. *Or* the increase in A could reduce land's share, triggering the opposite reaction. There are a lot of possibilities, but none will be of much quantitative importance unless the elasticity of substitution between land and labor is far from one.

Discussion

There is a historical tradition that the origins of cities and civilized life can be traced to an "agricultural surplus." Certainly it is the case that if not everyone in a society is engaged in producing food, then those who do produce food must provide for those who do not. But if the term surplus is taken to mean that it was technical changes in agriculture that generated the surplus, then a fallacy is surely involved. The surplus described in the private property equilibrium of this chapter, as compared to the equilibrium in the hunter-gatherer society, is generated not by change in physical methods of production but rather by a change in property rights. A society of hunters that succeeded in establishing and enforcing private property in hunting territories would generate a "hunting surplus" without any changes in hunting technology. (Indeed, the privatization of hunting rights or gathering rights must have preceded or at least paralleled the development of agriculture. Who would undertake to domesticate an animal if everyone else has the right to kill and eat it?)

In the two examples discussed in this chapter, the reason the private property equilibrium has a higher per capita income than the equilibrium that arises when resources are held in common has nothing to do with differences in technology. The income difference arises solely out of the concern people have for the well-being of their descendants, combined with a change in property rights that permits people to pass productive resources on to these descendants. The interaction of these forces produces an income "surplus" in the egalitarian society of independent farmers studies in this chapter. In the next chapter, we will see that this continues to be the case when we consider a society organized along class lines.

Does the proposition that "property rights matter" carry over to the case where $\nexists$ altruism? ($\beta = 0$)

4. THE ROLE OF CLASS IN THE CLASSICAL THEORY

The egalitarian economies of hunter-gatherers and small farmers described in the last chapter are possible ways for a society to be organized, and one can think of examples of economies that are reasonably well approximated by such models. Such representative agent systems are very much in the spirit of modern macroeconomics, in which questions of distribution are typically kept to one side in order to focus on questions of production and investment. But they are very far from the spirit of classical economics, in which the idea of *class* plays a central role, certainly in exposition and, I believe, in substance as well.

In the analysis of an economy with privately held land in the last chapter, I began with collection of households in identical positions, sought a symmetric evolution of the system, and found one. In this chapter, I ask whether asymmetric steady state equilibria, in which land is unequally distributed and remains so, may also exist for exactly the agricultural economy set out in Chapter 3. I begin with the analysis of equilibrium possibilities in a system, much closer to Ricardo's, in which an asymmetric equilibrium is simply *imposed* by the assumption of a particular kind of class structure. I then turn to the question of whether the same kind of inequality might also arise in a competitive equilibrium.

An Equilibrium with Two Classes

Consider, then, an economy with the technology and preferences of Chapter 3, but with a class structure *imposed* on the equilibrium by the assumptions that workers do not (or cannot) acquire land and that landowners do not work. In such an equilibrium, landowners simply live off rental income, and their fertility behavior will determine the number of families $N_{\ell t}$ (say) over which society's total land rents will be distributed. Production will depend on the quantity of land L and the number of workers N_{wt} (say) and not at all on $N_{\ell t}$. In these circumstances, a worker household is in exactly the same propertyless position as were all the households in the hunter-gatherer economy studied in Chapter 3. The equilibrium conditions are given by the system (2)-(3) in Chapter 3, except that the state variable x_t must be interpreted as the ratio $z_t = L/N_{wt}$ of land to the number of workers only (not total population), and income per worker $f(x_t)$ must be replaced by wage income per worker, $f(z_t) - z_t f'(z_t)$.

In a steady state, worker consumption will be c_m, the unique solution to

$$m(c) = k, \tag{1}$$

where $m(c)$ is the steady state marginal rate of substitution function defined in the last chapter. The equilibrium wage rate w_m (say) must equal $c_m + k$, so that

$$w_m = c_m + k = f(z) - zf'(z). \tag{2}$$

Since c_m is given by (1), (2) determines the steady state land-labor ratio, $z = L/N_w$. This ratio in turn determines the steady state land rent, $r = f'(z)$. The population of landowners, their preferences, and the nature of their decision problem all play no role in determining any of these quantities and prices, either in a steady state or along a transition path to the steady state. This implication is, I think, the essence of the labor theory of value.

In a steady state, the Bellman equation for a landowning, non-working dynasty is

$$\varphi(x) = \max_{c,n} W(c, n, \varphi(x/n)) \tag{3}$$

subject to

$$c + kn \leq rx, \tag{4}$$

where the state variable x is the dynasty's land holding and where the land rental rate r is a parametrically given price. We specialize the first order and envelope conditions for the problem (1) to the case $n = 1$. With the discount rate function $\rho(c)$ defined as in the last chapter, these conditions imply that a landowner's steady state consumption must satisfy

$$m(c) = k + \frac{rx}{\rho(c)}.$$

Using the budget constraint (4) to eliminate rx, this implies

$$m(c) = k + \frac{c+k}{\rho(c)}. \tag{5}$$

We solve (5) for landowner consumption, $c_\ell > c_m$ (see Figure 6). Now go back to the budget constraint (4) to find the equilibrium land holding per landowning dynasty, x:

$$c_\ell + k = f'(z)x.$$

Given z, there are $N_\ell = L/x$ landowning families in equilibrium.

Note the recursive structure of this two-class system. We get the real wage by Malthusian reasoning, which sets the land-labor ratio via marginal productivity, which sets the rental rate on land, which determines landowner fertility, which determines the number of landowners. It is a Ricardian round robin, in which no simultaneous equations need to be solved. Our first general equilibrium theorist needed no help from Brouwer![11]

In the the log utility-Cobb Douglas production case, worker consumption is

$$c_m = \frac{1-\beta}{\eta}k,$$

[11] The assumption used here that workers and landowners have the same utility function, which comes naturally to a modern economist, would not have occurred to Ricardo. It plays no role in the argument of this section and can obviously be dropped.

as in the hunter-gatherer example, and landowner consumption is

$$c_\ell = \frac{k}{\eta - \beta}$$

provided $\eta > \beta$. (If $\beta > \eta$, the number of children is a "bad," and a landowning parent would choose to have an arbitrarily small number of children, each endowed with arbitrarily high wealth! This is why the assumption $\eta > \beta$ was imposed above, and why I think it should be maintained.)

The population of workers is determined by $z = L/N_w$ and equation (2):

$$c_m + k = (1-\alpha)Az^\alpha.$$

The population of landowners is then given by the equality of total land rents and total landowner consumption (including child raising expenses):

$$N_w \alpha A z^\alpha = N_\ell(c_\ell + k).$$

Thus

$$N_\ell/N_w = \frac{\alpha}{1-\alpha}\frac{\eta-\beta}{\eta}.$$

Oddly, *average* consumption in the steady state is

$$\frac{N_w}{N_w+N_\ell}c_m + \frac{N_\ell}{N_w+N_\ell}c_\ell = \frac{1-\beta+\alpha\beta}{\eta-\alpha\beta}k,$$

which is exactly average consumption in the representative agent economy of Chapter 3. (This finding must be peculiar to the parametric example.)

Modern economists (certainly including me) are skeptical of theories that depend on large numbers of people acting in concert in ways that may serve their joint, "class" interests but not their interests as individuals. For this reason, the assumptions that workers do not save and that landowners do not work are not particularly appealing as positive economics. One might prefer to think of the equilibrium here calculated as an allocation engineered through state-enforced taxation of land rents.

For example, if we think of imposing taxation of land rents on the egalitarian equilibrium of Chapter 3, the agents in that model will rely increasingly on wage income, and as land tax rates increase to maximum levels, their fertility and consumption behavior will approach Malthusian levels. The state, in effect, becomes a single, large landowner. Equivalently, I suppose, we could say that a landowner class becomes the state.

Inequality in Competitive Equilibrium

What happens if people w/i each class unite so that the relevant concept is a bargaining eqm?

It is worth repeating that the equilibrium characterized in the last section is *not* a competitive equilibrium in the usual sense. At the prices I have calculated, workers were not permitted to save and acquire land and landowners were not allowed to earn wage income. Might there be assumptions on preferences under which there could exist a competitive equilibrium steady state in which families with large land holdings and high consumption coexist with other families with less land and lower consumption? In such an equilibrium, the wealthier families would devote more resources to children than poor families do, but these additional resources would take the form of higher "quality" per child, brought about by granting each child more inherited land. This possibility is explored in this section.

asymmetric case

In a steady state equilibrium in which each household takes as given a constant wage rate w, land rental r, and land price q, the Bellman equation for each household is:

$$\psi(x) = \max_{c,n,y} W(c, n, \psi(y/n)) \tag{6}$$

subject to

$$c + kn + q(y - x) = w + rx. \tag{7}$$

Here the dynasty begins with one unit of labor and x units of land. The labor yields w units of goods in wage income; the land yields rx of goods in rental income.

The household can then add to, or subtract from, its disposable income $w + rx$ by selling $x - y$ units of land at the market price q. The unsold land y is bequeathed to children at y/n units per child. The egalitarian allocation of the last chapter, the solution (c, u, x, q) to (17)-(20), is one such equilibrium with the associated prices $w = f(x) - xf'(x)$ and $r = f'(x)$.

Now we ask whether there may also be equilibria with constant prices (w, r, q), not necessarily equal to those associated with the symmetric equilibrium, at which different, price-taking households maintain different long-run land holdings. To characterize any such steady state equilibrium, obtain the first order and envelope conditions for the problem (6), specialize to the situation in which $n = 1$ and $y = x$, and define the functions $m(c)$ and $\rho(c)$ as in the last chapter. Then the equilibrium conditions include

$$m(c) = k + \frac{rx}{\rho(c)}, \tag{8}$$

$$\rho(c) \geq \frac{r}{q} \quad \text{with equality if} \quad x > 0, \tag{9}$$

and

$$c + k = w + rx. \tag{10}$$

In the log utility case, to begin with a familiar example, the left side of (9) is constant at the value $\rho = \beta^{-1} - 1$, and (9) fixes the ratio r/q. Eliminating rx between (8) and (10) gives

$$(1 - \frac{\eta}{\beta})c = w - \frac{k}{\beta}.$$

Under the assumption, imposed above, that $\eta > \beta$, there is a unique consumption level for any wage w, and since everyone faces the same wage in equilibrium this implies that the egalitarian solution is the only possible steady state. Of course, if land were initially unequally distributed, inequality would persist in a transition to a steady state but, if the system were stable, it would disappear in the long run. In short, under log utility the two-class equilibrium calculated in the last section does not have a competitive market interpretation.

