## 1 SUMMARY

To **solve one or more sets of sparse symmetric linear unassembled finite-element equations, $\mathbf{AX}=\mathbf{B}$, by the frontal method,** optionally holding the matrix factor out-of-core in direct access files. The package is primarily designed for positive-definite matrices since numerical pivoting is not performed. Use is made of high-level BLAS kernels. The coefficient matrix **A** must of the form

$$\mathbf{A}=\sum_{k=1}^{m}\mathbf{A}^{(k)}, \tag{1}$$

with $\mathbf{A}^{(k)}$ nonzero only in those rows and columns that correspond to variables in the $k$-th element.

The frontal method is a variant of Gaussian elimination and involves the factorization

$$\mathbf{A}=\mathbf{PLD}(\mathbf{PL})^{\mathrm{T}},$$

where **P** is a permutation matrix, **D** is a diagonal matrix, and **L** is a unit lower triangular matrix. The solution process is completed by performing the forward elimination

$$(\mathbf{PL})\mathbf{DY}=\mathbf{B},$$

followed by the back substitution

$$(\mathbf{PL})^{\mathrm{T}}\mathbf{X}=\mathbf{Y}.$$

MA62 stores the reals of the factors and their indices separately. A principal feature of MA62 is that, by holding the factors out-of-core, large problems can be solved using a predetermined and relatively small amount of in-core memory. At an intermediate stage of the solution, $l$ say, the 'front' contains those variables associated with one or more of $\mathbf{A}^{(k)}$, $k=1,2,...,l$, which are also present in one or more of $\mathbf{A}^{(k)}$, $k=l+1,...,m$. For efficiency, the user should order the $\mathbf{A}^{(k)}$ so that the number of variables in the front (the 'front size') is small. For example, a very rectangular grid should be ordered pagewise parallel to the short side of the rectangle. The elements may be preordered using the Harwell Subroutine Library routine MC63.

MA62 uses reverse communication.

**ATTRIBUTES — Version:** 1.0.0. (12 July 2004) **Types:** Real (single, double). **Calls:** _AXPY, _GER, _GEMV, _TPSV, _TRSV, _GEMM, _TRSM. **Helpful:** MC63. **Language:** Fortran 77. **Original date:** April 1997. **Origin:** I.S. Duff and J.A. Scott, Rutherford Appleton Laboratory.

## 2 HOW TO USE THE PACKAGE

### 2.1 Argument lists and calling sequences

There are six entries:

(a) The subroutine MA62I/ID must be called to initialize the parameters that control the execution of the package. This subroutine must be called once prior to calling other routines in the package.

(b) MA62A/AD must be called for each element to specify which variables are associated with it. This subroutine determines in which element each variable appears for the last time.

(c) MA62J/JD must be called for each element. This subroutine uses the information from MA62A/AD to determine the amount of real and integer storage required for the factorization.

(d) The use of MA62P/PD is optional. If direct access files are to be used, MA62P/PD must be called once prior to calling MA62B/BD.

(e) MA62B/BD must be called for each element to specify the nonzeros of $\mathbf{A}^{(k)}$ and, optionally, the corresponding element right-hand side(s) $\mathbf{B}^{(k)}$. MA62B/BD uses the information generated by MA62A/AD and MA62J/JD in the factorization of the matrix (1) and, if $\mathbf{B}^{(k)}$ are specified, MA62B/BD solves the equations $\mathbf{AX}=\mathbf{B}$ with right-hand side(s) $\mathbf{B}=\sum_{k=1}^{m}\mathbf{B}^{(k)}$.

(f) The use of MA62C/CD is optional. MA62C/CD uses the factor computed by MA62B/BD to solve for further right-hand sides. Several calls to MA62C/CD may follow a single call to MA62B/BD.

### 2.1.1 The initialization subroutine

To initialize control parameters, the user must make a single call of the following form:

*The single precision version*

```
CALL MA62I(ICNTL,CNTL,ISAVE)
```

*The double precision version*

```
CALL MA62ID(ICNTL,CNTL,ISAVE)
```

ICNTL is an INTEGER array of length 15 that need not be set by the user. This array is used to hold control parameters. On exit, ICNTL contains default values. If the user wishes to use values other than the defaults, the corresponding entries in ICNTL should be reset after the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1.

CNTL is a REAL (DOUBLE PRECISION in the D version) array of length 5 that need not be set by the user. This array is used to hold control parameters. On exit, CNTL contains default values. If the user wishes to use values other than the defaults, the corresponding entries in CNTL should be reset after the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1.

ISAVE is an INTEGER array of length 50 that need not be set by the user. This array is used to hold parameters that must be unchanged between calls to routines in the MA62 package.

### 2.1.2 Specification of which variables belong in each element

A call of the following form must be made for each element.

*The single precision version*

```
CALL MA62A(NVAR,IVAR,NDF,LAST,LENLST,ICNTL,ISAVE,INFO)
```

*The double precision version*

```
CALL MA62AD(NVAR,IVAR,NDF,LAST,LENLST,ICNTL,ISAVE,INFO)
```

NVAR is an INTEGER variable that must be set by the user to the number of variables in the element. This argument is not changed by the routine. **Restriction:** NVAR $\geq$ 1.

IVAR is an INTEGER array of length NVAR that must be set by the user to contain the indices of the variables associated with the element. These indices need not be in increasing order but must be distinct. This argument is not changed by the routine. **Restrictions:** 1 $\leq$ IVAR(I) $\leq$ LENLST and IVAR(I) $\neq$ IVAR(J), I, J = 1, 2, ..., NVAR.

NDF is an INTEGER variable that need not be set by the user. On each exit, it will be set to the largest integer so far used to index a variable. It must not be changed by the user between calls to MA62A/AD nor prior to subsequent calls to MA62J/JD and MA62B/BD. Note that, if the variables are not indexed contiguously, NDF will exceed the number of variables in the problem (see INFO(3) in Section 2.2.2).

LAST is an INTEGER array of length LENLST that need not be set by the user. On each exit from MA62A/AD, LAST(I) indicates the element in which the variable with index I last appeared or, if it has not appeared, LAST(I) is

zero. On exit from the final call, if I has been used to index a variable, LAST(I) is the element at which variable I is fully summed and is zero otherwise. The first NDF entries of this array must not be changed between calls to MA62A/AD nor prior to subsequent calls to MA62J/JD and MA62B/BD.

LENLST is an INTEGER variable that must be set by the user to the dimension of array LAST. LENLST must be at least as large as the largest integer used to index a variable and must not be changed between calls to MA62A/AD. This argument is not changed by the routine. **Restriction:** LENLST ≥ 1.

ICNTL is an INTEGER array of length 15 that must be set by the user to hold control parameters. Default values are set by the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1. Only ICNTL(I), I = 1, 2, and 8, are used by the routine. This argument is not changed by the routine.

