Harmonic Optimization of Multilevel Converters Using Genetic Algorithms

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***Abstract*—In this letter, a genetic algorithm (GA) optimization technique is applied to determine the switching angles for a cascaded multilevel inverter which eliminates specified higher order harmonics while maintaining the required fundamental voltage. This technique can be applied to multilevel inverters with any number of levels. As an example, in this paper a seven-level inverter is considered, and the optimum switching angles are calculated offline to eliminate the fifth and seventh harmonics. These angles are then used in an experimental setup to validate the results.**

***Index Terms*—Genetic algorithms, harmonics, multilevel inverters, optimization.**

I. INTRODUCTION

MULTILEVEL inverters have drawn increasing attention in recent years, especially in the distributed energy resources area, because several batteries, fuel cells, solar cells, or rectified wind turbines or microturbines can be connected through a multilevel inverter to feed a load or interconnect to the ac grid without voltage balancing problems. In addition, multilevel inverters have a lower switching frequency than standard PWM inverters and thus have reduced switching losses.

The output waveforms of multilevel inverters are in a stepped form resulting in reduced harmonics compared to a square-wave inverter. To reduce the harmonics further, different multilevel sinusoidal PWM and space-vector PWM schemes are suggested in the literature [1], [2]; however, PWM techniques increase the control complexity and the switching frequency. Another approach to reduce the harmonics is to calculate the switching angles in order to eliminate certain order harmonics. Chiasson *et al.* [3]–[5] used the mathematical theory of resultants to compute the optimum switching angles. These expressions were high order polynomials that could not be solved when the number of levels in the multilevel converter became large.

In this letter, a general genetic algorithm (GA) approach will be presented. This solves the same problem with a simpler formulation and with any number of levels without extensive derivation of analytical expressions. GA is a search method to find the maximum of functions by mimicking the biological evolutionary processes. There are a few examples of GA applications for power electronics in the literature [6]–[8], but only recently has GA been applied to multilevel inverters [9]. In [10] and [11], other alternative optimal harmonic elimination techniques are introduced.

II. CASCADED MULTILEVEL INVERTERS

The cascaded multilevel inverter is one of several multilevel configurations. It is formed by connecting several single-phase, H-bridge converters in series as shown in Fig. 1 for a seven-level inverter. Each converter generates a square-wave voltage waveform with different duty ratios. Together, these form the output voltage waveform, as shown in Fig. 2. A three-phase configuration can be obtained by connecting three of these converters in Y or Δ. For harmonic optimization, the switching angles θ_1, θ_2, and θ_3 (for a seven-level inverter) shown in Fig. 2 have to be selected so that certain order harmonics are eliminated.

III. GENETIC ALGORITHM (GA)

A genetic algorithm is a computational model that solves optimization problems by imitating genetic processes and the theory of evolution. It imitates biological evolution by using genetic operators referred to as *reproduction*, *crossover*, *mutation*, etc. [12]. To minimize a function $f(x_1, x_2, \ldots, x_k)$ using GA, first, each x_i is coded as a binary or floating-point string of length m. In this letter, a binary string is preferred, e.g.,

$$\begin{aligned} x_1 &= [10001\ldots01001] \\ x_2 &= [00101\ldots11110] \\ &\cdots\cdots\cdots\cdots\cdots \\ x_k &= [11110\ldots01011]. \end{aligned} \tag{1}$$

The set $\{x_1, x_2, \ldots, x_k\}$ is called a *chromosome* and x_i are *genes*.

IV. FORMULATING THE PROBLEM

The GA methodology is the same for any application. There are only a few parameters to be set for a GA to work. The steps for formulating a problem and applying a GA are as follows.

1) Select binary or floating point strings.
2) Find the number of variables specific to the problem; this number will be the number of genes in a chromosome. In this application, the number of variables is the number

