SOLVING THE PARALLEL TASK SCHEDULING PROBLEM BY MEANS OF A GENETIC APPROACH

ESQUIVEL S.C., GATICA C. R., GALLARD R.H.

Proyecto UNSL-338403[1]
Departamento de Informática
Universidad Nacional de San Luis (UNSL)
Ejército de los Andes 950 - Local 106
5700 - San Luis, Argentina.
E-mail: {esquivel, crgatica, rgallard}@unsl.edu.ar
Phone: + 54 2652 420823
Fax : +54 2652 430224

Abstract

A parallel program, when running, can be conceived as a set of parallel components (tasks) which can be executed according to some precedence relationship. In this case efficient scheduling of tasks permits to take full advantage of the computational power provided by a multiprocessor or a multicomputer system. This involves the assignment of partially ordered tasks onto the system architecture processing components.

This paper shows the problem of allocating a number of non-identical tasks in a multiprocessor or multicomputer system. The model assumes that the system consists of a number of identical processors and only one task may execute on a processor at a time. All schedules and tasks are non-preemptive. The well-known Graham's [8] list scheduling algorithm (LSA) is contrasted with an evolutionary approach using a proposed indirect-decode representation.

Key words: Parallel task allocation, Genetic algorithm, List Scheduling Algorithm, Optimization.

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1. INTRODUCTION

A parallel program is a collection of tasks, some of which must be completed before than others begin. The precedence relationships between tasks are commonly delineated in a directed acyclic graph known as the *task graph*. Nodes in the graph identify tasks and their duration and arcs represent the precedence relationship. Factors, such as number of processors, number of tasks and task precedence make harder to determine a good assignment. The problem to find an schedule on $m > 2$ processors of equal capacity, that minimizes the whole processing time of independent tasks has been shown as belonging to the NP-complete class (Horowitz and Sanhi 1976 [9]).

Task scheduling can be classified as *static* and *dynamic*. Some strong reasons make static scheduling desirable. First, static scheduling sometimes results in lower execution times than dynamic scheduling. Second static scheduling allows only one process per processor, reducing process creation, synchronization and termination overhead. Third, static scheduling can be used to predict the speedup that can be achieved by a particular parallel algorithm on a target machine, assuming that no preemption of processes occur.

2. A DETERMINISTIC MODEL

In a deterministic model, the execution time for each task and the precedence relations between them are known in advance. This information is depicted in a directed graph, usually known as the task graph. In Fig. 1 we have seven tasks with the corresponding duration and their precedence relations

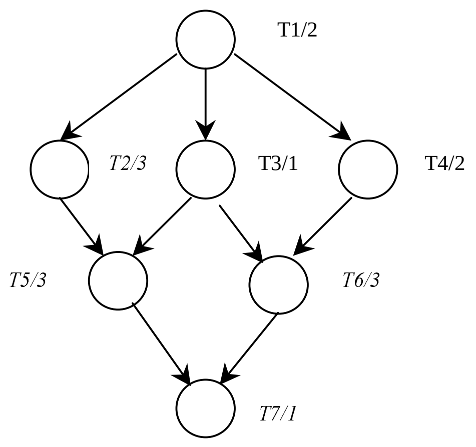


Fig. 1 . The model task graph

Even if the task graph is a simplified representation of a parallel program execution, ignoring overheads due to interrupts for accessing resources, communication delays, etc., it provides a basis for static allocation of processors. A schedule is an allocation of tasks to processors which can be depicted by a Gantt chart.

In a Gantt chart, the initiation and ending times for each task running on the available processors are indicated and the makespan (total execution time of the parallel program) of the schedule can be easily derived.

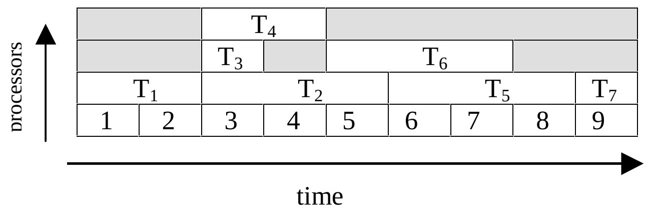


Fig. 2. Scheduling 7 tasks onto 3 processors by LSA

Figure 2 shows a Gantt chart corresponding to one possible schedule of the parallel tasks of the task graph of figure 1 onto three processors. By simple observation we notice a makespan of 9 and a utilization of a 100% for processor $P_{1,}$ 44.4% for P_2 and 22.2% for $P_{3.}$ Also an speed-up of 1.67 can be determined.

