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Study on Hemispherical Soft-Fingered Handling for Fine Manipulation by Minimum D.O.F. Robotic Hand

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***Abstract*— This paper provides that an elastic force and an elastic potential energy due to the deformation of soft fingers, which are previously derived, can be calculated into straightforward equations in an analytical way. These formulae lead to the fact that the potential energy of a soft fingertip is a function of two variables, and has a local minimum through the elastic rolling of a contacting object. We formulate four geometric constraints in grasping motions of a rigid object by means of two degrees of freedom robotic hand that has two rotational joints. We define a basic motion including translational and rolling motions when two fingers rotate by infinitesimal angle, and propose a quasi-static manipulation and its algorithm by using the local minimum of elastic potential energy (LMEE) of soft fingers with geometric constraints. In this theory, we define an energy function included in the LMEE algorithm. By solving that function we simulate the path of the center of gravity and the change of orientation of the grasped object, and compare those values with measurements experimentally obtained from a CCD camera equipped above the manipulated object. Finally, we confirm the effectiveness of the quasi-static manipulation theory based on the LMEE algorithm from experiments.**

***Index Terms*— Soft fingertip, Manipulation, Grasping, Elastic force, Local minimum, Rolling constraint.**

I. Introduction

Recently several researches associated with the grasping using a soft fingertip have been studied. Xydas and Kao *et al.* [1]–[3] have shown an exact deformation shape of a hemispherical soft fingertip by using FEM analysis. These above studies, however, have focused only on deriving the more exact deformation model, and have proposed only the vertical contact deformation of the soft fingertip. In addition, that paper have not mentioned any equations of motion of the grasped object, which are required for the analysis of manipulation stability. Nguyen *et al.* [4] have proposed a simple deformation model of a soft fingertip in order to use analytical mechanics theory in control. The deformation model, however, assumes that all the elastic forces acting on the soft fingertip face toward the origin of the fingertip. Therefore, it is difficult to represent the contact model of the fingertip and equations of motion of the object relating to the plane-contact. Arimoto *et al.* [5] and Doulgeri *et al.* [6], [7] have proposed a control law for pinching motion of a grasped object using soft fingertips based on the concept of stability on a manifold. Unfortunately, the proposed control law needs a superfluous control input to control an elastic rolling motion of the object due to the lack of a concept of local minimum of elastic potential energy (LMEE) caused by deformation of the soft fingertip.

In this paper, we shortly redescribe our static elastic model and its elastic potential energy equation in straightforward forms, and newly propose a quasi-static manipulation theory using an LMEE numerical algorithm detailed in this paper. Finally, we demonstrate the effectiveness of our manipulation theory based on the LMEE algorithm by performing handling experiments of a grasped object.

II. Local Minimum of Elastic Potential Energy (LMEE)

We have previously proposed the static elastic model and its elastic potential energy equation on a hemispherical soft fingertip which occur in the case of large deformation of the fingertip. We have numerically described a local minimum of elastic potential energy (LMEE) which arises in that deformation. These equations, however, remain double integration forms which are difficult to intuitively understand the existence of the LMEE. In this section, we show these physical quantities in straightforward forms as follows:

$$||\mathbf{F}|| = \frac{\pi E d^2}{\cos\theta_p}, \tag{1}$$

$$P = \frac{\pi E d^3}{3\cos^2\theta_p}, \tag{2}$$

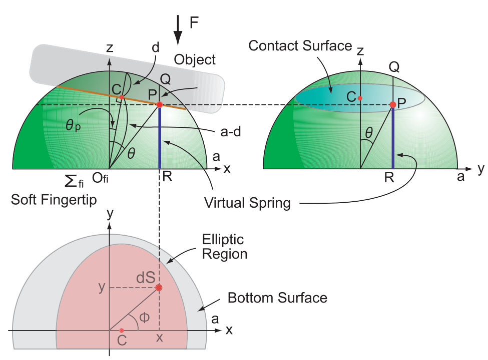


Fig. 1. Elastic force and potential energy models

where E means the Young's modulus of a soft fingertip material, and d and θ_p denote the maximum displacement of the soft fingertip and the orientation angle of a contacting object respectively, as shown in Fig.1. The derivation process of eq.(2) is detailed in Appendix.