More generally, if the function ρ satisfies $\rho'(c) > 0$—the condition Lucas and Stokey (1984) call *increasing marginal impatience*—then (4) implies a unique steady state consumption level at any given prices, and then (5) implies unique equilibrium land holdings x. Again, any steady state must be egalitarian.[12] I turn, then, to an examination of the possibilities when increasing marginal impatience does not obtain. Again, we focus on a two-class situation, in which one class consists of landless workers.

Consider steady states for such a two-class economy, restricting attention to equilibria in which behavior *within* each class is identical. Our task is to determine equilibrium prices (w, r, q), the populations of the two classes, N_w and N_ℓ, the consumption levels of each class, and the average land holdings x of dynasties that do hold land. If we proceed in the right order, this is no harder to carry out than was the Ricardian analysis of the last section.

In a steady state, the consumption of anyone without rental income—a "worker"—will be c_m, the unique solution to (1). The equilibrium wage rate is again equal to $c_m + k$, so the economy-wide land-labor ratio

$$z = \frac{L}{N_w + N_\ell} \tag{11}$$

is determined by (2). This ratio in turn determines the steady state land rent, $r = f'(z)$. So far, then, we have determined both equilibrium prices without any reference to the resources in the system, L, to the population of landowners, or to their preferences and the nature of their decision problem.

Given the prices w and r, the behavior of landowner families is determined by

[12] The case $\rho'(c) < 0$ is not symmetrically interesting, because the steady state at which (9) holds with equality would not be stable. See Lucas and Stokey (1984).

(8)-(10). Eliminating rx between (8) and (10) yields

$$m(c) = k + \frac{c + k - w}{\rho(c)}. \quad (12)$$

From (1) and (2), one solution to (12) is c_m, but this cannot correspond to an equilibrium in which someone earns positive land rents. Suppose, then, that preferences W, and hence the functions $m(c)$ and $\rho(c)$, are consistent with Figure 6. Interpret the larger solution on the figure, c_ℓ say, as landowner consumption. Then average land holdings per landowning dynasty x can be obtained from the budget constraint (10), and the number of landowning families is $N_\ell = L/x$. Finally, (11) determines the number of landless families, N_w. Given z, there are $N_\ell = L/z$ landowning families in equilibrium.

[INSERT FIGURE 6 HERE]

The equilibrium condition (9) has not been used in this construction. Since (9) must hold with equality for the wealthy class, the interest rate and hence the price of land are determined by

$$\rho(c_\ell) = \frac{r}{q} = \frac{f'(z)}{q}.$$

Then if the construction just described is in fact an equilibrium, one in which landless families can acquire land at the price q but choose not to do so, (9) implies that

$$\rho(c_m) \geq \rho(c_\ell). \quad (11)$$

Discussion

The central feature of all of the models developed in this chapter and the last is a steady state income of a worker, $c_m + k$, that is determined independently of the technology or of the levels of population and resources. It depends only on the goods cost of raising children, k, and on parental attitudes toward children and child raising. Ricardo credits Malthus with this population theory, but it is worth

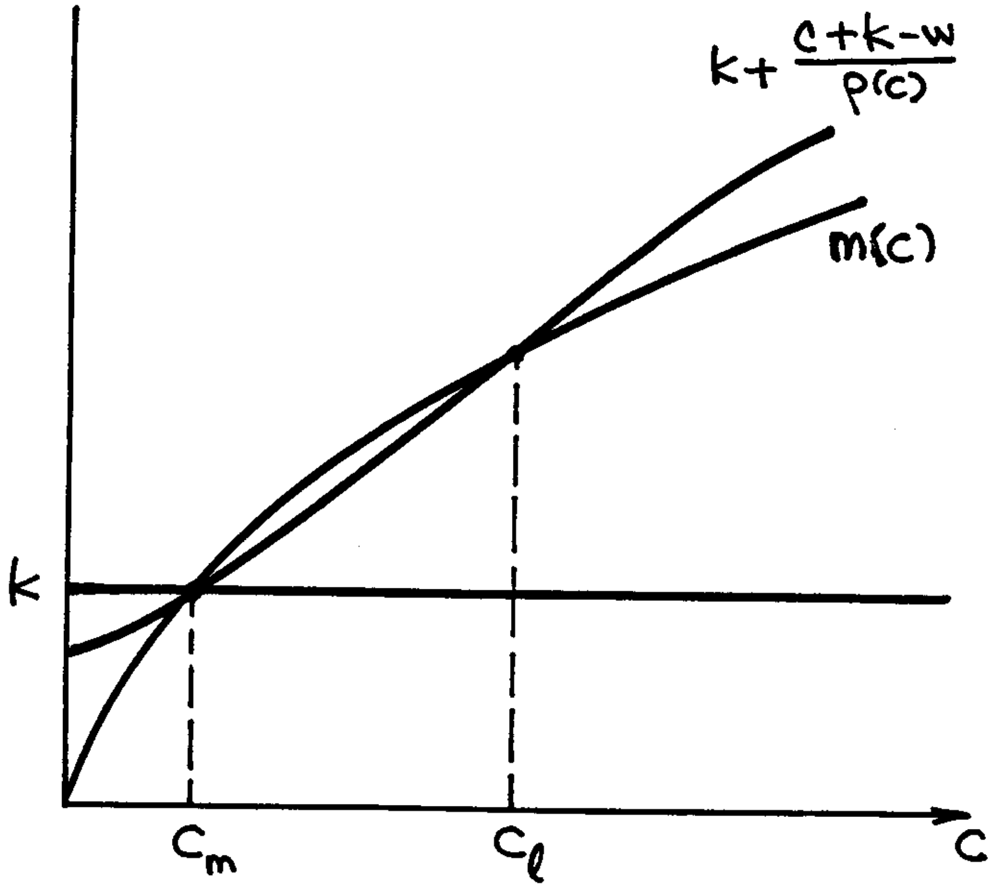


FIGURE 6

stressing that what he takes from Malthus is the sensible idea of a dynamically stable steady state income level, not the famous nonsense about geometric versus arithmetic growth rates. As Ricardo documents, this same idea is present in Smith, though there with the important qualification that the income level that is determined by fertility behavior alone is "the wages of the inferior classes of workmen." (Ricardo (1817), p.215) In short, all of the classical economists were "Malthusians," and none of them saw the potential for reductions in fertility to make possible sustained growth in living standards.

Ricardo (p.93) calls the income level c_m+k "the natural price of labour," defining it, as I just have, as a steady state: "that price which is necessary to enable the labourers...to subsist and to perpetuate their race, without either increase or diminution." What I have done here is to deduce this level and its stability using utility theory, unknown to the classics but entirely consistent with their discussion of the determinants of this "natural price." In my formulation, it is unclear whether the parameter k and perhaps the function W should be viewed as describing tastes or technology. The question-begging "necessary" in Ricardo's definition reflects the same ambiguity. He later refers to "those comforts which custom renders absolute necessaries," but then later still adds: "It is not to be understood," he adds (p. 96), "that the natural price of labour, estimated even in food and necessaries, is absolutely fixed and constant. It varies at different times in the same country, and very materially differs in different countries. It essentially depends on the habits and customs of the people."

The idea that tastes determine living standards has both possibilities and dangers. In the First Edition of *Principles,* Ricardo proposed : "Give to the Irish labourer a taste for the comforts and enjoyments which habit has made essential to the English labourer, and he would be then content to devote a further portion of his time to industry, that he might be enabled to obtain them." To this George Ensor asked, reasonably enough, "But how are these tastes to be excited in the Irish labourers? Is it supposed that they are not like other human creatures? but that they make choice of privations?" In later editions, Ricardo dropped the suggestion that taste differences could be used to account for English-Irish income differentials.[13] Since Ricardo retreated from his attempt at a cultural account of income differences, innumerable other cultural explanations have been proposed, only to be forced into retreat by historical fact.

Classical arguments over differences among countries' living standards and over what the causes of such differences might be were certainly of legitimate interest. The same issues continue to engage economic historians today. In a world in which Malthusian dynamics dictate that any society have an average income of, say, \$ 600 ± 200 1985 U.S. it is obviously of considerable interest whether a given economy is at \$800 or \$400. But even if differences of this magnitude between 18th century incomes in Europe and Asia can be defended, it certainly does not follow that such differences were in any sense a key to the origins of the industrial revolution. What kinds of economic forces transform small initial differences in income levels into sustained differences in rates of growth?

[13]This exchange is described by Sraffa in Ricardo (1817), p. 100.

5. CAPITAL ACCUMULATION AND FERTILITY

Despite their awareness of the importance of both physical and human capital accumulation, the classical economists lacked a method that could let them see how its possibility would affect equilibrium values. An operational theory of capital is entirely a development of the twentieth century. An advantage of re-stating the classical theory of production in explicit, neoclassical terms is that it becomes straightforward to incorporate reproducible capital into the theory. It will be useful to carry out this modification explicitly, to see how it is related to both classical theory and to modern growth theory, to see how a theory of fertility can be added to modern growth theory, and to verify once again Solow's (1956) conclusion that physical capital accumulation alone does not suffice to transform a static economy into a perpetually growing one.[14] With these conclusions established, we will be in a position to consider the preconditions for sustained income growth.

We could think of introducing reproducible capital with a class structure, introducing a capitalist class to go along with the workers and landowners in the model of Chapter 4. Such an approach would be in the Ricardian spirit, but its maintenance would entail strictures that neither workers nor landowners could own capital, analogous to the requirement in the last chapter than workers cannot acquire land. In view of the way capitalist economies have evolved in the last 200 years, however, assumptions designed to maintain rigid class lines have become increasingly dubious empirically as well as hard to reconcile with individual rationality. I will deal here

[14]The models discussed in this chapter descend directly from Razin and Ben Zion (1975). Intellectual debts are also acknowledged to Ahituv (1995), Benhabib and Nishimura (1993), Ehrlich and Lui (1997), Galor and Weil (1996), Nerlove, Razin, and Sadka (1987), and Razin and Sadka (1995).

with models of classless societies only.[15]

We also need to decide whether to introduce reproducible capital in place of or in addition to the land input. So as not to stray too far from the Ricardian set-ups of Chapters 3 and 4, let us begin by retaining the land input and consider the representative agent economy of Chapter 3 with reproducible capital added as a third factor of production. Let x be per land per household, as in Chapter 3, and let z be the stock of reproducible capital per household. Then a pair (x, z) of the two kinds of assets is a complete description of a household's situation, and we seek to formulate a Bellman equation for its value function $v(x, z)$.