Connected with the makespan, an optimal schedule is one such that the total execution time is minimized. Other performance variables, such as individual processor utilization or evenness of load distribution can be considered. As we can see some simple scheduling problems can be solved to optimality in polynomial time while others can be computationally intractable.

As we are interested in the scheduling of arbitrary tasks graphs onto a reasonable number of processors we would be content with polynomial time scheduling algorithms that provide good solutions even though optimal ones can not be guaranteed.

3. THE LIST SCHEDULING ALGORITHM (LSA)

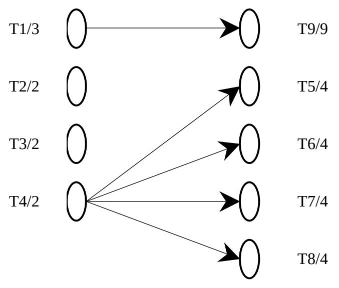
For a given list of tasks ordered by priority, it is possible to assign tasks to processors by always assigning each available processor to the first unassigned task on the list whose predecessor tasks have already finished execution.

Let be:
$T=\{T_1,....,Tn\}$ a set of tasks,
$\mu: T \rightarrow (0, \infty)$ a function which associates an execution time to each task,
$\leq$ a partial order in *T* and
L a priority list of tasks in *T*.

Each time a processors is idle, it immediately removes from *L* the first ready task; that is, an unscheduled task whose ancestors under $\leq$ have all completed execution. In the case that two or more processors attempt to execute the same task, the one with lowest identifier succeed and the remaining processors look for another adequate task. The Gantt chart of Fig. 2, resulted of applying the list scheduling algorithm to the task graph of Fig. 1, with the priority list $L = [T_1, T_2, T_3, T_4, T_5, T_6, T_7]$.

3.1 LSA ANOMALIES

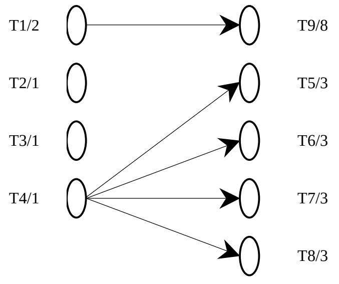
Using this heuristic, contrary to the intuition, some anomalies can happen. For example, as shown in Fig. 3, increasing the number of processors, decreasing the execution times of one or more tasks, or eliminating some of the precedence constraints can actually increase the makespan. In his work Graham presented the following examples using the same priority task list L = [T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8].



P_3	T_3		▨	▨	T_6				T_8			
P_2	T_2		T_4		T_5				T_7			
P_1	T_1			T_9								
ts	1	2	3	4	5	6	7	8	9	10	11	12

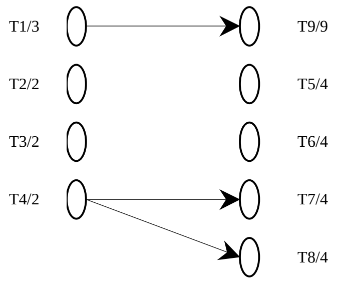
P_4	T_4		T_7				▨	▨	▨	▨	▨	▨	▨	▨	▨
P_3	T_3		T_6				▨	▨	▨	▨	▨	▨	▨	▨	▨
P_2	T_2		T_5				T_9								
P_1	T_1			T_8				▨	▨	▨	▨	▨	▨	▨	▨
ts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(a)



P_3	T_3	▨	T_7			▨	▨	▨	▨	▨	▨	▨	▨
P_2	T_2	T_4	T_6			T_9							
P_1	T_1		T_5			T_8			▨	▨	▨	▨	▨
ts	1	2	3	4	5	6	7	8	9	10	11	12	13

(b)



P_3	T_3		T_5				T_8				▨	▨	▨	▨	▨	▨
P_2	T_2		T_4		T_7				▨	▨	▨	▨	▨	▨	▨	▨
P_1	T_1			T_6				T_9								
ts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

(c)

Fig. 3. Increasing number of processors, decreasing tasks durations and eliminating precedence constraints can increase makespan using Graham's heuristic.

4. Using evolutionary algorithms to provide near-optimal solutions

The task allocation problem has been investigated by many researchers [3], [4], [5], [6], [7], [10], [11]. Several heuristics methods has been proposed, such as mincut-based heuristics, orthogonal recursive bisection, simulated annealing, genetic algorithms and neural networks.

From the representation perspective many evolutionary computation approaches to the general scheduling problem exists. With respect to solution representation these methods can be roughly categorized as *indirect* and *direct* representations (Bagchi et al, 1991 [1]).

