Towards Convolutional Neural Networks Compression via Global Error Reconstruction

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Abstract

In recent years, convolutional neural networks (CNNs) have achieved remarkable success in various applications such as image classification, object detection, object parsing and face alignment. Such CNN models are extremely powerful to deal with massive amounts of training data by using millions and billions of parameters. However, these models are typically deficient due to the heavy cost in model storage, which prohibits their usage on resource-limited applications like mobile or embedded devices. In this paper, we target at compressing CNN models to an extreme without significantly losing their discriminability. Our main idea is to explicitly model the output reconstruction error between the original and compressed CNNs, which error is minimized to pursuit a satisfactory *rate-distortion* after compression. In particular, a global error reconstruction method termed GER is presented, which firstly leverages an SVD-based low-rank approximation to coarsely compress the parameters in the fully connected layers in a layer-wise manner. Subsequently, such layer-wise initial compressions are jointly optimized in a global perspective via back-propagation. The proposed GER method is evaluated on the ILSVRC2012 image classification benchmark, with implementations on two widely-adopted convolutional neural networks, *i.e.*, the AlexNet and VGGNet-19. Comparing to several state-of-the-art and alternative methods of CNN compression, the proposed scheme has demonstrated the best rate-distortion performance on both networks.

1 Introduction

In recent years, convolutional neural networks have demonstrated impressive performance in various computer vision applications, for example, image classification [A. Krizhevsky and Hinton, 2012; Y. Lecun and Haffner, 1998; Simonyan and Zisserman, 2014; C. Szegedy and Rabinovich, 2015; Zeiler and Fergus, 2014; Y. Jia and Darrell, 2014; K. He and Sun, 2015], object detection [R. Girshick and Malik, 2014; K. He and Sun, 2014] and image retrieval [Y. Gong and Lazebnik, 2014]. Despite the long history of neural network research in the literature [Fukushima, 1980], the significant success of CNNs is mainly driven by the advanced computing resources available nowadays. For instance, to train a discriminative CNN model like AlexNet [A. Krizhevsky and Hinton, 2012] or VGGNet [Simonyan and Zisserman, 2014], it is typically required to set hundreds of millions of parameters, which are tuned using massive labelled or unlabelled data via approximated optimization (*e.g.* stochastic gradient descent) through GPU or distributed settings [J. Deng and Li, 2009]. To that effect, various implementations of CNNs are introduced in the literature, *e.g.*, AlexNet [A. Krizhevsky and Hinton, 2012], VGGNet [Simonyan and Zisserman, 2014], GoogleNet [C. Szegedy and Rabinovich, 2015] *etc.* Despite the state-of-the-art performances reported in challenging tasks like ImageNet ILSVRC [J. Deng and Li, 2009], the storage cost of CNNs is essentially huge, which typically require a large number of parameters ($\sim 10^8$) [A. Krizhevsky and Hinton, 2012; Zeiler and Fergus, 2014; P. Sermanet and LeCun, 2013]. For instance, an 8-layer-AlexNet with 600,000 nodes costs 240MB storage, while a 19-layer-VGGNet with 1.5M nodes costs 548MB. Under such a circumstance, the existing CNNs cannot be directly deployed on devices that require compact storage, such as mobile phones or embedding devices. On the contrary, it is shown that CNNs with million-scale parameters typically tend to be heavily over-parameterized [M. Denil and Freitas, 2013]. Therefore, *not all* parameters and structures are essentially necessary in producing a discriminative CNN model. On the other hand, it is quantitatively shown in [Ba and Caruana, 2014] that, neither shallow nor simplified CNNs provide comparable performance to deep CNNs with million-scale parameters. Therefore, a natural thought is to discover and discard the parameter redundancy in deep CNNs without significantly decreasing the classification accuracy.

The compression of CNNs has attracted a few research attentions very recently, which can be further categorized into three groups, *i.e.*, parameter sharing, parameter pruning and

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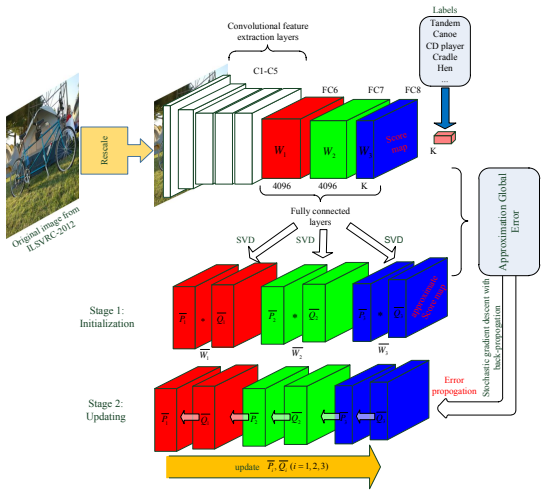


