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Epistemic Boolean Games Based on a Logic of Visibility and Control

Andreas Herzig[1], Emiliano Lorini[1], Faustine Maffre[1], Francois Schwarzentruber[2]
[1]IRIT, University of Toulouse, France
[2]IRISA, ENS Rennes, France

Abstract

We analyse epistemic boolean games in a computationally grounded dynamic epistemic logic. The agents' knowledge is determined by what they see, including higher-order visibility: agents may observe whether another agent observes an atom or not. The agents' actions consist in modifying the truth values of atoms. We provide an axiomatisation of the logic, establish that the model checking problem is in PSPACE, and show how one can reason about equilibria in epistemic boolean games.

1 Introduction

Modeling knowledge and decision making are central in multi-agent systems. A key issue is to provide models that are natural and easy to build. Boolean games [Harrenstein *et al.*, 2001; Bonzon *et al.*, 2006; Dunne *et al.*, 2008] were designed with that motivation. There, actions are determined by the agents' control of propositional variables. In that sense, boolean games are easy to build. More recently, they were extended with epistemic goals [Ågotnes *et al.*, 2013] where knowledge is determined by the agents' observation of propositional variables. Several approaches to reasoning about knowledge building on such propositional observability exist in the literature, e.g. [Lomuscio *et al.*, 2000; Su *et al.*, 2007; van der Hoek *et al.*, 2012; van Benthem *et al.*, 2015]. There, epistemic accessibility relations are built in a natural way: agent i cannot distinguish w from w' when all the variables i sees have the same truth value at w and w'. A drawback of these approaches is that who sees what is common knowledge: for every variable p and agent i, if i sees p then it is common knowledge that i knows whether p is true; if i does not see p then i's ignorance about p is common knowledge.

There exist only few attempts avoiding this. Some of them suppose that observation comes from geometrical constraints: an agent must physically see another agent in the space [Balbiani *et al.*, 2013a; Gasquet *et al.*, 2014]. A more recent attempt is Dynamic Epistemic Logic of Propositional Assignment and Observation DEL-PAO [Herzig *et al.*, 2015]. It is not grounded on geometrical constraints and thereby allows more flexibility in the description of higher-order visibility. Its language contains special *higher-order visibility atoms*: the atom $S_i p$ expresses that i sees the value of p; $S_i S_j p$ expresses that i sees whether j sees p; and so on. It also contains dynamic operators that are based on assignments of propositional variables. The resulting programs modify not only the world, but also the agents' knowledge.

We extend DEL-PAO by adding further special atoms: *control atoms*. The atom $C_i p$ is true if agent i controls the value of p; $C_i C_j p$ is true if i controls whether agent j controls p; etc. We allow nesting: $C_i S_j p$ means that i controls j's visibility of the truth value of p; $S_i C_j p$ that i sees whether j controls p; and so on. A given set of visibility and control atoms determines a Kripke structure in a natural way. This allows to interpret epistemic operators $K_i \varphi$ ("i knows that φ") and strategic operators $\Diamond_J \varphi$ ("agents in J can change values of variables they control while the other agents do not act so that φ is true"). The latter is an operator of "ceteris paribus strategic ability" in the sense of [van der Hoek and Wooldridge, 2005; Herzig, 2015]. We claim that it is easier for a modeler to describe a situation with visibility and control atoms instead of designing a Kripke structure from scratch. We call our logic DEL-PAOC: Dynamic Epistemic Logic of Propositional Assignment, Observation and Control. In DEL-PAOC, epistemic and strategic operators can be reduced to programs, leading to a sound and complete axiomatization by reduction axioms. Our logic allows to capture epistemic boolean games. While control in such games is both *exclusive and exhaustive*—each atom is controlled by exactly one agent—we do not impose this in our logic, but can nevertheless reason about it.

The paper is organized as follows. Sections 2 and 3 introduce language and semantics. Sections 4 and 5 provide an axiomatization and complexity results. Section 6 applies DEL-PAOC to epistemic boolean games. Section 7 concludes.

2 Language

Let $Prop$ be a countable non-empty set of propositional variables and let Agt be a finite non-empty set of agents. Atomic formulas of our language are sequences of visibility and control operators followed by propositional variables.

The set of *observability operators* is

$$OBS = \{S_i : i \in Agt\} \cup \{JS\},$$

where S_i stands for individual visibility of agent i and JS stands for joint visibility of all agents. The set of *control operators* is

$$CTRL = \{C_i : i \in Agt\},$$

where C_i stands for control of agent i. The set of all sequences of visibility and control operators is noted $(OBS \cup CTRL)^*$ and the set of all non-empty sequences is noted $(OBS \cup CTRL)^+$. We use $\sigma, \sigma', \dots$ for elements of $(OBS \cup CTRL)^*$. Finally, the set of atomic formulas is:

$$ATM = \{\sigma\, p : \sigma \in (OBS \cup CTRL)^*, p \in Prop\}.$$

The language of programs and formulas of DEL-PAOC is defined by the following grammar:

$$\pi ::= +\alpha \mid -\alpha \mid (\pi;\pi) \mid (\pi \sqcup \pi) \mid \varphi?$$
$$\varphi ::= \alpha \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_i\varphi \mid CK\varphi \mid \Diamond_J\varphi \mid [\pi]\varphi$$