ISAVE is an INTEGER array of length 50 that is used to hold parameters that must be unchanged between calls to routines in the MA62 package. This argument is changed by the routine.

INFO is an INTEGER array of length 20 that need not be set by the user. On each successful exit, INFO(1) is set to 0. Negative values of INFO(1) indicate a fatal error has been detected (see Section 2.3). If an error is detected, INFO(2) holds additional information concerning the error. INFO(I), I ≥ 3, are not accessed by the routine. This array must not be altered by the user.

### 2.1.3 Symbolic factorization of A

To determine the amount of real and integer storage required by the factorization, a call of the following form must be made for each element. The elements must have the same index lists and be in exactly the same order as when MA62A/AD was called. All the calls to MA62A/AD must be completed before MA62J/JD is called. Note that the storage is dependent on the control parameter ICNTL(5). If the user wishes to compute the storage required by different values of ICNTL(5), it is not necessary to recall MA62A/AD before repeating the sequence of calls to MA62J/JD.

*The single precision version*

```
CALL MA62J(NVAR,IVAR,NDF,LAST,ICNTL,ISAVE,INFO,RINFO)
```

*The double precision version*

```
CALL MA62JD(NVAR,IVAR,NDF,LAST,ICNTL,ISAVE,INFO,RINFO)
```

NVAR, IVAR are as in the corresponding calls to MA62A/AD but MA62J/JD does not check IVAR for duplicate indices. NVAR and IVAR are not changed by the routine.

NDF is an INTEGER variable which must be unchanged since the final call to MA62A/AD. This argument is not changed by the routine.

LAST is an INTEGER array of length NDF which must be unchanged since the final call to MA62A/AD. This argument is not changed by the routine.

ICNTL is an INTEGER array of length 15 that must be set by the user to hold control parameters. Default values are set by the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1. Only ICNTL(I), I = 1, 2, 5, and 8, are used by the routine. This argument is not changed by the routine.

ISAVE is an INTEGER array of length 50 that is used to hold parameters that must be unchanged between calls to routines in the MA62 package. This argument is changed by the routine.

INFO is an INTEGER array of length 20 that need not be set by the user. On successful exit, INFO(1) is set to 0. Negative values of INFO(1) indicate a fatal error has been detected (see Section 2.3). If an error is detected, INFO(2) holds additional information concerning the error. On exit from the final call, INFO(I), I = 3, 4, 5, 6, contain information about the factorization and INFO(7) and INFO(8) are set to zero. Full details are given in Section 2.2.2. INFO(I), I ≥ 9, are not accessed by the routine. This array must not be altered by the user.

RINFO is a REAL (DOUBLE PRECISION in the D version) array of length 20 that need not be set by the user. On exit from the final call, RINFO(I), I = 1, 2 contain information about the symbolic factorization. Details are given

in Section 2.2.2. RINFO(I), I ≥ 3, are not accessed by the routine. This array must not be altered by the user.

#### 2.1.4 To set up direct access files

If the user wishes to keep in-core memory requirements low by using direct access files for the factors, a single call of the following form must be made. Note that for very large problems, using direct access files is recommended. If not, it may be necessary to use 64-bit integer arithmetic.

*The single precision version*

```
CALL MA62P(ISTRM,FILNAM,LENBUF,ICNTL,ISAVE,INFO)
```

*The double precision version*

```
CALL MA62PD(ISTRM,FILNAM,LENBUF,ICNTL,ISAVE,INFO)
```

ISTRM is an INTEGER array of length 2. ISTRM(1) and ISTRM(2) must be set by the user to specify the unit numbers of the direct access files for the reals in the factors and the indices of the variables in the factors, respectively. This argument is not changed by the routine. **Restrictions:** ISTRM(I) must lie in the range [1, 99], ISTRM(I) ≠ 6, ICNTL(1), or ICNTL(2) (I = 1, 2), and ISTRM(1) ≠ ISTRM(2).

FILNAM is a CHARACTER*128 array of length 2. If ICNTL(6) is reset by the user to a nonzero value, the user must set FILNAM(1) and FILNAM(2) to the filenames for the direct access files for the reals in the factors and the indices of the variables in the factors, respectively. If ICNTL(6) = 0 (the default), FILNAM is not accessed by the routine. This argument is not changed by the routine.

LENBUF is an INTEGER array of length 2. LENBUF(1) must be set by the user to the length, in REAL (DOUBLE PRECISION in the D version) words, of the in-core buffer (workspace) associated with the direct access file for the reals in the factors (including the corresponding right-hand sides) and LENBUF(2) must be set by the user to the length, in INTEGER words, of the buffer associated with the direct access file for the indices of the variables in the factors. LENBUF(I) (I = 1, 2) have a crucial effect on the in-core memory requirements of MA62B/BD and MA62C/CD (see arguments LW and LIW in Sections 2.1.5 and 2.1.6). If NRHSB is the number of right-hand sides to be input to MA62B/BD, LENBUF should be chosen so that RINFO(1) + NDF∗NRHSB = $k_1$∗LENBUF(1) and RINFO(2) = $k_2$∗LENBUF(2) with $k_1$, $k_2 \geq 1$ as small as available space permits (RINFO(1) and RINFO(2) as output from the final call MA62J/JD). LENBUF is not changed by the routine. **Restrictions:** LENBUF(I) > 0, I = 1, 2.

ICNTL is an INTEGER array of length 15 that must be set by the user to hold control parameters. Default values are set by the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1. ICNTL(I), I = 1, 2, 3, 4, 6, and 8, are used by the routine. This argument is not changed by the routine. **Restrictions:** ICNTL(3) > 0 and ICNTL(4) > 0.

ISAVE is an INTEGER array of length 50 that is used to hold parameters that must be unchanged between calls to routines in the MA62 package. This argument is changed by the routine.

INFO is an INTEGER array of length 20 that need not be set by the user. On successful exit, INFO(1) is set to 0. Negative values of INFO(1) indicate a fatal error has been detected (see Section 2.3). If an error is detected, INFO(2) holds additional information concerning the error. INFO(I), I ≥ 3, are not accessed by the routine. This array must not be altered by the user.

#### 2.1.5 To factorize A and optionally solve AX = B

A call of the following form must be made for each element. The elements must have the same index lists and be in exactly the same order as when MA62A/AD and MA62J/JD were called.

Note that all the calls to MA62J/JD for a particular problem must be completed before calling MA62B/BD.