Figure 1: Framework of our compression for CNN via global error reconstruction

matrix decomposition. As for parameter sharing, Gong *et al*. [Y. Gong and Bourdev, 2014] employed vector quantization over parameters to reduce the redundancy in the parameter space. Chen *et al*. [W. Chen and Chen, 2015] proposed a HashedNets model which uses a low-cost hash function to group weights between two connected layers into hash buckets to share parameters. Cheng *et al*. [Y. Cheng and Chang, 2015] proposed to replace the conventional linear projection in fully connected layers with the circulant projection, which reduces the storage cost and enables the use of Fast Fourier Transform to accelerate the computation. As for parameter pruning, Srinivas and Babu [Srinivas and Babu, 2015] explored the redundancy among neurons, and proposed a data-free pruning to remove redundant neurons. Han *et al*. [S. Han and Dally, 2015] aimed at reducing the total amount of parameters and operations in the entire network. The above pruning approaches can give significant reductions in both parameter size and computation workload. As for matrix decomposition, Denil *et al.* [M. Denil and Freitas, 2013] adopted low-rank decomposition to compress the weights in the fully connected layers in a layer-by-layer manner. Novikov *et al*. [A. Novikov and Vetrov, 2015] converted the dense weight matrices of the fully connected layers to the Tensor Train format, such that the number of parameters is reduced by huge factor while preserving the expressive power of the layer.

However, the state-of-the-art methods [M. Denil and Freitas, 2013; Y. Gong and Bourdev, 2014; Srinivas and Babu, 2015] still rely on a layer-wise parameter compression, which do not provide an explicit modeling to the overall loss of classification accuracy. In other words, such works can be regraded as a layer-wised, "implicit" and "local" compression for CNNs. In terms of the "implicit" compression, the existing works only considered approximating the parameters $\mathbf{W}$ between fully connected layers with $\hat{\mathbf{W}}$ by minimizing their Euclidean distance $\left\|\mathbf{W}-\hat{\mathbf{W}}\right\|_F^2$. This setting is indeed problematic, which does not directly recover the output of CNNs (*a.k.a.*, the learned deep features) used for classification. In terms of "local" compression, a more optimal solution is indeed to preserve the classification accuracy in a *global* manner, *i.e.*, compressing all parameters across the entire fully connected layers[1]. Meanwhile, the global correlations among weights of inter-layers are simply ignored in [M. Denil and Freitas, 2013; Y. Gong and Bourdev, 2014; Srinivas and Babu, 2015]. In particular, due to the nonlinear activation functions (*e.g.* sigmoid, tahn [Y. LeCun and Müller, 2012], or rectifier linear unit (ReLU) [Nair and Hinton, 2010]), small quantization error between $\mathbf{W}$ and $\hat{\mathbf{W}}$ in each layer might magnified and propagated in the network, leading to large generalization error as shown in our experiments.

In this paper, we present a novel framework towards an "explicit" and "global" compression of CNNs, whose structure is shown in Fig. 1. Our key innovation lies in introducing a **Global Error Reconstruction** algorithm, which explicitly models the reconstruction error between the outputs of both the original and the compressed CNNs in a global way. In such a manner, both inter-layer and intra-layer relationships of weight parameters are jointly compressed. Meanwhile, instead of minimizing the reconstruction error between the original and approximated parameters layer-by-layer, GER directly establishes an objective function to recover the outputs of the compressed CNNs, which includes the influences of nonlinear activation functions within and across the fully connected layers.

In practice, we conduct an initial compression of the weights in the fully connected layers by an SVD-based low-rank decomposition, which can relax the constrained term of the optimization function to being trackable. Subsequently, such layer-wise, coarse compressions are further jointly optimized among layers via back propagation to minimize the global error, which is done by a novel optimization method that using the stochastic gradient descent to well solve the non-convex optimization problem.

The proposed method is evaluated on the ILSVRC2012 image classification benchmark, with testing on two widely-adopted CNNs, *i.e.*, AlexNet [A. Krizhevsky and Hinton, 2012] and VGGNet-19 [Simonyan and Zisserman, 2014]. We have demonstrated that the proposed GER compression scheme can lead to the state-of-the-art rate-distortion[2] performance by comparing with several state-of-the-art and alternative schemes in CNN compression [M. Denil and Freitas, 2013; Y. Gong and Bourdev, 2014; X. Zhang and Sun, 2015]. The main contribution of this paper is three-fold:

- We introduce an explicit objective function to directly minimize the reconstruction error of outputs before and after network compression, which differs from the ex-

[1]In this paper, we focus on the compression of the fully connected layer, as it occupies the largest proportion in the entire CNN model. For instance, above 80% storage is cost in the fully connected layers comparing to 20% in the convolutional layers of the AlexNet [A. Krizhevsky and Hinton, 2012].

[2]Rate-distortion is an evaluation protocol of model compression in which the *rate* and *distortion* represent the compression rate of model and classification precision, respectively.

isting works that indirectly minimize the difference between the original and compressed parameters.