As the accumulation of capital of various kinds receives greater emphasis, it becomes more critical whether one thinks of the costs of children as taking the form of *time* or of *goods*. For now, we continue with the assumption of Chapters 3 and 4 that each child entails an outlay of k units of goods. Assume that goods production per household is a constant returns to scale function of the capital, labor, and land inputs, and with the labor input fixed at unity, write $f(x, z)$ for the production of a one person household. Let end-of-period capital per child be y (so that total capital is yn) and assume, purely for simplicity, that there is no physical depreciation of capital. We adopt a one sector approach: There is a single produced good, which may be used either as consumption or capital. Then a household's resource constraint is

$$c + (k + y)n \leq f(x, z) + z. \tag{1}$$

Its Bellman equation is

$$v(x, z) = \max_{c,n,y} W\left(c, n, v(y, x/n)\right) \tag{2}$$

subject to (1). Equation (2) reflects the distinction between reproducible capital, accumulated by investment out of goods production, and non-reproducible land, where

[15] Of course, a "classless" society is not the same thing as an equal income society.

the stock x is simply divided equally among the n children, each receiving x/n.

The first-order and envelope conditions for the problem (2) are:

$$W_n(c,n,u') = (k+y)W_c(c,n,u') + W_u(c,n,u')v_x(y, x/n)x/n^2,$$

$$W_u(c,n,u')v_z(y, x/n) = nW_c(c,n,u'),$$

$$v_z(x,z) = W_c(c,n,u')[f_z(x,z)+1],$$

$$v_x(x,z) = W_c(c,n,u')f_x(x,z) + W_u(c,n,u')v_x(y, x/n)\frac{1}{n},$$

where

$$u' = v(y, x/n).$$

In a steady state, in which $n = 1$, $z = y$, and $u = u' = v(x,z)$, this system reduces to

$$u = W(c,1,u), \tag{3}$$

$$c + k = f(x,z), \tag{4}$$

$$W_n(c,1,u) = (k+z)W_c(c,1,u) + W_u(c,1,u)v_x(x,z)x, \tag{5}$$

$$W_u(c,1,u)v_z(x,z) = W_c(c,1,u), \tag{6}$$

$$v_z(x,z) = W_c(c,1,u)[f_z(x,z)+1], \tag{7}$$

$$v_x(x,z) = W_c(c,1,u)f_x(x,z) + W_u(c,1,u)v_x(x,z). \tag{8}$$

Use the envelope conditions (7) and (8) to eliminate the two derivatives of the value function. Then the first-order conditions (5) and (6) can be restated as

$$\frac{W_n(c,1,u)}{W_c(c,1,u)} = k + z + \frac{W_u(c,1,u)}{1-W_u(c,1,u)}f_x(x,z)x, \tag{9}$$

$$W_u(c,1,u)[f_z(x,z)+1] = 1. \tag{10}$$

View (3), (4), (9), and (10) as four equations in the steady state values of u, c, x, and z.

Following the procedure used in Chapter 3, solve (3) for $u = g(c)$ and define the marginal rate of substitution function $m(c)$ and the discount rate function $\rho(c)$ as in Chapter 3. In terms of these functions, (9) and (10) can be restated as

$$m(c) = k + z + \frac{f_x(x,z)x}{\rho(c)}, \tag{11}$$

$$f_z(x,z) = \rho(c). \tag{12}$$

Now we solve (4), (11), and (12) for the steady state values of c, x, and z. Given the equilibrium amount of land per person, z, the given quantity of land determines the equilibrium population.

With the per capita values c and x constant, these equations obviously do not describe an economy undergoing sustained growth. But is this system Malthusian, in the sense that steady state consumption and welfare are independent of the technology? The case of Cobb-Douglas production is again helpful, though not conclusive. Let $f(x,z) = Ax^\alpha z^\nu$, $\alpha + \nu < 1$, so that (4), (11) and (12) become

$$c + k = Ax^\alpha z^\nu, \tag{13}$$

$$m(c) = k + z + \frac{\alpha A x^\alpha z^\nu}{\rho(c)}, \tag{14}$$

and

$$\nu A x^\alpha z^{\nu-1} = \rho(c). \tag{15}$$

Use (15) to eliminate x and substitute into (13) and (14) to obtain

$$c + k = z\frac{\rho(c)}{\nu} \tag{16}$$

and

$$m(c) = k + (\frac{\alpha+\nu}{\nu})z. \tag{17}$$

Eliminating z between these equations gives

$$m(c) = k + (\alpha+\nu)\frac{c+k}{\rho(c)}, \tag{18}$$

which replicates (8) in Chapter 3 for the land-only economy.

In a steady state with no capital depreciation, then, land and reproducible capital enter fully symmetrically. The system can again be solved recursively, one-variable-at-a-time. Equation (18) yields steady state consumption, given child raising costs k and the production function parameters α and ν. The production function intercept A does not affect this level. Given c, the equilibrium discount rate $\rho(c)$ is determined, and the level of reproducible capital z can be obtained from (16): It, too, is independent of A. Finally, land per household x can be obtained from (15). With total land quantity given, this amounts to determining the equilibrium population. Increases in A induce increases in population only, exactly as in the economies of Chapters 3 and 4.[16]

Two Variations

The equilibrium just developed is fragile, in the sense that some of its most essential features depend critically on specific assumptions. One of these is the presence (1) no land input of land as an input, along side capital. If we were to introduce capital *in place of* land as an input, so that the value function depends on the single state variable z, the system of first-order and envelope conditions is replaced by the three conditions

$$W_n(c,n,u') = (k+y)W_c(c,n,u'),$$

$$W_u(c,n,u')v'(y) = nW_c(c,n,u'),$$

$$v'(z) = W_c(c,n,u')(f'(z)+1).$$

The steady state equation system, analogous to (3), (4), (9), and (10), becomes

$$u = W(c,1,u), \tag{19}$$

[16]As in Chapters 3 and 4, the sharp distinction between the effects of changes in the production function intercept A and of changes in the share parameters α and ν is very much specific to the Cobb-Douglas assumption. Away from this borderline case, other possibilities emerge.

$$c + k = f(z), \tag{20}$$

$$\frac{W_n(c,1,u)}{W_c(c,1,u)} = k + z, \tag{21}$$

$$W_u(c,1,u)(f'(z)+1) = 1. \tag{22}$$

But (19)-(22) are four equations in the three unknowns c, u, and z: the system is over-determined!

In the absence of land as an essential input, the analogue to a steady state equilibrium is found by leaving the fertility level n free and solving the system

$$u = W(c,n,u), \tag{23}$$

$$c + (k+z)n = f(z) + z, \tag{24}$$

$$\frac{W_n(c,n,u)}{W_c(c,n,u)} = k + z, \tag{25}$$

$$W_u(c,n,u)(f'(z)+1) = n. \tag{26}$$

A solution (c, u, n, z) to the system (23)-(26) corresponds to a kind of balanced growth path, on which population is growing (or declining!) at the constant rate $n-1$, and per capita capital and consumption are constant. We can think of such an equilibrium path as the counterpart to the original growth models of Solow (1956) and Cass (1965), without exogenous technical change, but with the non-zero population growth rate determined by the model rather than simply assumed. As in Razin and Ben-Zion (1975), we obtain a theory of sustained growth in total production, but not in living standards.

It is also essential to the existence of a steady state population *level* in the basic model of this chapter that the cost of bearing children take the form of an amount of goods rather than simply of *time*. To see this, we pursue a second variation. We reintroduce land as a factor of production, but assume that raising n children requires using kn units of the household's unit *time* endowment and no goods. That is, we reformulate the resource constraint as

$$c + yn \leq f(x, 1 - kn, z) + z, \tag{27}$$

where $f(x, \ell, z)$ is a constant returns function of the three inputs: land, x, labor, ℓ, and reproducible capital, z. The household's Bellman equation in this case is

$$v(x, z) = \max_{c,n,y} W\left(c, n, v(y, x/n)\right) \tag{28}$$

subject to (27).

We will study this problem under the assumptions that W is homogeneous of degree one in the pair (c, u), for any fixed n, and that production is Cobb-Douglas: $f(x, \ell, z) = Ax^{\alpha}\ell^{1-\alpha-\nu}z^{\nu}$. Under these conditions, the change of variable

$$w = x^{\alpha}z^{\nu-1}$$

(similar to that used by Caballe and Santos (1993)) is useful in reducing the dimension of the state space from two to one. In terms of the pair (x, w) the production function is

$$f(w, \ell, z) = A\ell^{1-\alpha-\nu}zw,$$

and the resource constraint (27) becomes

$$c + yn \leq A(1 - kn)^{1-\alpha-\nu}zw + z. \tag{29}$$

With the state variables (z, w), the household's Bellman equation is:

$$\psi(z, w) = \max_{c,n,y} W\left(c, n, \psi(y, wn^{-\alpha}(\frac{y}{z})^{\nu-1})\right), \tag{30}$$

subject to (29).

Let $\omega = c/z$ and $\gamma = y/z$. Then it is a reasonable conjecture that (30) has a solution of the form $\psi(z, w) = z\varphi(w)$, where the function φ satisfies

$$\varphi(w) = \max_{\omega,n,\gamma} W[\omega, n, \gamma\varphi(wn^{-\alpha}\gamma^{\nu-1})] \tag{31}$$

subject to

$$\omega + n\gamma \leq A(1-kn)^{1-\alpha-\nu}w + 1 - \delta. \tag{32}$$

We formalize this idea as the

Lemma. If $\varphi(w)$ solves (31) then $\psi(x,w) \equiv x\varphi(w)$ solves (30).

Proof. Suppose φ satisfies (31). Fix w and let (ω, n, γ) attain (31). Then (32) is satisfied and the triple $(c, n, y) = (\omega z, n, \gamma z)$ satisfies (29). Then if $(z\hat{\omega}, \hat{n}, z\hat{\gamma})$ is any triple that satisfies (29), $(\hat{\omega}, \hat{n}, \hat{\gamma})$ also satisfies (32) and we have

$$\begin{aligned}\psi(z,w) = z\varphi(w) &= zW[\omega, n, \gamma\varphi(wn^{-\alpha}\gamma^{\nu-1})] \\ &\geq zW[\hat{\omega}, \hat{n}, \hat{\gamma}\varphi(w\hat{n}^{-\alpha}\hat{\gamma}^{\nu-1})] \\ &= W[z\hat{\omega}, \hat{n}, z\hat{\gamma}\varphi(w\hat{n}^{-\alpha}(\frac{z\hat{\gamma}}{z})^{\nu-1})] \\ &= W[z\hat{\omega}, \hat{n}, \psi(z\hat{\gamma}, w\hat{n}^{-\alpha}(\frac{z\hat{\gamma}}{z})^{\nu-1})].\end{aligned}$$

Note that the next-to-last step uses the homogeneity of W. □.