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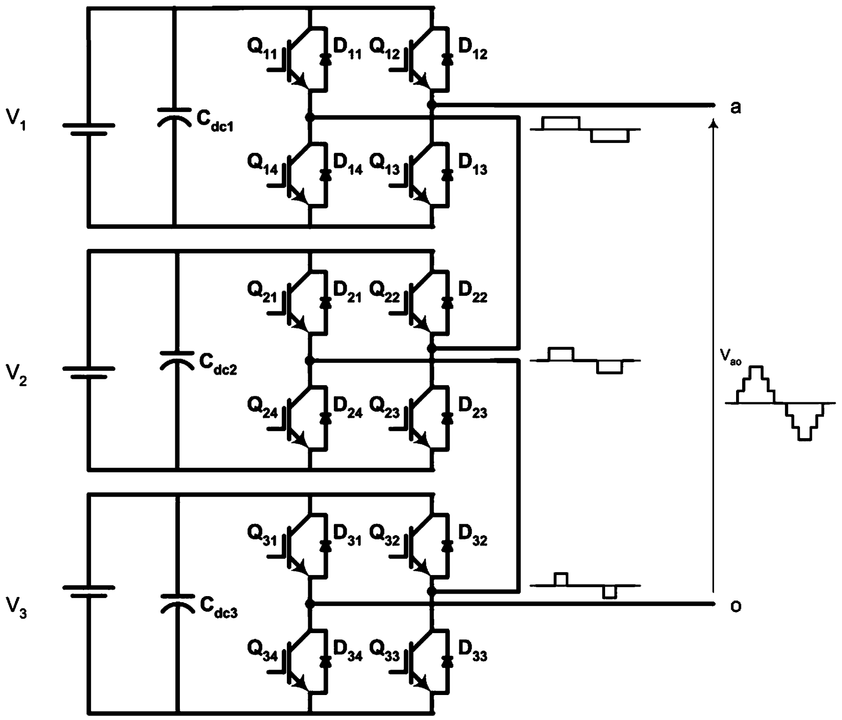

Fig. 1. Seven-level cascaded multilevel inverter.

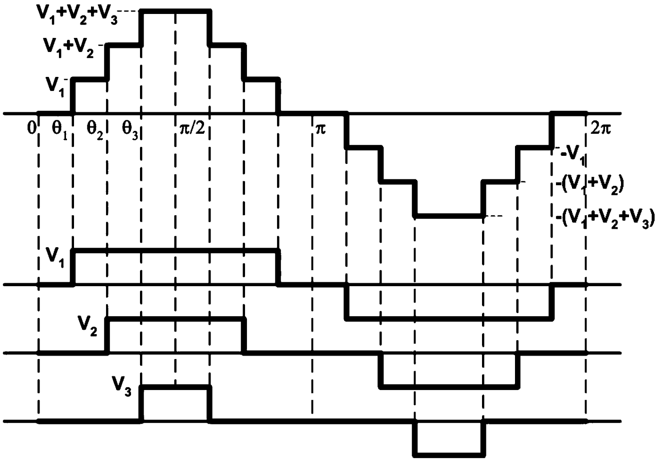

Fig. 2. Seven-level cascaded multilevel inverter waveform generation.

of controllable switching angles, which is the number of H-bridges in a cascaded multilevel inverter. A seven-level inverter requires three H-bridges; thus, each chromosome for this application will have three switching angles, i.e., $\{\theta_1, \theta_2, \theta_3\}$.

3) Set a population size and initialize the population. Higher population might increase the rate of convergence, but it also increases the execution time. The selection of an optimum-sized population requires some experience in GA. The population in this paper has 20 chromosomes, each containing three switching angles. The population is initialized with random angles between 0° and 90° taking into consideration the quarter-wave symmetry of the output voltage waveform.

4) The most important item for the GA to evaluate the fitness of each chromosome is the cost function. The objective of this study is to minimize specified harmonics; therefore the cost function has to be related to these harmonics. As an example, assume that the fifth and seventh harmonics at the output of a seven-level inverter are to be minimized. Then, the cost function f can be selected as the sum of these two harmonics normalized to the fundamental,

$$f(\theta_1, \theta_2, \theta_3) = 100 \times \frac{|V_5| + |V_7|}{|V_1|} \tag{2}$$

where θ_i are the switching angles and V_n are the nth order voltage harmonics. For each chromosome, a multilevel output voltage waveform (Fig. 2) is created using the switching angles in the chromosome, and the required harmonic magnitudes are calculated using FFT techniques.

Typically, the GA algorithm is formulated as a maximization problem rather than a minimization problem. In cases where minimization is required, the negative or the reciprocal of the function to be optimized is used. Using this formulation, the fitness value FV is calculated for each chromosome using

$$FV(\theta_1, \theta_2, \theta_3) = -100 \times \frac{|V_5| + |V_7|}{|V_1|}. \tag{3}$$

The switching angle set producing the maximum FV is the best solution of the first iteration.