Eqs.(1) and (2) indicate that the total elastic force is proportional to the square of maximum displacement and in inverse proportion to the cosine of orientation angle of the object. Also, the elastic potential energy is proportional to the cube of maximum displacement and in inverse proportion to the square of cosine of the object angle. Note that both equations have a local minimum when the orientation angle takes zero.

III. Geometric Constraints

In this paper, we propose a new manipulation theory based on LMEE algorithm by considering the handling motion of a grasped object in a quasi-static condition. First, we categorize geometric constraints into two directions in terms of contacting surfaces: vertical and tangential directions, and formulate these constraints in the following.

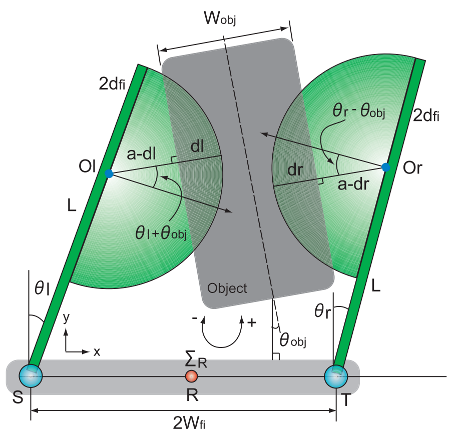
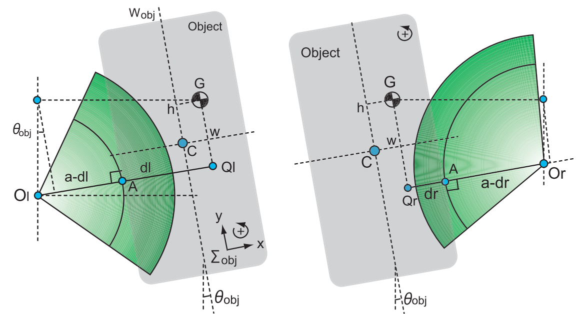


Fig. 2. Soft-fingered manipulation

A. Normal Constraint

As illustrated in Fig.2, we consider a soft-fingered handling by means of minimum D.O.F. robotic hand that has two rotational joints. We assume that the robotic hand grasps a planar rigid object in two-dimentional plane, and ignore the gravitational force in this system.

Let a be the radius, d_r and d_l be the right and left maximum displacements of soft fingertips, L be the length of fingers, and W_{obj} be the object width of a grasped object, respectively. Let $2W_{fi}$ be the width between foots of both fingers, and $2d_{fi}$ be the thickness of that finger. Furthermore, let θ_r and θ_l be the rotational angle of both fingers, O_r and O_l be the fingertip origin, and R be the origin of reference coordinate system Σ_R. Letting θ_{obj} be the orientation angle of the object with respect to Σ_R, Each coordinate of points O_r and O_l is described with respect to Σ_R as follows:



(a) Left fingertip (b) Right fingertip

Fig. 3. Normal constraints

$$\mathbf{O}_r = \begin{bmatrix} W_{fi} - L\sin\theta_r - d_{fi}\cos\theta_r \\ L\cos\theta_r - d_{fi}\sin\theta_r \end{bmatrix}, \tag{3}$$

$$\mathbf{O}_l = \begin{bmatrix} -W_{fi} + L\sin\theta_l + d_{fi}\cos\theta_l \\ L\cos\theta_l - d_{fi}\sin\theta_l \end{bmatrix}, \tag{4}$$

where θ_{obj} takes positive value in counter-clock-wise direction. As shown in Fig.3-(a), let Σ_{obj} be the object coordinate system having the origin C, and $G(x_{obj}, y_{obj})$ be the center of gravity of the grasped object with respect to Σ_R, whose coordinate equals to (w, h) in Σ_{obj}. Letting A be the center of contacting circular area, and Q_l be the foot of a perpendicular droped on a line O_lA, the length of O_lQ_l can be represented as follows:

$$O_lQ_l = (x_{obj} - O_{lx})\cos\theta_{obj} + (y_{obj} - O_{ly})\sin\theta_{obj}. \tag{5}$$

Thus, we derive a geometric normal constraint in the left fingertip when the plane contact occurs with the maximum displacement d_l.

$$(x_{obj} - O_{lx})\cos\theta_{obj} + (y_{obj} - O_{ly})\sin\theta_{obj} = (a - d_l) + \frac{W_{obj}}{2} + w. \tag{6}$$

As well as the left fingertip relation, we obtain another equation as illustrated in Fig.3-(b).

$$-(x_{obj} - O_{rx})\cos\theta_{obj} - (y_{obj} - O_{ry})\sin\theta_{obj} = (a - d_r) + \frac{W_{obj}}{2} - w, \tag{7}$$

where (O_{rx}, O_{ry}) and (O_{lx}, O_{ly}) denote xy-coordinates in eqs.(3) and (4).

