Unknown Input High Gain Observer for Fault Detection and Isolation of Uncertain Systems

Sharifuddin Mondal*, G. Chakraborty and K. Bhattacharyya

Abstract **—An unknown input high gain observer (UIHGO) based component fault detection and isolation (FDI) technique is presented. First, a reduced order UIHGO is derived for a linear system whose parameters are uncertain to some extent. The observer gain is determined by solving the well-known algebraic Riccati equation (ARE). Then, using a bank of such observers, a FDI algorithm is devised to detect and isolate the component fault (i.e., parametric fault) of an uncertain system. The FDI algorithm consists of two steps. In the first step, the detection of fault and the isolation of faulty region are accomplished and in the next step, the faulty parameter is isolated from the faulty region. Effectiveness of the proposed observer as well as the FDI technique is shown with the help of a numerical example.**

Index Terms— **Unknown input high gain observer; component fault; fault detection and isolation; uncertain system; parameter estimation.**

I. INTRODUCTION

With the rising demands of high reliability and safety of advanced processes like avionics, nuclear power stations, automobiles etc have led to increasing requirements of developing new methods of supervision and monitoring as a part of overall process control scheme. Different fault detection and isolation (FDI) schemes have been developed for avoiding failure of the plants. Model based fault detection techniques (like Kalman filter or observer based) have received increasing attention following the pioneering work of Beard [1].

The FDI concept using observers or Kalman filters is devised based on the assumption that the mathematical model of a system is perfectly known. In reality, however this assumption does not hold because the parameters of a process are in general uncertain or time varying. Again the characteristics of disturbances or noise are not completely known; hence they cannot be perfectly modeled. There is always a mismatch between the actual process model and its mathematical model (even if there is no fault in the process), which sometimes produces false alarms corrupting the performance of the FDI technique. To avoid false alarms, the FDI method should be robust i.e., insensitive to modeling uncertainties. But the algorithm should not be too robust to ignore the fault i.e., a significantly large variation of the parameter values.

Over the years, various kinds of robust fault detection and isolation techniques have been developed to diagnose different types of faults like sensors, actuators or components [2-4, 7, 9, 10, 12, 16-20]. Frank [8], in a survey paper, described different types of observer based robust fault diagnosis techniques. Patton and Chen [15] discussed various robustness issues related observer based fault diagnosis techniques. Linear matrix inequality (LMI) based robust fault detection techniques for uncertain systems have been developed in [19, 20]. The identification based FDI techniques have been used by many researchers [2, 11, 13] to detect parameter faults. Daley and Wang [5] used a high gain observer, which was developed by Petersen and Hollot [14], as a tool for sensor fault detection.

In the present work, an unknown input high gain observer (UIHGO) based component (i.e., parameter) fault detection and isolation technique is derived. First, an unknown input high gain observer is developed for a linear uncertain system. Such type of unknown input observers has wide applications in modern control systems where the uncertainties (modeling or parametric or both) are unavoidable. Next, using a bank of such observers, a parameter fault detection and isolation technique is devised for a parametrically uncertain system on the assumptions that sensors and actuators are fault free. Since the high gain observer [5, 14] is robust against parameter uncertainties to some extent, the FDI technique is also robust against the uncertainties. The FDI algorithm works in two steps. In step-1, the detection of fault and isolation of faulty zone is accomplished using a bank of UIHGOs. In the next step, faulty parameter is isolated by parameter isolation method. In the present work, a part of the system parameters (i.e., the parameters of the faulty subsystem) is estimated in step-2 and only when a fault occurs in the system. In this respect the complexity of fault isolation is drastically reduced in comparison with standard parameter identification technique [2, 11, 13] where all the parameters of a system are estimated at every time step irrespective of the occurrence of any fault and the estimated values are compared with their nominal values. A numerical example is presented to demonstrate the effectiveness of the proposed observer as well as the FDI technique.

The basic methodology of designing the unknown input high gain observer for an uncertain system is discussed in

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section II. The fault detection and isolation algorithm is explained in section III. In section IV, a numerical example is presented to demonstrate the performances of the proposed methods. The concluding remarks are included in section V.

II. UNKNOWN INPUT HIGH GAIN OBSERVER

In this section, an unknown input high gain observer is developed for a linear uncertain system. The sufficient conditions for the existence of the observer are provided.

Consider a linear time-invariant system with unknown inputs

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t) \quad (1)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t)\,. \quad (2)$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ - the state vector, $\boldsymbol{u}(t) \in \boldsymbol{R}^m$ - the measurable input vector, $\boldsymbol{y}(t) \in \boldsymbol{R}^p$ - the output vector and $\boldsymbol{d}(t) \in \boldsymbol{R}^q$ - the unknown input vector. The matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ and $\boldsymbol{E}$ of suitable dimensions are known. The matrices $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$ are the uncertainties of the system and input matrices respectively. These may be constant or time varying depending on the system. It is assumed that $(\boldsymbol{A} + \Delta\boldsymbol{A})$ is always asymptotically stable for all $\Delta\boldsymbol{A}$. If this condition is not satisfied then first a controller is to be designed to stabilize the system.

It is also assumed that the system satisfies the rank condition: $rank(\boldsymbol{CE}) = rank(\boldsymbol{E})$. This is a basic assumption for designing any unknown input observer.