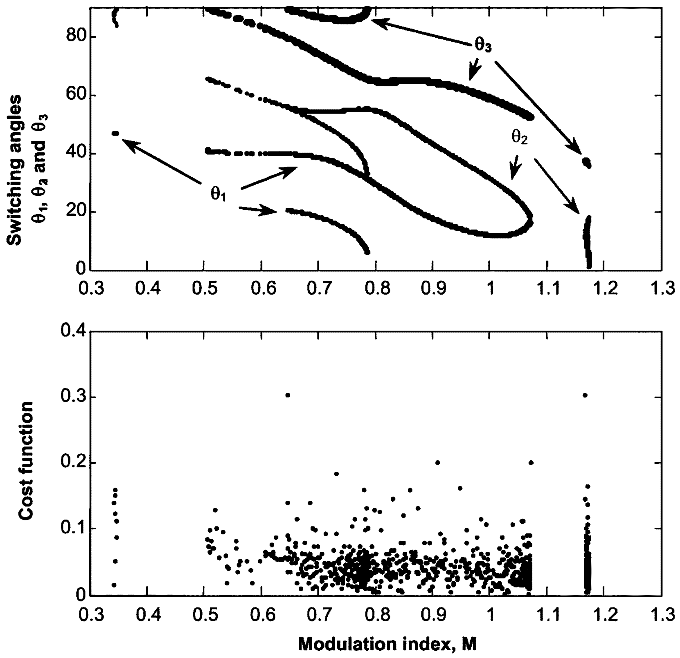

Fig. 3. Solutions for θ_1, θ_2, and θ_3 versus M and the cost function versus M.

5) The GA is usually set to run for a certain number of iterations (100 in this case) to find an answer. After the first iteration, FV's are used to determine new offspring. These go through crossover and mutation operations and a new population is created which goes through the same cycle starting from FV evaluation. Sometimes, the GA can converge to a solution well before 100 iterations are completed. To save time, in this paper, the iterations have been stopped when the absolute value of the cost function goes below 1, in which case the sum of the fifth and the seventh harmonics is negligible compared to the fundamental. As seen in Fig. 3, the GA resulted in cost functions even smaller than 0.4. This was the original set value to eliminate the answers with higher sums including the fifth and the seventh harmonics. Note that after these iterations, the GA finds one solution; therefore, it has to be run as many times as the number of solutions required to cover the whole modulation index range.

The MATLAB GA Optimization Toolbox was used for this work [12]. A complete source code for the GA used in this paper is given in [9, Appendix]. This code can find the switching angle solutions for a multilevel inverter with any number of levels and for the elimination of any number of harmonics.

V. EXPERIMENTAL RESULTS

For the seven-level inverter, switching angles that minimize the fifth and seventh harmonics are shown in Fig. 3. Note that this plot is similar to the one in [3] but has more solutions. In [3], the solution only includes angles that result in zero fifth and seventh order harmonics. In this paper, however, as seen in the bottom plot of Fig. 3, any solution that yields a cost function less than 1 is accepted. This means that if low harmonics are tolerable, a wider solution space is available.

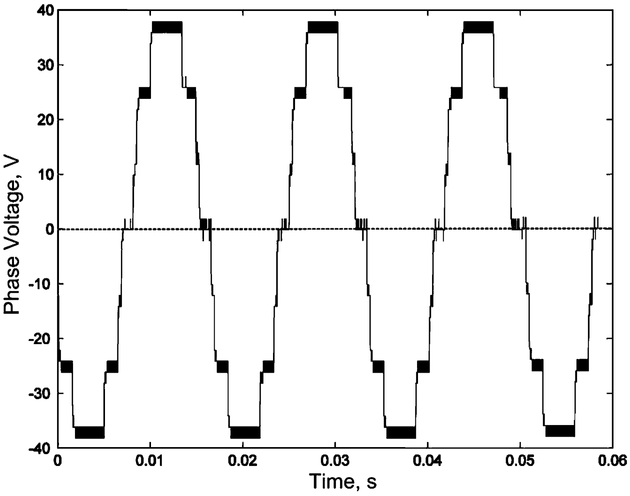

Fig. 4. Experimental output voltage waveform.

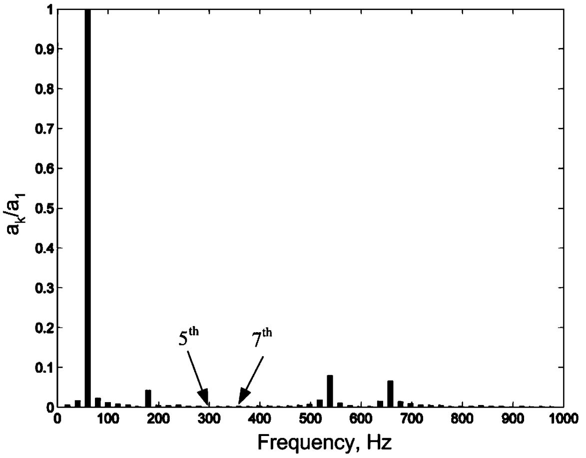

Fig. 5. Normalized (with respect to the fundamental) FFT versus frequency.

In Fig. 3, for certain modulation indices, several sets of solutions are available. Either of these solutions can be used to minimize the selected harmonics. Another possibility [3] is to calculate THD's for each solution set and use the set that gives the lowest THD. As can be observed in Fig. 3, for some modulation indices, no solution sets are available. This means that for those modulation indices, either there is not a solution or the GA could not find one. The former reason is more of a possibility than the latter.