B. Rolling Constraint

In order to formulate rolling constraints in terms of the tangential direction on both fingertips, we assume that each fingertip does not slip along the surface of the grasped object. As illustrated in Fig.4-(a), let

$\theta_{obj}^{n}, \theta_{obj}^{n-1}, \cdots, \theta_{obj}^{k}, \cdots, \theta_{obj}^{1}, \theta_{obj}^{0}$ be the object orientation when the object iterates n-times rolling motions on the fingertip, and let us use θ_{obj} instead of θ_{obj}^{n} as a matter of convenience in the following. Let us draw a straight line passing points O_l and A, let P be the foot of a perpendicular dropped on the center line passing point C from the center of gravity G, Q_l be the intersection on that perpendicular, and each point is defined as $P(w,0)$ and $Q_l(w,s_l)$ with respect to the coordinate system Σ_{obj}. Here, s_l is a relative position with respect to the tangential direction between point C and the fingertip, as shown in Fig.4-(a). That is, s_l corresponds to a physical quantity including the past history of object rolling distance. Consequently, when the object rolls on the fingertip with radius $a-d_l$, the length GQ_l is updated by adding the rolling distance AA' as follows:

$$GQ_l = h - s_l - (a-d_l)\cdot(\theta_{obj} - \theta_{obj}^{n-1}), \tag{8}$$

where the counter-clock-wise direction associated with the object orientation θ_{obj} is positive as illustrated in Fig.4. At the same time, a geometric relationship on the tangential direction shown in Fig.3-(a) is described as below:

$$GQ_l = -(x_{obj} - O_{lx})\sin\theta_{obj} + (y_{obj} - O_{ly})\cos\theta_{obj}. \tag{9}$$

Finally, from eqs.(8) and (9) the rolling constraint on left fingertip is expressed as follows:

$$\begin{aligned} -(x_{obj} - O_{lx})\sin\theta_{obj} + (y_{obj} - O_{ly})\cos\theta_{obj} \\ = h - s_l - (a-d_l)\cdot(\theta_{obj} - \theta_{obj}^{n-1}). \end{aligned} \tag{10}$$

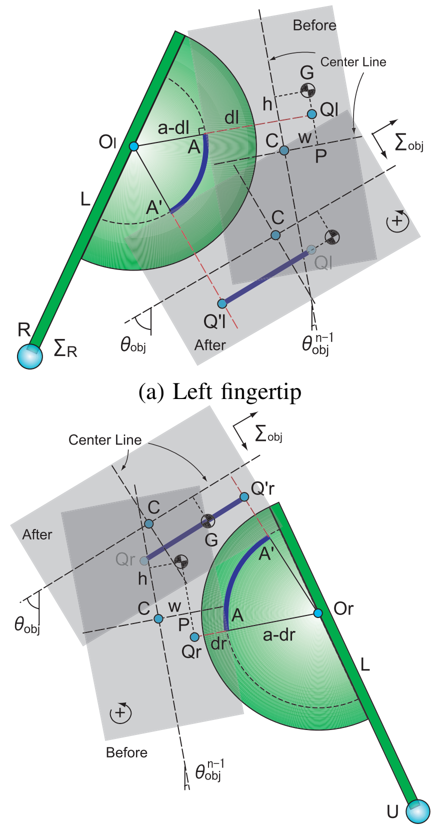
As well as the above equation, the rolling constraint of the object on the right fingertip is written by

$$\begin{aligned} -(x_{obj} - O_{rx})\sin\theta_{obj} + (y_{obj} - O_{ry})\cos\theta_{obj} \\ = h - s_r + (a-d_r)\cdot(\theta_{obj} - \theta_{obj}^{n-1}). \end{aligned} \tag{11}$$

Here s_r denotes y-coordinate of point $Q_r(w,s_r)$ with respect to the coordinate system Σ_{obj}, and is a physical quantity including the past rolling distance on the right fingertip as well as s_l shown in Fig.4-(b). Eqs.(10) and (11) are geometric tangential constraints due to the object rolling on both fingertips.