Now, using a state transformation matrix $\boldsymbol{T}$, the states are redefined as $\boldsymbol{z}(t) = \boldsymbol{T}\boldsymbol{x}(t)$ such that $\boldsymbol{TE} = \begin{bmatrix} \phi_{(n-r)\times q} \\ \bar{\boldsymbol{E}}_2 \end{bmatrix}$ where $\bar{\boldsymbol{E}}_2$ is $r \times q$ dimensional matrix with $rank(\bar{\boldsymbol{E}}_2) = rank(\boldsymbol{E})$ and ϕ is a null matrix.

The system and output equations can be recast as follows

$$\begin{Bmatrix} \dot{\boldsymbol{z}}_1 \\ \dot{\boldsymbol{z}}_2 \end{Bmatrix} = \begin{bmatrix} \bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_{12} \\ \bar{\boldsymbol{A}}_{21} & \bar{\boldsymbol{A}}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{Bmatrix} + \begin{bmatrix} \Delta\bar{\boldsymbol{A}}_{11} & \Delta\bar{\boldsymbol{A}}_{12} \\ \Delta\bar{\boldsymbol{A}}_{21} & \Delta\bar{\boldsymbol{A}}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{Bmatrix} + \begin{bmatrix} \bar{\boldsymbol{B}}_1 \\ \bar{\boldsymbol{B}}_2 \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} \Delta\bar{\boldsymbol{B}}_1 \\ \Delta\bar{\boldsymbol{B}}_2 \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} \phi_{(n-r)\times q} \\ \bar{\boldsymbol{E}}_2 \end{bmatrix} \boldsymbol{d} \quad (3)$$

$$\boldsymbol{y}(t) = \begin{bmatrix} \bar{\boldsymbol{C}}_1 & \bar{\boldsymbol{C}}_2 \end{bmatrix} \begin{Bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{Bmatrix}. \quad (4)$$

Now it is assumed that the measurement signals are such that the following rank condition is satisfied: $rank(\boldsymbol{C}) > rank(\boldsymbol{E})$. This is a necessary condition for designing this observer as the extra measurement signals are used to design the reduced order observer after decoupling the unknown inputs.

This condition allows the rearrangement of the output equation in the following form with the help of a transformation $\begin{bmatrix} \bar{\boldsymbol{y}}_1 \\ \bar{\boldsymbol{y}}_2 \end{bmatrix} = \boldsymbol{V}\boldsymbol{y}$, where $\boldsymbol{V}$ is a nonsingular matrix, as

$$\begin{Bmatrix} \bar{\boldsymbol{y}}_1 \\ \bar{\boldsymbol{y}}_2 \end{Bmatrix} = \begin{bmatrix} \bar{\boldsymbol{C}}_{11} & \phi \\ \bar{\boldsymbol{C}}_{21} & \bar{\boldsymbol{C}}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{Bmatrix}. \quad (5)$$

Now the equations (3) and (5) can be written in expanded form as follows

$$\dot{\boldsymbol{z}}_1 = \bar{\boldsymbol{A}}_{11}\boldsymbol{z}_1 + \bar{\boldsymbol{A}}_{12}\boldsymbol{z}_2 + \Delta\bar{\boldsymbol{A}}_{11}\boldsymbol{z}_1 + \Delta\bar{\boldsymbol{A}}_{12}\boldsymbol{z}_2 + \bar{\boldsymbol{B}}_1\boldsymbol{u} + \Delta\bar{\boldsymbol{B}}_1\boldsymbol{u} \quad (6)$$

$$\dot{\boldsymbol{z}}_2 = \bar{\boldsymbol{A}}_{21}\boldsymbol{z}_1 + \bar{\boldsymbol{A}}_{22}\boldsymbol{z}_2 + \Delta\bar{\boldsymbol{A}}_{21}\boldsymbol{z}_1 + \boldsymbol{A}_{22}\boldsymbol{z}_2 + \bar{\boldsymbol{B}}_2\boldsymbol{u} + \Delta\bar{\boldsymbol{B}}_2\boldsymbol{u} + \bar{\boldsymbol{E}}_2\boldsymbol{d} \quad (7)$$

$$\bar{\boldsymbol{y}}_1 = \bar{\boldsymbol{C}}_{11}\boldsymbol{z}_1 \quad (8)$$

$$\bar{\boldsymbol{y}}_2 = \bar{\boldsymbol{C}}_{21}\boldsymbol{z}_1 + \bar{\boldsymbol{C}}_{22}\boldsymbol{z}_2\,. \quad (9)$$

Eliminating $\boldsymbol{z}_2$ from the equation (6) using the equation (9), one gets

$$\dot{\boldsymbol{z}}_1 = \bar{\boldsymbol{A}}_{11}\boldsymbol{z}_1 + \bar{\boldsymbol{A}}_{12}\bar{\boldsymbol{C}}_{22}^{-1}(\bar{\boldsymbol{y}}_2 - \bar{\boldsymbol{C}}_{21}\boldsymbol{z}_1) + \bar{\boldsymbol{B}}_1\boldsymbol{u} + \boldsymbol{E}_u\boldsymbol{d}_u\,. \quad (10)$$

where $\boldsymbol{E}_u\boldsymbol{d}_u = \Delta\bar{\boldsymbol{A}}_{11}\boldsymbol{z}_1 + \Delta\bar{\boldsymbol{A}}_{12}\boldsymbol{z}_2 + \Delta\bar{\boldsymbol{B}}_1\boldsymbol{u}$ with $\boldsymbol{E}_u$- known matrix and $\boldsymbol{d}_u$ - unknown signal.