where α ranges over ATM, i over Agt and J over 2^{Agt}. Atomic programs are assignments to atoms: $+\alpha$ makes α true and $-\alpha$ makes α false. Complex programs are constructed with dynamic logic operators: $\pi;\pi'$ is sequential composition, $\pi \sqcup \pi'$ is nondeterministic choice, and $\varphi?$ is test. $[\pi]\varphi$ reads "after all executions of π, φ holds"; $K_i\varphi$ and $CK\varphi$ read "i knows that φ on the basis of what she observes" and "all agents commonly know that φ on the basis of what they jointly observe"; $\Diamond_J\varphi$ reads "agents in J can achieve φ by modifying variables they control if other agents do not act", which is ceteris paribus strategic ability [van der Hoek and Wooldridge, 2005; Herzig, 2015].

The length of formulas φ and programs π, noted $|\varphi|$ and $|\pi|$, is the number of symbols used to write them down, where we do not count [,], and parentheses and consider that JS, CK, agent names and propositional variables have length 1. The set of atomic formulas of ATM occurring in the formula φ is noted $ATM(\varphi)$; the set $ATM(\pi)$ is defined similarly.

The other boolean operators $\top$, $\bot$, $\vee$, $\rightarrow$ and $\leftrightarrow$ are defined as usual. Moreover, $\widehat{K}_i\varphi$ abbreviates $\neg K_i \neg\varphi$, $\Box_J\varphi$ abbreviates $\neg\Diamond_J\neg\varphi$ and $\langle\pi\rangle\varphi$ abbreviates $\neg[\pi]\neg\varphi$. The program *skip* abbreviates $\top?$ and *fail* abbreviates $\bot?$.

3 Semantics

Our semantics are based on valuations, or worlds, that are noted w, w', u, v, etc. They are subsets of the set of atomic formulas ATM. Then the (unique) model $W = 2^{ATM}$ contains every possible valuation. As it is unique we keep it implicit and only mention the current valuation w in the truth conditions. We write $w(\alpha) = w'(\alpha)$ when α has the same value in w and w', i.e., when either $\alpha \in w$ and $\alpha \in w'$, or $\alpha \notin w$ and $\alpha \notin w'$.

3.1 Strategic Relations

The relation R_J is an accessibility relation for the ceteris paribus strategic operator $\Diamond_J$. It is defined as follows:

$$wR_Jw' \text{ iff } C_i\alpha \notin w \text{ for all } i \in J \text{ implies } w(\alpha) = w'(\alpha).$$

Intuitively, only atoms controlled by at least one agent in J can be modified by the coalition J between w and w'.

3.2 Epistemic Relations and Introspection

We now define accessibility relations $\rightsquigarrow_i$ and $\rightsquigarrow_{Agt}$ allowing to interpret the epistemic operators K_i and CK. These relations are determined by visibility information: two valuations w and w' are related by $\rightsquigarrow_i$ if every α that i sees at w has the same value, and similarly for $\rightsquigarrow_{Agt}$. More formally:

$$w \rightsquigarrow_i w' \text{ iff } S_i\alpha \in w \text{ implies } w(\alpha) = w'(\alpha)$$
$$w \rightsquigarrow_{Agt} w' \text{ iff } JS\alpha \in w \text{ implies } w(\alpha) = w'(\alpha)$$

The relations $\rightsquigarrow_i$ and $\rightsquigarrow_{Agt}$ are reflexive, but they are neither transitive nor symmetric.[1] Transitivity and symmetry are however standard requirements for a logic of knowledge. We will see in the sequel how these constraints can be guaranteed by means of appropriate introspection constraints.

A valuation $w \in 2^{ATM}$ is *introspective* if and only if the following hold, for every $\alpha \in ATM$ and $i \in Agt$:

$$S_iS_i\alpha \in w \quad \text{(C1)}$$
$$JS\,JS\alpha \in w \quad \text{(C2)}$$
$$JSS_iS_i\alpha \in w \quad \text{(C3)}$$
$$\text{if } JS\alpha \in w, \text{ then } S_i\alpha \in w \quad \text{(C4)}$$
$$\text{if } JS\alpha \in w, \text{ then } JSS_i\alpha \in w \quad \text{(C5)}$$

The set of all introspective valuations is noted $INTR$.

Together, the last two constraints ensure that when $JS\alpha \in w$ then $\sigma\,\alpha \in w$ for $\sigma \in OBS^+$. This motivates the following relation of *introspective consequence* between atoms:

$$\alpha \Rightarrow \beta \text{ iff either } \alpha = \beta, \text{ or } \alpha = JS\alpha' \text{ and } \beta = \sigma\,\alpha' \text{ for some } \sigma \in OBS^+.$$

Introspective valuations can be characterized as valuations that are closed under introspective consequence.

An atom $\alpha \in ATM$ is *valid in INTR* if and only if α belongs to every valuation in $INTR$. It is shown in [Herzig *et al.*, 2015] that α is valid in $INTR$ iff α is of the form either $\sigma\, S_iS_i\alpha$ with $\sigma \in OBS^*$, or $\sigma\, JS\alpha$ with $\sigma \in OBS^+$. It is further shown there that the relation $\rightsquigarrow_{Agt}$ and every relation $\rightsquigarrow_i$ are equivalence relations on $INTR$.