IV. Quasi-Static Handling Motion

In this paper, we do not consider the dynamics of fingers, fingertips and the grasped object, and confine to a quasi-static handling motion. We define that an instantaneous single process for manipulating motions be a minimum basic motion in which both infinitesimal rolling and translational motions of the object are combined during the handling process. In the following section, by using the concept of LMEE due to the large deformation of soft fingertips, we propose a quasi-static handling algorithm in which physical variables $(x_{obj}, y_{obj}, \theta_{obj})$ are determined uniquely after minimum basic motions.



(a) Left fingertip

(b) Right fingertip

Fig. 4. Tangential constraints

A. Quasi-Static Handling Algorithm with LMEE

We develop the concept of LMEE represented in eq.(2) to a minimum D.O.F. robotic hand with two rotational joints. As shown in Fig.2, the total elastic potential energy due to the deformation of both fingertips are represented as follows:

$$P = \frac{\pi E}{3}\left\{\frac{d_r^3}{\cos^2(\theta_r - \theta_{obj})} + \frac{d_l^3}{\cos^2(\theta_l + \theta_{obj})}\right\}. \tag{12}$$

We find (d_r, d_l) and $(x_{obj}, y_{obj}, \theta_{obj})$ such that eq.(12) reaches to its local minimum, where all variables are uniquely determined by using the rolling constraints eqs.(10) and (11) and the normal constraints eqs.(6) and (7). The procedure of the quasi-static handling algorithm proposed in this paper is summarized as follows:

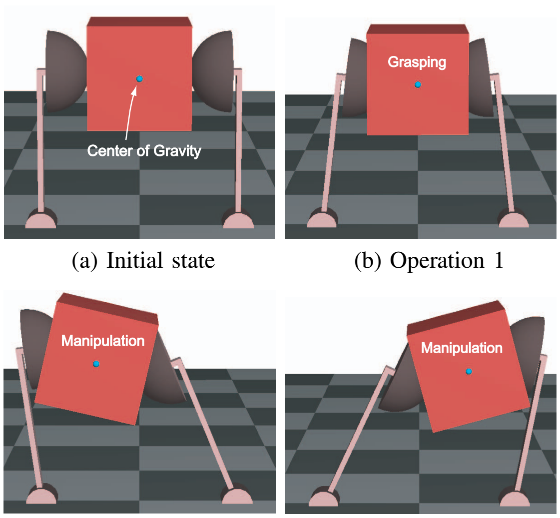
1) Finger joint angles (θ_r, θ_l) are arbitrarily given as input angles.
2) An LMEE of eq.(12) is computed.
3) The center of gravity of a grasped object (x_{obj}, y_{obj}) and its orientation angle θ_{obj} are determined from the LMEE value.

4) Each maximum displacement of both fingertips (d_r, d_l) is calculated from the above computed values $(x_{obj}, y_{obj}, \theta_{obj})$.

Here the grasped object is rigid and its dimension is an arbitrary rectangular solid. We assume that the slip motion between the object and the soft fingertip is ignorable and all manipulation and grasping processes in this system are confined to the two-dimensional plane without the gravity effect.

V. Simulation

We search for a local minimum of eq.(12) by using a numerical algorithm, *Nelder-Mead Method*, in every infinitesimal change of the finger rotation.



(a) Initial state (b) Operation 1

(c) Operation 2 (d) Operation 3

Fig. 5. Quasi-static manipulation process

We summarize the simulation procedure in terms of quasi-static handling motion as follows:

1) As shown in Fig.5-(a) and (b), both fingers rotate by a certain degree to the inner side from an initial condition $(\theta_r = \theta_l = d_r = d_l = 0)$ (Operation1). In this paper we perform manipulation tests for five different grasping angles in the operation 1, where it takes 2.4 deg up to 7.2 deg at the interval of 1.2 deg.
2) Both fingers rotate toward the counter-clock-wise (CCW) direction by 20 deg as shown in Fig.5-(c) (Operation2).
3) Both fingers rotate toward the clock-wise (CW) direction by 40 deg as shown in Fig.5-(d) (Operation3).
4) Both fingers rotate toward the CCW direction by 40 deg.
5) Operation 3 and 4 iterate three times.
6) Both fingers rotate toward the CW direction by 20 deg and return back to the operation 1.
7) Both fingers go back to the initial state.