*The single precision version*

```
      CALL MA62B(NVAR,IVAR,NDF,LAST,LAVAR,AVAR,NRHSB,RHS,LX,X,
     *           LENBUF,LW,W,LIW,IW,ICNTL,CNTL,ISAVE,INFO,RINFO)
```

*The double precision version*

```
      CALL MA62BD(NVAR,IVAR,NDF,LAST,LAVAR,AVAR,NRHSB,RHS,LX,X,
     *            LENBUF,LW,W,LIW,IW,ICNTL,CNTL,ISAVE,INFO,RINFO)
```

`NVAR`, `IVAR`, `NDF`, `LAST` are as in the corresponding calls to `MA62J/JD`. `NVAR` and `NDF` are not changed by the routine. On exit, the data in `IVAR` may have been permuted. Between calls to `MA62B/BD`, `LAST` is used as workspace and will be changed but on exit from the final call (or on an error return), `LAST` will have been restored to its original value.

`LAVAR` is an `INTEGER` variable that must be set by the user to the first dimension of the arrays `AVAR` and `RHS`. This argument is not changed by the routine. **Restriction:** `LAVAR` ≥ `NVAR`.

`AVAR` is a `REAL` (`DOUBLE PRECISION` in the D version) array of dimensions `LAVAR` by `NVAR`. On entry, `AVAR(I,J)` must contain the contribution to entry (`IVAR(I)`, `IVAR(J)`) in the matrix **A** from the current element (`I`, `J` = `1`, `2`,..., `NVAR`, `J` ≥ `I`). Contributions to the same entry from different elements are summed. This argument is changed by the routine.

`NRHSB` is an `INTEGER` variable that must be set by the user to the number of right-hand sides and must not be changed between calls to `MA62B/BD`. If the user does not wish to solve for any right-hand sides, `NRHSB` should be set to `0`. This argument is not changed by the routine. **Restriction:** `NRHSB` ≥ `0`.

`RHS` is a `REAL` (`DOUBLE PRECISION` in the D version) array with leading dimension `LAVAR`. If `NRHSB` = `0`, this array is not accessed. Otherwise on entry, the first `NRHSB` columns of `RHS` must be set by the user so that `RHS(I,J)` contains the contribution to component `IVAR(I)` of the `J`-th right-hand side from the current element (`I` = `1`, `2`,..., `NVAR`, `J` = `1`, `2`,..., `NRHSB`). Contributions to the same component from different elements are summed. This argument is changed by the routine.

`LX` is an `INTEGER` variable that must be set by the user to the first dimension of the array `X`. This argument is not changed by the routine. **Restriction:** If `NRHSB` ≥ `1`, `LX` ≥ `NDF`.

`X` is a `REAL` (`DOUBLE PRECISION` in the D version) array with leading dimension `LX` that need not be set by the user. If `NRHSB` = `0`, this array is not accessed. Otherwise, the second dimension of `X` must be at least `NRHSB` and, on successful exit from the final call to `MA62B/BD`, if `I` has been used to index a variable, `X(I,J)` holds the solution for variable `I` to system `J` and is set to zero otherwise (`I`=`1`,`2`,...,`NDF`, `J`=`1`,`2`,..., `NRHSB`).

`LENBUF` is an `INTEGER` array of length `2`. If the user is using direct access files, `LENBUF` must be unchanged since the call to `MA62P/PD`. Otherwise, `LENBUF(1)` must be set by the user to the length, in `REAL` (`DOUBLE PRECISION` in the D version) words, of the file for the reals in the factors (including the corresponding right-hand sides) and `LENBUF(2)` must be set by the user to the length, in `INTEGER` words, of the file for the indices of the variables in the factors. This array must not be changed between calls to `MA62B/BD`. This argument is not changed by the routine. **Restriction:** If direct access files are not being used, `LENBUF(1)` ≥ `RINFO(1)` + `NDF*NRHSB`, `LENBUF(2)` ≥ `RINFO(2)` (`RINFO(1)` and `RINFO(2)` as output from the last call to `MA62J/JD`).

`LW` is an `INTEGER` variable that must be set by the user to the dimension of array `W`. It must be unchanged between calls to `MA62B/BD`. This argument is not changed by the routine. **Restriction:** `LW` ≥ `LENBUF(1)` + `INFO(6)*(NRHSB` + `INFO(6))` + `3` (`INFO(6)` as output from the last call to `MA62J/JD`).

`W` is a `REAL` (`DOUBLE PRECISION` in the D version) array of length `LW` that is used as workspace by `MA62B/BD`. This array must be unchanged between calls to `MA62B/BD`. If direct access files are not being used (`MA62P/PD` not called), the first `LENBUF(1)` + `3` entries of `W` must be unchanged between the last call to `MA62B/BD` and any subsequent calls to `MA62C/CD`.

`LIW` is an `INTEGER` variable that must be set by the user to the dimension of array `IW`. It must not be changed

between calls to MA62B/BD. This argument is not changed by the routine. **Restriction:** LIW≥LENBUF(2) + 3*INFO(6) (INFO(6) as output from the last call to MA62J/JD).

IW is an INTEGER array of length LIW that is used as workspace by MA62B/BD. This array must be unchanged between calls to MA62B/BD. If direct access files are not being used (MA62P/PD not called), the first LENBUF(2) entries of IW must be unchanged between the final call to MA62B/BD and any subsequent calls to MA62C/CD.

ICNTL is an INTEGER array of length 15 that must be set by the user to hold control parameters. Default values are set by the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1. ICNTL(5) must be unchanged since calling MA62J/JD. ICNTL(I), I = 1, 2, 5, 7, 8, and 9 are used by the routine. This argument is not changed by the routine.

CNTL is a REAL (DOUBLE PRECISION in the D version) array of length 5 that must be set by the user to hold control parameters. Default values are set by the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1. Only CNTL(1) is used by the routine. This argument is not changed by the routine.

ISAVE is an INTEGER array of length 50 that is used to hold parameters that must be unchanged between calls to routines in the MA62 package. This argument is changed by the routine.

INFO is an INTEGER array of length 20 that need not be set by the user. On successful exit, INFO(1) is set to 0. Negative values indicate a fatal error. For nonzero values of INFO(1), see Section 2.3. For details of the information contained in the other components of INFO, see Section 2.2.2. This array must not be altered by the user.

RINFO is a REAL (DOUBLE PRECISION in the D version) array of length 20. RINFO(1) and RINFO(2) must be unchanged since the final call to MA62J/JD. The other components need not be set by the user. On exit from the final call, RINFO holds information on the factorization. Full details are given in Section 2.2.2. This array must not be altered by the user.

#### 2.1.6 To solve further systems AX = B

*The single precision version*

```
CALL MA62C(NRHSC,LX,X,LW,W,LIW,IW,ICNTL,ISAVE,INFO)
```

*The double precision version*

```
CALL MA62CD(NRHSC,LX,X,LW,W,LIW,IW,ICNTL,ISAVE,INFO)
```

NRHSC is an INTEGER variable that must be set by the user to the number of systems which are to be solved. This argument is not changed by the routine. **Restriction:** NRHSC≥1.