In the case of indirect representation of solutions the algorithm works on a population of encoded solutions. Because the representation does not directly provides a schedule, a schedule builder is necessary to transform a chromosome into a schedule, validate and evaluate it. The schedule builder guarantees the feasibility of a solution and its work depends on the amount of information included in the representation.

In direct representation (Bruns 93 [2]) a complete and feasible schedule is an individual of the evolving population. The only method that performs the search is the evolutionary algorithm because the represented information comprises the whole search space.

We devised different evolutionary computation approaches regarding to different direct and indirect representation schemes. In this paper we present a direct approach with the *as-soon-as-possible* crossover method and an *indirect-decode* representation. We will concentrate on results of this latter variant of indirect representation.

4.1. Direct representation of solutions

Here we propose to use a schedule as a chromosome. Suppose we have to different schedules, (a) and (b) (Fig. 4), for the model task graph of Fig. 1, represented by the following Gannt charts.

P_2	▨	▨	T_3	T_4		T_6			▨	▨
P_1	T_1		T_2			T_5			T_7	T_8
ts	1	2	3	4	5	6	7	8	9	10

Schedule (a)

P_2	▨	▨	T_3	▨	▨	T_5			▨	▨	▨	T_8
P_1	T_1		T_2			T_4		T_6			T_7	▨
ts	1	2	3	4	5	6	7	8	9	10	11	12

Schedule (b)

Fig. 4. Feasible schedules for the model task graph.

The precedence relation described in the task graph can be properly represented in the corresponding precedence matrix A, where element a_{ij} is set to 1 if task i precedes task j and it is set to 0 otherwise. A gene in the chromosome can be the following four-tuple:

<*task_id, proc_id, init_time, end_time* >

where,

task_id, identifies the task to be allocated.
proc_id, identifies the processor where the task will be allocated.
init_time, is the commencing time of the *task_id* in *proc_id.*
end_time, is the termination time of the *task_id* in *proc_id.*

With this structure the list of the corresponding predecessors tasks is easily retrieved by entering the column of A indexed by the *task_id* value.

The corresponding chromosomes C_a and C_b for schedules (a) and (b) are:

C_a :	1,1,0,2	2,1,2,5	3,2,2,3	4,2,3,5	5,1,5,8	6,2,5,8	7,1,8,9	8,1,9,10
C_b:	1,1,0,2	2,1,2,5	3,2,2,3	4,1,5,7	5,2,5,8	6,1,7,10	7,1,10,11	8,2,11,12

This representation has a problem. If we use conventional crossover such as one-point crossover invalid offspring can be created. For example, if we decide to apply this operator after the fifth position we would obtain two invalid chromosomes.

$C_{a'}$:	1,1,0,2	2,1,2,5	3,2,2,3	4,2,3,5	5,1,5,8	6,1,7,10	7,1,10,11	8,2,11,12
$C_{b'}$:	1,1,0,2	2,1,2,5	3,2,2,3	4,1,5,7	5,2,5,8	6,2,5,8	7,1,8,9	8,1,9,10

Both of them violate the restriction that a processor processes a task at a time. Genes 5 and 6 in $C_{a'}$ and $C_{b'}$ describe invalid schedules where the same processor (P_1 for the case of $C_{a'}$ and P_2 for the case of $C_{b'}$) processes two tasks at some time interval.

To remedy this situation it can be used penalty functions or repair algorithms. Penalty functions [12] of varied severity can be applied to invalid offspring in order to lower their fitness values but allowing them to remain in the population aiming to retain valuable genetic material. Repair algorithms attempt to build up a valid solution from an invalid one. This latter approach is embedded in the *knowledge-augmented crossover* operator proposed by Bruns. Here a *collision* occurs if an operation (task processing) inherited from one of the parents cannot be scheduled in the specified time interval on the assigned processor. In this case the processor assignment is unchanged and it is delayed into the future until the processor is available. In our example, this advanced crossover would generate the following chromosomes and corresponding feasible schedules (Fig. 5):

$C_{a''}$:	1,1,0,2	2,1,2,5	3,2,2,3	4,2,3,5	5,1,5,8	6,1,8,11	7,1,11,12	8,2,12,13
$C_{b''}$:	1,1,0,2	2,1,2,5	3,2,2,3	4,1,5,7	5,2,5,8	6,2,8,11	7,1,11,12	8,1,12,13

P_2	▨	▨	T_3	T_4		▨	▨	▨	▨	▨	▨	▨	T_8
P_1	T_1		T_2			T_5			T_6			T_7	▨
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	11	12	13

Schedule (a")

P_2	▨	▨	T_3	▨	▨	T_5			T_6			▨	▨
P_1	T_1		T_2			T_4		▨	▨	▨	▨	T_7	T_8
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	11	12	13

Schedule (b")

Fig 5. Feasible offspring schedules for the model task graph (Bruns).