All parameters used in this simulation equal to those of a robotic hand fabricated for handling experiments. Also, we set that the center of gravity of the object G and the geometric center C are indentical to each other for simplicity. Every simulation result is plotted in experimental results shown in the following chapter.

VI. Experiments

In this paper we conduct quasi-static handling experiments by means of a minimum D.O.F. robotic hand, and compute the center of gravity and object orientation by a CCD camera in real time. We validate the quasi-static handling theory based on LMEE by comparing simulations and experiments.

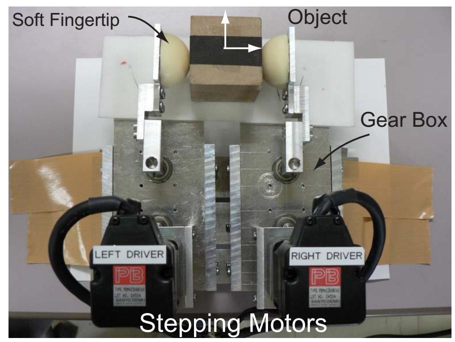


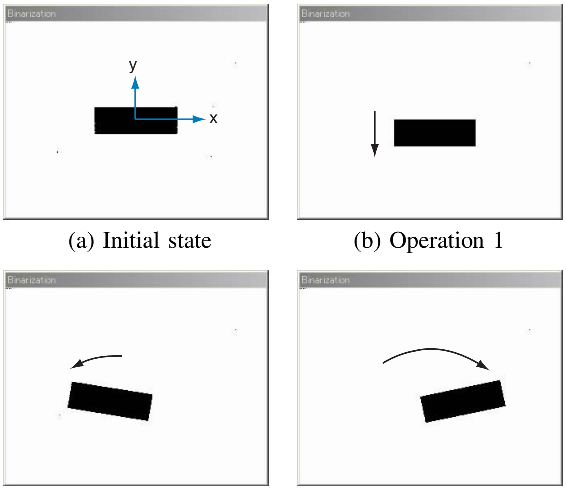
Fig. 6. Apparatus

A. Apparatus

As shown in Fig.6, we conduct the quasi-static handling experiment by using stepping motors and hemispherical soft fingertips mounted on two rotational fingers. The diameter of the fingertip is 40 mm and the grasped object has 49 mm square. We perform this experiment on the two-dimensional plane, and all parameters relating to the robotic hand are shown in Table I. The Young's modulus of soft fingers is measured by the tensile test of a polyurethane rubber.

B. Experimental Method

We attach a rectangular black drawing paper on the object, and compute the center of gravity and the orientation of the object by using a CCD camera equipped above the robotic hand. We compute the luminance center of the extracted binary images of the black drawing paper as a center of gravity of the object, and then obtain an object orientation angle by computing the second order moment of the binary images shown in Fig.7.