LX is an INTEGER variable that must be set by the user to the first dimension of the array X. This argument is not changed by the routine. **Restriction:** LX≥NDF (NDF as output from the final call to MA62A/AD).

X is a REAL (DOUBLE PRECISION in the D version) array of dimensions LX by NRHSC that must be set by the user so that if I has been used to index a variable, X(I,J) is the corresponding component of the right-hand side for the J-th system (J=1,2,..., NRHSC). On exit, if I has been used to index a variable, X(I,J) holds the solution for variable I to system J and is unchanged otherwise (J=1,2,..., NRHSC).

LW is an INTEGER variable that must be set by the user to the dimension of the array W. A sufficient value for LW is L1+L2, where L1 = LENBUF(1)+NRHSC*INFO(6). If direct access files are not being used (MA62P/PD was not called), L2 = 3, otherwise, L2 = INFO(5)*(INFO(6)+NRHSB). (INFO(5) and INFO(6) as output from the last call to MA62J/JD, and NRHSB as in the calls to MA62B/BD). This argument is not changed by the routine. **Restriction:** LW≥L1+L2.

W is a REAL (DOUBLE PRECISION in the D version) array of length LW. If direct access files are not being used (MA62P/PD was not called), the first LENBUF(1) + 3 entries of W must be unchanged since the last call to MA62B/BD and these entries are unchanged by MA62C/CD. Otherwise, W is used by MA62C/CD as workspace.

LIW is an INTEGER variable that must be set by the user to the dimension of the array IW. If direct access files are not being used (MA62P/PD was not called), LIW must be at least L1 = LENBUF(2). Otherwise, LIW must be at least L1 = LENBUF(2) + INFO(6) + 4. This argument is not changed by the routine. **Restriction:** LIW ≥ L1.

IW is an INTEGER array of length LIW. If direct access files are not being used (MA62P/PD was not called), the first LENBUF(2) entries of IW must be unchanged since the last call to MA62B/BD and these entries are unchanged by MA62C/CD. Otherwise, IW is used by MA62C/CD as workspace.

ICNTL is an INTEGER array of length 15 that must be set by the user to hold control parameters. Default values are set by the call to MA62I/ID. Details of the control parameters are given in Section 2.2.1. ICNTL(I), I = 1, 2, and 8, are used by the routine. This argument is not changed by the routine.

ISAVE is an INTEGER array of length 50 that is used to hold parameters that must be unchanged between calls to routines in the MA62 package. This argument is changed by the routine.

INFO is an INTEGER array of length 20 that need not be set by the user. On successful exit, INFO(1) is set to 0. Negative values of INFO(1) indicate a fatal error has been detected (see Section 2.3). If an error is detected, INFO(2) holds additional information concerning the error. INFO(I), I ≥ 3, are not accessed by the routine. This array must not be altered by the user.

### 2.2 Arrays for control and information

#### 2.2.1 Control parameters

The elements of the arrays ICNTL and CNTL control the action of MA62A/AD, MA62J/JD, MA62P/PD, MA62B/BD, and MA62C/CD. Default values are set by MA62I/ID.

ICNTL(1) is the stream number for error messages and has the default value 6. Printing of error messages is suppressed if ICNTL(1) < 0.

ICNTL(2) is the stream number for warning messages and diagnostic printing. It has the default value 6. Printing of such messages is suppressed if ICNTL(2) < 0.

ICNTL(3) is the length (in processor-dependent units) of an unformatted i/o record that holds a real. In Fortran 90, ICNTL(3) may be set using the statement INQUIRE(IOLENGTH=ICNTL(3)) R, where R is a REAL (DOUBLE PRECISION in the D version) variable; otherwise, the vendor's documentation should be consulted.

ICNTL(4) is the length (in processor-dependent units) of an unformatted i/o record that holds an integer. In Fortran 90, ICNTL(4) may be set using the statement INQUIRE(IOLENGTH=ICNTL(4)) I, where I is an INTEGER variable; otherwise, the vendor's documentation should be consulted.

ICNTL(5) has the default value 16. ICNTL(5) controls the minimum number of variables which are eliminated at any one stage (except the last stage, when fewer than ICNTL(5) variables may remain). ICNTL(5) is only accessed on the first call to MA62J/JD and the first call to MA62B/BD. The value of ICNTL(5) on the first call to MA62B/BD should be the same as on the first call to MA62J/JD. Increasing ICNTL(5) in general increases the number of floating-point operations and real storage requirements but allows greater advantage to be taken of Level 3 BLAS.

ICNTL(6) has the default value 0. If it is reset to a nonzero value, the user must supply names for the direct access data sets in the parameter FILNAM when calling MA62P/PD.

ICNTL(7) is the block size for the numerical factorization of the frontal matrix. It controls the trade-off between Level 2 and Level 3 BLAS. If ICNTL(7) = 1, Level 2 BLAS is used to form the Schur complement. If ICNTL(7) ≥ 1, the Level 3 BLAS routine _GEMM is used with internal dimension ICNTL(7). Increasing ICNTL(7) increases the number of flops since symmetry is not exploited as well. The optimal value for ICNTL(7) depends on the computer being used. A value of ICNTL(7) less than one is treated as one and, if at some stage of the factorization, ICNTL(7) has a value which is larger than the current front size, ICNTL(7) is treated as the front size. Typical range: 16 to 64. Default value: 16.

ICNTL(8) is used to control the printing of error, warning, and diagnostic messages in MA62. It has default value 2. Possible values are:

- 0 No messages are output.
- 1 Only error messages are output.
- 2 Error and warning messages output.
- 3 As for 2, plus scalar parameters, arrays of length 2, and the control parameters on the first entry to MA62A/AD, MA62J/JD, MA62P/PD, and MA62B/BD, and scalar parameters on entry to MA62C/CD.
- 4 As for 3, plus INFO(I, (I = 1, 2,..., 8) on exit from final call to MA62J/JD, and the arrays INFO and RINFO on on exit from final call to MA62B/BD.

ICNTL(9) controls whether zeros within the frontal matrix are exploited. If ICNTL(9) = 0, the frontal matrix is treated as a dense matrix and zeros within the front are ignored. If ICNTL(9) is nonzero, the code will look for zeros occurring within the frontal matrix and will try to avoid unnecessary operations using zeros. This option can increase the amount of data movement but can also give worthwhile savings in the computation and in the number of entries in the factors if some variables are involved in only a few elements and these elements are well separated in the element order. The default value is 1.