Steady states and balanced growth paths of this system, solutions that have the property that the state variable w is constant, will satisfy

$$w = wn^{-\alpha}\gamma^{\nu-1},$$

or

$$n = \gamma^{(\nu-1)/\alpha}. \tag{33}$$

Thus a steady state with constant population, $n = 1$, will have a constant level of physical capital as well, $\gamma = 1$. But the existence of such a steady state would be purely coincidental. In general n will differ from one, and (33) requires that capital grows at the rate that will maintain constancy of the marginal product of capital (proportional, in this Cobb-Douglas case, to the output-capital ratio). Suppose, for example, that $n > 1$, so that population is increasing forever. Then since the total quantity of land is constant, land per household is decreasing forever, and since land and capital are complements, this in itself would imply that the marginal product of capital is decreasing forever. To maintain constancy, and hence to keep (33) satisfied, capital per household and goods consumption per household have to be decreasing forever too. Imagine a world in which ever more numerous but smaller people cultivate ever smaller plots of land with ever decreasing quantities of capital!

The problem with these variations on the basic model of this chapter is that both of them, in different ways, break the link between resources and human physiology. In the first variation, neither land nor any other fixed resource is used in production, so the economics of the situation can determine no more than the ratio of people to reproducible capital. In the second variation, only adult *time* is used to produce a new adult. Land is needed to produce goods, but the consumption of goods per person can either grow or decline indefinitely. Later on, when we want to approximate a modern economy in which limits on land have become less and less important, something like this second variation will serve as an interesting and helpful approximation. But in the present context, where we are seeking theoretical possibilities for capital accumulation that have the potential to help a society to free itself from Malthusian demography, neither of the variations explored in this section is of much interest.

Discussion

None of the classical economists would have viewed the failure of the models of this chapter to produce sustained growth in per capita income as either a surprise or a deficiency. To see the aggregate behavior they were attempting to understand, cover up Figure 3 from 1817 (when Ricardo's *Principles* were first published) on. But by the middle of the 19th century, it was evident to Karl Marx and Friedrich Engels (at least) that something had happened in the wealthiest nations that was entirely new in human history. Recall their paragraph on the economic achievements of capitalism in *The Communist Manifesto*:[17]

> The bourgeoisie, during its rule of scarce one hundred years, has created more massive and more colossal productive forces than have all preceding generations together. Subjection of Nature's forces to man, machinery, application of chemistry to industry and agriculture, steam navigation, railways, electric telegraphs, clearing of whole continents for cultivation, canalization of rivers, whole populations conjured out of the ground—what earlier century had even a presentiment that such productive forces slumbered in the lap of social labor?

Marx believed that the industrial revolution described in this paragraph was a transition from one steady state to a new one, following a shift in technology—the "factory system"—that made capital-intensive production more profitable. He viewed it as an event that would be completed without permanently altering the living standards of the poor, not as the onset of sustained growth in living standards. Surely this was a defensible position in 1848: see Figure 3, again. We now scorn Marx for this unsuccessful forecast, though all of his contemporaries shared it (or would have if they had had the imagination to try to work out where the capitalist economies were heading). We should rather credit him for his empirical judgment in distinguishing the genuinely new from the familiar.

One can see from (18) that without a class structure an increase in the elasticity ν of capital will induce an increase in steady state consumption (and welfare). The same is true for an increase in the land elasticity α. Either change in the productivity of a kind of inheritable capital alters the terms of the quantity-quality tradeoff in favor of reduced fertility. Some modern observers attempt to reconcile the fact of sustained income growth with the idea of the industrial revolution as a transition

[17]Marx and Engels (1848), reprinted in Simon (1994), p.163.

from one steady state to another by thinking of a succession of technology shocks of this type. One reads of the first industrial revolution, the second, and so on, in stages that are either ~~either~~ simply numbered or given more colorful labels. There must be something to this—we do not get sustained growth by making the same invention over and over—but no one has had any empirical success with models based on a succession of discrete, widely separated, big shocks. I am inclined to take such popular usage as a reflection of lack of analytical technique rather than a substantive insight into the process of technical change and growth.

6. FERTILITY AND SUSTAINED GROWTH

We know that in a model in which population growth is taken as given, the accumulation of physical capital is not sufficient, by itself, to generate sustained growth in per capita incomes. Since adding a fertility decision to the theory does not provide an engine of growth, it is hardly surprising that this modification does not yield a theory of growth. In this chapter, I consider two modificatons that have been used to generate sustained growth in models without a fertility decision: the introduction of exogenously given technological change, and the introduction of endogenous human capital accumulation under a constant returns technology. We will see that these two potential engines of growth interact with the theory of fertility in very different ways.

A Model with Exogenous Human Capital Growth

Looking ahead to the construction of balanced growth equilibrium paths, it will be essential to maintain the assumption that the dynastic preference function W is homogeneous of degree one in the pair (c, u), for any fertility level n. This property holds for the example

$$W(c, n, u) = c^{1-\beta} n^{\eta} u^{\beta} \tag{1}$$

used repeatedly (in log version) in earlier discussion.

On the technology side, it will be simplest to begin with a constant returns, labor only technology for producing goods. Specifically, goods production per household is assumed to be $h_t u_t$, where u_t is the fraction of the household's unit time endowment that is devoted to goods production, and h_t is the human capital of the household head. In this example, I assume that h_t simply grows at the given, constant rate

$\gamma - 1$,

$$h_{t+1} = \gamma h_t.$$

The only other use of time is assumed to be child raising. The technology for *this* activity involves a fixed amount k of time (not goods) per child, so that a parent with n children spends $1 - kn$ units of time producing goods. The resource constraint of a typical household is then

$$c \leq h(1 - kn). \tag{2}$$

The household's Bellman equation is:

$$v(h) = \max_{c,n} W\left(c, n, v(\gamma h)\right), \tag{3}$$

subject to (2).

Equation (3) will have a solution of the form $v(h) = Bh$, where the constant B satisfies

$$Bh = \max_{n} W\left(h(1 - kn), n, B\gamma h\right) = \max_{n} hW\left(1 - kn, n, B\gamma\right)$$

by the first degree homogeneity property of W. Cancelling gives:

$$B = \max_{n} W\left(1 - kn, n, B\gamma\right). \tag{4}$$

The first-order condition for the problem (4) is:

$$W_n(1 - kn, n, B\gamma) = kW_c(1 - kn, n, B\gamma). \tag{5}$$

We can think of (5) and

$$B = W\left(1 - kn, n, B\gamma\right)$$

as two equations to be solved for n and B, independent of the technology level h.

For the example (1), the solution for n is given by

$$kn = \frac{\eta}{1 - \beta + \eta}, \tag{6}$$

analogous to the solution for the hunter-gatherer example of Chapter 3.[18] In this case, note that the rate of technological change γ does not affect fertility at all: The parameter B is an increasing function of γ (provided $\beta\gamma < 1$, as I assume), but neither B nor γ affects the solution for n.

More generally, as long as we retain the complementarity assumption (see Note 9) that an increase in future utility per child increases the marginal utility of children to the parent—that parents would rather bear happy children than unhappy ones—this theory implies a fertility level that is non-decreasing in the rate of technological change. I do not see how one can get a demographic transition out of such a theory. On the other hand, if we reverse the complementarity assumption we lose the stability of the Mathusian equilibrium in the models of pre-industrial societies.

A Model with Endogenous Human Capital Growth

Re-stating this labor-only model as a model of endogenous technical change completely resolves this impasse, as I will next show. Replace the exogenously given growth rate γ for human capital with the equation

$$h_{t+1} = h_t \varphi(r_t), \tag{7}$$

where r_t is the fraction of the household's unit time endowment that is devoted to investment in the human capital of one's children. I continue to assume that there is a minimum amount k of time (not goods, in this case) per child, so that the resource constraint of a typical household is

$$c \leq h(1 - (r + k)n). \tag{8}$$

[18]The analogy is natural, since that example and this one involve labor-only technologies; it is inexact because in the present example child-raising costs take the form of time rather than goods.

The household's Bellman equation is now written:

$$v(h) = \max_{c,n,r} W\left(c, n, v(h\varphi(r))\right), \tag{9}$$

subject to (8).

Once again, we can see that the homogeneity property of W implies that a solution to (9) will take the form $v(h) = Bh$, and that the constant of proportionality B must satisfy

$$B = \max_{n,r} W\left(1-(r+k)n, n, B\varphi(r)\right). \tag{10}$$

The two first order conditions are

$$W_n\left(1-(r+k)n, n, B\varphi(r)\right) = (r+k)W_c\left(1-(r+k)n, n, B\varphi(r)\right), \tag{11}$$

$$W_u\left(1-(r+k)n, n, B\varphi(r)\right)B\varphi'(r) = nW_c\left(1-(r+k)n, n, B\varphi(r)\right). \tag{12}$$

Specializing to the example (1), (11) becomes:

$$\frac{\eta}{n} = (r+k)\frac{1-\beta}{1-(r+k)n},$$

which can be rearranged to give the expression

$$(r+k)n = \frac{\eta}{1-\beta+\eta} \tag{13}$$

for the *total* time expenditure on children. Compare (13) to the solution (6) for the exogenous technical change case, but in contrast to (6), equation (13) determines neither fertility n nor human capital investment r separately.

In this parametric example, the first order condition (12) becomes

$$\beta\frac{\varphi'(r)}{\varphi(r)} = n\frac{1-\beta}{1-(r+k)n}.$$

Assuming the form

$$\varphi(r) = (Cr)^{\varepsilon}$$

for the human capital accumulation technology, we have

$$\beta\varepsilon = rn\frac{1-\beta}{1-(r+k)n}. \tag{14}$$

Now use (13) and (14) to solve for r and n separately:

$$n = \frac{1}{k}\frac{\eta - \beta\varepsilon}{1+\eta-\beta}.$$

and

$$r = \frac{\beta\varepsilon}{\eta - \beta\varepsilon}k.$$

In this particular, exponential example, then, a change in the parameter C of the human capital accumulation function does not affect behavior, but an increase in the exponent ε induces both an increase in the time r devoted to acquiring human capital and a decrease in fertility.[19] This is certainly not the first example I have discussed in which the quantity-quality tradeoff plays a role in determining fertility, but it is the first in which an improvement in technology—which an increase in ε represents—leads to a permanent reduction in fertility. This is the central idea in the Becker, Murphy, Tamura (1990) account of the demographic transition.

Some recent research on economic growth has assigned great importance to the assumption that the engine of growth—technological change—is *exogenous*. (Some writers even take this feature as the defining characteristic of a *neoclassical* model!) This emphasis seems to me badly misplaced: Surely technological change must arise from some time-consuming activity. The present example shows that to explain the observed behavior of fertility during a demographic transition, one needs to assign an important role to endogenous human capital accumulation, motivated by the private

[19] The functional form $\varphi(r) = (Cr)^\varepsilon$ facilitates the explicit solution in this example, but it does not make a lot of sense at all values of r: think of the case $r = 0$. I think of the parameter C as chosen so that equilibrium growth is positive, which requires $Cr > 1$. In this case, an increase in ε increases the growth rate of human capital for each level of r, and so unambiguously represents an improvement in technology.

rate of return to this activity. Of course, one need not assume equality of the private and social returns to investment in human capital. It may well be that new knowledge accrues to some who do not invest at all, in which case they might perceive it as "exogenous."