(a) Initial state (b) Operation 1

(c) Operation 2 (d) Operation 3

Fig. 7. Binary images

TABLE I
SYSTEM PARAMETERS

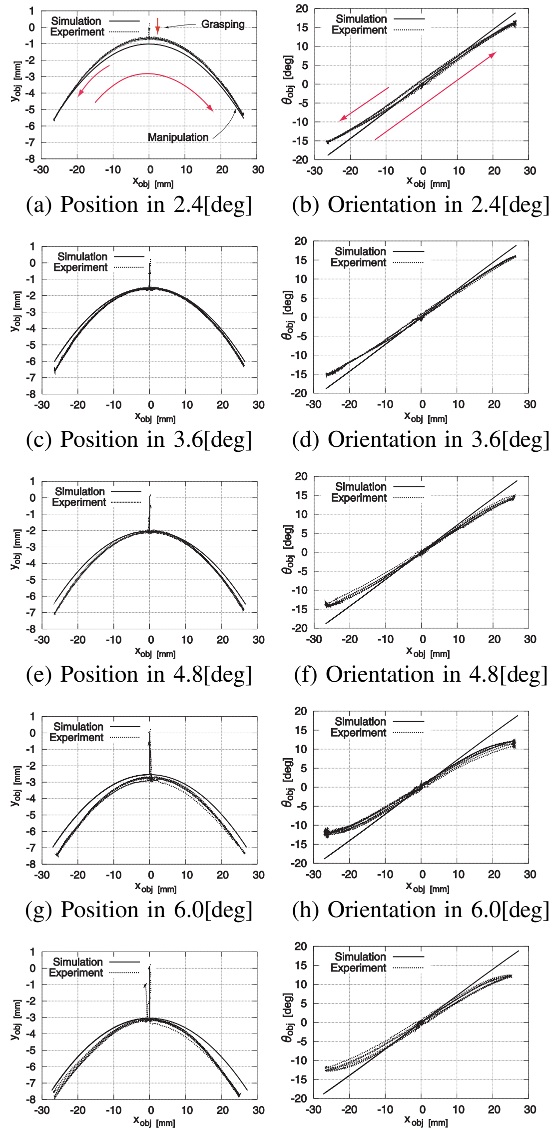
Parameters	Values
Finger length : L	76.2 [mm]
Fingertip radius : R	20 [mm]
Finger width : W_{fi}	48.5 [mm]
Object width : W_{obj}	49 [mm]
Finger thickness : d_{fi}	4 [mm]
Young's modulus : E	0.2032 [MPa]
Gear ratio	108
Input pulse	1[kHz]

C. Experimental results

Fig.8 shows experimental results in which simulation results are also plotted together. Left column shows the path of center of gravity (x_{obj}, y_{obj}) during grasping and manipulation, and right column shows the change of object orientation with respect to x_{obj}. The first row graphs correspond to the results of grasping angle 2.4 deg in the operation 1. Also, five different experimental results in terms of grasping angle up to 7.2 deg are shown in Fig.8. Here the origins in every graph are equivalent to those of the center of black drawing paper attached on the grasped object shown in Fig.6.

The position of the object shown in the left column goes downward largely, which results from the fact that the object follows to the large deformation of soft fingertips because of the frictional restriction of the contact area between the object and fingertips in the experiment. In the manipulating motion, we can find that both simulation and experimental results are identical to each other as well as the grasping motion.

On the other hand, the orientation results shown in the right column in Fig.8 indicate that a S-shaped curve appears explicitly in every graph in experiments. This results from the fact that each rolling distance on both fingertips increases as the rolling motion progresses and the rolling radii $(a-d_r, a-d_l)$ get larger.



(a) Position in 2.4[deg] (b) Orientation in 2.4[deg]

(c) Position in 3.6[deg] (d) Orientation in 3.6[deg]

(e) Position in 4.8[deg] (f) Orientation in 4.8[deg]

(g) Position in 6.0[deg] (h) Orientation in 6.0[deg]

(i) Position in 7.2[deg] (j) Orientation in 7.2[deg]

Fig. 8. Comparison with simulation and experiment

Fig.9 shows a comparison between all experimental results associated with the path of the object position. The most important point is that each end point of the path of manipulation processes conforms closely to all together at approximately ± 28 mm. In this experiment, every rotational angle of both fingers (θ_r, θ_l) after operation 2 and 3 is all different in five patterns of grasping angle

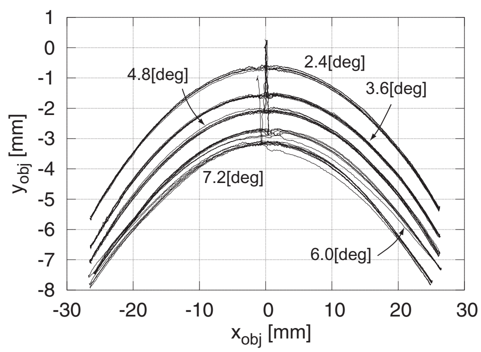


Fig. 9. Comparison of locational path in experiments

from 2.4 deg to 7.2 deg. This resultant consistency of x_{obj} implies that the x-coordinate of the object position does not depend on the magnitude of the grasping force. In other words, in order to transfer an object to a certain position along the x-axis, it is only necessary to activate the fingers by a certain degree that corresponds one-to-one with the position of the object irrespective of the elastic force level of soft fingertips.

