Proceedings of the 2004 Winter Simulation Conference
R .G. Ingalls, M. D. Rossetti, J. S. Smith, and B. A. Peters, eds.

A META-HEURISTIC BASED ON SIMULATED ANNEALING FOR SOLVING MULTIPLE-OBJECTIVE PROBLEMS IN SIMULATION OPTIMIZATION

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ABSTRACT

This paper presents a new meta heuristic algorithm based on the search method called simulated annealing, and its application to solving multi objective simulation optimization problems. Since the simulated annealing search method has been extensively applied as a modern heuristic to solve single objective simulation optimization problems, a modification to this method has been developed in order to solve multi objective problems. The efficiency of this new algorithm was tested on a real case problem modeled under discrete simulation.

1 INTRODUCTION

A subset of optimization theory deals with problems with mathematically-explicit objective functions. Furthermore, if the objective function and all constraints are linear expressions of the optimization variables, direct solution techniques and software packages may be applied in order to obtain a deterministic solution, in terms of an optimal array (vector) of variables that maximizes or minimizes the objective function of that specific problem. It is also possible in some cases to ensure that the solution found represents a global optimum (the final array of solution variables delivers the highest or lowest value from the objective function).

Also, optimization problems usually deal with one single mathematically explicit objective function, with deterministic variables an coefficients, in both restrictions as well as the objective function itself, and all of them related to a specific problem. If the objective function and the restrictions are linear expressions of the optimizing variables, the problem can be written as:

Min (Max) Z = c1*X1 + c2*X2 + ... +cn*Xn

s/t

a11*X1 + a12*X2 + ... + a1n*Xn <= b1
a21*X1 + a22*X2 + ... + a2n*Xn <= b2
.
.
am1xX1 + am2*X2 + … + amn*Xn <= bm

The expression above relates to problems called linear programming. There are also optimization problems with more than one objective function, and usually the improvement direction of one objective function deviates from another. Such problems can be written as:

Min (Max) Z1 = c11*X1 + c12*X2 + ... +c1n*Xn
Min (Max) Z2 = c21*X1 + c22*X2 + ... +c2n*Xn
.
:
Min (Max) Z1o = co1*X1 + co2*X2 + ... +con*Xn

The optimal solutions associated to each and every objective function generally do not converge into a single unique solution. Usually a multiple objective situation is faced, which may be improve in opposite directions from each other, and trade off among them has to be made, in order to sacrifice single objective optimality in order to reach a global pseudo optimal solution. The criteria used to define this trade off is not an objective one, and will depend upon conditions such as cost benefit relation of the output variables, etc.

An additional difficulty relies on the fact that the concept of optimality turns rather blurry when dealing with problems that concerns more than one objective, the so called multi-objective simulation optimization problems. This happens every time the convergence towards one solution in a particular direction tends to improve one objective, but at the same time drifts one or more other objectives towards poorer values. Hence, a trade off among all objective must be made in order to obtain a solution that suits all objectives up to an acceptable result. The criteria needed to balance all objectives is not absolute. It must be set by the investigator in order to match the overall specific requirements of each problem.

However, there are some other optimization problems where objective functions are not presented as a mathematically explicit relation among independent variables. One particular case is the one in which the objective function is a simulation model, with more than one output variable as an optimization objective. These sort of problems are called multi-objective simulation optimization problems. The independent variables are often positive integers with large ranges, which are expressed as arrays or vectors containing discrete numbers, each representing the value of a specific variable of the simulation model. These arrays are usually known as solutions of the optimization problem.

Since the simulation model cannot be expressed as an exact and deterministic mathematical expression, and also the independent variable domain is discrete, neither is it possible to be solved using direct methods. It is necessary to try feasible solutions and keep track of the best results obtained. Besides, there is a practical drawback in terms of time required to try and evaluate all possible solutions, which is the set that contains all possible combinations of solution arrays. Therefore, an efficient or "intelligent" method must be developed in order to guide a search that shall find an good, if not optimal, solution, in a rather short period of time. The latter involves the evaluation of the objective functions from an initial solution array, and the decision after every evaluation on how to modify that solution and therefore move to another solution array that delivers an improved objective function or an improved group of objective functions.

Several heuristic methods have been applied to solve simulation optimization problems, but not as many have dealt with the multi-objective situation. One meta heuristic technique that has not yet been applied to solve multi objective simulation optimization problems is simulated annealing. Here we explain the concepts needed to develop a new artificial intelligence tool based on simulated techniques for single-objective simulation optimization problems, in order to solve multi objective problems.