ICNTL(10) controls whether a packed triangular form is used. If ICNTL(10) = 0 (the default value), a packed triangular form is used. In this case, the Level 2 BLAS routine _TPSV is used to perform triangular solves. If ICNTL(10) is reset to a nonzero value, a packed triangular form is not used. This increases the real storage required for the factors but the advantage is that, for multiple right-hand sides, the Level 3 BLAS routine _TRSM is used.

ICNTL(11) controls whether BLAS are used by MA62C/CD when solving for a single right-hand side. If ICNTL(11) = 1 and NRHS = 1, BLAS are not used. Otherwise, BLAS are used. The default value is 0.

ICNTL(12) to ICNTL(15) are currently not used but are given a default value of zero and they may be used in a later release of the code.

CNTL(1) has the default value zero. If, during the factorization, the absolute value of any pivot is less than or equal to CNTL(1), the computation terminates and the matrix is declared to be not positive definite (see INFO(1) = –12).

CNTL(2) to CNTL(5) are currently not used but are given a default value of zero and they may be used in a later release of the code.

### 2.2.2 Information arrays

The entries of the arrays INFO and RINFO provide information on the action of MA62A/AD, MA62J/JD, MA62P/PD, MA62B/BD, and MA62C/CD.

INFO(1) is used as an error flag. If a call to a routine in the MA62 package is successful, on exit INFO(1) has value 0. A negative value of INFO(1) indicates an error has been detected and a value greater than zero indicates a warning has been issued (see Section 2.3). If an error is detected during a call to MA62J/JD, the information contained in INFO(I), $3 \leq I \leq 6$, and RINFO(1) and RINFO(2) will be incomplete. Likewise, if an error is detected during a call to MA62B/BD, the information contained in INFO(I), $I \geq 9$, and in RINFO(I), $I \geq 3$, will be incomplete.

INFO(2) holds additional information concerning the error (see Section 2.3).

INFO(3) holds, on successful exit from the final call to MA62J/JD, the total number of variables in the problem.

INFO(4) holds, on successful exit from the final call to MA62J/JD, the number of static condensation variables (a static condensation variable is one which appears in only one element).

INFO(5) holds, on successful exit from the final call to MA62J/JD, the largest number of variables eliminated at a

single stage (that is, the maximum order of a pivot block).

INFO(6) holds, on successful exit from the final call to MA62J/JD, the maximum front size.

INFO(7) and INFO(8) are currently not used but are initialised on the first call to MA62J/JD to zero and they may be used in a later release of the code.

INFO(9) holds, on successful exit from the final call to MA62B/BD, the number of negative pivots.

INFO(10) holds, on successful exit from the final call to MA62B/BD, the number of buffers used for the reals in the factors.

INFO(11) holds, on successful exit from the final call to MA62B/BD, the number of buffers used for the indices of the variables in the factors.

INFO(12) holds, on successful exit from the final call to MA62B/BD, the maximum number of buffers required to hold the reals in a block of pivot rows.

INFO(13) holds, on successful exit from the final call to MA62B/BD, the maximum number of buffers required to hold the indices of the variables in a block of pivot rows.

INFO(14) to INFO(20) are currently not used but are initialised on the first call to MA62B/BD to zero and they may be used in a later release of the code.

RINFO(1) holds, on successful exit from the final call to MA62J/JD, the length in REAL (DOUBLE PRECISION in the D version) words of the file required by the numerical factorization for the reals in the factors (no allowance is made for right-hand sides or for possible exploitation of zeros in the front).

RINFO(2) holds, on successful exit from the final call to MA62J/JD, the length in INTEGER words of the file required by the numerical factorization for the indices of the variables in the factors (no allowance is made for possible exploitation of zeros in the front).

RINFO(3) holds, on successful exit from the final call to MA62B/BD, the length in REAL (DOUBLE PRECISION in the D version) words of the file actually used during the factorization for the reals in the factors and the corresponding right-hand sides. If NRHSB = 0 and ICNTL(9) = 0, RINFO(3) = RINFO(1).

RINFO(4) holds, on successful exit from the final call to MA62B/BD, the number of entries in the factors that have value zero. If there a large number of zeros in the factors and zeros in the front have not been exploited (ICNTL(9) = 0), the user should try a smaller value of ICNTL(5), or reset ICNTL(9) to a nonzero value, or reorder the elements.

RINFO(5) holds, on successful exit from the final call to MA62B/BD, the number of entries (including zero entries) stored in the factors. If zeros in the front are not exploited (ICNTL(9) = 0), RINFO(5) = RINFO(1).

RINFO(6) holds, on successful exit from the final call to MA62B/BD, the number of integers actually used during the factorization to store the factors. If zeros in the front are not exploited (ICNTL(9) = 0), RINFO(6) = RINFO(2).

RINFO(7) holds, on successful exit from the final call to MA62B/BD, the natural logarithm of the modulus of the determinant of the matrix **A**.

RINFO(8) holds, on successful exit from the final call to MA62B/BD, the number of floating-point operations to perform the factorization. This count includes operations performed during static condensation.

RINFO(9) holds, on successful exit from the final call to MA62B/BD, the root-mean-squared front size.

RINFO(10) to RINFO(20) are currently not used but are initialised on the first call to MA62B/BD to zero and they may be used in a later release of the code.

### 2.3 Error diagnostics

On successful completion, the subroutines in the MA62 package will exit with the parameter INFO(1) set to 0. Other values for INFO(1) and the reasons for them are given below.

A negative value for INFO(1) is associated with a fatal error. If ICNTL(8) > 0 and ICNTL(1) > 0, a self-explanatory message is, in each case, output on unit ICNTL(1) (see Section 2.2.1). The negative values for INFO(1) are:

–1 LENLST ≤ 0 on entry to MA62A/AD. (MA62A/AD first entry only).

–2 NVAR ≤ 0 in the current element. (MA62A/AD, MA62J/JD, and MA62B/BD entries). This error is also returned if NVAR is greater than LAVAR (MA62B/BD entries only).

–3 An index of a variable in the current element is out of range. INFO(2) holds the index which is out of range. (MA62A/AD, MA62J/JD, and MA62B/BD entries).

–4 Duplicate occurrences of the same variable index found in the current element. INFO(2) holds the duplicated index. (MA62A/AD entries only).

–5 NRHSB ≥ 1 and the defined first dimension LX of the array X is less than NDF as output from the final call to MA62A/AD. INFO(2) holds the value of NDF output from MA62A/AD. (MA62B/BD first entry only). This error is also returned by MA62C/CD if the defined first dimension LX of the array X is less than NDF.

–6 Defined length LW of the real workspace array W violates the restrictions on LW. LW must be increased to at least INFO(2). (MA62B/BD first entry only and MA62C/CD entry).

–7 Defined length LIW of the integer workspace array IW violates the restrictions on LIW. LIW must be increased to at least INFO(2). (MA62B/BD first entry only and MA62C/CD entry).