We do not have any relations between visibility and control. One might e.g. require that control implies visibility. We will comment on that in the conclusion.

3.3 Truth Conditions and Validity

Given an introspective valuation w, our update operations add or remove atoms. This requires some care: we want the resulting valuation to be introspective. For example, removing S_iS_ip should be impossible. Another example is when w does not contain S_ip: then $w \cup \{JSp\}$ would violate (C4). So when adding an atom to w we also have to add all its *positive consequences*. Symmetrically, when removing an atom we also have to remove its *negative consequences*. Let us define the following:

$$Eff^+(\alpha) = \{\beta \in ATM : \alpha \Rightarrow \beta\}$$
$$Eff^-(\alpha) = \{\beta \in ATM : \beta \Rightarrow \alpha\}$$

Clearly, when w is introspective then both $w \cup Eff^+(\alpha)$ and $w \setminus Eff^-(\alpha)$ are so, too (unless α is valid in $INTR$).

Truth conditions are standard for boolean operators and:

$w \models \alpha \qquad$ iff $\alpha \in w$

[1] For example, $\emptyset \rightsquigarrow_i w$ for every $w \subseteq ATM$, while $w \rightsquigarrow_i \emptyset$ fails to hold as soon as there is a p such that p and S_ip are in w.

$w \models K_i\varphi$ iff $\forall w' \in INTR$ such that $w \rightsquigarrow_i w', w' \models \varphi$
$w \models CK\varphi$ iff $\forall w' \in INTR$ such that $w \rightsquigarrow_{Agt} w', w' \models \varphi$
$w \models \Diamond_J\varphi$ iff $\exists w' \in INTR$ such that wR_Jw' and $w' \models \varphi$
$w \models [\pi]\varphi$ iff $\forall w'$ such that $wR_\pi w', w' \models \varphi$

where R_π is a binary relation on valuations that is defined as:

$wR_{+\alpha}w'$ iff $w' = w \cup Eff^+(\alpha)$
$wR_{-\alpha}w'$ iff $w' = w \backslash Eff^-(\alpha)$ and α not valid in $INTR$
$wR_{\pi_1;\pi_2}w'$ iff there exists u such that $wR_{\pi_1}u$ and $uR_{\pi_2}w'$
$wR_{\pi_1 \sqcup \pi_2}w'$ iff $wR_{\pi_1}w'$ or $wR_{\pi_2}w'$
$wR_{\varphi?}w'$ iff $w = w'$ and $w \models \varphi$

Lemma 1 ([Herzig *et al.*, 2015]). *Let $w \in INTR$ and $wR_\pi w'$. Then $w' \in INTR$.*

Truth conditions are defined on any valuation, including non-introspective ones. However, the intended behavior of the relations R_π is not guaranteed in the latter case.

When $w \models \varphi$ we say that w is a *model* of φ. The set of (not necessarily introspective) models of φ is noted $\|\varphi\|$.

We generalise validity in $INTR$ from atoms to formulas: a formula φ is *valid in $INTR$* if $INTR \subseteq \|\varphi\|$. It is *plainly valid* if $\|\varphi\| = 2^{ATM}$. Satisfiability in $INTR$ and *plain* satisfiability are defined analogously. For example, $S_ip \leftrightarrow (K_ip \vee K_i\neg p)$ is $INTR$-valid, but not plainly so.

We note that our epistemic operators behave differently from the standard operators; for a discussion see [Herzig *et al.*, 2015; Charrier *et al.*, 2016].

4 Axiomatics

We are going to reduce epistemic and strategic operators to programs. We then show how to reduce dynamic operators.

4.1 Replacement of Equivalents

The rule of replacement of equivalents preserves validity (both, plain and introspective). It allows us to apply the reduction axioms that we are going to introduce in the rest of the section to the subformulas of a given formula.

Proposition 1. *Let φ' be obtained from φ by replacing some occurrence of χ in φ by χ'. Let w be a valuation. If $w \models \chi \leftrightarrow \chi'$ then $w \models \varphi \leftrightarrow \varphi'$.*

4.2 Eliminating Epistemic Operators

The epistemic operators can be eliminated with the help of the following programs:

$$\mathsf{varyIfNotSeen}(i, \alpha) = S_i\alpha? \sqcup (\neg S_i\alpha?; (+\alpha \sqcup -\alpha))$$
$$\mathsf{varyIfNotSeen}(i, A) = \mathsf{varyIfNotSeen}(i, \alpha_1); \cdots ; \mathsf{varyIfNotSeen}(i, \alpha_n)$$

where $A = \{\alpha_1, \ldots, \alpha_n\}$ is a finite set of atoms that we suppose ordered in some arbitrary way. The former program tests whether α is seen by agent i; if this is the case then nothing is done, otherwise the truth value of α is nondeterministically set to either true or false. Similarly for the set of all agents:

$$\mathsf{varyIfNotSeen}(Agt, \alpha) = JS\alpha? \sqcup (\neg JS\alpha?; (+\alpha \sqcup -\alpha))$$
$$\mathsf{varyIfNotSeen}(Agt, A) = \mathsf{varyIfNotSeen}(Agt, \alpha_1); \cdots ; \mathsf{varyIfNotSeen}(Agt, \alpha_n)$$

In both cases, the program is $skip$ when the set A is empty.