As expected both children have a larger makespan but still are feasible. In the proposed Bruns's *knowledge-augmented* crossover only a child is generated where the part taken from the first parent build a consistent schedule. Then the assignment of the missing tasks are chosen from the second parent maintaining the assignment order and the processor allocations to tasks. Timing adjustments are included if necessary. The latter decision can imply, as we showed, larger makespans for the children.

We adopted an *as-soon-as-possible* (ASAP) crossover approach similar to the Brun's proposal but modified because delays are avoided, moving the assignment to the earliest possible time, by random selection of one available processor at the ready time of the unassigned task. In this way no processor will remain idle if a task is available to be executed and the precedence constraints are satisfied. The available processor is selected as to minimize assignment changes in the second parent part of the offspring. In our example this decision provides only one alternative and would give us the following chromosomes and their corresponding schedules, which differs from their parents only in the assignments of tasks T_7 and T_8.

$C_{a'''}$:	1,1,0,2	2,1,2,5	3,2,2,3	4,2,3,5	5,1,5,8	6,2,5,8	7,1,8,9	8,2,9,10
$C_{b'''}$:	1,1,0,2	2,1,2,5	3,2,2,3	4,1,5,7	5,2,5,8	6,1,7,10	7,1,10,11	8,1,11,12

P_2	▨	▨	T_3	T_4		T_6			▨	T_8
P_1	T_1		T_2			T_5			T_7	▨
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*

Schedule (a''')

P_2	▨	▨	T_3	▨	▨	T_5			▨	▨	▨	▨
P_1	T_1		T_2			T_4		T_6			T_7	T_8
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	11	12

Schedule (b''')

Fig 6. Feasible offspring schedules for the model task graph (ASAP).

A similar operator was conceived for mutation. If the chromosome undergoes mutation then a search is done, from left to right, until one gene is modified either by choosing an alternative free processor or by moving the assignment to the earliest possible time. This would imply modifying subsequent genes of the chromosome to create a valid offspring.

4.2. INDIRECT REPRESENTATION OF SOLUTIONS

Under this approach a schedule is encoded in the chromosome in a way such that the task indicated by the gene position is assigned to the processor indicated by the corresponding allele, as shown in Fig. 7:

processor	→	*1*	*2*	*3*	*2*	*1*	*3*	*1*	*2*
task	→	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*

Fig. 7. Chromosome structure for the task allocation problem

The idea is to use a decoder. A *decoder* is a mapping from the space of representations that are evolved to the space of feasible solutions that are evaluated. Here the chromosome gives instructions to a decoder on how to build a feasible schedule. Regarding the task allocation problem, to build a feasible schedule, a decoder is instructed in the following way: By following the priority list, traverse the chromosome and assign the corresponding task to the indicated processor as soon as the precedence relation is fulfilled. Under this approach the restriction on avoiding processor idleness while a task is ready is relaxed. We believe that this less restrictive approach will contribute to population diversity. One advantage of decoders resides in their ability to create valid offspring even by means of simple conventional operators. One disadvantage is an slower evaluation of solutions. For the model task graph of Fig. 1, with three processors, parent chromosomes C_1 and C_2, and one point crossover after the fourth position we obtain the valid offspring $C_{1'}$ and $C_{2'}$.

C_1:	1	2	3	1	1	1	3	2
C_2:	3	1	2	2	3	1	2	3

$C_{1'}$:	1	2	3	1	3	1	2	3
$C_{2'}$:	3	1	2	2	1	1	3	2

The interpretation of these four decoders with priority task list *L* = [T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8] are the schedules of Fig. 5.

P_3	▨	▨	T_3	▨	▨	▨	▨	▨	▨	▨	T_7	▨
P_2	▨	▨	T_2			▨	▨	▨	▨	▨	▨	T_8
P_1	T_1		T_4		T_5			T_6			▨	▨
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	11	12

a) Parent C_1

P_3	T_1		▨	▨	▨	T_5			▨	T_8
P_2	▨	▨	T_3	T_4		▨	▨	▨	T_7	▨
P_1	▨	▨	T_2			T_6			▨	▨
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*

b) Parent C_2

P_3	▨	▨	T_3	▨	▨	T_5			▨	T_8
P_2	▨	▨	T_2			▨	▨	▨	T_7	▨
P_1	T_1		T_4		▨	T_6			▨	▨
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*

c) Offspring C'$_1$

P_3	T_1		▨	▨	▨	▨	▨	▨	▨	▨	▨	T_7	▨
P_2	▨	▨	T_3	T_4		▨	▨	▨	▨	▨	▨	▨	T_8
P_1	▨	▨	T_2			T_5			T_6			▨	▨
ts	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	11	12	13

d) Offspring C'$_2$

Fig 8. Parents and feasible offspring schedules using decoders

Also, simplest mutation operators can be implemented by a simple swapping of values at randomly selected chromosome positions or by a random change in the allele. The new allele value identifies any of the involved processors.