–8 A call to MA62J/JD has not been preceded by calls to MA62A/AD. (MA62J/JD first entry only).

–9 The number of right-hand sides is out of range. Either NRHSB < 0 (MA62B/BD first entry only) or the user has changed the number of right-hand sides between calls to MA62B/BD. INFO(2) holds the value of NRHSB on the first call to MA62B/BD. This error is also returned by MA62C/CD if NRHSC < 1.

–10 Either the order of the elements or the index list for one or more of the elements has been changed since the calls to MA62J/JD. (MA62B/BD entries only).

–11 A variable appears again after it has been fully summed (this happens if an index list for an element has been changed since MA62A/AD was called, or the order of the elements has been changed, or more elements have been entered than were entered to MA62A/AD). INFO(2) holds the index of the fully summed variable. (MA62J/JD and MA62B/BD entries).

–12 Attempt to use a pivot of absolute value less than or equal to CNTL(1). INFO(2) holds the call on which this error was encountered. (MA62B/BD entries only).

–13 The value of NDF has been changed since the final call to MA62A/AD. INFO(2) holds the value of NDF output from MA62A/AD. (MA62J/JD and MA62B/BD first entries only).

–14 The number of calls made to MA62J/JD differs from the number of calls made to MA62A/AD. This error is also returned if MA62B/BD is called without MA62J/JD being called. INFO(2) holds the number of calls made to MA62J/JD. (MA62B/BD first entry only).

–15 LENBUF(1) or LENBUF(2) violates the restrictions on it.
LENBUF(I) ≤ 0, I = 1 or 2 (MA62P/PD entry only), or
LENBUF(1) < RINFO(1) + NDF*NRHSB, or LENBUF(2) < RINFO(2), (MA62B/BD first entry only, direct access files not in use), or
LENBUF(I), I = 1 or 2, has been changed between the call to MA62P/PD and the first call to MA62B/BD.

–16 ISTRM(1)=ISTRM(2) or ISTRM(I) lies out of range, or is equal to 6, ICNTL(1), or ICNTL(2) (I = 1 or 2). (MA62P/PD entry only).

–17 Error encountered in Fortran OPEN statement. INFO(2) holds the IOSTAT parameter (the IOSTAT parameter is a parameter which, after an input/output operation is completed, is set to zero if no error was detected and to a positive integer otherwise). (MA62P/PD entry only).

–18 ICNTL(I) ≤ 0 for I = 3 or 4. (MA62P/PD entry only).

–19 Error detected when reading a direct access file. INFO(2) holds the IOSTAT parameter. (MA62B/BD and MA62C/CD entries).

–20 Error detected when writing to a direct access file. INFO(2) holds the IOSTAT parameter (MA62B/BD entries only).

Warning messages are associated with a positive value for INFO(1). If ICNTL(8) > 1 and ICNTL(2) > 0, a self-explanatory message is, in each case, output on unit ICNTL(2) (see Section 2.2.1). The warnings are:

+1 On entry to MA62J/JD, ICNTL(5) is less than or equal to zero. The default value 16 is used. (MA62J/JD first entry only).

+2 On entry to MA62B/BD, ICNTL(5) is not equal to the value used by MA62J/JD. The value used by MA62J/JD is used. (MA62B/BD first entry only).

+4 On entry to MA62B/BD or MA62C/CD, ICNTL(1) (or ICNTL(2)) has a value equal to ISTRM(1) or ISTRM(2). The default value 6 is used. (MA62B/BD first entry only and MA62C/CD entry). This warning can only be issued if MA62P/PD has been called.

+x x = 3, 5, 6, 7 are combinations of the above warnings corresponding to summing the constituent warnings.

## 3 GENERAL INFORMATION

**Use of common:** None.

**Other routines called directly:** The BLAS routines SAXPY/DAXPY, SGER/DGER, SGEMV/DGEMV, STPSV/DTPSV, STRSV/DTRSV, SGEMM/DGEMM, STRSM/DTRSM. Subroutines internal to the package are MA62D/DD, MA62E/ED, MA62F/FD, MA62G/GD, MA62H/HD, MA62L/LD, MA62M/MD, MA62N/ND, MA62O/OD.

**Workspace:** Workspace is provided by the arrays:
W(LW) (MA62B/BD and MA62C/CD).
IW(LIW) (MA62B/BD, and MA62C/CD).
LAST(NDF) is used locally as workspace (MA62B/BD only).
ISAVE(50) is a work array that must be unchanged between calls to routines in the MA62 package.

**Input/output:** In the event of errors, diagnostic messages are printed. The output streams for these messages are controlled by the variables ICNTL(1) and ICNTL(2), and the level of printing is controlled by ICNTL(8) (see Section 2.2.1). Stream ICNTL(1) is used for error messages (INFO(1) < 0) and stream ICNTL(2) for warnings (INFO(1) > 0) and diagnostic printing.

**Restrictions:**

MA62A/AD:

NVAR ≥ 1.

LENLST ≥ 1.

1 ≤ IVAR(I) ≤ LENLST and IVAR(I) ≠ IVAR(J), I, J, = 1, 2, ..., NVAR.

MA62J/JD:

NVAR ≥ 1.

1 ≤ IVAR(I) ≤ NDF, I = 1, 2, ..., NVAR.

MA62P/PD:

ISTRM(1) and ISTRM(2) lie in the range [1, 99] and do not equal 6, ICNTL(1), or ICNTL(2).

ISTRM(1) ≠ ISTRM(2).

ICNTL(I) > 0, I = 3, 4.

LENBUF(I) > 0, I = 1, 2.

MA62B/BD:

NVAR ≥ 1.

1 ≤ IVAR(I) ≤ NDF, I = 1, 2, ..., NVAR.

LAVAR ≥ NVAR.

NRHSB ≥ 0.

If NRHSB ≥ 1, LX ≥ NDF.

If MA62P/PD is not called, LENBUF(1) ≥ RINFO(1) + NDF*NRHSB, LENBUF(2) ≥ RINFO(2).

LW ≥ LENBUF(1) + INFO(6)*(INFO(6) + NRHSB) + 3

LIW ≥ LENBUF(2) + 3*INFO(6).

MA62C/CD:

NRHSC ≥ 1.

LX ≥ NDF.

If MA62P/PD is not called,
LW ≥ LENBUF(1) + INFO(6)*NRHSC + 3
LIW ≥ LENBUF(2) + INFO(6) + 4
otherwise,
LW ≥ LENBUF(1) + INFO(6)*NRHSC + INFO(5)*(INFO(6)+NRHSB).
LIW ≥ LENBUF(2).

## 4 METHOD

The method used is a modification of the unsymmetric frontal code of Duff and Scott (1993, 1996).