Proposition 2. *Let φ be without epistemic operators. The following equivalences are valid in $INTR$:*

$$K_i\varphi \leftrightarrow [\mathsf{varyIfNotSeen}(i, ATM(\varphi))]\varphi$$
$$CK\varphi \leftrightarrow [\mathsf{varyIfNotSeen}(Agt, ATM(\varphi))]\varphi$$

Proof. The program $\mathsf{varyIfNotSeen}(i, ATM(\varphi))$ is the syntactic counterpart of the definition of $\rightsquigarrow_i$, restricted to relevant atomic variables $ATM(\varphi)$. Lemma 1 ensures that we do not leave the set of introspective valuations. □

4.3 Eliminating Strategic Operators

Strategic operators $\Diamond_J$ are reduced to programs in a similar way:

$$\mathsf{varyIfCtrl}(J, \alpha) = \left(\left(\bigvee_{i \in J} C_i\alpha\right)?; (+\alpha \sqcup -\alpha)\right) \sqcup \neg\left(\bigvee_{i \in J} C_i\alpha\right)?$$
$$\mathsf{varyIfCtrl}(J, A) = \mathsf{varyIfCtrl}(J, \alpha_1); \ldots ; \mathsf{varyIfCtrl}(J, \alpha_n)$$

where $A = \{\alpha_1, \ldots, \alpha_n\}$ is a finite set of atoms that we suppose ordered such that $k \leq l$ implies that $|\alpha_k| \leq |\alpha_l|$.[2]

Again, the program is $skip$ when A is empty.

Proposition 3. *Let φ be without epistemic operators. The following equivalence is valid in $INTR$:*

$$\Diamond_J\varphi \leftrightarrow \langle \mathsf{varyIfCtrl}(J, ATM(\varphi))\rangle\varphi$$

Proof. The program $\mathsf{varyIfCtrl}(J, ATM(\varphi))$ is the syntactic counterpart—restricted to relevant atomic variables $ATM(\varphi)$—of the relation R_J between valuations that is used in order to interpret $\Diamond_J\varphi$. Lemma 1 ensures that we do not leave the set of introspective valuations. □

For example, the formula

$$\Diamond_{\{i\}}(p \vee q) \leftrightarrow \langle \mathsf{varyIfCtrl}(\{i\}, \{p, q\})\rangle(p \vee q)$$

is $INTR$-valid, where $\mathsf{varyIfCtrl}(\{i\}, \{p, q\})$ is the program

$$\big((C_ip?; (+p \sqcup -p)) \sqcup \neg C_ip?\big);$$
$$\big((C_iq?; (+q \sqcup -q)) \sqcup \neg C_iq?\big).$$

So epistemic and strategic operators can be removed, starting from the innermost and using propositions 2 and 3. The result is a formula containing only dynamic operators. The next series of axioms will allow to reduce these.

4.4 Reduction Axioms for Programs

Reduction axioms for programs are standard; we only have to deal with the special case of removing an $INTR$-valid atom.

[2] This is necessary: suppose C_iC_ip is true, and both C_iC_ip and C_ip are in A. Then when $\mathsf{varyIfCtrl}(\{i\}, C_ip)$ is executed, it will modify C_ip non-deterministically, thus being problematic when testing it in $\mathsf{varyIfCtrl}(\{i\}, p)$ afterwards.

Proposition 4. *The following equivalences are plainly valid.*

$$[\pi;\pi']\varphi \leftrightarrow [\pi][\pi']\varphi \qquad (Red_;)$$

$$[\pi \sqcup \pi']\varphi \leftrightarrow [\pi]\varphi \land [\pi']\varphi \qquad (Red_\sqcup)$$

$$[\varphi?]\varphi' \leftrightarrow \varphi \to \varphi' \qquad (Red_?)$$

$$[+\alpha]\neg\varphi \leftrightarrow \neg[+\alpha]\varphi \qquad (Red_{+\alpha,\neg})$$

$$[-\alpha]\neg\varphi \leftrightarrow \begin{cases} \top \text{ if } \alpha \text{ is valid in } INTR \\ \neg[-\alpha]\varphi \text{ otherwise} \end{cases} \qquad (Red_{-\alpha,\neg})$$

$$[+\alpha](\varphi\land\varphi') \leftrightarrow [+\alpha]\varphi \land [+\alpha]\varphi' \qquad (Red_{+\alpha,\land})$$

$$[-\alpha](\varphi\land\varphi') \leftrightarrow [-\alpha]\varphi \land [-\alpha]\varphi' \qquad (Red_{-\alpha,\land})$$

$$[+\alpha]\beta \leftrightarrow \begin{cases} \top & \text{if } \alpha \Rightarrow \beta \\ \beta & \text{otherwise} \end{cases} \qquad (Red_{+\alpha})$$

$$[-\alpha]\beta \leftrightarrow \begin{cases} \bot & \text{if } \alpha \text{ is not valid in } INTR \text{ and } \beta\Rightarrow\alpha \\ \top & \text{if } \alpha \text{ is valid in } INTR \\ \beta & \text{otherwise} \end{cases} \qquad (Red_{-\alpha})$$