Elaborations with Physical Capital

When physical capital is added to this model, one obtains essentially the theory of Lucas (1988) and Caballe and Santos (1993), modified to include the fertility decision. Assume, to be specific, that goods production per household is a constant returns function of physical capital z and the labor input h. (This entails the assumption that one person with human capital h is the productive equivalent of two people with $h/2$ each.) Let physical capital depreciate at the rate δ, so a household in state (z, h) that produces $f(z, h)$ has $f(z, h)+(1-\delta)z$ units of goods at its disposal, to be divided between consumption c, capital left over for its children, yn, and child raising costs $(r+k)hn$. Finally, assume the human capital technology (7). The resource constraint of a typical household is then

$$c + [y + (r + k)h]n \leq f(z, h) + (1 - \delta)z, \tag{15}$$

and the household's Bellman equation is:

$$v(z, h) = \max_{c,n,r,y} W\left(c, n, v(y, h\varphi(r))\right), \tag{16}$$

subject to (15).

The change of variable similar to that used in the last example of Chapter 5 can reduce this problem to one with a single state variable. Let $w = z/h$ be the *ratio* of physical to human capital, let $\theta = c/h$ be the ratio of consumption to human capital, and let $\gamma = y/x$, so that $\gamma - 1$ is the growth rate of physical capital per capita. The resource constraint (15), restated in terms of the pair (w, h), is then:

$$\theta + (\gamma w + r + k)n \leq f(w, 1) + (1 - \delta)w. \tag{17}$$

It is a reasonable conjecture that if the function $\phi(w)$ solves

$$\phi(w) = \max_{\theta,n,r,\gamma} W\left(\theta, n, \varphi(r)\phi(\frac{\gamma w}{\varphi(r)})\right), \tag{18}$$

subject to (18), then $h\phi(z/h)$ will solve (16). We formalize this as a

Lemma. If $\phi(w)$ satisfies (18), then $h\phi(x/h)$ satisfies (16).

The proof, an application of the fact that $W(c, n, u)$ is homogeneous of degree one in the pair (c, u), is essentially the same as that of the Lemma in Chapter 5.

Without working out all the details, we can see that the system of first-order and envelope conditions for the problem (18) can be made consistent with a balanced equilibrium path, on which consumption and the two stocks of capital z and h all grow at the constant, common rate $\gamma = \varphi(r)$, and the ratio $w = z/h$ is constant: essentially the model of Section 4 of Lucas (1988). On such a balanced path, the fertility level n and the level of human capital investment in each child, r, are both constant.

Conclusions

Models of sustained per capita income growth can be based on exogenous improvements in technology, or knowledge, or human capital, or they can be based on economic decisions to invest in activities that produce such improvements. With the population growth rate taken as fixed, such models can be hard to distinguish on the basis of aggregative time series. Once fertility is viewed as an economic decision, however, these two classes of models have sharply different predictions. Theories of exogenously given technological change imply that higher growth should be associated with higher fertility: People prefer to bring more children, not fewer, into a world that offers them a more prosperous life. Theories in which higher growth is viewed as a response to increases in the return to investment in human capital, in contrast, can imply that increases in growth are associated with reductions in fertility. This is the case in the second example of this section. Under the technology of this example, a family that wants to take advantage of an increase in the return to investment in knowledge does so, in part, by reducing the number of children so as to devote more time and resources to each child. Only this second classes of theories, those based on endogenous human capital growth, is consistent with the demographic transition.

Human capital is a broad term, encompassing cognitive achievements that range from basic scientific discoveries to a child's learning how to read or how to plow behind a horse. Which particular activities are we thinking of if we center our view of economic growth and the industrial revolution on human capital accumulation? When Paul Romer stresses "knowledge capital" as "blueprints," he is thinking of human capital at the most abstract and ethereal end of the spectrum: Important additions to a society's human capital that are, for almost all of us, events that happen to us without our doing anything to bring them about. When another economist stresses improvements in literacy, he is thinking of human capital at the other end of the spectrum, far from Science with a capital S, capital that is accumulated only if many people devote their time and energy to doing so. In any actual society, the accumulation of knowledge takes both these extreme forms, and the range of possibilities in between, but it is *only* the second kind that can simultaneously also help us to explain the reduction in fertility that is essential to defeat the logic of Malthusian theory.

The demographic transition necessarily involves more than the accumulation of knowledge by a privileged class or subset of people. Such accumulation has taken place for centuries, inducing technical change, improved living standards, and increases in population, but ultimately leading to a return to the living standards of earlier ages. The new element that must have been involved in the demographic transition was an increase in the return to human capital accumulation that affected *everyone*, and hence every family's fertility choices. The industrial revolution required a change in the way people viewed the possibilities for the lives of their children that was widespread enough to reduce fertility across economic classes, affecting propertied and propertyless people alike.

The fact that the colonial system failed to bring sustained income growth to the colonized societies of Asia and Africa seems to me consistent with such an economically democratic view of the industrial revolution. Certainly the flow of European ideas brought increases in productivity and (hence) in population to British India and the Dutch East Indies. What colonialism did *not* bring to these societies were increases in the return to investment in children of ordinary familes, increases with the potential to change the quantity-quality tradeoff faced by these families and to induce a demographic transition. Independence (I conjecture) brought about the removal of the limits on upward mobility imposed by the colonial system, altering the terms of this tradeoff and opening the way for growth in per capita as well as total production.

7. CONCLUSIONS

Defined as the onset of sustained income growth, the industrial revolution was not exclusively, or even primarily, a technological event. Important changes in technology have occurred throughout history, yet the sustained growth in living standards is an event of the last 200 years. The invention of agriculture, the domestication of animals, the invention of language, writing, mathematics and printing, the utilization of the power of fire, wind and water, all led to major improvements in the ability to produce goods and services, and these and many other inventions made possible enormous growth in population. Depending on where such inventions occurred, some of them induced important shifts in the relative power of different societies. By the seventeenth century, indeed, their ability to generate new technology had enabled the Europeans to conquer much of the world. Yet none of these inventions led to *any* substantial increase in the living standards of ordinary people, Europeans or others. All of this is precisely as the theory of Malthus and Ricardo explicitly predicted, or more likely, as it was designed to explain.

Of course this is not to say that prior to the last two centuries everyone lived at a level of subsistence, even where subsistence is given an economic as opposed to a biological definition, as I have done. Wherever property rights in land and other resources have been established, property owners have enjoyed incomes in excess of, often far in excess of, subsistence.[20] Virtually everything we think of as civilization

[20] According to Johnson (1948), the share of farm income accruing to land in the U.S., 1910-1946, was between 0.30 and 0.35. (Table IV, p.734). This fraction is apparently fairly stable across different levels of income. McEvedy and Jones (1978), estimate that Egypt's population at the time of the conquest by Rome was 4m. At a per capita income of $600 1985 U.S., this corresponds to a GDP of $2.4b., almost all of which was farm income. Multiplying by 0.3 gives an estimate of land

has been sustained by land's share in production, and most of political and military history is the history of conflicts over how this share shall be disposed. The existence and indeed the persistence over centuries of wealthy landowning families need not conflict with the Malthusian or Ricardian fertility theory, for parents with any kind of inheritable property are faced with a Beckerian tradeoff between the quantity of children and the "quality" (that is, utility) of each child.

Our common sense tells us that the kind of quantity-quality tradeoff entailed in the heritability of land cannot generate sustained income growth. A redistribution of land can reduce the fertility of some families and increase the fertility of others, but as in the example worked out in Chapter 4, such a redistribution need not even affect the long run *level* of average income and surely it cannot affect the long term growth rate under any reasonable assumptions. The accumulation of reproducible capital adds new possibilities, but for the reasons familiar from Solow's (1956) original paper, diminishing returns prevent physical capital from serving as an engine of growth. This leaves human capital.

That the accumulation of human capital can serve as an engine of sustained growth is by now well known, and this idea has been embedded in a wide variety of tractable models. But human capital is a very broad term, so broad indeed that human capital based growth theories can be little more than older theories with key terms re-labelled. I have argued in the last chapter that incorporating a fertility decision into theories of growth can help to sharpen our thinking considerably on the kind of human capital growth that is essential to income growth. What is needed are opportunities for human capital investment that face the mass of households in a society with a quantity-quality tradeoff. A small group of leisured aristocrats

rental income of $720m. Some of this annual income flow must have gone to those who enforced and administered Roman rule, but even so it is not hard to see how some very comfortable lifestyles and impressive monuments were financed.

can produce Greek philosophy or Portuguese navigation, but this is not the way the industrial revolution came about.

Transition

I have said what I can about the industrial revolution, mostly by means of explicit models that seem to me to illuminate various aspects of it. But I have not offerred a model capable of unifying classical and modern growth theory in the sense of my introductory chapter, a model that can be simulated to exhibit a transition from per capita income stagnation to sustainned growth. In my view, the essentials of such a transition would include a shift in the return to human capital accumulation that alters the trade-offs individual families face, and an account of why the onset of sustained growth is associated with a dramatic reduction in the importance of land as a factor of production. I hope the models I *have* developed illustrate the central importance of these two issues: Compare any of the classical models of Chapters 3 and 4 to the endogenous growth models of Chapter 6.

Becker, Murphy, and Tamura (1990) have addressed the first of these transitional issues, in a way that has stimulated much of these lectures. Their model exhibits two possibilities for long run behavior, one with high fertility and low growth and another with low fertility and high growth. But their low growth equilibrium is not really Malthusian or classical in any sense: Land plays no role and the population *level* is not determined by the theory. Laitner (1994) has modelled the changing role of land as a factor of production over the course of the industrial revolution in a way that is technically promising and closely grounded in evidence. Let me simply say that I have spent many days trying to integrate the ideas in these exciting papers into the models presented above, and continue to believe that a satisfactory theory of the transition to the industrial revolution will have to do this successfully.

Diffusion

An understanding of the onset of the industrial revolution would presumably also yield an improved understanding of its gradual *diffusion* to country after country. Tamura (1991), (1996) proposes a mechanism for the diffusion of the industrial revolution, based on the idea that the return to accumulating human capital in any economy is an increasing function of the level of human capital in the world economy as a whole. According to this model, once any economy has entered onto a phase of sustained human capital growth, the world stock is destined to grow without bound, and this growth eventually leads, via an external effect, to an increase in the return on human capital investment sufficient to trigger a demographic transition in every other economy. The model fits our sense of the diffusion of knowledge, and also our sense that the industrial revolution is a single event, not many independent events. The fact that human-capital based models of growth lend themselves, with external effects, to such diffusion is, I think, yet another point in their favor.