2 METHODOLOGY

This research involve the following methodological steps:

- Information and state of the art review.
- Definition of the new meta heuristic algorithm based on simulated annealing.
- Construction and programming of the new interface between the simulation model and the meta heuristic algorithm.
- Selection of a previously solved multi objective simulation optimization problem, to serve as pattern for comparison.
- Design of experiments.
- Output data analysis.

3 BACKGROUND

In case of dealing with multi-objective simulation optimization problems, it is not possible to directly use a meta-heuristic technique such as simulated annealing, since the algorithm is not programmed to sort which of the multiple objectives apply the acceptance function to. Another difficulty is related to how this simulated annealing algorithm could balance good solutions that satisfy all objectives simultaneously, and what criteria should it use. All the above, during each step of the search. It is important to remind, as stated before, that most of the time, convergence towards one solution in a particular direction tends to improve one objective, but at the same time drifts one or more other objectives towards poorer values. This is one the main aspects a multi-objective algorithm has to take into consideration.

Thus, to apply a simulated annealing algorithm to the resolution of multi objective simulation optimization problems, a new meta-heuristic method must be developed. This new simulated annealing based algorithm shall consider the multi objective nature of the problem, and all the decisions that have to be made along the search to deal with the completion of a good overall result that matches all objectives as required by the specifications of the particular problem.

Simulated annealing has been tested on several empirical single-objective simulation optimization problems (as well as genetic algorithms and taboo search meta-heuristic methods), but to date no information has been reviewed in order to suggest that a new simulated annealing based algorithm has been developed to try and test these kind of problems.

3.1 Literature Review

A review on the state of the arte shows that not much empirical investigation has been carried out specifically on the multi objective simulation optimization topic, with solving methods based on simulated annealing. Tuytiens et al. (2000) proposed a multi objective simulated annealing based method, aimed a two objective classical linear assignment problem. Hota, Chakrabarti, and Chattopadhyay (2000) approached a three objective optimization problem, but simulated annealing was used only after the problem was reduced to a single objective type by means of goal attainment defined by the decision maker. On the other had, on conventional simulation or single objective simulation optimization, authors Bulgak and Sanders (1988), Haddok and Mittentahal (1992), Lacksonen and Anussornnitisarn (1995), and Brady and MacGarvey (1998) have researched on the topic, but considering the conventional simulated annealing algorithm, which allows to search solutions for single objective problems. On the multi objective field, other authors have researched the subject, focusing on topics like meta models, gradient based models, and genetic algorithms. Among the authors that have studied the multi objective subject using genetic algorithms, mention can be made of Baesler (2000), using meta models, Boyle (1996), and using gradient based models, Mollaghasemi (1994), Mollaghasemi and Evans (1994), and Mollaghasemi, Evans, and Biles (1991).

3.2 Simulated Annealing Meta Heuristic

Simulated annealing is a meta-heuristic technique that has proved to be effective as a solving solution for many problems, among them, simulation optimization problems.

There is a close analogy between the simulated annealing meta-heuristic method and the thermodynamic process of annealing in physics, and it was indeed this analogy which originally motivated the development of the method.

Simulated annealing is a technique made popular in the 1980's, and since then has become a useful tool for solving many problems. In particular, it can be applied in the resolution of simulation optimization problems.

Simulated annealing works by means of searching and evaluating a set of feasible solutions, reducing the possibility of finding a solution that might turn out to be a local optimum. This means it avoids converging to a local optimum solution at early stages of the search. This is obtained allowing to evaluate or "explore" solutions in a neighborhood which bears a lower quality than the previously evaluated (this is, the output of the objective function delivers worse solutions), based upon a probability to accept those solutions, which is calculated from a mathematical function called acceptance function. In particular, the evaluation of a lower quality solution X' as compared with another solution X from its neighborhood, that delivers a variation in the objective function C'-C, will permit to keep exploring the neighborhood of the lower quality solution X', only if the condition settled by the acceptance function is fulfilled.

The acceptance function can be written as:

$$\exp[(C'-C)/T] < R \tag{1}$$

Where T is a control parameter or "temperature" and R is IID random number in the range [0,1]. The parameter T decreases with time, as the search goes on, so at every step it becomes more difficult to accept lower quality solutions by the acceptance function in order to explore new neighborhoods (the acceptance function turns stricter). This is the aspect of the algorithm that allows to avoid converging to local optimal solutions at early stages of the search. The function that relates the decrease of the temperature parameter T with time is called cooling curve.