5. EXPERIMENTS AND RESULTS

The preliminary experiments implemented a generational GA with *indirect-decode representation* of chromosomes, randomized initial population of size fixed at 50 individuals. Twenty series of ten runs each were performed on five testing cases, using elitism, one point crossover and big creep mutation. The maximum number of generations was fixed at 100, but a stop criterion was used to accept convergence when after 20 consecutive generations, mean population fitness values differing in $\varepsilon \leq 0.001$ were obtained. Probabilities for crossover and mutation were fixed at 0.65 and 0.001, respectively. The testing cases corresponded to:

Case 1: Task graph of Fig. 1 (7 tasks and 3 processors)
Case 2: Task graph of Fig. 3.a (9 tasks and 3 processors)
Case 3: Task graph of Fig. 3.a (9 tasks and 4 processors)

Case 4: Task graph of Fig. 3.b (9 tasks and 3 processors, decreasing task's duration)
Case 5: Task graph of Fig. 3.c (9 tasks and 3 processors, eliminating precedence constraints)

The following performance variables were considered to contrast the genetic approach (GA) versus the LSA:

Alt: Number of alternative solutions. It is the mean number of distinct alternative solutions found by the algorithm including optimum and non-optimum solutions.
Opt: Number of optimal solutions. It is the mean number of distinct optimum solutions found by the algorithm.
Topt: Total number of optima. It is the total number of distinct optimal solutions found by the algorithm throughout all the runs.

Case	Alt		Opt		Topt	
	GA	LSA	GA	LSA	GA	LSA
1	4.2	1	4.0	1	39	1
2	1.8	1	1.2	1	11	1
3	4.2	1	4.2	-	42	-
4	2.8	1	2.7	-	26	-
5	1.4	1	1.2	-	12	-

Table 1. GA versus LSA, comparative performance

The stop criterion allowed to run the GA for a number of generations between 40 to 80. In some of the alternative solutions, one or more processors remained idle (no tasks allocated to them). As the permutation of processors provides new alternative solutions, all the allocation list of an involved processor can be switched to an idle one. Consequently a fault tolerance scheme can be implemented when the GA provides schedules with idle processors.

By observing table 1 the following comparison can be done:

- The genetic approach found more than a single optimal solution for any case.

- All the anomalies observed with LSA do not hold when GA is applied, because:

    - When the number of processors is increased the minimum (optimum) makespan is also found.
    - When the duration of tasks is reduced this reduction is reflected in a reduced optimum makespan.
    - When the number of precedence restrictions is reduced the optimum makespan is preserved.

A more detailed analysis on each run detected that in most of the cases alternative solutions do not include, or include a low percentage, of non-optimal alternative solutions. That means that the final population is composed of many replicas of the optimal solutions due to a loss of diversity. This fact stagnates the search and further improvements are difficult to obtain. To avoid this behaviour it would be necessary to continue experimentation with different parameter settings and recombination approaches.

6. Conclusions

The allocation of a number of parallel tasks in parallel supporting environments, multiprocessors or multicomputers, is a difficult and important issue in computer systems. In this paper we approached allocation attempting to minimize makespan. Other performance variables such as individual processor utilization or evenness of load distribution can be considered.

As we are interested in scheduling of arbitrary tasks graphs onto a reasonable number of processors, in many cases we would be content with polynomial time scheduling algorithms that provide good solutions even though optimal ones can not be guaranteed. The list scheduling algorithm (LSA) satisfies this requirement.

A genetic approach was undertaken to contrast its behaviour against the LSA. Preliminary results on the selected test suite showed two important facts. Firstly, GA provides not a single but a set of optimal solutions, providing fault tolerance when system dynamics must be considered. Secondly, GA is free of the LSA anomalies. These facts do not guarantee finding optimal solutions for any arbitrary task graph but show a better approach to the problem. Consequently, further research is necessary to investigate potentials and limitations of the GA approach under more complex test suites, different representations, and convenient genetic operators. The above mentioned *as-soon-as-possible* (ASAP) crossover approach oriented to direct representation is now being implemented.

7. Acknowledgements

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