4.5 Axiomatization

The axiomatization of DEL-PAOC is given by:

- the axioms of CPL (Classical Propositional Logic);
- the reduction axioms for epistemic, strategic, and dynamic operators of propositions 2, 3, and 4;
- the introspection axioms respectively noted (Vis_{C1})-(Vis_{C5}): $S_iS_i\alpha$, $JSJS\alpha$, $JSS_iS_i\alpha$, $JS\alpha \to S_i\alpha$ and $JS\alpha \to JSS_i\alpha$;
- the rule of Modus Ponens and the rules of inference for K_i, CK, and $[\pi]$:

$$\frac{\varphi \leftrightarrow \varphi'}{K_i\varphi \leftrightarrow K_i\varphi'} \qquad \frac{\varphi \leftrightarrow \varphi'}{CK\varphi \leftrightarrow CK\varphi'} \qquad \frac{\varphi \leftrightarrow \varphi'}{[\pi]\varphi \leftrightarrow [\pi]\varphi'}$$

Theorem 1. *The axiomatization of* DEL-PAOC *is sound and complete w.r.t. the set of introspective valuations.*

Proof. The reduction axioms for epistemic, strategic, and dynamic operators allow to eliminate all modal operators. The resulting formula is $INTR$-equivalent to the original formula. It is $INTR$-valid if and only if it is a logical consequence in classical propositional logic of the introspection axioms. □

5 Complexity

We define the relevant atoms of a formula φ and a program π by structural induction as follows:

$$RATM(\alpha) = \{\alpha\}$$
$$RATM(K_i\varphi) = RATM(\varphi) \cup \{S_i\alpha : \alpha \in RATM(\varphi)\}$$
$$RATM(CK\varphi) = RATM(\varphi) \cup \{JS\alpha : \alpha \in RATM(\varphi)\}$$
$$RATM(\Diamond_J\varphi) = RATM(\varphi) \cup \{C_i\alpha : \alpha \in RATM(\varphi)\}$$

and homomorphic otherwise. We observe that the cardinality of $RATM(\varphi)$ can be exponential in the length of φ: for example, the cardinality of the set of atoms $RATM(K_{i_1} \dots K_{i_n}p)$ is in 2^n.

Given a set of atoms A, a valuation w is *introspective w.r.t.* A if (i) if α is valid in $INTR$ and $\alpha \in A$ then $\alpha \in w$ and (ii) if $\alpha \in w$ and $\alpha' \in Eff^+(\alpha) \cap A$ then $\alpha' \in w$.

Proposition 5. *For every* DEL-PAOC *formula* φ, $w \models \varphi$ *iff* $w \cap RATM(\varphi) \models \varphi$.

The DEL-PAOC model checking problem is then defined for valuations that are introspective w.r.t. the relevant atoms of the formula under concern:

- input: a DEL-PAOC-formula φ and a finite valuation w that is introspective w.r.t. $RATM(\varphi)$;
- output: yes if $w \models \varphi$, no otherwise.

Theorem 2. *The* DEL-PAOC *model checking problem is* PSPACE*-complete.*

Proof. For the upper bound, we give a reduction in polynomial time from the DEL-PAOC-model checking problem to the model checking problem of Dynamic Logic of Propositional Assignments DL-PA [Herzig *et al.*, 2011; Balbiani *et al.*, 2013b; 2014]. The reduction transforms a instance (w, φ) of our model checking problem into a instance $(w, tr(\varphi))$ of the DL-PA model checking problem. The translation tr is defined as follows: a DEL-PAOC-formula φ is transformed into a DL-PA-formula $tr(\varphi)$, that is, a DEL-PAOC-formula that does not contain any occurrence of K_i, CK and $\Diamond_J$ operators. We replace in φ any innermost subformula of the form $K_i\psi$, $CK\psi$ and $\Diamond_J\psi$ by, respectively,

$$[\mathsf{varyIfNotSeen}(i, ATM(\psi))]\psi,$$
$$[\mathsf{varyIfNotSeen}(Agt, ATM(\psi))]\psi,$$
$$\langle\mathsf{varyIfCtrl}(J, ATM(\psi))\rangle\psi.$$

We keep replacing until the resulting formula $tr(\varphi)$ does not contain any occurrence of K_i, CK and $\Diamond_J$ operators. By propositions 2 and 3, $tr(\varphi)$ is equivalent to φ.

We define the size of $\langle w, \varphi\rangle$ as the cardinality of w plus the length of φ. Note that the size of $tr(\varphi)$ is exponential in the size of φ but is polynomial in the size of the input $\langle w, \varphi\rangle$; actually the reduction can be implemented in polynomial time. As the model checking problem of DL-PA is in PSPACE [Balbiani *et al.*, 2014], the DEL-PAOC-model checking problem is in PSPACE, too.