- We globally model the inter-layer correlation during the network compression, which can address the issue of layer-wise accumulated compression error.
- We introduce an effective optimization method to solve the corresponding non-convex optimization, which firstly uses an SVD-based low-rank decomposition to relax the constrained term, and then adopts a stochastic gradient descent to learn optimal parameters.

2 Initial Compression of CNN Based on Low-rank Decomposition

2.1 Preliminaries

We define a feature matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n] \in \Re^{d \times n}$ as the input to compress a fully-connected CNN, where d is the dimension of feature vectors, and n is the number of feature vectors which are the outputs of the last convolutional layer in the original CNN-like AlexNet [A. Krizhevsky and Hinton, 2012]. The forward propagation of the l-th layer in a compressed fully-connected CNN can be written as:

$$a_i^{l+1} = f(z_i^{l+1}), where\ z_i^{l+1} = \sum_{j=0}^{n^l} W_{ij}^l a_j^l, \tag{1}$$

where W_{ij}^l is the element of weight matrix $\mathbf{W}^l$. The vectors $\mathbf{z}, \mathbf{a} \in \Re^{n^l}$ denote the activation units before and after transformation $f(\cdot)$. Typically, $f(\cdot)$ is a non-linear transition, for instance, the rectifier linear unit (ReLU), sigmoid or tanh *etc.*

2.2 Layer-wise Low-rank Approximation of Linear Response

We first consider the low-rank approximation $\hat{\mathbf{W}}^l$ of the original weights $\mathbf{W}^l$ between the l-th and $l+1$-th layers. To find an approximated low-rank subspace, we minimize the reconstruction error of the neuron responses as below:

$$\begin{aligned} &\min_{\hat{\mathbf{W}}^l} \|\mathbf{W}^l\mathbf{X} - \hat{\mathbf{W}}^l\mathbf{X}\|_F^2 \\ &s.t. \quad rank(\hat{\mathbf{W}}^l) \le k, \end{aligned} \tag{2}$$

where $\mathbf{W}^l, \hat{\mathbf{W}}^l \in \Re^{d^{l+1} \times d^l}$. With respect to the error of two linear transforms on the same input signal $\mathbf{X}$, Eq. 2 can be rewritten as:

$$\begin{aligned} &\min_{\hat{\mathbf{W}}^l} \|\mathbf{W}^l - \hat{\mathbf{W}}^l\|_F^2 \\ &s.t. \quad rank(\hat{\mathbf{W}}^l) \le k. \end{aligned} \tag{3}$$

We solve Eq. 3 by Singular Vector Decomposition (SVD) [Golub and Loan, 2012], *a.k.a.*, $\hat{\mathbf{W}}^l = \hat{\mathbf{U}}_k^l \hat{\mathbf{S}}_k^l \hat{\mathbf{V}}_k^{l^\top}$, where $\hat{\mathbf{U}}_k^l \in \Re^{d^{l+1} \times k}$ and $\hat{\mathbf{V}}_k^l \in \Re^{d^l \times k}$ are two submatrices that correspond to the top k singular vectors in $\mathbf{U}^l$ and $\mathbf{V}^l$. The diagonal elements in $\hat{\mathbf{S}}_k^l \in \Re^{k \times k}$ correspond to the k largest singular values in $\mathbf{S}^l$, which is a diagonal matrix by running SVD over $\mathbf{W}^l$. Then we obtain the decomposition of $\hat{\mathbf{W}}^l$ as $\hat{\mathbf{P}}^l \hat{\mathbf{Q}}^{l^\top}$, where $\hat{\mathbf{P}}^l = \hat{\mathbf{U}}_k^l \hat{\mathbf{S}}_k^{l\frac{1}{2}}$ and $\hat{\mathbf{Q}}^l = \hat{\mathbf{V}}_k^l \hat{\mathbf{S}}_k^{l\frac{1}{2}}$.

Algorithm 1 Alternating optimization layer-by-layer

Input: Training data points $\mathbf{X}$; the output $\mathbf{Z}^l$ between the l-th and $l+1$-th layers before non-linear transformation; the input $\mathbf{C}^l$ between the l-th and $l+1$-th layers; the restricted rank k, the maximum iterations T.
Output: The approximated low-rank matrix $\hat{\mathbf{W}}^l$.
1: Initialize $\mathbf{C}^0$ by $\mathbf{X}$, the auxiliary variables $\mathbf{Y}^l$ by $\mathbf{Z}^l$, and $t = 1$.
2: **repeat**
3: **Step I:** Fix $\mathbf{Y}^{l+1}$ and update $\hat{\mathbf{W}}^l$ via GSVD
4: **Step II:** Fix $\hat{\mathbf{W}}^l$ and update $\mathbf{Y}^{l+1}$ by analyzing each element y_{ij}^{l+1}, and solve 1-dimensional optimization Eq. 9 via Eq. 10 and Eq. 11.
5: $t := t + 1$;
6: **until** convergence or reach maximum iterations T