How do these external effects occur? What is the process by which knowledge in one society affects its rate of accumulation in another? Geography is obviously part of the answer: One can see industrialization diffusing in space in the historical data. But physical proximity cannot be the key to anything: Albania did not share in Italy's postwar miracle, nor North Korea in Japan's. Geographical proximity must matter because it increases *economic* proximity, through trade and the interchange of ideas that trade stimulates. Stokey (1988), (1991) and Young (1991) have formalized a connection between trade and human capital growth. Chuang (1993) and Irwin and Klenow (1994) have demonstrated the empirical importance of learning spillovers. Lucas (1993) argues for their central importance, and I interpret Parente and Prescott (1994) as a complementary view.

Prediction

Modern theories of sustained growth, such as those of Chapter 6, abstract from considerations of land supply and limited resources generally. Such theories can, and do, fit long stretches of economic time series fairly well, but obviously they cannot fit forever. At some point, population will attain an upper bound and either stay there or begin to decline. Popular discussion has emphasized limits imposed by finite supplies of fuel, but predictions of fuel exhaustion—especially exhaustion that catches everyone by surprise!—have not received much support from economic analysis. Limited land supply may ultimately show up in rising relative prices of food, though there is certainly no sign of this yet, or in prices of residential land. Residential space has a high income elasticity, so it may be that if per capita incomes continue to grow without bound population growth will ultimately turn negative. We may become an ever less numerous race, with ever more palatial estates! Obviously none of the models I have explored here would be of much help in thinking about these questions about the very distant future, but this does not mean economic reasoning could not usefully be applied to them.[21]

If it is clear that population must eventually be bounded, no analogous reasoning applies to per capita incomes. There are a number of theoretical models that relate population growth rates to the growth rate of knowledge, with the property that zero population growth implies zero growth in productivity, but this feature seems accidental, readily modified, and not supported by evidence.[22] Economies with rapidly growing populations do not have systematically higher rates of productivity growth than other economies (Backus, Kehoe, and Kehoe (1992)). On the other hand, there is enough mystery surrounding the discovery of new knowledge that it is hard to be

[21]Cohen (1995) contains a stimulating, but entirely non-economic, discussion of the estimation of very long term population.

[22]The earliest is Arrow (1962).

sure than 20th century rates of growth will continue into the indefinite future. All one can say is that no tendency for scientists and others to run out of unsolved problems has yet been observed. The 19th century idea that Science would get to the bottom of things has not fared well in the 20th.

Even if it is impossible to tell whether productivity growth in the wealthiest economies will accelerate or diminish, the evolution of *relative* productivities has a clear and theoretically intelligible pattern. The industrial revolution began when a few economies entered onto a phase of sustained growth, leaving the rest of the world in a Malthusian equilibrium. By the mid nineteenth century, this process of sustained growth in some countries and stagnation in others had led to perhaps a factor of 1.7 difference in living standards between Europe and mainly European occupied countries and the rest of the world; by the beginning of the 20th century, this difference had grown to something like a factor of 3.2.[23] Throughout the first half of this century this gap continued to grow, leading to a factor of 5.7 difference between European and non-European societies by 1950, and to factor of 15 to 25 differences in income between the richest and poorest societies.

In the period since World War II and the end of the colonial age, measures of income inequality across countries have stayed fairly stable. During this period, many societies have joined the industrial revolution, which has tended to reduce measured inequality, but others have continued to stagnate, increasing inequality. It is hard to read the future in data from 1960 to 1990, but I think it is becoming increasing clear that the enormous inequality of the postwar period is at its all-time peak, and will decline in the future until something like the relative incomes of 1800 are restored. Within Europe, and between Europe and America, there been a dramatic equalization since 1950. One non-European nation after another has followed Japan into rapid

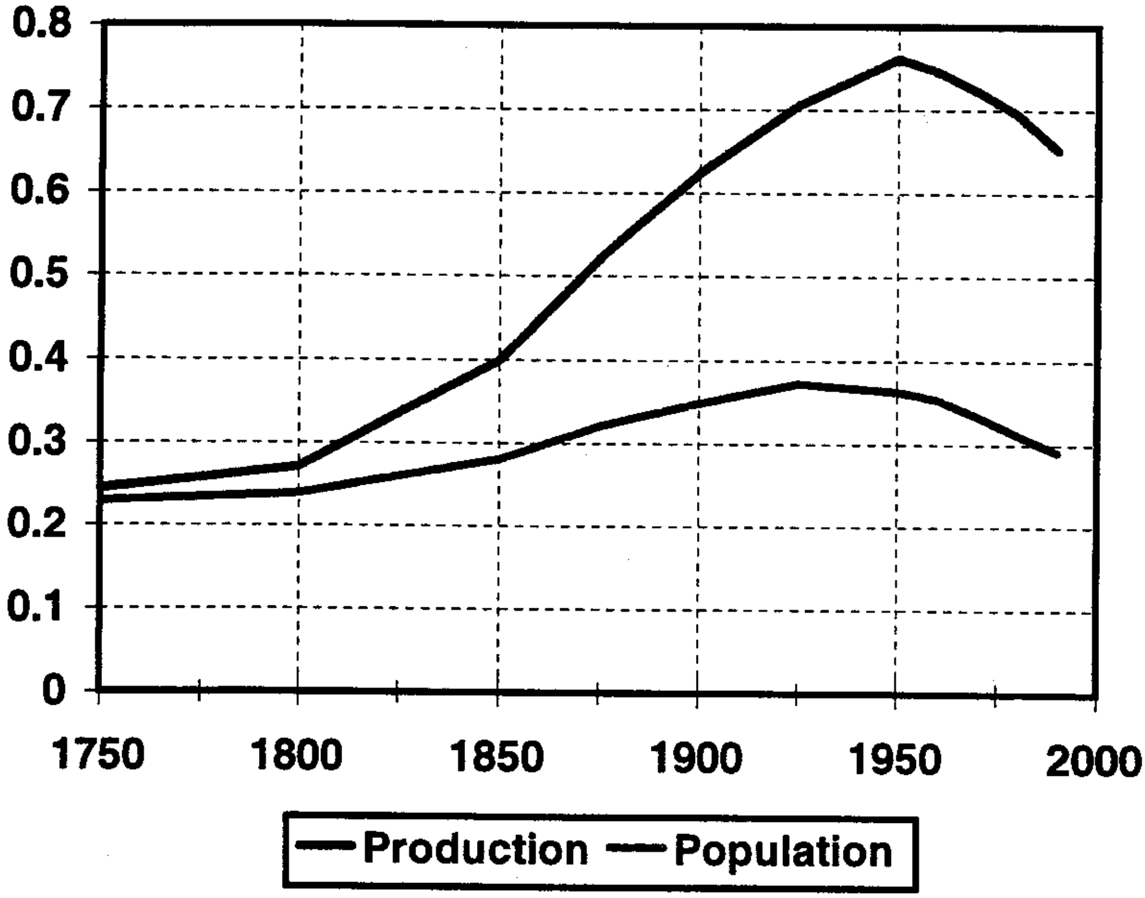
[23] By "European and mainly European occupied countries" I mean Regions I, III, and IV as defined in Figure 3.

income growth, and no one can see anything but unstable politics and mercantilist trade policies that keep the rest from doing so.

At the turn of the last century, many sophisticated observers were persuaded that northern Europeans must have racial or cultural advantages that accounted for the income differentials that had emerged during the 19th century. Now, at the end of another century, it is abundantly clear that full participation in the economic benefits of the industrial revolution is open to countries of all races and cultural backgrounds. Figure 7 plots the share of the total world population in European dominated economies, and also the European share in total product, from 1750 to the present. These shares were roughly equal, at about .24, in 1750, reflecting approximate equality in per capita production at that date. The share of world population in countries of mainly European populations peaked in the 1920s, at about .37. It was just under .30 in 1850 and has returned to that level today. The share of production in the European economies reached a peak of .76 in 1950, and is now .65. Does anyone doubt where these two time series are headed? The legacy of economic growth that we have inherited from the industrial revolution is an irreversible gain to humanity, of a magnitude that is still unknown. It is becoming increasingly clear, I think, that the legacy of inequality, the concomitant of this gain, is a historical transient.

[INSERT FIGURE 7 HERE]

Figure 7: European Shares of World Production and Population



25-Jan-1998

Data Appendix

This appendix describes the sources and procedures used to construct Figures 1-5 in the text. All of these figures are based on the population and per capita GDP data provided in Tables 1 and 2. Table 1 provides population estimates for 21 regions of the world, for the 15 indicated years from 1500 to 1990. Table 2 provides estimates of per capita GDP in 1985 U.S. dollars for the 11 indicated years from 1750 to 1990. I will first describe the sources and general procedures used for obtaining the population estimates, then the sources and procedures for the production estimates, and finally, define the 21 regions and discuss any special problems that arise with them.

[INSERT TABLES 1 AND 2 HERE]

Unless otherwise noted, the population estimates in Table 1 from 1960 on are from version PWT5.6 of the Penn World Tables described in Summers and Heston (1991). I abbreviate this source as S&H. Population figures from 1950 and before for all regions are from McEvedy and Jones (1978), which I abbreviate M&J. Post-war population figures for countries not found in either of these sources are usually taken from the UN Statistical Yearbooks, abbreviated U.N. Other sources are cited as pertinent. All these secondary sources use primary census data and official estimates, wherever available, so differences among them are minor. Comparison of population data for 1950 from M&J and S&H shows very close agreement for most regions.