3.3 Multi Objective Simulation Optimization with Meta Heuristic Solution

In the case of multi-objective simulation-optimization problems, simulated annealing cannot be applied directly. This is because the search direction depends on the objective function chosen. Therefore, it is necessary to modify conventional simulated annealing so that it accounts for the multi-objective nature of the problem. This is one of the main goals of this research.

4 GUIDE LINES TO DEVELOPING A NEW SIMULATED ANNEALING BASED MULTI OBJECTIVE ALGORITHM

An algorithm based on simulated annealing should focus on how to guide the search for an optimum solution that satisfies all objectives simultaneously, according to a specific criteria established by the investigator or the engineer.

The main issue to be handled by the algorithm during each step of the search is how to make the decision on which of the objectives, or what kind of output from the objectives, to consider in order to evaluate the acceptance function. This can be achieved by selecting a specific objective at each step of the search, or evaluating the output of all objectives and calculate a weighted average or some other collective objective output function. The latter is very similar to transforming a multi objective problem into a single-objective one by means of gathering and transforming all objectives into one explicit objective function. This approach may seem tempting at first glance. It is fairly simple to develop this approach, and does not require programming any additional artificial intelligence into the simulated annealing meta heuristic method. But the disadvantage of not considering any fluctuations in the search path that may guide to a better overall solution, since the criteria for moves are fixed from the beginning.

On the other hand, selecting specific objectives along the search path, at each step, taking into consideration the trend the approach is taking, could result in better solutions in the long term.

At any step of the path, upon evaluation of all objectives, three different scenarios may be faced. All objectives improve, all objectives get worse, or some improve and some others get worse.

In case all objectives improve, there is no major trouble in deciding how to continue the search, since the last solution is better than the preceding one, in the first place.

In case all objectives get worse, it is clear that the last solution must be evaluated by some sort of acceptance function.

If some objectives improve and some others get worse, a decision has to be made. One possible decision is to choose among all objectives one to lead that specific step of the search. If that objective improves or gets worse, a consequent move should follow, whether it is to accept the solution if that particular objective has gotten better, or to evaluate some sort of particular acceptance function, if the case is the contrary.

The central idea underlying all of these criteria is not to make it such an obvious issue as to just incorporate a random selection of the lead objective in the algorithm, for each step of the search. It would be interesting that the selection of the lead objective would take into consideration not only a random or fixed probability, but also the amount of improvement, or optimization performance, shown on the last move, as well as since the beginning of the search. This individual performance of each objective optimization, both historical and immediate, should up to some extent influence the probability of selection of a lead objective, in terms of allowing the objectives with the poorest behaviors, both historical and immediate, to have a better chance to be selected.

Thus, a simulated annealing based algorithm could take into consideration a new concept, the selection function, in addition to the acceptance function.

5 NEW SIMULATED ANNEALING BASED META HEURISTIC

The new modified simulated annealing based algorithm is designed in such a way so that it can guide the search in order to satisfy all objectives simultaneously.

The new algorithm presented here contains, unlike conventional simulated annealing, several cooling curves instead of one: one global cooling curve and one particular cooling curve for each objective. The working mechanism of the algorithm hinges on deciding which of the multiple objectives should become a reference reference objective, every time the evaluation of the objective functions delivers improved and non improved objectives. This is done by the selection function, explained further below.

In the first case, if all objectives improve at a given step of the search, no further decision is needed, and the search should be continued in the present direction.

In the second case, since all objectives get worse, some sort of global acceptance function must evaluate the last multiple solution, using a global or general cooling curve, but considering a reference objective, and the decision of acceptance or rejection of the solution is treated the same way as in conventional simulated annealing.

Finally, in the third case, as some objectives improve while the rest get worse, the following decision has to be made: Choose one of the objectives as a reference and carry out the next movement according to the performance of that particular objective at that stage of the search. The selection function is called, and a reference objective is selected. If the reference objective improves in terms of the objective function, the solution must be accepted and the search continues in that direction. On the other hand, if the reference objective gets worse in terms of the objective function, a particular acceptance function is evaluated for that specific objective, but considering the particular cooling curve for the reference objective instead of the general or global cooling curve.