2.3 Extending to Non-Linear Responses

For non-linear transition that commonly occurs in CNNs, the result of approximated matrix is not equivalent to that of the original one. Therefore, the effect of non-linear transition should be taken into account when designing low-rank approximation of parameter matrix $\mathbf{W}$. Taking the widely used ReLU transition for instance, ReLU is defined as $f(\cdot) = \max(\cdot, 0)$. To minimize the reconstruction error of ReLU-like response, we have:

$$\begin{aligned} &\min_{\hat{\mathbf{W}}^l} \sum_i^n \|f(\mathbf{W}^l\mathbf{a}_i^l) - f(\hat{\mathbf{W}}^l\hat{\mathbf{a}}_i^l)\|_2^2 \\ &s.t. \quad rank(\hat{\mathbf{W}}^l) \le k, \end{aligned} \tag{4}$$

where the first term $\mathbf{a}_i^l$ is the non-approximated input of the l-th layer (*i.e.* output of the $l-1$-th layer), and the second term $\hat{\mathbf{a}}_i^l$ approximates the input of the l-th layer. The optimization problem in Eq. 4 can be solved by a layer-wise optimization with an alternating solver. More specifically, we consider this optimization layer-to-layer in a bottom up manner: Taking Eq. 4 for example, we fix $\hat{\mathbf{a}}_i^l$ as a constant which denotes $\mathbf{c}_i^l$, and use $\mathbf{z}_i^{l+1}$ instead of $\mathbf{W}^l\mathbf{a}_i^l$ in optimization. Then the optimization of each layer in Eq. 4 can be rewritten as

$$\begin{aligned} &\min_{\hat{\mathbf{W}}^l} \sum_i^n \|f(\mathbf{z}_i^{l+1}) - f(\hat{\mathbf{W}}^l\mathbf{c}_i^l)\|_2^2 \\ &s.t. \quad rank(\hat{\mathbf{W}}^l) \le k. \end{aligned} \tag{5}$$

Unfortunately, Eq. 5 is hard to be solved due to the coexistence of the non-linearity and the low-rank constraint. To obtain a feasible solution, we relax Eq. 5 to:

$$\begin{aligned} &\min_{\hat{\mathbf{W}}^l, \{\mathbf{y}_i^{l+1}\}} \sum_i^n \|f(\mathbf{z}_i^{l+1}) - f(\mathbf{y}_i^{l+1})\|_2^2 + \lambda\|\mathbf{y}_i^{l+1} - \hat{\mathbf{W}}^l\mathbf{c}_i^l\|_2^2 \\ &s.t. \quad rank(\hat{\mathbf{W}}^l) \le k, \end{aligned} \tag{6}$$

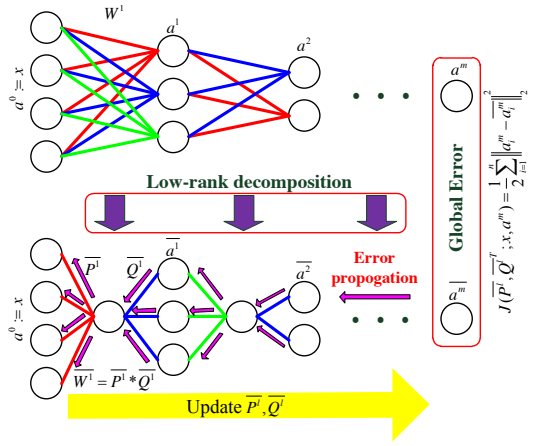


Figure 2: GER scheme for optimizing a global objective function across all CNN layers

where λ is a penalty parameter and $\mathbf{y}_i^{l+1}$ is a set of auxiliary variables of the same size as $\mathbf{z}_i^{l+1}$. If $\lambda \to \infty$, the solution to Eq. 6 will converge to the solution to Eq. 5. To solve Eq. 6, we further employ an alternating solver that alternatively fixes $\{\mathbf{y}_i^{l+1}\}$ and solves $\hat{\mathbf{W}}^l$, and vice versa. We detail this alternative optimization as below:

Alternating Step I: **Fix $\{\mathbf{y}_i^{l+1}\}$ and update $\hat{\mathbf{W}}^l$**, *i.e.*,

$$\min_{\hat{\mathbf{W}}^l} \sum_i^n \|\mathbf{y}_i^{l+1} - \hat{\mathbf{W}}^l \mathbf{c}_i^l\|_2^2 \quad s.t. \quad rank(\hat{\mathbf{W}}^l) \leq k. \tag{7}$$