It can be seen that $\bar{\boldsymbol{C}}_{22}^{-1}$ should be full rank matrix, which will be always so as $rank(\boldsymbol{CE}) = rank(\boldsymbol{E})$.

Now, the equation (10) can be re-written in simplified form as

$$\dot{\boldsymbol{z}}_1 = \bar{\boldsymbol{A}}_s\boldsymbol{z}_1 + \bar{\boldsymbol{B}}_s\bar{\boldsymbol{u}} + \boldsymbol{E}_u\boldsymbol{d}_u\,, \quad (11)$$

where $\bar{\boldsymbol{A}}_s = \bar{\boldsymbol{A}}_{11} - \bar{\boldsymbol{A}}_{12}\bar{\boldsymbol{C}}_{22}^{-1}\bar{\boldsymbol{C}}_{21}$, $\bar{\boldsymbol{B}}_s = \begin{bmatrix} \bar{\boldsymbol{B}}_1 & \bar{\boldsymbol{A}}_{12}\bar{\boldsymbol{C}}_{22}^{-1} \end{bmatrix}$ and $\bar{\boldsymbol{u}} = \begin{Bmatrix} \boldsymbol{u} \\ \bar{\boldsymbol{y}}_2 \end{Bmatrix}$.

For designing an observer, the system should satisfy the observability condition: $rank(O(\boldsymbol{A}, \boldsymbol{C})) = n$.

Now one can design an observer for the systems (11) and (8) to estimate the state $\hat{\boldsymbol{z}}_1$ as

$$\dot{\hat{\boldsymbol{z}}}_1 = \bar{\boldsymbol{A}}_s\hat{\boldsymbol{z}}_1 + \bar{\boldsymbol{B}}_s\bar{\boldsymbol{u}} + \boldsymbol{K}(\bar{\boldsymbol{y}}_1 - \hat{\bar{\boldsymbol{y}}}_1) \quad (12)$$

$$\hat{\bar{\boldsymbol{y}}}_1 = \bar{\boldsymbol{C}}_{11}\hat{\boldsymbol{z}}_1\,. \quad (13)$$

The observer gain matrix $\boldsymbol{K}$ is determined by solving the following algebraic Riccati equation (ARE) [5, 14]

$$\bar{\boldsymbol{A}}_s\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}}_s^T + \boldsymbol{Q} + \frac{q^2\boldsymbol{E}_u\boldsymbol{E}_u^T}{\sigma} - \boldsymbol{P}\bar{\boldsymbol{C}}_{11}^T\bar{\boldsymbol{C}}_{11}\boldsymbol{P} + \frac{\boldsymbol{P}\boldsymbol{E}_u\boldsymbol{E}_u^T\boldsymbol{P}}{q^2\sigma} = 0 \quad (14)$$

with $\boldsymbol{K} = \boldsymbol{P}\bar{\boldsymbol{C}}_{11}^T$, (15)

where $\boldsymbol{Q}$ is a pre-chosen positive definite matrix and the constants q & σ are specified numbers. It was shown in [14] that for any $\sigma > 0$, there exists q such that gain obtained from the above equations will lead to $\| \bar{\boldsymbol{C}}_{11}(jw\boldsymbol{I} - \bar{\boldsymbol{A}}_s + \boldsymbol{K}\bar{\boldsymbol{C}}_{11})\boldsymbol{E}_u \| < \sigma$ for $\forall\, w \in \boldsymbol{R}^1$ where w is the frequency. This condition implies that the effect of unknown signal $\boldsymbol{d}_u$ becomes very small in error dynamics for an appropriate value of σ.

Now the state $\hat{\boldsymbol{z}}_2$ is estimated from equation (9) using the estimated state $\hat{\boldsymbol{z}}_1$ as

$$\hat{\boldsymbol{z}}_2 = \bar{\boldsymbol{C}}_{22}^{-1}(\bar{\boldsymbol{y}}_2 - \bar{\boldsymbol{C}}_{21}\hat{\boldsymbol{z}}_1)\,. \quad (16)$$

Finally using $\hat{\boldsymbol{z}} = \left\{ \hat{\boldsymbol{z}}_1^T \quad \hat{\boldsymbol{z}}_2^T \right\}^T$, the estimated states $\hat{\boldsymbol{x}}$ are found out as $\hat{\boldsymbol{x}} = \boldsymbol{T}^{-1}\hat{\boldsymbol{z}}$. With this, observer design process completes.

III. FAULT DETECTION AND ISOLATION ALGORITHM

In this section, a component fault detection and isolation technique for a linear uncertain system is derived. It consists of two steps. In the first step, a set of residuals is generated with the help of a bank of unknown input high gain observers (UIHGOs) to detect the fault and isolate the faulty region. In the second step, faulty parameter is isolated from the faulty region.