The elements are assembled into an in-core frontal matrix one at a time. A variable which has appeared for the last time (i.e. does not occur in future elements) is fully summed and is used as a pivot in Gaussian elimination, provided it is of absolute value at least CNTL(1) and there are at least ICNTL(5) fully summed variables. Once all possible eliminations for the current element have been performed, the pivot columns are written to in-core buffers and later, if requested, to direct access files. To prevent the amount of in-core memory required becoming too large, the user should order the elements so that the same variable does not occur in elements that are widely apart in the ordering. Thus, for example, in a problem with a narrow pipe geometry, the elements should be ordered across the cross-section of the pipe rather than along its length. An efficient element ordering can be obtained using the Harwell Subroutine Library routine MC63.

**References.**

Duff, I.S. and Scott, J.A. (1993). MA42 – A new frontal code for solving sparse unsymmetric systems. Report RAL-93-064, Rutherford Appleton Laboratory.

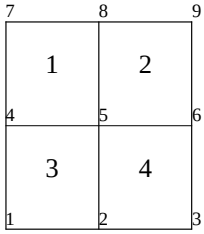
Duff, I.S. and Scott, J.A. (1996). The design of a new frontal code for solving sparse unsymmetric systems. *ACM Trans. Math. Softw.*, **22**, 30-45.

Duff, I.S. and Scott, J.A. (1997). MA62 – A frontal code for sparse positive-definite symmetric systems. Technical Report RAL-TR-97-012, Rutherford Appleton Laboratory.

## 5 EXAMPLE OF USE

We give an example of the code required to solve a set of symmetric finite-element equations using the MA62 package. The example illustrates the full calling sequence for the MA62 package. In this example, we wish to solve $\mathbf{AX}=\mathbf{B}$. We supply one right-hand side with the elements and then use MA62C/CD to solve for a further two right-hand sides. Direct access files are used to hold the factors.

We wish to solve the following simple finite-element problem in which the finite-element mesh comprises four 4-noded quadrilateral elements with one degree of freedom at each node i, $1 \le i \le 6$ (the nodes 7, 8, and 9 are assumed constrained).




The four element matrices $\mathbf{A}^{(k)}$ $(1 \le k \le 4)$ are

$$\begin{matrix}4\\5\end{matrix}\begin{pmatrix}2. & 1.\\ 1. & 7.\end{pmatrix} \qquad \begin{matrix}5\\6\end{matrix}\begin{pmatrix}3. & 2.\\ 2. & 8.\end{pmatrix} \qquad \begin{matrix}4\\5\\1\\2\end{matrix}\begin{pmatrix}4. & 3. & 2. & 3.\\ 3. & 1. & 3. & 2.\\ 2. & 3. & 6. & 1.\\ 3. & 2. & 1. & 5.\end{pmatrix} \qquad \begin{matrix}5\\6\\2\\3\end{matrix}\begin{pmatrix}2. & 1. & 8. & 3.\\ 1. & 3. & 2. & 2.\\ 8. & 2. & 2. & 5.\\ 3. & 2. & 5. & 4.\end{pmatrix},$$

where the variable indices are indicated by the integers before each matrix (columns are identified symmetrically to rows). The corresponding element vectors $\mathbf{b}^{(k)}$ $(1 \le k \le 4)$ are

$$\begin{pmatrix}3.\\ 8.\end{pmatrix} \qquad \begin{pmatrix}5.\\ 10.\end{pmatrix} \qquad \begin{pmatrix}12.\\ 9.\\ 12.\\ 11.\end{pmatrix} \qquad \begin{pmatrix}14.\\ 8.\\ 17.\\ 14.\end{pmatrix}.$$