VII. Concluding Remarks

In this paper we have proposed a quasi-static handling theory based on LMEE algorithm due to the large deformation of a hemishperical soft fingertip. Also, we have two normal constraints and two rolling constraints including variable rolling radius on a soft fingertip, and shown that each of $(x_{obj}, y_{obj}, \theta_{obj})$ of a grasped object is uniquely determined by numerical analysis using the LMEE algorithm.

Appendix

As illustrated in Fig.1, since the elastic potential energy caused by the deformation of soft fingertip is represented as a double integration on xy-plane, it takes the following equation by using the spring constant k of a virtual linear spring QR:

$$P = \frac{1}{2}\iint_S kPQ^2 = \frac{1}{2}\iint_S k(QT^2 + PT^2). \tag{13}$$

Using the geometric relationship of the triangle PQT illustrated in Fig.10, the above equation is transformed into the following equation:

$$P = \frac{1}{2\cos^2\theta_p}\iint_S kQT^2. \tag{14}$$

As shown in Fig.11, the numerical analysis of the double integration in eq. (14) shows that $\iint_S kQT^2$ is constant with respect to the object orientation, the integration area can therefore be transformed into a circular region as follows:

$$P = \frac{1}{2\cos^2\theta_p}\iint_C kQT^2, \tag{15}$$

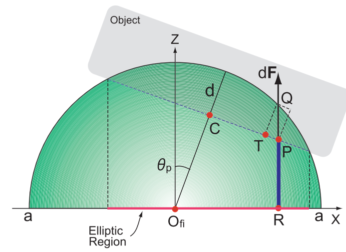


Fig. 10. Decomposition of elastic force

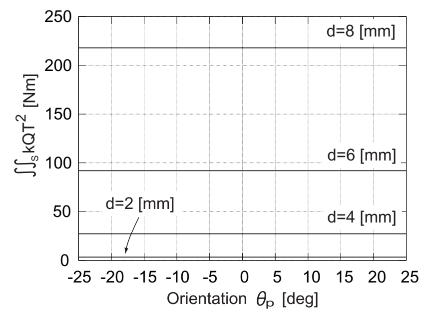


Fig. 11. Numerical analysis of $\iint_S kQT^2$ in eq.(14)

where C denotes the circular region. The above form can finally be calculated in an analytical way as eq.(2).

References

[1] N.Xydas and I.Kao, "Modeling of Contact Mechanics and Friction Limit Surfaces for Soft Fingers in Robotics, with Experimental Results", *Journal of Robotics Research*, Vol.18, No.8, pp.941-950, 1999.

[2] I.Kao and F.Yang : "Stiffness and Contact Mechanics for Soft Fingers in Grasping and Manipulation", *IEEE Trans. on Robotics and Automation*, Vol.20, No.1, pp.132-135, 2004.

[3] N.Xydas, M.Bhagavat, and I.Kao, "Study of Soft-Finger Contact Mechanics Using Finite Elements Analysis and Experiments", *Proc. IEEE Int. Conf. on Robotics and Automation*, pp.2179-2184, 2000.

[4] P.Nguyen and S.Arimoto, "Performance of Pinching Motions of Two Multi-DOF Robotic Fingers with Soft-Tips", *Proc. IEEE Int. Conf. on Robotics and Automation*, pp.2344-2349, 2001.

[5] S.Arimoto, K.Tahara, M.Yamaguchi, P.Nguyen, and H.Y.Han, "Principle of Superposition for Controlling Pinch Motions by means of Robot Fingers with Soft Tips", *Robotica*, Vol.19, pp.21-28, 2001.

[6] Z.Doulgeri, J.Fasoulas, and S.Arimoto, "Feedback Control for Object Manipulation by a pair of Soft Tip Fingers", *Robotica*, Vol.20, pp.1-11, 2002.

[7] J.Fasoulas and Z.Doulgeri, "Equilibrium Conditions of a Rigid Object Grasped by Elastic Rolling Contacts", *Proc. IEEE Int. Conf. on Robotics and Automation*, pp.789-794, 2004.