Consider a linear time invariant system as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) \tag{17}$$

where the significance of the matrices and vectors are same as described in section II.

Suppose a fault occurs in a component of the plant. The detection and isolation of the fault are carried out in two steps as follows.

Step-1: Detection and partial isolation of fault

The faulty system is written as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A}_f + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B}_f + \Delta\boldsymbol{B})\boldsymbol{u}(t)\,, \tag{18}$$

where $\Delta\boldsymbol{A}_f$ and $\Delta\boldsymbol{B}_f$ are the faulty parts of the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively. It can be emphasized that the magnitude of faults (i.e., $\Delta\boldsymbol{A}_f$ and $\Delta\boldsymbol{B}_f$) should be significantly larger compared to the magnitude of uncertainties (i.e., $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$). The state equation (18) can now be rearranged as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t)\,, \tag{19}$$

where $\boldsymbol{E}$ is a known matrix and $\boldsymbol{d}(t) \in \boldsymbol{R}^q$ is the unknown input satisfying the relation

$$\boldsymbol{E}\boldsymbol{d}(t) = \Delta\boldsymbol{A}_f\boldsymbol{x}(t) + \Delta\boldsymbol{B}_f\boldsymbol{u}(t)\,. \tag{20}$$

Now the system is divided into N numbers of subsystems with each characterized by a few parameters. The choice of subsystems is quite arbitrary but there should not have any common elements between them. In a physical system, the subsystems are chosen based on the physical proximity of different parameters. Assume that the fault has been occurred in the *i*-th subsystem.

The system equation considering the fault in the *i*-th subsystem is written as

$$\dot{\boldsymbol{x}}_{(i)}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}_{(i)}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}_{(i)}\boldsymbol{d}_{(i)}(t) \tag{21}$$

where the subscript (*i*) indicates that the fault has been considered in the *i*-th subsystem.

The output equation for this system is written as

$$\boldsymbol{y}_{(i)}(t) = \boldsymbol{C}_{(i)}\boldsymbol{x}_{(i)}(t)\,, \tag{22}$$

The equations (21) and (22) are similar to the equations (1) and (2). Now, following the procedure discussed in section II, an unknown input high gain observer is designed to estimate the states $\hat{\boldsymbol{x}}_{(i)}(t)$.

Once the states $\hat{\boldsymbol{x}}_{(i)}(t)$ are estimated, the residuals are calculated as

$$\boldsymbol{r}_{(i)}(t) = \boldsymbol{y}_{(i)}(t) - \hat{\boldsymbol{y}}_{(i)}(t) = \boldsymbol{y}_{(i)}(t) - \boldsymbol{C}_{(i)}\hat{\boldsymbol{x}}_{(i)}(t)\,. \tag{23}$$

Now, an unknown input observer, if properly designed, can estimate the states irrespective of unknown inputs. So the residual $\boldsymbol{r}_{(i)}(t)$, calculated from the equation (23), converges within a bounded value known as threshold value if the fault occurs in the *i*-th subsystem or there is no fault in the system as the effect of possible faults in *i*-th subsystem is considered as unknown inputs. In ideal case, i.e., in the absence of noise and parameter uncertainties, the residuals should converge to zero (though a small threshold value is always set to take care of errors due to the numerical limitations) whereas in the present case the convergence takes place within a threshold value, which again depends on the amount of uncertainties and input signal applied to the system. In this way, one can detect a fault and isolate the faulty subsystem using N numbers of residuals calculated with the help of that numbers of UIHGOs. However $(N-1)$ such observers will be sufficient to isolate a faulty subsystem when $N > 2$ because once $(N-1)$ subsystems are found fault free, the remaining subsystem is automatically identified as the faulty one. A decision table (as shown in table 1) is drawn to isolate the faulty subsystem from observation of $(N-1)$ residuals.

Table 1: Decision table for isolation of faulty subsystem

Observation cases	For given residuals ($\boldsymbol{r}_{(i)}$) and threshold values ($\varepsilon_{(i)}$): if $\boldsymbol{r}_{(i)} > \varepsilon_{(i)}$ then use '1' and if $\boldsymbol{r}_{(i)} \le \varepsilon_{(i)}$ then use '0'.					Decision
	$\boldsymbol{r}_{(1)}$	$\boldsymbol{r}_{(1)}$	$\boldsymbol{r}_{(1)}$	……..	$\boldsymbol{r}_{(N-1)}$	
Case-0	0	0	0	….....	0	No fault
Case-1	0	1	1	….....	1	Fault: SS1
Case-2	1	0	1	……..	1	Fault: SS2
Case-3	1	1	0	……..	1	Fault: SS3
. . . .	. . . .	. . . .	. . . .	…….. …….. …….. ……..	. . . .	. . . .
Case-(*N*-1)	1	1	1	……..	0	Fault: SS (*N*-1)
Case-*N*	1	1	1	….....	1	Fault: SS (*N*)

Step-2: Total isolation of fault

Once the faulty subsystem is isolated, the faulty parameter in the faulty subsystem is identified in this step. First, the effect of the faulty subsystem is simulated as an unknown input signal, say $F_u(t)$. The relationship between $F_u(t)$, the parameters of the faulty subsystem, say s, and the states $\boldsymbol{x}(t)$ are known and can be written as

$$F_u(t) = f(s, \boldsymbol{x}(t))\,, \tag{24}$$

where the function 'f' is linear for a linear system.