For the lower bound, it suffices to observe that the model checking for DEL-PAO can be trivially reduced to the model checking for DEL-PAOC. □

6 Epistemic Boolean Games

We first recall boolean games and epistemic boolean games. We then show how to express in DEL-PAOC that a given valuation is a Nash equilibrium and whether a Nash equilibrium exists.

6.1 Background

Boolean games [Harrenstein *et al.*, 2001; Bonzon *et al.*, 2006; Dunne *et al.*, 2008] are games in which each player wants to achieve a certain goal that is represented by a propositional formula. They correspond to the specific subclass of normal-form games in which agents have binary preferences (i.e., payoffs are either 0 or 1) and are widely accepted as a useful and natural abstraction for reasoning about social interaction in multi-agent systems.

A boolean game is a tuple $(Agt, Prop^f, (\Psi_i)_{i\in Agt}, (\gamma_i)_{i\in Agt})$ where $Prop^f$ is a finite subset of $Prop$, each $\Psi_i \subseteq Prop^f$ is the set of variables agent i controls, and γ_i is a formula of the propositional language such that $ATM(\gamma_i) \subseteq Prop^f$. The latter expresses i's personal goal, i.e., the state of affairs i wants to achieve. *Exclusive and exhaustive control* is assumed: the sets Ψ_i partition the set of variables $Prop^f$. So the two conditions (i) $\Psi_i \cap \Psi_j = \emptyset$ if $i \neq j$ and (ii) $\bigcup_{i\in Agt} \Psi_i = Prop^f$ are both satisfied.

A *strategy* for an agent i, noted s_i, is an interpretation of variables controlled by i, that is to say, $s_i \subseteq \Psi_i$. The set of all possible strategies of agent i is denoted by Σ_i. Given a strategy s_i for each member of a coalition J, the induced strategy for J is $s_J = \bigcup_{i\in J} s_i$. A strategy s_{Agt} for Agt is called a *strategy profile*. It can be seen as an interpretation of the set of variables $Prop^f$.

Agent i's *utility function* maps every strategy profile to i's reward, depending on on the truth value of i's goal in s_{Agt}:

$$U_i(s_{Agt}) = \begin{cases} 1 & \text{if } s_{Agt} \models \gamma_i; \\ 0 & \text{otherwise.} \end{cases}$$

We can express solution concepts such as Nash equilibrium in a boolean game in our logic. Let $\langle s_i, s_{Agt\setminus\{i\}}\rangle$ be the strategy profile composed of the strategy s_i of i and of the strategy $s_{Agt\setminus\{i\}}$ of i's opponents. We say that s_i is a *best response* to $s_{Agt\setminus\{i\}}$ if and only if, for every strategy $s'_i \in \Sigma_i$, $U_i(\langle s'_i, s_{Agt\setminus\{i\}}\rangle) \leq U_i(\langle s_i, s_{Agt\setminus\{i\}}\rangle)$ holds. That is, every other strategy of i against the same strategy of opponents would not increase i's reward.

A strategy profile s_{Agt} is a *Nash equilibrium*, if and only if, for every $i \in Agt$, s_i is a best response to $s_{Agt\setminus\{i\}}$. This means every agent cannot get a bigger reward by choosing another strategy if the others do not change theirs.

6.2 Epistemic Boolean Games in DEL-PAOC

An interesting aspect of our logic DEL-PAOC is that it allows us to generalise boolean games to epistemic goals, in the sense of being goals about (own and other agents') knowledge. Agents' goals may now be *epistemic formulas*: DEL-PAOC formulas that are built from the propositional variables by the epistemic operators K_i and CK. For example, we can represent the fact that i wants j to know that p is true if and only if agent z knows that p is true by: $\gamma_i = K_j p \leftrightarrow K_z p$. Another example is i's goal that the agents have common knowledge that p, formally: $\gamma_i = CKp$.

We define ATM_{OBS} to be the subset of ATM which only contains propositional variables and atoms about observability of other atoms, that is, atoms beginning with a S_i:

$$ATM_{OBS} = Prop \cup \{S_i\sigma p \in ATM : i \in Agt, \sigma \in OBS^*\}.$$

An *epistemic boolean game* is a tuple $\mathcal{B} = (Agt, ATM^f_{OBS}, (\Psi_i)_{i\in Agt}, (\gamma_i)_{i\in Agt})$ where ATM^f_{OBS} is a finite subset of ATM_{OBS}, the sets $(\Psi_i)_{i\in Agt}$ partition ATM^f_{OBS}, and every γ_i is an epistemic formula such that $ATM(\gamma_i) \subseteq ATM^f_{OBS}$. Strategies of players and coalitions, utilities over strategy profiles, as well as the concept of best response and Nash equilibrium are defined exactly as in the context of standard boolean games.

6.3 Nash Equilibrium in DEL-PAOC

The Nash equilibria of a given epistemic boolean game can be characterised in the language of DEL-PAOC.

Proposition 6. *Let $\mathcal{B} = (Agt, ATM^f_{OBS}, (\Psi_i)_{i\in Agt}, (\gamma_i)_{i\in Agt})$ be an epistemic boolean game and let*

$$\mathsf{Nash} = \bigwedge_{i\in Agt} (\Diamond_{\{i\}}\gamma_i \to \gamma_i).$$

Then s_{Agt} is a Nash equilibrium for $\mathcal{B}$ if and only if

$$s_{Agt} \cup \{C_i\alpha : i \in Agt, \alpha \in \Psi_i\} \models \mathsf{Nash}.$$

The following proposition provides a characterization in the logic DEL-PAOC of the existence of a Nash equilibrium in a certain epistemic boolean game.