Fig. 4 shows the experimental seven-level voltage waveform for $M = 1.061$ (M is the modulation index defined by $|V_1|/4kV_{dc}$ where k is the number of dc sources and V_{dc} is the voltage supplied by one dc source). Fig. 5 shows the first 15 harmonics of the waveform in Fig. 4. As seen in this figure, the fifth and the seventh harmonics of the voltage waveform are negligible.

Fig. 6 shows the optimum switching angles when this technique is applied to an 11-level inverter (five H-bridges, five switching angles) to minimize the fifth, seventh, 11th, and 13th harmonics. Figs. 7 and 8 show the experimental phase voltage waveform and its harmonic spectrum, respectively, for $M = 0.64$. Each

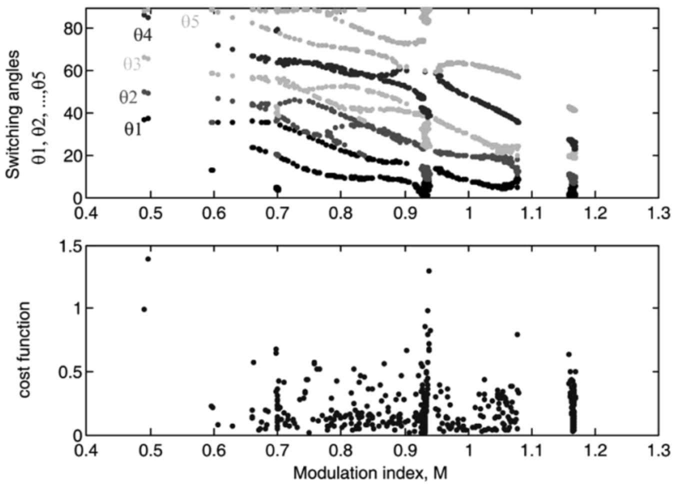

Fig. 6. Solutions for θ_1, θ_2, θ_3, θ_4, and θ_5 and the cost function.

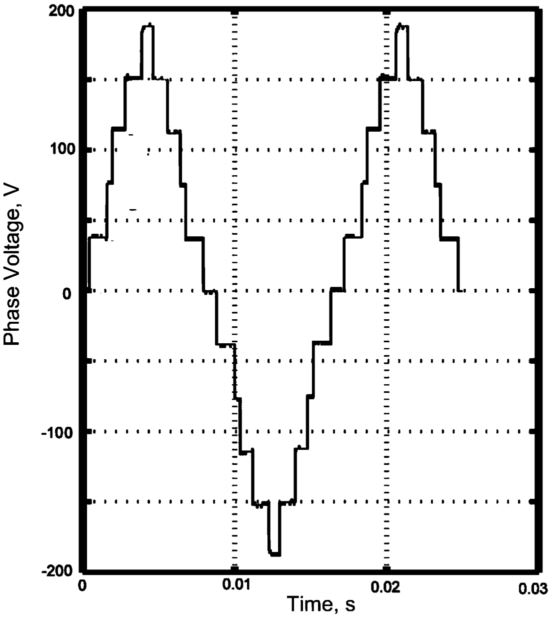

Fig. 7. Experimental output voltage waveform.

GA iteration takes around 15 s on a personal computer with 2.5 GHz Pentium 4 processor and 1 GB memory.

VI. CONCLUSIONS

The comparison of the results in this letter to similar work in the literature shows that the GA approach for the harmonic optimization of multilevel inverters works properly. As in this approach, GA can be applied to any problem where optimization is required; therefore, it can be used in many applications in power electronics. A MATLAB GA optimization toolbox [12] is available for GA optimization requiring only basic MATLAB programming.

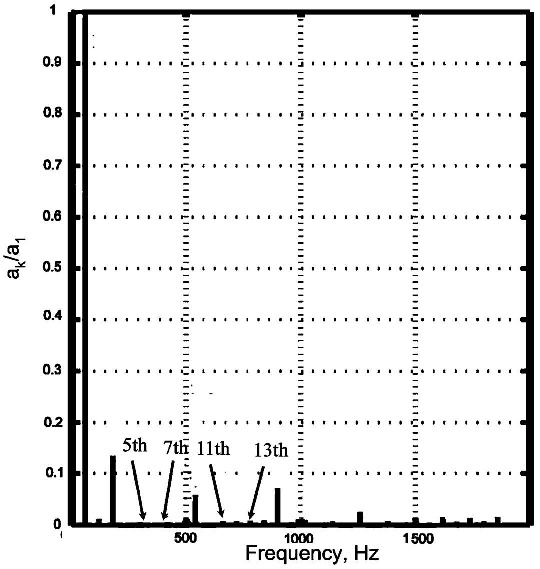

Fig. 8. Normalized (with respect to the fundamental) FFT versus frequency.

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