The system equation for this case becomes

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t)\,, \tag{25}$$

where $\boldsymbol{d}(t) = F_u(t)$. With a measurement matrix $\boldsymbol{C}$, an observer is then designed to estimate the states. Knowing the states, the unknown input signal is estimated from the state equations neglecting the uncertainties using the nominal values of the parameters of the other non-faulty subsystems.

The estimated signal $\hat{F}_u(t)$ is now used to estimate the parameters s's from the relation (24), which is rewritten as

$$\hat{F}_u(t) = f(s, \hat{\boldsymbol{x}}(t)). \tag{26}$$

All the elements of equation (26) excepting the parameters s are known. Different parameter estimation techniques can be used to estimate s from equation (26). However, a very simple logical approach is applied in the present work in order to isolate the faulty element.

Let us consider the *k*-th parameter s_k as the faulty one. From the above relation, s_k can be estimated using nominal values of rest of the parameters. Mathematically,

$$\hat{s}_k = g(s_1, s_2, \ldots, s_{k-1}, s_{k+1}, \ldots s_l, \hat{\boldsymbol{x}}(t), \hat{F}_u(t)) \tag{27}$$

where l is the number of parameters of the faulty subsystem and *g* is a functional.

It is observed that in steady state, the estimated values fluctuate very less if the assumption is correct. The moving averages technique can be used to smoothen the fluctuation of the estimated values due to uncertainties. If the assumption is wrong, the estimated values vary significantly large. Now, as the single fault case is being considered, there will be only one case when the estimated parameter will vary less. The particular parameter for which it happens is the faulty one. In this way, the faulty parameter is isolated. In the same way, any parametric fault of any subsystem can be detected and isolated following the above two steps. The FDI technique can be summarized in a block diagram as shown figure 1.

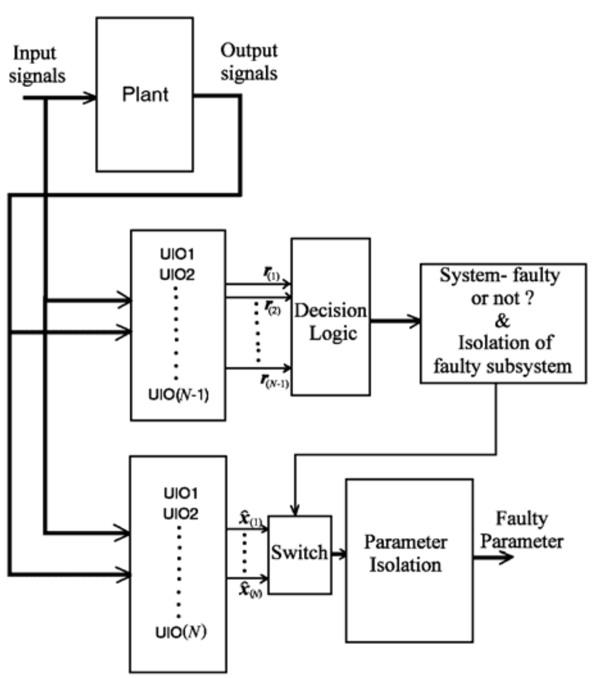

Figure 1: Structure of the FDI algorithm

IV. NUMERICAL EXAMPLE

Consider a mechanical system (as shown in figure 2) that consists of two mass elements and three sets of springs and dampers. The state space model of the system can be written as follows:

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t),$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{(K_1+K_2)}{M_1} & \dfrac{K_2}{M_1} & -\dfrac{(C_1+C_2)}{M_1} & \dfrac{C_2}{M_1} \\ \dfrac{K_2}{M_3} & -\dfrac{(K_2+K_3)}{M_3} & \dfrac{C_2}{M_3} & -\dfrac{(C_2+C_3)}{M_3} \end{bmatrix},$$

$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 & (1/M_3) \end{bmatrix}^T$, $\boldsymbol{x} = \{X_1 \quad X_2 \quad \dot{X}_1 \quad \dot{X}_2\}^T$, $\boldsymbol{u}(t) = F(t)$, where X_i and $\dot{X}_i$ are the displacement and velocity of the mass element M_i respectively, K_j - the stiffness element and C_j - the damping coefficient (*i*=1,2 and *j*=1, 2, 3). The matrices $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$ are the uncertainties of the system and input matrices respectively.

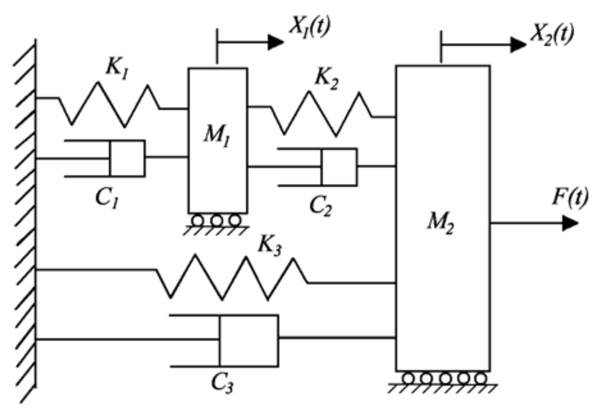

Figure 2: Mechanical system having two masses and three sets of spring-damper

The numerical values of the system parameters are $M_1 = 870\,\mathrm{kg}$, $M_3 = 1550\ \mathrm{kg}$, $K_1 = 280000\ \mathrm{N/m}$, $K_2 = 370000\ \mathrm{N/m}$, $K_3 = 340000\ \mathrm{N/m}$, $C_1 = 3500\ \mathrm{Ns/m}$, $C_2 = 3000\ \mathrm{Ns/m}$ and $C_3 = 5675\ \mathrm{Ns/m}$.