Proposition 7. *Let $\mathcal{B} = (Agt, ATM^f_{OBS}, (\Psi_i)_{i\in Agt}, (\gamma_i)_{i\in Agt})$ be an epistemic boolean game. Then $\mathcal{B}$ has at least one Nash equilibrium if and only if*

$$\{C_i\alpha : i \in Agt, \alpha \in \Psi_i\} \models \Diamond_{Agt}\mathsf{Nash}.$$

The preceding two propositions together with Theorem 2 about complexity of model checking for DEL-PAOC provide a complexity result both for membership problem and for existence problem of Nash equilibria in epistemic boolean games, respectively the problem of checking whether a certain strategy profile is a Nash equilibrium of a given epistemic boolean game and the problem of checking whether a given epistemic boolean game has at least one Nash equilibrium.

Theorem 3. *Membership problem and existence problem of Nash equilibria in epistemic boolean games are both in* PSPACE.

Proof. From propositions 6 and 7, both problems polynomially reduce to the DEL-PAOC model checking problem. ☐

Let us illustrate epistemic boolean games with an example of coordination game. We note that it cannot be expressed in the framework of [Ågotnes *et al.*, 2013].

Example 1. Suppose we have two agents 1 and 2 each of which knows a bit, respectively noted b_1 and b_2, that the other agent does not know. Both 1 and 2 have the same goal: $K_1\mathsf{b}_2 \leftrightarrow K_2\mathsf{b}_1$, that is, 1 wants 2 to know her secret only if 1 knows the secret of 2, and similarly for 2. Each agent can either talk or keep quiet; in other words, each agent has control of the other agent's visibility of her bit. The corresponding epistemic boolean game is $\mathcal{B}_\mathsf{b} = (Agt, ATM^f_{OBS}, (\Psi_i)_{i\in Agt}, (\gamma_i)_{i\in Agt})$ where $Agt = \{1, 2\}$, $ATM^f_{OBS} = \{\mathsf{b}_1, \mathsf{b}_2, S_2\mathsf{b}_1, S_1\mathsf{b}_2\}$, $\Psi_1 = \{\mathsf{b}_1, S_2\mathsf{b}_1\}$, $\Psi_2 = \{\mathsf{b}_2, S_1\mathsf{b}_2\}$ and $\gamma_1 = \gamma_2 = K_1\mathsf{b}_2 \leftrightarrow K_2\mathsf{b}_1$.

Two interesting Nash equilibria are the following. Either they both share their secrets: $s_{Agt} = \{\mathsf{b}_1, \mathsf{b}_2, S_2\mathsf{b}_1, S_1\mathsf{b}_2\}$; or they both remain silent: $s_{Agt} = \{\mathsf{b}_1, \mathsf{b}_2\}$.

Indeed, we can show that

$$\{\mathsf{b}_1, \mathsf{b}_2, S_2\mathsf{b}_1, S_1\mathsf{b}_2, C_1\mathsf{b}_1, C_2\mathsf{b}_2, C_1S_2\mathsf{b}_1, C_2S_1\mathsf{b}_2\} \models (\Diamond_{\{1\}}\gamma_1 \to \gamma_1) \wedge (\Diamond_{\{2\}}\gamma_2 \to \gamma_2)$$

and $\{\mathsf{b}_1, \mathsf{b}_2, C_1\mathsf{b}_1, C_2\mathsf{b}_2, C_1S_2\mathsf{b}_1, C_2S_1\mathsf{b}_2\} \models$

$$(\Diamond_{\{1\}}\gamma_1 \rightarrow \gamma_1) \wedge (\Diamond_{\{2\}}\gamma_2 \rightarrow \gamma_2).$$

Intuitively, each agent can only modify the other agent's visibility of her secret; if only one of them changes her strategy then it will break the equivalence of the goal.

6.4 Capturing Exclusive and Exhaustive Control in DEL-PAOC

Our logic DEL-PAOC does not impose that control is exclusive. However, the valuations in propositions 6 and 7 where we check whether Nash is true have exclusive and exhaustive control. In this section, we investigate how exclusive and exhaustive control can be built into DEL-PAOC.

First, the set of valuations with *exclusive and exhaustive control* is:

$$\begin{aligned} EX = \{w \in 2^{ATM} : & \text{ for every } \alpha \in ATM, \\ & \text{there is an } i \in Agt \text{ such that } C_i\alpha \in w \\ & \text{and for all } j \in Agt \setminus \{i\}, C_j\alpha \notin w\}. \end{aligned}$$

The set EX is therefore the set of valuations where

$$\Phi_{EX}(\alpha) = \Big(\bigwedge_{i,j \in Agt, j \neq i} \neg(C_i\alpha \wedge C_j\alpha) \Big) \wedge \Big(\bigvee_{i \in Agt} C_i\alpha \Big)$$

is true for every $\alpha \in ATM$.