We rewrite Eq. 7 as a Reduced Rank Regression problem: [Gower and Dijksterhuis, 2004; Takane and Hwang, 2007],

$$\min_{\hat{\mathbf{W}}^l} \|\mathbf{Y}^{l+1} - \hat{\mathbf{W}}^l \mathbf{C}^l\|_F^2 \quad s.t. \quad rank(\hat{\mathbf{W}}^l) \leq k. \tag{8}$$

Here$\|\cdot\|_F$ is the Frobenius norm. let $\bar{\mathbf{W}}^l = \mathbf{Y}^{l+1}\mathbf{C}^{l^T}(\mathbf{C}^l\mathbf{C}^{l^T})^{-1}$, Eq. 8 can be solved by the Generalized Singular Vector Decomposition (GSVD) [Takane and Jung, 2006]. *i.e.*,

1. GSVD decomposes $\bar{\mathbf{W}}^l$ as $\widetilde{\mathbf{W}}^l = \mathbf{U}^l\mathbf{S}^l\mathbf{V}^l$.
2. The solution $\hat{\mathbf{W}}^l$ to Eq. 8 is given by $\hat{\mathbf{W}}^l = \mathbf{U}_k^l\mathbf{S}_k^l\mathbf{V}_k^{l^T}$, where $\mathbf{U}_k^l$ and $\mathbf{V}_k^l$ are the first k columns of $\mathbf{U}^l$ and $\mathbf{V}^l$, and $\mathbf{S}_k^l$ are the k largest singular value in $\mathbf{S}^l$.
3. We obtain the decomposition of $\hat{\mathbf{W}}^l = \mathbf{P}^l\mathbf{Q}^{l^\top}$, where $\mathbf{P}^l = \mathbf{U}_k^l\mathbf{S}_k^{l\frac{1}{2}}$ and $\mathbf{Q}^l = \mathbf{V}_k^l\mathbf{S}_k^{l\frac{1}{2}}$.

Alternating Step II: **Fix $\hat{\mathbf{W}}^l$ and update $\{\mathbf{y}_i^{l+1}\}$**. Each element y_{ij}^{l+1} of each vector $\mathbf{y}_i^{l+1}$ is independent to any other. We then rewrite Eq. 6 as a 1-dimensional optimization process:

$$\min_{y_{ij}^{l+1}} \left(f(z_{ij}^{l+1}) - f(y_{ij}^{l+1})\right)^2 + \lambda(y_{ij}^{l+1} - \bar{z}_{ij}^{l+1}), \tag{9}$$

Algorithm 2 Global error Reconstruction for compressing CNN

Input: Training data points $\mathbf{X}$; the corresponding output $\mathbf{A}$ after softmax function; the original weights $\mathbf{W}^l$ in each fully-connected layers; the restricted rank k; the maximum number of epoch T; the number of fully connected layers m.

Output: The low-rank matrices $\hat{\mathbf{P}}^l$ and $\hat{\mathbf{Q}}^l$.

1: Initialize $\hat{\mathbf{P}}^l$, $\hat{\mathbf{Q}}^l$ by low-rank decomposition of $\mathbf{W}^l, l = 0, 1, \cdots, m-1, t = 1$.
2: **repeat**
3: Compute $\nabla_{\hat{\mathbf{P}}^l} J$ by (17);
4: Compute $\nabla_{\hat{\mathbf{Q}}^{l^\top}} J$ by (18);
5: Update $\hat{\mathbf{P}}^l$ and $\hat{\mathbf{Q}}^l$ by SGD;
6: $t := t + 1$.
7: **until** convergence or $t > T$.

where $\bar{z}_{ij}^{l+1}$ is the j-th entry of $\hat{\mathbf{W}}^l\mathbf{c}_i^l$. We separately take $\bar{z}_{ij}^{l+1} \geq 0$ and $\bar{z}_{ij}^{l+1} \leq 0$ into account due to the restriction of ReLU. Then we derive the solution of Eq. 9:

$$\bar{y}_{ij}^{l+1} = \min(0, \bar{z}_{ij}^{l+1}) \tag{10}$$

$$\widetilde{y}_{ij}^{l+1} = \max(0, \frac{\lambda\bar{z}_{ij}^{l+1} + f(z_{ij}^{l+1})}{\lambda + 1}). \tag{11}$$

Note that $y_{ij}^{l+1} = \bar{y}_{ij}^{l+1}$ if $\bar{y}_{ij}^{l+1}$ have a smaller value in Eq. 9 than $\widetilde{y}_{ij}^{l+1}$ and otherwise $y_{ij}^{l+1} = \widetilde{y}_{ij}^{l+1}$. We use the gradient descent to solve the above 1-dimensional, non-linear least square problem.

The above alternating optimization is further shown in Algorithm 1. The initialization of $\hat{\mathbf{W}}^l$ is given by the solution to the linear case of Eq. 3. Theoretically speaking, λ should be gradually increase to being infinity [J. Wang and Gong, 2010]. However, if λ is too large, it is difficult for the iterative solver to take effects. As a trade-off and to perform more iterations, we first increase λ to 1, and then we obtain the result $\hat{\mathbf{W}}^l$ after convergence, which is adopted as the initial (coarse) compression among all the fully connected layers.