The system response with arbitrary initial conditions is simulated using MATLAB-SIMULINK toolbox. The parametric uncertainties are simulated in such a way that the elements of the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ differ maximum of ± 5 % from their nominal values. A fault is now introduced in the spring of stiffness K_2 at $t = 50\,\mathrm{sec}$. The new value of K_2 is set to 185000 N/m. Now using the FDI algorithm, discussed in section III, the fault is detected and the faulty element (here K_2) is isolated as follows.

Step-1: Detection and partial isolation of the fault

First, the system is divided into three subsystems as follows: SS1: K_1, C_1 & M_1; SS2: K_2 & C_2 and SS3: K_3, C_3 & M_3.

The uncertainties are introduced as follows: $\Delta\boldsymbol{A} = \boldsymbol{M}\Sigma\boldsymbol{N}_1$ and $\Delta\boldsymbol{B} = \boldsymbol{M}\Sigma\boldsymbol{N}_2$ with $\boldsymbol{M} = \boldsymbol{I}_n$, $\boldsymbol{N}_1 = 0.05\times\boldsymbol{A}_{\boldsymbol{u}}$, $\boldsymbol{N}_2 = 0.05\times\boldsymbol{B}_{\boldsymbol{u}}$ and $\Sigma = \Sigma_0\sin(w_1 t)$ where $\Sigma_0 = 0.25\times\boldsymbol{I}$ and $w_1 = 0.05$ rad/s. The matrices $\boldsymbol{A}_{\boldsymbol{u}}$ and $\boldsymbol{B}_{\boldsymbol{u}}$ are same as $\boldsymbol{A}$ and $\boldsymbol{B}$ excepting the elements containing constant terms are replaced with zeros. The sinusoidal variation in system parameters is introduced in simulation. The following input signal is applied for this case: $\boldsymbol{u} = u_0\sin(wt)$ with $u_0 = 100$ N and $w = 1$ rad/s.

As the system is divided into three subsystems, so two UIHGOs are sufficient as a part of step-1. The observers are designed for SS1 and SS3. The unknown input matrices $\boldsymbol{E}$'s and unknown input signals $\boldsymbol{d}$'s for those observers are given below

$\boldsymbol{E}_{(1)} = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}^T$, $\boldsymbol{d}_{(1)} = \Delta\boldsymbol{A}_{(1)}\boldsymbol{x}_{(1)} + \Delta\boldsymbol{B}_{(1)}\boldsymbol{u}_{(1)}$

$\boldsymbol{E}_{(3)} = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T$, $\boldsymbol{d}_{(3)} = \Delta\boldsymbol{A}_{(3)}\boldsymbol{x}_{(3)} + \Delta\boldsymbol{B}_{(3)}\boldsymbol{u}_{(3)}$.

The output matrices are

$$\boldsymbol{C}_{(1)} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \text{ and } \boldsymbol{C}_{(3)} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\end{bmatrix}.$$

Now applying the method discussed in section-II, two $\boldsymbol{E}_u$'s are found out as $\boldsymbol{E}_{u(1)} = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$ and $\boldsymbol{E}_{u(3)} = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$. The values of the tuning parameters σ and q are considered as $\sigma_{(1)} = 0.05$, $\sigma_{(3)} = 0.05$, $q_{(1)} = 15$ and $q_{(3)} = 15$. The value of $\boldsymbol{Q}$ is chosen as $\boldsymbol{Q} = 5\boldsymbol{I}_{n_1}$ for both the observers.

The gain matrices are calculated by solving the equations (14) and (15). The values of the observer gains for the above observers are $\boldsymbol{K}_{(1)} = \begin{bmatrix}0.7253 & 4.7760 & 14.8934\end{bmatrix}^T$ and $\boldsymbol{K}_{(3)} = \begin{bmatrix}0.4368 & 2.9387 & -13.1569\end{bmatrix}^T$ respectively. Two high gain observers are then designed for the above systems. Finally the residuals are calculated and plotted in figure 3 and figure 4.

In ideal situation (i.e., fault free and in the absence of parameter uncertainties), the residuals should be zero. However in the present case these will not be zero due to presence of parameter uncertainties. Hence two small constant threshold values $\varepsilon_{(1)} = \left\{1.5\times10^{-5} \quad 1\times10^{-20}\right\}^T$ and $\varepsilon_{(3)} = \left\{3\times10^{-9} \quad 3\times10^{-20}\right\}^T$ units are chosen as the simulation is carried out applying fixed input signals. In real situation, adaptive threshold values [15] should be chosen as input signals vary depending on operating conditions. Here the threshold values are calculated in normal operating condition i.e., when there is no fault in the system.