The following program is used to solve this problem. In this program, we read the element data into arrays ELTPTR (location of first entry of element), ELTVAR (variable indices), VALUE (numerical values), and RHSVAL (right-hand sides). This method of storing the element data is used here for illustrative purposes; the user may prefer, for example, to read in the element data from a direct access file.

```
C Example to illustrate the use of MA62.
C
C     .. Parameters ..
      INTEGER MAXE,LIWMAX,LRHS,NZMAX,MELT,LENLST,LWMAX,MAXVL,MAXRVL,
     +        NDFMAX
      PARAMETER (MAXE=4,LIWMAX=120,LRHS=2,NZMAX=30,MELT=4,LENLST=6,
     +          LWMAX=120,MAXVL=30,MAXRVL=15,NDFMAX=9)
C     ..
C     .. Local Scalars ..
      INTEGER I,IELT,J,JSTRT,K,KSTRT,LIW,LW,LX,NDF,NELT,NFRONT,LAVAR,
     +        NPIV,NRHSB,NRHSC,NVAR,NZ,RHSCRD,VALCRD
C     ..
C     .. Local Arrays ..
      DOUBLE PRECISION AVAR(MAXE,MAXE),CNTL(5),
     +                 RHS(MAXE,LRHS),RHSVAL(MAXRVL),RINFO(20),
     +                 VALUE(MAXVL),W(LWMAX),X(NDFMAX,LRHS)
      INTEGER ELTPTR(MELT+1),ELTVAR(NZMAX),ICNTL(15),
     +        INFO(20),ISAVE(50),ISTRM(2),IVAR(MAXE),IW(LIWMAX),
     +        LAST(LENLST),LENBUF(2)
      CHARACTER FILNAM(2)*128
```

```
C     ..
C     .. External Subroutines ..
      EXTERNAL MA62AD,MA62BD,MA62CD,MA62ID,MA62JD,MA62PD
C     ..
C
C*** Call to MA62ID
      CALL MA62ID(ICNTL,CNTL,ISAVE)

C Read in the element data.
C NELT is number of elements.
C ELTVAR contains lists of the variables belonging to the finite
C elements, with those for element 1 preceding those for element
C 2, and so on. ELTPTR points to the position in ELTVAR
C of the first variable in element I. NZ is the total number
C of entries in the element lists.

      READ (5,FMT=*) NELT
      READ (5,FMT=*) (ELTPTR(I),I=1,NELT+1)
      NZ = ELTPTR(NELT+1) - 1
      READ (5,FMT=*) (ELTVAR(I),I=1,NZ)

C Calls to MA62AD to establish when each variable is fully assembled
      DO 20 IELT = 1,NELT
         NVAR = ELTPTR(IELT+1) - ELTPTR(IELT)
         JSTRT = ELTPTR(IELT)
         DO 10 J = 1,NVAR
            IVAR(J) = ELTVAR(JSTRT+J-1)
   10    CONTINUE
C*** Call to MA62AD
         CALL MA62AD(NVAR,IVAR,NDF,LAST,LENLST,ICNTL,ISAVE,INFO)
         IF (INFO(1).LT.0) GO TO 100
   20 CONTINUE

C Calls to MA62JD to determine file sizes
      DO 40 IELT = 1,NELT
         NVAR = ELTPTR(IELT+1) - ELTPTR(IELT)
         JSTRT = ELTPTR(IELT)
         DO 30 J = 1,NVAR
            IVAR(J) = ELTVAR(JSTRT+J-1)
   30    CONTINUE
C*** Call to MA62JD
         CALL MA62JD(NVAR,IVAR,NDF,LAST,ICNTL,ISAVE,INFO,RINFO)
         IF (INFO(1).LT.0) GO TO 100
   40 CONTINUE
      WRITE (6,FMT=9090)
      WRITE (6,FMT=9040) (INFO(I),I=1,6)
      WRITE (6,FMT=9070) (RINFO(I),I=1,2)

C Call to MA62PD to open direct access data sets.
C Choose stream numbers and file sizes.
      ISTRM(1) = 8
      ISTRM(2) = 9
      LENBUF(1) = 30
      LENBUF(2) = 30
C*** Call to MA62PD
      CALL MA62PD(ISTRM,FILNAM,LENBUF,ICNTL,ISAVE,INFO)
      IF (INFO(1).LT.0) GO TO 100

C Input elemental matrices and right-hand sides.
C VALCRD is the number of numerical values to be input.
C VALUE contains lists of the numerical values in the elemental
```

```
C matrices, with element 1 preceding element 2, and so on.
C Since the elemental matrices are symmetric only the upper
C triangular part is stored.

      READ (5,FMT=*) VALCRD
      READ (5,FMT=*) (VALUE(I),I=1,VALCRD)
C
C RHSCRD is the number of right-hand side numerical values to
C be input.
C RHSVAL contains lists of the right-hand side numerical values
C corresponding to each of the elements in order.
C
      READ (5,FMT=*) RHSCRD
      READ (5,FMT=*) (RHSVAL(I),I=1,RHSCRD)
C
C Prepare to call MA62BD.
      LAVAR = MAXE
      NRHSB = 1
      LX = NDFMAX
      NFRONT = INFO(6)
      LW = 3 + LENBUF(1) + NFRONT* (NFRONT+NRHSB)
      LIW = LENBUF(2) + 3*NFRONT
      IF (LW.GT.LWMAX .OR. LIW.GT.LIWMAX) GO TO 110
C
      KSTRT = 1
      DO 70 IELT = 1,NELT
         NVAR = ELTPTR(IELT+1) - ELTPTR(IELT)
         JSTRT = ELTPTR(IELT)
         DO 60 J = 1,NVAR
            IVAR(J) = ELTVAR(JSTRT+J-1)
            DO 50 K = J,NVAR
               AVAR(J,K) = VALUE(KSTRT)
               KSTRT = KSTRT + 1
   50       CONTINUE
            RHS(J,1) = RHSVAL(JSTRT+J-1)
   60    CONTINUE
C*** Call to MA62BD
         CALL MA62BD(NVAR,IVAR,NDF,LAST,LAVAR,AVAR,NRHSB,RHS,LX,X,
     +               LENBUF,LW,W,LIW,IW,ICNTL,CNTL,ISAVE,INFO,RINFO)
         IF (INFO(1).LT.0) GO TO 100
   70 CONTINUE
C
C  Solution is in first NDF locations of X
      WRITE (6,FMT=9100)
      WRITE (6,FMT=9000)
      WRITE (6,FMT=9010) (X(I,1),I=1,NDF)
      WRITE (6,FMT=9050) INFO(1),INFO(2),(INFO(I),I=9,13)
      WRITE (6,FMT=9080) (RINFO(I),I=3,9)

C Now read in further right-hand sides
C
      NRHSC = 2
      DO 80 J = 1,2
         READ (5,FMT=*) (X(I,J),I=1,NDF)
   80 CONTINUE
      NPIV = INFO(5)
      LW = LENBUF(1) + NFRONT*NRHSC + NPIV*(NFRONT+NRHSC)
      LIW = LENBUF(2) + NFRONT + 4
      IF (LW.GT.LWMAX .OR. LIW.GT.LIWMAX) GO TO 100
C*** Call to MA62CD
      CALL MA62CD(NRHSC,NDFMAX,X,LW,W,LIW,IW,ICNTL,ISAVE,INFO)
```

```
      IF (INFO(1).LT.0) GO TO 100
C
C  Solution for J-th right-hand side is in X(.,J), J=1,NRHS
      DO 90 J = 1,NRHSC
         WRITE (6,FMT=9060) J
         WRITE (6,FMT=9010) (X(I,J),I=1,NDF)
   90 CONTINUE
      GO TO 110
C Print appropriate fatal error diagnostic
  100 WRITE (6,FMT=9020)
      WRITE (6,FMT=9030) INFO(1)
  110 STOP
C
 9000 FORMAT (' The solution is:')
 9010 FORMAT (/6G12.4)
 9020 FORMAT (/ ,'Error return')
 9030 FORMAT (' INFO(1) = ',I3)
 9040 FORMAT (/' INFO(1:6)    = ',/10I5)
 9050 FORMAT (/' INFO(1,2,9:13)    = ',/10I5)
 9060 FORMAT (/3X,'The solution for right hand side number',I2,' is:')
 9070 FORMAT (/' RINFO(1:2)    = ',/4(1PD15.3))
 9080 FORMAT (/' RINFO(3:9)    = ',/4(1PD15.3))
 9090 FORMAT (/' On exit from MA62JD:')
 9100 FORMAT (/' On exit from MA62BD:')
      END
```

The input data used for this problem is:

```
 4
 1    3    5    9   13
 4    5    5    6    4    5    1    2    5    6    2    3
26
 2.   1.   7.   3.   2.   8.   4.   3.   2.   3.   1.   3.
 2.   6.   1.   5.   2.   1.   8.   3.   3.   2.   2.   2.
 5.   4.
12
 3.   8.   5.  10.  12.   9.  12.  11.  14.   8.  17.  14.
-6.  -4.   0.   3.  -2.   8.
31. 104.  49.  52. 131.  91.
```

This produces the following output:

```
 On exit from MA42JD:

 INFO(1:6)    =
    0    0    6    2    4    4

 RINFO(1:2)    =
      1.800D+01      2.500D+01

 On exit from MA42BD:
 The solution is:

   1.000       1.000       1.000       1.000       1.000       1.000

 INFO(1,2,9:13)    =
    0    0    1    1    1    1    1

 RINFO(3:9)    =
      2.400D+01      0.000D+00      1.800D+01      2.500D+01
      1.035D+01      6.400D+01      3.651D+00

   The solution for right hand side number 1 is:

  -1.000       1.000      -1.000       1.000      -1.000       1.000

   The solution for right hand side number 2 is:

   1.000       2.000       3.000       4.000       5.000       6.000
```