Per capita production data are presented in Table 2, for these same 21 regions. Where both GDP and GNP are available, GDP is used. Otherwise, GNP is used. In all cases, we refer to the measure as GDP. Unless otherwise noted, all GDP estimates from 1960 on (from 1950 on, where available) are from S&H. The entries in bold face in Table 2 are the earliest figures taken from S&H. I use Variable 2 (RGDPCH) from S&H as the per-capita GDP estimate. As described in the source, RGDPCH is a

Table 1: World Population (in millions) 1500-1990

	1500	1600	1650	1700	1750	1800	1850	1875	1900	1925	1950	1960	1970	1980	1990
1 Africa	38	44	48	52	56	60	68	76	88	111	162	235	301	381	510
2 North Africa & Middle East	28	35	34	33	34	36	44	52	63	77	111	143	188	242	303
3 United States	1	1	1	1	2	6	24	44	76	115	150	181	205	228	250
4 Canada	0.2	0.2	0.2	0.2	0.3	0.5	2	4	5	10	14	18	21	24	27
5 Mexico	5	4	4	4	5	6	8	9	14	15	27	38	53	67	82
6 Southern Cone	1	1	1	1	1	1	3	5	9	16	25	31	36	42	49
7 Rest of Latin America	7	6	6	7	8	11	22	29	43	67	111	149	197	250	307
8 Japan	17	22	25	29	29	28	32	36	45	60	84	94	104	117	124
9 China	110	160	140	160	225	329	433	413	472	527	582	668	820	983	1136
10 East Asia	4	5	5	7	8	10	12	13	15	23	39	47	64	79	91
11 Southeast Asia	20	22	24	26	29	33	44	57	85	122	180	219	283	362	442
12 Indian Subcontinent	105	135	150	165	175	190	230	255	290	330	445	553	700	887	1108
13 United Kingdom	5	5	6	7	8	11	21	29	38	45	50	53	56	56	57
14 France	15	18	21	22	24	29	36	38	41	40	42	46	51	54	57
15 Germany	9	12	11	13	15	18	27	33	43	55	70	72	78	78	79
16 Low Countries	2	3	4	4	4	5	8	9	12	16	19	21	23	24	25
17 Scandinavia	2	2	3	3	4	5	8	10	12	16	18	20	21	22	23
18 Rest of Western Europe	22	29	27	31	37	46	60	67	77	88	108	116	125	134	139
19 Former Soviet Union	17	21	23	27	34	45	74	93	124	168	181	214	243	266	289
20 Eastern Europe	13	17	17	18	23	31	43	53	68	82	87	98	106	115	120
21 Australia & New Zealand	0.2	0.3	0.3	0.3	0.4	0.4	1	2	4	8	10	13	15	18	20

Table 2: Per Capita GDP in 1985 U.S. Dollars, 1750 - 1990

	1750	1800	1850	1875	1900	1925	1950	1960	1970	1980	1990
1 Africa	455	455	455	466	489	525	636	**763**	961	1116	966
2 North Africa & Middle East	542	542	542	571	634	742	1129	**1647**	2574	3583	3494
3 United States	837	870	1519	2581	3943	6034	**8772**	9895	12963	15295	18054
4 Canada	821	854	1279	1923	3095	4254	**6380**	7258	10124	14133	17173
5 Mexico	826	858	922	1105	1373	1646	**2198**	2836	3987	6054	5827
6 Southern Cone	826	858	958	1262	1756	2313	**3594**	4028	4987	5720	4600
7 Rest of Latin America	817	858	907	1041	1228	1409	1757	**1983**	2664	3853	3598
8 Japan	636	636	625	681	1025	1401	**1430**	2954	7307	10072	14331
9 China	630	630	630	630	630	630	500	**568**	697	973	1325
10 East Asia	630	630	630	630	630	630	630	**1004**	1812	3458	6807
11 Southeast Asia	630	630	630	630	630	630	630	**696**	915	1403	1954
12 Indian Subcontinent	630	630	630	630	630	630	630	**779**	873	935	1296
13 United Kingdom	805	840	1864	2633	3527	4362	**5395**	6823	8537	10167	13217
14 France	665	752	1207	1612	2152	3110	**4045**	5823	9200	11756	13904
15 Germany	652	738	1048	1488	2179	2974	3122	**5843**	8415	11005	13543
16 Low Countries	665	752	1274	1795	2464	3564	**4518**	5851	8844	11222	13158
17 Scandinavia	692	783	979	1373	2044	2879	**4930**	6541	9450	11798	14444
18 Rest of Western Europe	665	753	1126	1408	1840	2301	2813	**3951**	6622	8694	10729
19 Former Soviet Union	578	620	697	815	991	1114	1713	**2397**	4088	6119	7741
20 Eastern Europe	578	620	709	797	911	1078	1340	**1823**	2621	3986	3970
21 Australia & New Zealand	837	870	1862	2650	3672	3984	**6676**	7815	10505	12143	13962

Note: Bold face indicates earliest estimate from Summers and Heston.

chain index series that values a country's real gross product in international prices of a base year. The listed figures are in 1985 international prices.

For the years 1950 and before, I used Bairoch (1981), which I abbreviate B, as the source of per capita GDP. (Maddison (1983), (1991), and (1995) also provides per capita GDP estimates for various countries and parts of the world.) These estimates are in 1960 U.S. dollars. I converted to 1985 dollars using the ratio of deflators for 1960 and 1985 from S&H, which entails simply multiplying all Bairoch estimates by 3.5.

Where overlapping estimates exist, generally for 1960, there are differences between S&H and Bairoch estimates. When the difference was 4 percent or less, I simply multiplied all Bairoch estimates again by the ratio of the S&H to Bairoch estimate for the overlapping year. These multiplicative factors are listed in the region-specific sections below, for all regions where this procedure was applied.

When the difference between overlapping year estimates exceeded 4 percent, I used a different procedure. For these cases, I used Bairoch's estimates of income *levels* for 1800, and used his estimates for later years to estimate rates of growth from this base. I then multiplied all these growth rates by a constant chosen to make the adjusted Bairoch estimates and the S&H estimates agree for 1960 (1950, when available in S&H). The details differ from region to region, and are provided below in the region-specific sections.

We next define the 21 regions, and provide data sources for each.

1. *Africa.* Includes all of Africa except for the northern tier of countries: Morocco, Algeria, Tunisia, Libya, and Egypt.

Population: The populations of the Maghreb (M&J, p. 221), Libya (p. 225), and Egypt (p. 227) were substracted from Africa as a whole (p. 206). Some interpolations were used for 1650, 1750, 1875, and 1925. Livi-Bacci (1992), Table 1.3, also provides population data for the entire continent of Africa. These estimates are consistently higher than the M&J estimates for Africa as a whole, substantially so for the earlier years. (For example, the 1500 estimates from these two sources are 46 and 87 million respectively.)

GDP: Estimates from 1960 on are from S&H. Estimates for 1950 and earlier are based on the Bairoch, Table 1.7, estimates for Africa as a whole, provided for the years 1800, 1860, 1913, 1938, 1950, and 1960. The 1800 estimate, $455 = 130 \times 3.5$, was applied to 1750 as well. Estimates for later years were interpolated and used to estimate the annual growth rates given for Region 1 in Table A1. As indicated in Row 1 of Table A.1, these growth rates were then multiplied by the constant .957, chosen so that the 1960 per capita income estimate agrees with the S&H estimate, and then used to interpolate per capita GDP for the indicated years from 1850-1950.

2. *North Africa and the Middle East.* Includes Morocco, Algeria, Tunisia, Libya, and Egypt, and Turkey, Syria, Lebanon, Palestine (Israel, Jordan), Arabia (Kuwait, Bahrain, the UAE, Saudi Arabia, Oman, Yemen, Qatar), Iran, Iraq, and Afghanistan.

Population: To the populations of the Maghreb, Libya, and Egypt (above, **1**) were added the populations of Turkey-in-Europe (M&J, p.113), Turkey-in-Asia (p. 134), Syria and Lebanon (p. 138), Palestine and Jordan (p. 143), Arabia (p. 144), Iraq (p. 150), Iran (p. 152), and Afghanistan (p. 154). Some interpolations are used for 1650, 1750, 1850, 1875, and 1925. For comparison, M&J report 112m. for 1950, while S&H (supplemented by the UN yearbooks) report 118m.

GDP: Data from 1960 on are from S&H, with minor extrapolations. Data before 1960 are from B, Table 1.7. The average of the Asia and Africa estimates are used for 1800: 542 1985 U.S. dollars. This figure is applied to 1750 as well. The years between 1800 and 1960 are interpolated, using the assumption that the growth rates were proportional to those given for Africa in Table 1.A. The necessary constant of proportionality in this case is 2.1.

3. *USA*

Population: M&J, pp. 287 and 290. Hawaiian population is given separately, pp. 334-336.

GDP: B, Table 1.4 has per capita figures for the U.S. and Canada separately from 1830. North American figures from Table 1.6 are used for 1750 and 1800, for both U.S. and Canada. S&H data are used from 1950 on. Estimates before 1960 are multiplied by 1.04, the ratio of the S&H to Bairoch estimates for 1950.

4. *Canada.*

Population: M&J, p. 285.

GDP: B, Table 1.4 is used after 1830. North American figures from Table 1.6 are used for 1750 and 1800. S&H data are used from 1950 on. Estimates before 1960 are multiplied by 1.02, the ratio of the S&H to Bairoch estimates for 1950.

5. *Mexico.*

Population: M&J, p. 293. M&J note that the population figure for 1500 is controversial, with estimates as different as 5 million and 30 million. M&J choose the lower. Interpolation is needed for 1650.

GDP: Estimates from 1950 on are from S&H. Estimates for 1925 and earlier are based on the Bairoch, Table 1.7, estimates for "Third World America" as a whole, provided for the years 1800, 1860, 1913, 1928, 1938, 1950. The 1800 estimate is 858. This figure was extrapolated to 1750 as well, keeping the ratio to the U.S. fixed: This gave $(858/870) \times 837 = 825$. Estimates for later years were based on interpolations of the estimates in Bairoch, Table 1.7, which imply the annual growth rates given in Row 5 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality 1.45.

6. *South American Cone.* Argentina, Chile, and Uruguay.

Population: M&J, pp. 315-317.

GDP: Data for 1950 and later years are from S&H. The estimate for 1800 is from Bairoch, Table 1.7 for all of Latin America. It was extrapolated back to 1750 with the same procedure (and the same resulting number) as with Mexico. Incomes for the indicated years from 1850-1950 were interpolated, assuming that Southern Cone growth rates were proportional to those in Row 6 of Table A.1, with the constant of proportionality 2.2.

7. *Rest of Latin America and the Caribbean.* Includes the Caribbean Islands, Central America (Guatemala, Belize, El Salvador, Honduras, Nicaragua, Costa Rica, and Panama), Colombia, Venezuela, the Guyanas, Suriname, Brazil, Ecuador, Peru, Bolivia, and Paraguay.

Population: M&J, pp. 295-311. Interpolations are used for 1650, 1750, 1875, and 1925. From 1950 on the population figures for individual countries from S&H are used, where available, and where not, data are from the UN yearbooks.

GDP: From 1960 on, S&H data are used, with minor exceptions. Before 1960, data from B, Table 1.7, are used, with exactly the same procedures applied in 5 and 6. These implied the growth rates given in Row 7 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality 1.1.

8. *Japan.*

Population: M&J, p.181.

GDP: Estimates before 1950 are from B, Table 1.6. S&H is used for the remaining years. Estimates from Bairoch are multiplied by 1.01, the ratio of the S&H to Bairoch estimates for 1950.

9. *China.* Includes Mongolia and Tibet, but excludes Taiwan and Hong Kong.

Population: Starting with the estimate given in M&J, p.167, the populations of Taiwan (p. 175) and Hong Kong are substracted, and the population of Mongolia (p. 164) is added. The Hong Kong population was negligible until 1900. The figures of 0.37m. in 1901, 0.71m in 1925, and 1.93m in 1950 (the last two are interpolated) from the UN demographic report on Hong Kong are subtracted for the later years. S&H provide no data on China for 1950, so the M&J estimate is used. The 1975

S&H estimate, 916m., is considerably higher than the M&J figure of 815m.

