MATH GU4053: INTRODUCTION TO ALGEBRAIC TOPOLOGY

Homework 6
Due: 03/06/20 by 5pm

(1) Hatcher 2.1.9

(2) Hatcher 2.1.11. Use this exercise to conclude that any inclusion $i : S^k \hookrightarrow S^n, k < n$, does not have a retract.

(3) Hatcher 2.1.14

(4) Hatcher 2.1.15

(5) a) Hatcher 2.1.20 (assume that X is a Δ-complex and prove this by finding a Δ-complex for SX in terms of a Δ-complex for X; or use the long exact sequence)

b) Suppose M is a Δ-complex. Construct a Δ-complex on $Cone(M)$ and use this to show that $\tilde{H}_n(Cone(M)) = 0$ for all n.

6. Consider a chain complex C given by $0 \to \mathbb{Z}^k \stackrel{\varphi}{\to} \mathbb{Z}^n \stackrel{\psi}{\to} \mathbb{Z}^m \to 0$ that is exact; so φ is injective, ψ is surjective, and $Image(\varphi) = Kernel(\psi)$. Find an chain homotopy between the identity chain map $Id : C \to C$ and the zero chain map $0 : C \to C$.

7. a) Is every chain map $B_* \to C_*$ so that $B_k \to C_k$ is surjective for all k induce a surjective map $H_k(B) \to H_k(C)$ on homology? If not, give a counterexample.
b) Is every chain map $B_* \to C_*$ so that $B_k \to C_k$ is injective for all k induce a injective map $H_k(B) \to H_k(C)$ on homology? If not, give a counterexample.

8) Optional:

The simplicial chain complex for a point p is $C_n(p) = \mathbb{Z}$ if $n = 0$ and $C_n(p) = 0$ otherwise; the boundary map is zero for all n. The singular chain complex for p is $D_n(p) = \mathbb{Z}$ for all n (since there is a single, constant map $\Delta^n \to p$ for all n); the boundary map $\partial_n : D_n(p) \to D_{n-1}(p)$ is the identity map if n is even and the zero map if n is odd. These two chain complexes have the same homology (later we will prove that simplicial and singular homology always agree).
Find chain maps $f_* : C_* \to D_*$ and $g_* : D_* \to C_*$, and chain homotopies $h_1 : C_* \to C_{*+1}, h_2 : D_* \to D_{*+1}$ so that $g_* \circ f_* : C_* \to C_*$ is chain homotopic to the identity chain

map $Id_* : C_* \to C_*$ (via the chain homotopy h_1) and $f_* \circ g_* : D_* \to D_*$ is chain homotopic to the identity chain map $Id_* : D_* \to D_*$ (via the chain homotopy h_2).