3 Cross-Layer Compression via Global Error Reconstruction

The initial compression of CNN using low-rank decomposition is to coarsely approximate $\hat{\mathbf{W}}^l$ of each layer in a bottom up manner. As discussed above, the compression errors are accumulated layer-by-layer, resulting to large overall error in the output layer of CNN. To address this issue, the proposed global error reconstruction (GER) targets at jointly optimizing among layers as shown in Fig. 2.

In particular, if the original CNN model has m fully connected layers, we minimize the global reconstruction error of ReLU-like non-linear responses as follows:

$$\min_{\hat{\mathbf{W}}^l, l=0,1,\cdots,m-1} \sum_i^n \|\mathbf{a}_i^m - f(\hat{\mathbf{W}}^{m-1}\hat{\mathbf{a}}_i^{m-1})\|_2^2 \quad s.t. \quad rank(\hat{\mathbf{W}}^l) \leq k, \quad l = 0, 1, \cdots, m-1, \tag{12}$$

where $\mathbf{a}_i^m$ is the non-approximated output, and $\hat{\mathbf{a}}_i^{m-1}$ contains $m-1$ weights in the hidden layers, which is written as:

$$\hat{\mathbf{a}}_i^{m-1} = \underbrace{f \circ \hat{\mathbf{W}}^{m-2} \circ \cdots \circ f \circ \hat{\mathbf{W}}^{0}}_{m-1} \mathbf{x}_i^0. \tag{13}$$

To find a feasible solution, we use the solution of Eq. 4 to relax the constrain term of Eq. 12, and let $\mathbf{A}^m$ be the matrix concatenating the vectors of $\mathbf{a}_i^m$. The objective function of Eq. 12 can be rewritten as:

$$J(\hat{\mathbf{P}}^l, \hat{\mathbf{Q}}^{l^\top}; \mathbf{X}, \mathbf{A}^m) = \frac{1}{2}\|\mathbf{A}^m - \hat{\mathbf{A}}^m\|_F^2, \tag{14}$$

where $l = 0, 1, \cdots, m-1$, and $\hat{\mathbf{A}}^m$ can be written as:

$$\hat{\mathbf{A}}^m = \underbrace{f \circ (\hat{\mathbf{P}}^{m-1}\hat{\mathbf{Q}}^{m-1^\top}) \circ \cdots \circ f \circ (\hat{\mathbf{P}}^0\hat{\mathbf{Q}}^{0^\top})}_{m} \mathbf{X}^0. \tag{15}$$

In Eq. 15 $\hat{\mathbf{P}}^l$ and $\hat{\mathbf{Q}}^{l^\top}$ are the approximate decomposition to $\mathbf{W}$ by solving Eq. 4. To learn the parameters $\{\hat{\mathbf{P}}^l\}$ and $\{\hat{\mathbf{Q}}^{l^\top}\}$, a stochastic gradient descent algorithm is employed via back-propagation, which needs to calculate the gradient of the objective function with respect to all weights. Therefore, the error signals of the cost function in Eq. 14 is obtained via:

$$\delta^{\hat{\mathbf{A}}^{l+1}} = \frac{\partial J}{\partial \hat{\mathbf{A}}^{l+1}} = \begin{cases} -(\mathbf{A}^m - \mathbf{A}^{l+1}), & l = m-1, \\ \delta^{\hat{\mathbf{Z}}^{l+2}} \cdot \hat{\mathbf{P}}^{l+1}\hat{\mathbf{Q}}^{l+1^\top}, & otherwise. \end{cases} \tag{16}$$

$$\delta^{\hat{\mathbf{Z}}^{l+1}} = \frac{\partial J}{\partial \mathbf{Z}^{l+1}} = \delta^{\hat{\mathbf{A}}^{l+1}} \cdot f'(\hat{\mathbf{Z}}^{l+1}), \tag{17}$$

where $\hat{\mathbf{Z}}^{l+1} = \hat{\mathbf{P}}^l\hat{\mathbf{Q}}^{l^\top} \cdot \hat{\mathbf{A}}^l$, $l = m-1, m-2, \cdots, 0$. After obtaining $\hat{\mathbf{A}}^{l+1}$, we compute two gradients of objective function with respect to the parameters as follows:

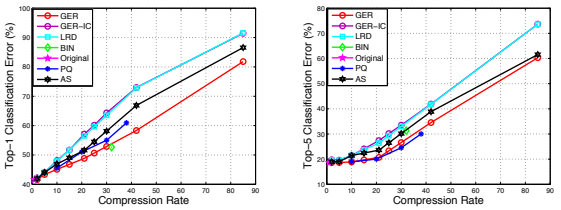
$$\nabla_{\hat{\mathbf{P}}^l} J(\hat{\mathbf{P}}^l, \hat{\mathbf{Q}}^{l^\top}; \mathbf{X}, \mathbf{A}^m) = \frac{\partial J}{\partial \hat{\mathbf{P}}^l} = \delta^{\hat{\mathbf{A}}^{l+1}} \cdot \hat{\mathbf{A}}^{l^\top}\hat{\mathbf{Q}}^l \tag{18}$$

$$\nabla_{\hat{\mathbf{Q}}^{l^\top}} J(\hat{\mathbf{P}}^l, \hat{\mathbf{Q}}^{l^\top}; \mathbf{X}, \mathbf{A}^m) = \frac{\partial J}{\partial \hat{\mathbf{Q}}^{l^\top}} = \hat{\mathbf{P}}^{l^\top}\delta^{\hat{\mathbf{A}}^{l+1}} \cdot \hat{\mathbf{A}}^{l^\top}, \tag{19}$$

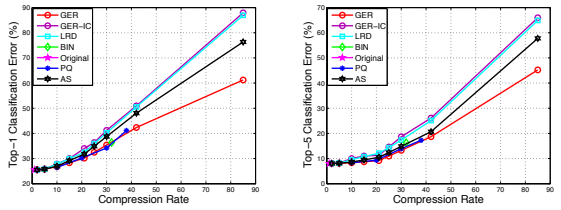
where $l = m-1, m-2, \cdots, 0$. It is worth to note that the stochastic gradient descent can reduce the accumulative errors. Algorithm 2 presents the detailed optimization algorithm for GER.

4 Experiments

To evaluate the performance of GER scheme, we conduct comprehensive experiments on ILSVRC2012 image classification benchmark. We deploy the proposed GER on two widely used CNNs (*a.k.a.* AlexNet [A. Krizhevsky and Hinton, 2012] and VGGNet-19 [Simonyan and Zisserman, 2014]), with comparisons to the state-of-the-art scheme proposed very recently [M. Denil and Freitas, 2013; Y. Gong and Bourdev, 2014; X. Zhang and Sun, 2015].



(a) Comparison of different methods for compressing AlexNet.



(b) Comparison of different methods for compressing VGGNet.

Figure 3: Comparison of methods on AlexNet and VGGNet. Values are averaged over 5 runs.

4.1 Experimental Setting

Dataset. We test the proposed GER based CNN compression on the ILSVRC2012 image classification benchmark. The dataset contains more than 1 million training images from 1,000 object classes. It also has a validation set of 50,000 images, where each object class contains 50 images. We randomly select 100,000 images (100 from each class) from the training set for training, and test on the validation set.

Implementation Details. We implement GER on two CNNs *i.e.*, AlexNet [A. Krizhevsky and Hinton, 2012] and VGGNet-19 [Simonyan and Zisserman, 2014]. The VGGNet-19 contains 16 convolutional layers and 3 fully connected layers, and the AlexNet contains 5 convolutional layers and 3 fully connected layers. The compressed networks are trained using Caffe [Y. Jia and Darrell, 2014] and run on NVIDIA GTX TITAN X graphics card with 12GB. The learning rate starts at 0.01 and is halved every 10-epochs; the weight decay is set to 0.0005 and the momentum is set to 0.9.

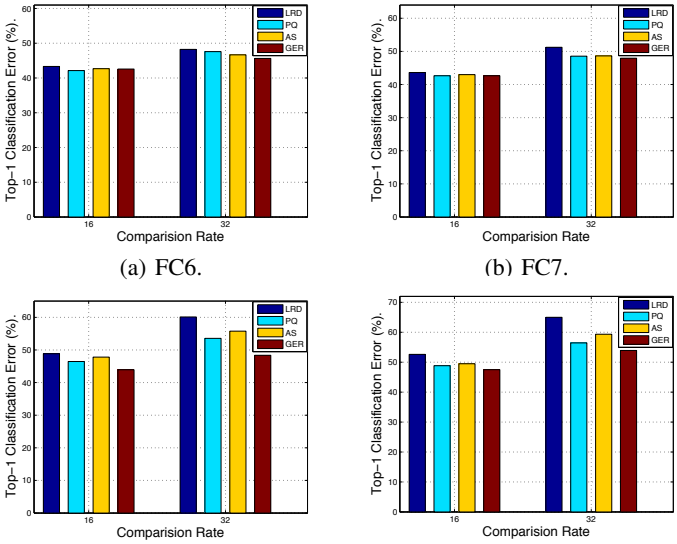
Baselines. We compare the proposed GER scheme with 4 state-of-the-art approaches published very recently, including PQ-based compression (PQ) [Y. Gong and Bourdev, 2014], Low-rank decomposition (LRD)[M. Denil and Freitas, 2013], Layer-wise optimization by alternating solver (AS) [X. Zhang and Sun, 2015], and Binary-based compression (BIN) [Y. Gong and Bourdev, 2014]. As for the alternative approach, we compare GER with GER-IC, which is the SVD-based initial compression based only on Sec.2.