GDP: Estimates from 1960 on are from S&H. B, Table 1.7, contains estimates for earlier years, but the Bairoch estimate for 1960 is 1.41 times the S&H estimate for that year, and even the Bairoch estimate for 1800 is 1.29 times the S&H estimate for 1960! Rather than work from these numbers, I used 500 for 1950 (reflecting some of the 1950 to 1960 growth recorded in B, Table 1.7) and then 630 for *all* earlier years. See the discussion in **10**, **11**, and **12** below.

10. *East Asia.* Includes Taiwan, South and North Korea, and Hong Kong.

Population: M&J, pp. 174-178 for Taiwan and Korea. Interpolations are used for 1650, 1750, and 1875. As mentioned in section **9**, Hong Kong's population for 1900 to 1950 is taken from the UN demographic report on Hong Kong; population before 1900 is taken to be negligible. S&H are used from 1960. North Korean figures from 1960 onward are from the UN yearbooks.

GDP: Estimates for before 1950 are based on the B, Table 1.7, estimates for all of Asia. These estimates indicate no growth between 1800 and 1950. The 1800 estimate of $630 = 180 \times 3.5$ was is used for 1750 and for all years through 1950. Data from 1960 on are from S&H. Extrapolation was used for North Korea.

11. *Southeast Asia.* Includes Burma, Thailand, Indo-China (Laos, Vietnam, Khmer Republic/Cambodia), Malaysia, Singapore, Indonesia, and the Philippines. Oceania is also included: Melanesia (Papua New Guinea, Bismarcks, Solomon Islands, New Hebrides, Fiji) and Polynesia (Tonga, Samoa, Tahiti, Cook Islands).

Population: M&J, pp. 190-203 and 330-336. Interpolations are used for 1650 and 1750.

GDP: As discussed in section **10**, the B, Table 1.7 estimate of 630 is used for all of the years 1750-1950. Data from 1960 on are from S&H, with the following amendments. Appendix B of S&H estimates Vietnamese per-capita GDP to be 3.1% of that of the US in 1985. We use this ratio for all of Indo-China from 1960 to 1990.

12. *Indian Subcontinent.* Includes Pakistan, India, Bangladesh, Sri Lanka, Nepal, Sikkim (now part of India) and Bhutan.

Population: M&J, p.183. Interpolations are used for 1650 and 1750.

GDP: As discussed in section **10**, the B, Table 1.7 estimate of 630 is used for all of the years 1750-1950. Data from 1960 on are from S&H. We assume that the per capita GDP of Bhutan is the same as that of Bangladesh.

13. *United Kingdom.*

Population: M&J, pp. 43, 47.

GDP: Estimates from 1950 on are from S&H. Estimates for 1750 and 1800 are taken from the "most developed" column in B, Table 1.3, multiplied by 3.5. Growth rates from 1800 to 1950 were estimated using interpolations of the estimates for the U.K. in B, Table 1.4, which imply the annual growth rates given Row 13 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality 1.06.

14. *France.*

Population: M&J, pp.57-59.

GDP: Estimates from 1950 on are from S&H. Estimates for 1750 and 1800 are taken from the "Western Europe" column in B, Table 1.6, multiplied by 3.5. Growth rates from 1800 to 1950 were estimated using interpolations of the estimates for France in B, Table 1.4, which imply the annual growth rates given in Row 14 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality 1.052.

15. Germany. Includes West and East Germany when they were separate states.

Population: M&J, pp. 69. East Germany (including East Berlin) for 1950, 1960, and 1990 are taken from the UN yearbooks.

GDP: Estimates from 1970 on are from S&H. For 1960, the estimate for West Germany is from S&H and the estimate for East Germany is interpolated using the assumption that the 1960 ratio of West to East German GDP per capita equals the 1970 ratio (from S&H). Estimates for 1750 and 1800 are taken from the "Western Europe" column in B, Table 1.6, multiplied by 3.5. Income levels from 1800 to 1960 were estimated using interpolations of the estimates for Germany in B, Table 1.4. For 1950 I used .72 times Bairoch's estimate for West Germany plus .28 times his East Germany estimate. For 1960 I used .77 times Bairoch's estimate for West Germany plus .23 times his East Germany estimate. These are population weights from S&H. All estimates from Bairoch are multiplied by .98, the ratio of the 1960 S&H estimates to those from Bairoch.

16. *The Low Countries.* Includes Belgium, Luxemburg, and The Netherlands.

Population: M&J, pp. 62-64.

GDP: Estimates from 1950 on are from S&H. Estimates for 1750 and 1800 are taken from the "Western Europe" column in B, Table 1.6, multiplied by 3.5. Estimates for Belgium and the Netherlands in B, Table 1.4 are aggregated, using the population weights .46 and .54 from S&H for 1950. Growth rates from 1800 to 1950 were estimated using interpolations of these estimates, which imply the annual growth rates given in Row 16 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality 1.055.

17. *Scandinavia.* Includes Denmark, Sweden, Norway, and Finland.

Population: M&J, p. 51.

GDP: Estimates from 1950 on are from S&H. Estimates for 1750 and 1800 are taken from the "Western Europe" column in B, Table 1.6, multiplied by 3.5. Estimates for Denmark, Finland, Norway, and Sweden in B, Table 1.4 are aggregated, using the population weights .23, .22, .18, and .37 from S&H for 1950. Income levels from 1800 to 1950 were estimated using interpolations of these estimates. All estimates from Bairoch are multiplied by 1.04, the ratio of the 1950 S&H estimates to those from Bairoch.

18. *Rest of Western Europe.* Includes Ireland, Switzerland, Austria, Spain, Portugal, Italy, Albania, Greece, Cyprus, Malta, Iceland, Greenland, and the islands that are now part of Spain and Portugal.

Population: M&J, pp. 47, 87, 89, 105, 107, 113, and 119. Interpolations are used for 1650, 1750 and 1875. Data from 1950 on are from S&H, except for Albania (all years) and Malta (1950 and 1990), whose population figures are from the UN yearbooks.

GDP: Estimates from 1960 on are from S&H. GDP per capita in Albania was estimated at .26 times the level in Yugoslavia for the years 1960, 70, 80, and 90, based on the 1990 figures given in the World Almanac and Book of Facts. Estimates for 1750 and 1800 are taken from the "Western Europe" column in B, Table 1.6, multiplied by 3.5. Growth rates from 1800 to 1960 were estimated using interpolations of the estimates for Western Europe in B, Table 1.6, which imply the annual growth rates given in Row 18 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality .89.

19. *Former Soviet Union.*

Population: M&J estimates for Russia-in-Europe (p.79), Caucasia (p.158), Siberia (p.161), and Russian Turkestan (p.163) were added. Interpolations were used for 1650, 1750, and 1875. S&H is used for 1960, 1970, and 1980. The UN yearbooks were used for 1950 and 1990. There is a sizeable discrepancy between the 1950 estimates of M&J (181m.) and the 1951 UN yearbook (193m.). Livi-Bacci (Table E.1) reports180m. We use the M&J figure.

GDP: Estimates from 1960 on are from S&H. Estimates for 1750 and 1800 are taken from the "Eastern Europe" column in B, Table 1.6, multiplied by 3.5. The growth rates from 1800 to 1830 were taken from the estimates for Eastern Europe in B, Table 1.6. Growth rates from 1830 to 1960 were estimated using interpolations of the estimates for the Soviet Union in B, Table 1.4. These sources imply the annual growth rates given in Row 19 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality .78.

20. *Eastern Europe.* Includes Poland, Czechoslovakia, Hungary, Romania, the former Yugoslavia, and Bulgaria.

Population: M&J estimates for "present day Poland" (p.75), Czechoslovakia (p.85), Hungary (p.92), Romania (p.97), Yugoslavia (p.113) and Bulgaria (p.113) were added. Interpolations were used for Yugoslavia and Bulgaria for 1750. For 1950 and later, estimates from S&H were used, with the exceptions of Czechoslovakia (1950), Romania (1950 and 1990), Bulgaria (1950,60, and 70), and Poland (1950 and 60), which came from the UN yearbooks.

GDP: Estimates from 1960 on are from S&H, with the following exceptions. Bulgaria per capita GDP for 1960 was interpolated using the assumption that the ratio to U.S. income for these years equalled the 1980 ratio. Hungary and Poland for 1960 were interpolated using the assumption that the ratios to U.S. income for that year equalled the 1970 ratios. Estimates for 1750 and 1800 are taken from the "Eastern Europe" column in B, Table 1.6, multiplied by 3.5. Growth rates from 1800 to 1960 were estimated using interpolations of the estimates for Eastern Europe in B, Table 1.6, which imply the annual growth rates given in Row 20 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality .67.

21. *Australia and New Zealand.*

Population: M&J, pp. 329 and 339. Interpolations are used for 1650, 1700, 1750, and 1875.

GDP: Estimates from 1950 on are from S&H. Estimates for 1750 and 1800 are taken equal to the U.S. (Population is neglible for these years.) Income levels from 1800 to 1950 were estimated using interpolations of the estimates for Australia in B, Table 1.4, supplemented by U.S. income for years before 1860. These imply the annual growth rates given in Row 21 of Table A.1. My estimated growth rates are proportional to these, with the constant of proportionality 1.087.

[INSERT TABLE A.1 HERE]

Table A.1: Relative Growth Rates by Region

Region	1800-50	1850-75	1875-1900	1900-25	1925-50	1950-60	Source	Constant
1	0	0.001	0.002	0.003	0.008	0.019	Table 1.7	0.957
2	0	0.001	0.002	0.003	0.008	0.019	Table 1.7	2.1
5	0.001	0.005	0.006	0.005	0.008	0.011	Table 1.7	1.45
6	0.001	0.005	0.006	0.005	0.008	0.011	Table 1.7	2.2
7	0.001	0.005	0.006	0.005	0.008	0.011	Table 1.7	1.1
13	0.015	0.013	0.011	0.008	0.008		Table 1.4	1.06
14	0.009	0.011	0.011	0.014	0.01		Table 1.4	1.052
16	0.01	0.013	0.012	0.014	0.009		Table 1.4	1.055
18	0.009	0.01	0.012	0.01	0.009	0.038	Table 1.6	0.89
19	0.003	0.008	0.01	0.006	0.022	0.043	Table 1.4	0.78
20	0.004	0.007	0.008	0.01	0.013	0.046	Table 1.6	0.67
21	0.014	0.013	0.012	0.003	0.019		Table 1.4	1.087

Source indicates table in Bairoch (1981) on which most growth rates are based.

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