Just as for introspective valuations, we would like to stay in valuations with exclusive control when executing our programs. Direct addition or removal of atoms of the form $C_i\alpha$ may lead to worlds outside of EX. We therefore require that control changing programs take the form

$$i \overset{\alpha}{\rightsquigarrow} j = C_i\alpha?; -C_i\alpha; +C_j\alpha,$$

reading "i passes the control of α to j".

Lemma 2. *Let π be a program built from nondeterministic choice, sequences, tests and programs of the form $i \overset{\alpha}{\rightsquigarrow} j$. Let $w \in EX$ and $wR_\pi w'$. Then $w' \in EX$.*

Proof. When $i \overset{\alpha}{\rightsquigarrow} j$ is executed, α becomes controlled by j only and control of the other atoms remains unchanged. □

In epistemic boolean games, exclusive and exhaustive control should not only be preserved by the execution of programs, it should also be common knowledge. We can tune DEL-PAOC to make it common knowledge that control is exclusive and exhaustive. We modify the semantics of DEL-PAOC so that we restrict to valuations in EX:

$w \models K_i\varphi$ iff $\forall w' \in INTR \cap EX$ s.th. $w \rightsquigarrow_i w', w' \models \varphi$

$w \models CK\varphi$ iff $\forall w' \in INTR \cap EX$ s.th. $w \rightsquigarrow_{Agt} w', w' \models \varphi$

We then have to add a test to the programs of Section 4.2 reducing the epistemic operators.

Proposition 8. *Let $\Phi_{EX}(\varphi) = \bigwedge_{\alpha \in RATM(\varphi)} \Phi_{EX}(\alpha)$. Then for every $w \in INTR \cap EX$:*

$w \models CK\ \Phi_{EX}(\alpha)$, *for every atom α;*

$w \models K_i\varphi \leftrightarrow [\mathsf{varyIfNotSeen}(i, ATM(\varphi)); \Phi_{EX}(\varphi)?]\varphi$;

$w \models CK\varphi \leftrightarrow [\mathsf{varyIfNotSeen}(Agt, ATM(\varphi)); \Phi_{EX}(\varphi)?]\varphi$.

Proof. The test $\Phi_{EX}(\varphi)?$ ensures we stay in EX. □

Moreover, under the new truth conditions all the S5 axioms are valid in $INTR \cap EX$. In contrast, plain validity fails for formulas such as the instance of the truth axiom $K_i\Phi_{EX}(\alpha) \rightarrow \Phi_{EX}(\alpha)$.

6.5 Discussion

In [Ågotnes *et al.*, 2013], an alternative concept of epistemic boolean game is introduced and both membership and existence problems for Nash equilibrium in this class of games are studied. However, there is a major difference with our epistemic boolean games: in our class of epistemic boolean games, agents can (i) affect the truth value of propositional variables, and (ii) modify the visibility conditions of propositional variables, including higher-order visibility. For instance, an agent can play a strategy to ensure that another agent can see the truth value of a given propositional variable p or to ensure that another agent can see whether another agent can see the truth value of p, and so on. In Ågotnes et al.'s class, agents can only affect the truth value of propositional variables and, indirectly, the knowledge of those agents' who can see the truth value of these variables; but agents cannot modify the visibility conditions of propositional variables. In other words, in Ågotnes et al.'s class of epistemic boolean games visibility conditions remain static, whereas in our class they can change. From this perspective, their class of epistemic boolean games can be seen as the subclass of ours where the condition $ATM^f_{OBS} \subseteq Prop$ holds (i.e., agents can only control propositional variables but cannot control visibility of propositional variables).

Ågotnes et al. show that for their class of epistemic boolean games, membership and existence of Nash equilibria are PSPACE-complete. In Section 6.3, we have given a complexity result for a more general class of epistemic boolean games.

7 Conclusion

We have studied a dynamic epistemic logic of visibility and control in which one can deduce strategic abilities of coalitions of agents. It accounts for concepts from boolean games such as the existence of a Nash equilibrium that can be extended to epistemic boolean games in a straightforward way. While the complexity of DEL-PAOC satisfiability is open, we have proved that model checking is in PSPACE, which involves reducing the infinite models to finite models involving only the relevant atoms. It follows that the membership problem and existence problem of a Nash equilibria in epistemic boolean games are both in PSPACE. It remains to establish the lower bound.

We claim that our framework can be used in practice easily since visibility information and control information corresponds to what an agents obtain from their sensors.

An interesting variant involving interaction between knowledge and strategies is the requirement 'control implies visibility'. One way of guaranteeing this is to change the condition $S_i\alpha?$ in the program $\mathsf{varyIfNotSeen}(i, \alpha) = S_i\alpha? \sqcup (\neg S_i\alpha?; (+\alpha \sqcup -\alpha))$ of Proposition 2 by $(C_i\alpha \wedge S_i\alpha)?$; other constraints can be implemented similarly. A more principled way is to further restrict the set of introspective valuations by imposing constraints such as $S_iC_i\alpha$, for every i and α. However, the difficulty here is to redefine $INTR$ such that Lemma 1 still holds. In future work we plan to extend our analysis to iterated boolean games and more generally to repeated interaction.

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