Evaluation protocol. The classification error on the validation set was employed as the evaluation protocol. We used both the *top-1 classification error* and *the top-5 classification error* to evaluate different compression methods. Then we measure the compression performance in terms of the *rate distortion*, which reflects the balance between compression rate and classification error.

Rate-Distortion Comparison. We use a different rank k

Table 1: Classification error rate (in %) with compression rate 16 and 32. "∼" represents that it is unable to obtain the number, since BIN quantizes weights from float to binary format.

Compression rate		AlexNet on ILSVRC					VGGNet-19 on ILSVRC				
		GER	LRD	BIN	PQ	AS	GER	LRD	BIN	PQ	AS
Top1	Compression rate = 16	**47.51**	52.63	∼	48.83	49.49	**28.63**	30.37	∼	28.66	29.59
	Compression rate = 32	53.92	65.01	**52.79**	56.45	59.34	36.52	41.95	36.37	**35.94**	40.29
Top5	Compression rate = 16	20.95	25.24	∼	**19.33**	23.01	**8.74**	11.25	∼	8.85	9.47
	Compression rate = 32	27.97	34.67	31.32	**25.93**	31.46	**14.11**	18.19	16.29	14.88	15.91



(a) FC6. (b) FC7.

(c) FC8. (d) global classification error.

Figure 4: Top-1 classification error for compressing only the single layer error of AlexNet. "FC6, FC7, FC8" represent the first, second, and third fully connected layer, respectively. Values are averaged over 5 runs.

from 2^5 to 2^{10} to achieve different compression rates. For PQ, we fix the number of centers to 256 (8 bits) and vary the segment dimension $s = 1, 2, 4, 8$. For LRD and Layer-wise optimization by alternating solver , we use the same compression criterion with GER that varies k from 2^5 to 2^{10}. For BIN, which has no parameter to tune, the compression rate is fixed as 32.

Both the top-1 and top-5 classification errors are shown in Fig. 3, which show a consistent trend in rate-distortion. As for intra-layer approximation, GER-IC achieves a similar classification error to LRD. Instead, by explicitly modeling the reconstruction error in a global way, GER greatly improves rate-distortion by comparing to GER-IC. GER also performs much better than LRD and AS for compressing the fully connected layers. To explain, GER merits in its "explicit" compression, which effectively combines the initial layer-wise compression and cross-layer global compression, while LRD and AS are "implicit" compression which only consider the local intra-layer relationship. Note that PQ has achieved better performance comparing to that of LRD and AS. However, it is shown in Fig. 3 that PQ is hard to achieve high compression rate, which might be due to the limited codebook size. In contrast, GER achieves the best rate-distortion by comparing to other baselines. Finally, as discovered by Gong *et al.* [Y. Gong and Bourdev, 2014], the simplest BIN works well when we fixed the compression rate to 32. Therefore, The base binary quantization is also a good choice when the goal is to compress data very aggressively. However, it is hard to be adaptive when we want to control the compression rate, which in turn is one of the key advantages for our scheme. We further show that the classification error with a fixed compression rate in Tab.1 which also shows GER can achieve the best performance by comparing to other baselines especially, when implementing on VGGNet-19.

On the Single Layer Error. we further analyze the classification error by compressing each single layer while fixing the rest layers as the original uncompressed version. The results are reported in Fig. 4. We found using all the baselines to compress the first two fully connected layers (*i.e.* "FC6" and "FC7") does not lead to the decreasing of accuracy. In contrast, compressing the last fully connected layer leads to large classification error for all the baselines, except for the proposed GER. We explain this advantage by the fact that GER can automatically adjust the inter-layer error by tuning and refining across all layers.

5 Conclusion

In this paper, we propose to compress the convolutional neural networks to reduce the model storage by a novel Global Error Reconstruction scheme, which can facilitate emerging applications in mobile or embedding devices with limited storage. GER firstly uses an SVD-based low-rank approximations to coarsely compress the parameters in the fully connected layers. Such layer-wise initial compressions are further jointly optimized among layers in a global way via back-propagation. Unlike previous approaches that only considered recovering the internal weight parameters, GER also explicitly models the reconstruction error between the outputs of both the original and compressed CNNs, which we significantly reduces the accumulated reconstruction error caused by the nonlinear activation. We have demonstrated that the proposed GER scheme can lead to state-of-the-art rate-distortion performance by comparing to several very recent schemes in CNN compression [M. Denil and Freitas, 2013; Y. Gong and Bourdev, 2014; X. Zhang and Sun, 2015]. In our future work, we would extend and evaluate our compression scheme from the fully connected layer to the convolutional layer, and at the same time, further accelerate computation of convolutional layers.

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