The bottom line is that one single objective, the reference objective, leads the search at that specific step of the search. The selection of one of the objectives as the reference one is made by a selection function. The selection function delivers which objective will perform as reference objective whenever a third case of evaluation of objectives is faced, based on three aspects of the objective performances up to that stage of the search. The selection function takes into consideration the following criteria: a) a random selection; b) the performance of each objective in terms of improvement or non improvement at the current step of the search (this is, the immediate performance); and c) the performance of each objective in terms of improvement or non improvement during the overall search as a whole, this is, the "historical" performance). The above criteria can be added up by means of individual weights so as to construct the selection function.

The selection function can be constructed in many ways. As stated before, the selection function takes into consideration the following criteria: a) a random selection; by which the selection of an objective can be made randomly, by generating a IID [0,1] random number. This function is called F1; b) the performance of each objective in terms of improvement or non improvement at the current step of the search (this is, the immediate performance), and can be calculated as an improvement percentage, giving a proportionally higher chance of selection to those objectives with smaller improvements, avoiding some low performance objectives to "stay behind". This function can be called F2; and c) the performance of each objective in terms of improvement or non improvement during the overall search as a whole, this is, the "historical" performance), which is given by the cooling record of each particular objective, and can be calculated as an percentage ratio between the actual cooling temperature and the maximum temperature for each objective, giving a proportionally higher chance of selection to those objectives with higher temperature or higher temperature percentage ratios, avoiding some high temperature objectives to "stay behind". This function can be called F3. Therefore, the selection function can be constructed as a weighted summation of the three functions mentioned above, F1, F2 and F3, resulting the following expression:

$$SF = w1*F1 + w2*F2 + w3*F3 \tag{2}$$

$$F1 = \text{constant probability.} \tag{3}$$

$$F2 = PT/\sum(PT) \tag{4}$$

$$F3 = 1 - \Delta C\%/\sum(\text{abs}(\Delta C\%)) \tag{5}$$

Similar to the annealing of metal, which gives the name simulated annealing to the conventional meta heuristic search method, the new algorithm presented here has been given the name Parallel Time-Space Phase Equilibrium Simulated Annealing, ParT-SPEq-SimAnn for short. The latter, due to the analogy of multiple objectives with different metals that are being annealed, forming different phases in thermodynamic equilibrium, only not as a whole to form one alloy, but as hyperspace alloy that is being annealed in parallel dimensions of time and space. As stated before, some new concepts related to this new algorithm are: global cooling curve, particular cooling curve, selection function, global cooling temperature and particular cooling temperature. The algorithm can be describe in the following steps.

1. Set initial solution X0.
2. Build neighborhood to X0.
3. Run simulation model. Evaluate objectives F(X0).
4. Select solution X1 from neighborhood of X0.
5. Run simulation model. Evaluate objectives F(X1).
6. If F(X1) is better than F(X0) for all objectives, do X0 = X1 and go to 2.
7. If F(X1) is worse than F(X0) for all objectives, evaluate selection function FS. Get reference objective RO. Go to 9.
8. If F(X1) is worse or better than F(X0) just for some objectives, evaluate selection function FS. Get reference objective RO. Go to 13.
9. Get temperature T from global cooling curve GCC.
10. Evaluate global acceptance function GAF for reference objective RO.
11. If GAF rejects solution, go to 4.
12. If GAF accepts solution, do X0 = X1 and go to 2.
13. If RO improves, do X0 = X1 and go to 2.
14. If RO gets worse, get temperature T from particular cooling curve PCC of RO.
15. Evaluate particular acceptance function PAF for reference objective RO.
16. If PAF rejects solution, go to 4.
17. If PAF accepts solution, do X0 = X1 and go to 2.

6 FUTURE RESEARCH

The next step in this research is to apply the simulated annealing algorithm to test cases in order to evaluate its performance. The algorithm will be calibrated based on the results of these cases in order to finally use the algorithm in a real life simulation model. The results obtained will be compared to other multi objective simulation optimization techniques.

7 CONCLUSIONS

Several meta-heuristics, such as simulated annealing, tabu search, and genetic algorithms, have been used for solving single-objective simulation-optimization problems. For problems with multiple objectives, one needs a suitable modification of the meta-heuristic. An algorithm that selects a lead objective function in each iteration is worthy of further research investigation since the history of the algorithm can be incorporated into the behavior of such an approach. Apart from the quality of the final solution generated, it will be interesting to analyze the actual search path adapted by the solution approach presented here. Results from our empirical work will be showcased at the conference when the paper is presented.

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