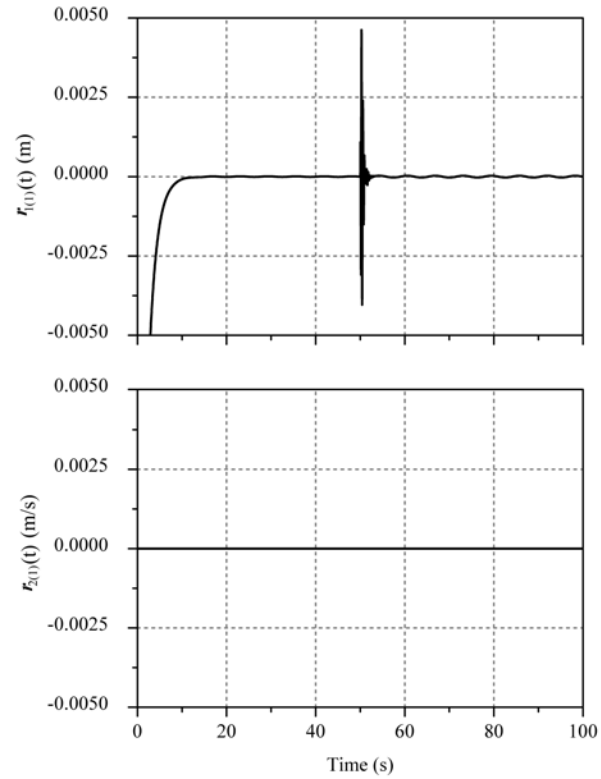

Figure 3: Components of the residual $\boldsymbol{r}_{(1)}(t)$

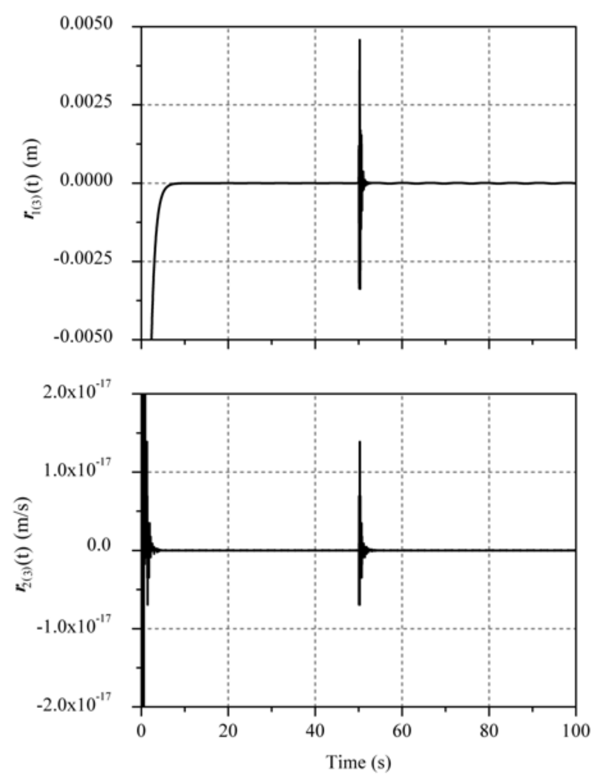

Figure 4: Components of the residual $\boldsymbol{r}_{(3)}(t)$

As both the residuals cross the threshold values, the existence of a fault is confirmed. To isolate the faulty subsystem a decision table (table 2) is constructed as shown below.

Table 2: Decision table for isolation of faulty subsystem

Observation	Is $\boldsymbol{r}_{(i)} > \boldsymbol{\varepsilon}_{(i)}$ (use '1') or $\boldsymbol{r}_{(i)} \le \boldsymbol{\varepsilon}_{(i)}$ (use '0') ?		Decision
	$\boldsymbol{r}_{(1)}$	$\boldsymbol{r}_{(3)}$	
Case	1	1	Fault: SS2.

From table-2, it is seen that the fault is in subsystem 2. So the next step (i.e., step-2) is carried out to isolate the faulty parameter.

Step-2: Total isolation of the fault

Here the faulty subsystem (SS2) is first replaced with an unknown force $F_u(t)$ as

$$F_u(t) = K_2(X_2 - X_1) + C_2(\dot{X}_2 - \dot{X}_1).$$

Then the system is remodeled as follows

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t)$$

where $\boldsymbol{E} = \begin{bmatrix} 0 & 0 & 1/M_1 & -1/M_3 \end{bmatrix}^T$ and $\boldsymbol{d}(t) = F_u(t)$. It can be noticed that the parameter uncertainties for this system are only in 3rd and 4th rows of $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices, which indicate $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$ have non-zero elements in 3rd and 4th rows only. Again the matrix $\boldsymbol{E}$ contains non-zero elements in the same rows. For this similarity here the system equation is remodeled combining the unknown inputs and uncertainties as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}_c\boldsymbol{d}_c(t)$$

where $\boldsymbol{E}_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\boldsymbol{d}_c = \begin{Bmatrix} \Delta\boldsymbol{A}_{3,1-4}\boldsymbol{x} + \Delta\boldsymbol{B}_{3,1}\boldsymbol{u} + F_u / M_1 \\ \Delta\boldsymbol{A}_{4,1-4}\boldsymbol{x} + \Delta\boldsymbol{B}_{4,1}\boldsymbol{u} - F_u / M_3 \end{Bmatrix}$.

This is a special case, which may not appear for all systems. Now a full order unknown input observer [6] is designed with output matrix $\boldsymbol{C} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ to estimate the states $\hat{\boldsymbol{x}}(t)$. Using the estimated states $\hat{\boldsymbol{x}}(t)$ and the nominal values of the parameters of subsystem-1, the unknown force $F_u(t)$ is estimated from the following relationship

$$\hat{F}_u(t) = M_1\dot{\hat{\boldsymbol{x}}}_3 + K_1\hat{\boldsymbol{x}}_1 + C_1\hat{\boldsymbol{x}}_3,$$

where $\dot{\hat{\boldsymbol{x}}}_3$ is calculated taking the derivative of $\hat{\boldsymbol{x}}_3$ with respect to time.

Finally the faulty parameters are estimated using the following relation

$$\hat{F}_u(t) = K_2(\hat{\boldsymbol{x}}_2 - \hat{\boldsymbol{x}}_1) + C_2(\hat{\boldsymbol{x}}_4 - \hat{\boldsymbol{x}}_3).$$

First, the fault is assumed to reside in the stiffness element K_2 and using the nominal value of C_2=3000 Ns/m, K_2 is estimated. The moving averages are taken to reduce the effect of uncertainties and numerical errors to the estimated values. The initial moving average is taken with a time window of 15 (5-20 seconds of data) sec. Initial data are not taken to reduce the transition effect that comes due to initial conditions. Now with an increment of 1 sec, the moving averages are calculated upto 320 sec of time span and the estimated values are plotted in figure 5. The plot shows that estimated values vary very less (maximum variation of 1.5 % from its mean value). Now C_2 is assumed to be faulty and using the nominal value of K_2=370000 N/m, C_2 is estimated. The estimated values after taking moving averages in similar manner as in case of K_2 are plotted in figure 6. The plot shows that the estimated values of C_2 vary widely (as high as 175 % from its mean value), which is because of the wrong assumption. This

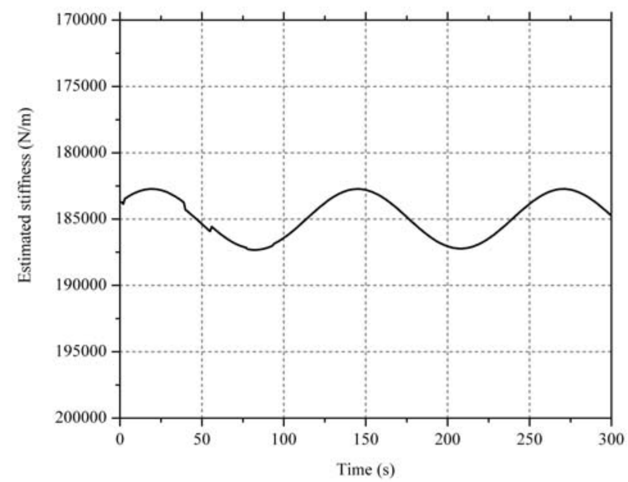

Figure 5: Estimated stiffness

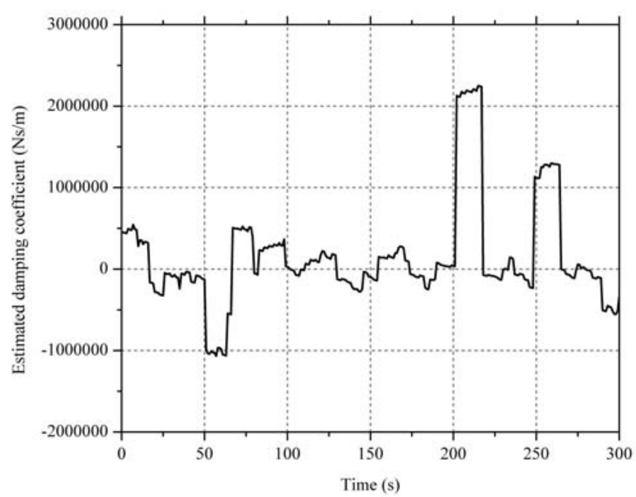

Figure 6: Estimated damping coefficient

confirms that faulty element is K_2 and thereby the fault isolation process completes.

Thus it is seen that the FDI scheme works well for the occurrence of a fault in subsystem 2. It can be shown easily that the method works with equal ease for the occurrence of any parameter fault in any other subsystem.

V. CONCLUSIONS

An unknown input high gain observer (UIHGO) based component fault detection and isolation (FDI) scheme is presented. First an UIHGO for a linear uncertain system is derived. These types of observers have wide applications in robust control and fault diagnosis. Then, using a bank of such observers, a FDI technique is devised. The advantage of the FDI algorithm is that it is capable of estimating faults even if the parameters are coupled in the system matrix. It also reduces the complexity of estimating all the parameters at every time instant unlike existing identification based parameter fault diagnosis techniques. The same FDI technique can also be used to detect a fault in a noisy system or a nonlinear system provided other types of unknown input estimators capable of handling noise or nonlinearity should be used.

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